Neural Networks and Biological Modeling

Lecture 6 – Networks of Neurons and **Associative Memory**

-Introduction
-Associative memory and Classification by similarity
-Detour: magnetic materials
-Associative Memory
-Hopfield Model
-Memory Capacity

Wulfram Gerstner, EPFL

Systems for computing and information processing

Brain

Computer

Distributed architecture

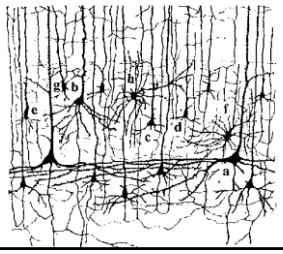
(10^{10} proc. Elements/neurons)
No separation of processing and memory

memory
CPU
input

Von Neumann architecture
1 CPU
(10^{10} transistors)

1mm

10 000 neurons
3 km wires

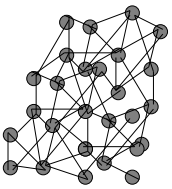


Systems for computing and information processing

Brain

1mm

10 000 neurons
3 km wires



Distributed architecture

10^{10} neurons
10^{4} connections/neurons

No separation of processing and memory

Associations, Associative Memory

*Read this text **NOW**!*

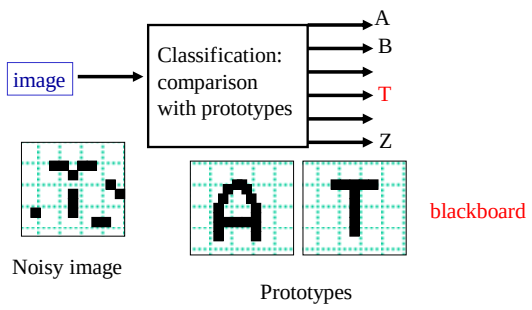
pattern completion/word recognition

brai* → atom, brave, brain, brass → brain

Noisy word | List of words | Output the closest one

Your brain fills in missing information: 'associative memory'

- Classification by similarity:
pattern recognition

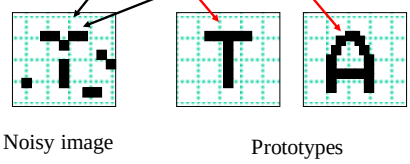


- recognize/understand images:
pattern recognition

Blackboard:

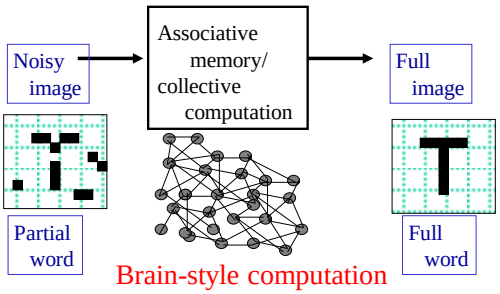
Classification by closest prototype

$$|x - p^T| \leq |x - p^A|$$



Aim: Understand Associative Memory

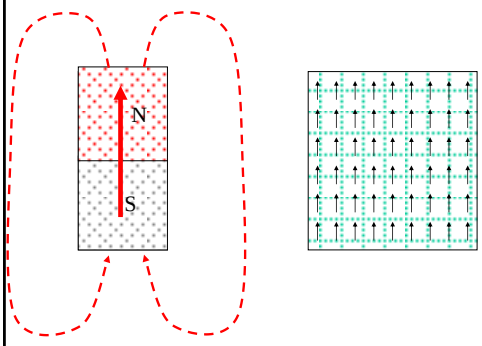
Pattern recognition/Pattern completion



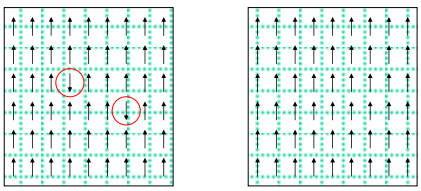
Lecture 5 – Network of neurons
and associative memory

√ -Introduction
- Associative Memory and Classification
-Detour: magnetic materials
-Associative Memory
-Hopfield Model
-Dense networks (mean-field)

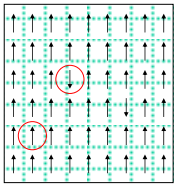
Detour: magnetism



Detour: magnetism



Detour: magnetism



Elementary magnet

↑ $S_i = +1$

↓ $S_i = -1$

Blackboard: example

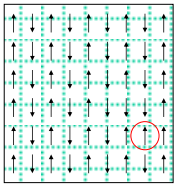
dynamics

$$S_i(t+1) = \mathrm{sgn}\left(\sum_j S_j\right)$$

Sum over all interactions with i

Detour: magnetism

Anti-ferromagnet



Elementary magnet

↑ $S_i = +1$

↓ $S_i = -1$

↑↑ $w_{ij} = +1$

↑↓ $w_{ij} = -1$

dynamics

$$S_i(t+1) = \mathrm{sgn}\left(\sum_j w_{ij} S_j\right)$$

blackboard

Sum over all interactions with i

Lecture 5 – Network of neurons and associative memory

-Introduction

-Associative Memory and Classification by similarity

-Detour: magnetic materials

-Associative Memory

-Hopfield Model

-Dense networks (mean-field)

Associative memory

Elementary pixel

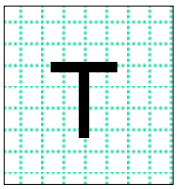
■ $S_i = +1$

□ $S_i = -1$

■↔■ $w_{ij} = +1$

□↔□ $w_{ij} = +1$

■↔□ $w_{ij} = -1$



dynamics

$$S_i(t+1) = \mathrm{sgn}\left(\sum_j w_{ij} S_j\right)$$

blackboard

Sum over all interactions with i

Hopfield model

Exercise 1: Associative memory (1 pattern)

Next lecture at 10h30

Elementary pixel

■ $S_i = +1$

□ $S_i = -1$

■↔■ $w_{ij} = +1$

□↔□ $w_{ij} = +1$

■↔□ $w_{ij} = -1$

dynamics

$$S_i(t+1) = \mathrm{sgn}\left(\sum_j w_{ij} S_j\right)$$

Sum over all interactions with i

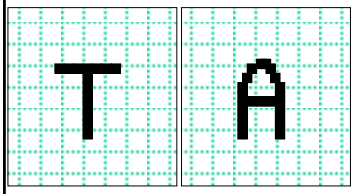
9 neurons

- define appropriate weights
- what happens if one neuron wrong?
- what happens if *n* neurons wrong?

Associative memory – many patterns

Hopfield Model

Associative memory – many patterns



Prototype $\vec{p}^1$ Prototype $\vec{p}^2$

interactions

$$w_{ij} = \sum_{\mu} p_i^{\mu} p_j^{\mu}$$

Sum over all prototypes

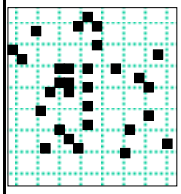
dynamics

$$S_i(t+1) = \mathrm{sgn}\left(\sum_j w_{ij} S_j\right)$$

Sum over all interactions with i

Hopfield model

Associative memory – many patterns



Prototype $\vec{p}^1$

interactions

$$w_{ij} = \sum_{\mu} p_i^{\mu} p_j^{\mu}$$

Sum over all prototypes

This rule is very good for **random** patterns

It does not work well for correlated patters

dynamics

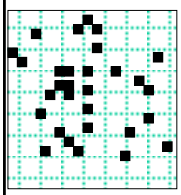
$$S_i(t+1) = \mathrm{sgn}\left(\sum_j w_{ij} S_j\right)$$

Sum over all interactions with i

DEMO

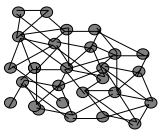
Random patterns, fully connected:
Hopfield model

Associative memory – many patterns



Prototype $\vec{p}^1$

Interacting neurons



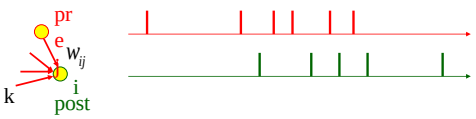
Finds the closest prototype i.e. maximal overlap (similarity) m^{μ}

Computation
- without CPU,
- without explicit memory unit

Hopfield model

Where do the connections come from?

Hebbian Learning

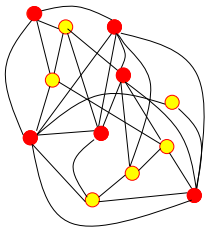


When an axon of cell **j** repeatedly or persistently takes part in firing cell **i**, then j's efficiency as one of the cells firing i is increased

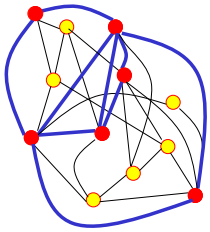
Hebb, 1949

- local rule
- simultaneously active (correlations)

Hebbian Learning



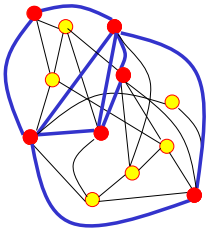
Hebbian Learning



item memorized

Hebbian Learning – Associative Recall

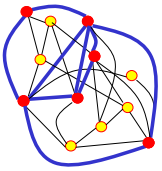
Recall:
Partial info



item recalled

Associative Recall

Tell me the object **shape**
Tell me the **color**
the following list of 5 items:
the following list of 5 items:

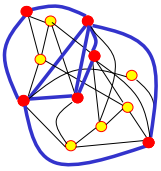


be as fast as possible:

time

Associative Recall

Tell me the **color**
the following list of 5 items:

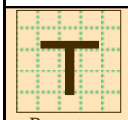


be as fast as possible:

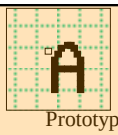
time

Stroop effect:
Slow response: hard to work
Against natural associations

Exercises 2 at home: learning of prototypes



Prototype $\vec{p}^1$



Prototype $\vec{p}^2$

interactions

(1) $w_{ij} = \sum_{\mu} p_i^{\mu} p_j^{\mu}$

Sum over all prototypes

a) Show that (1) corresponds to a rate learning rule

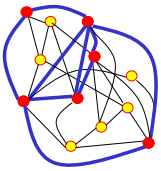
(2) $\frac{d}{dt} w_{ij} = a_2^{corr} (\nu_j^{pre} - \vartheta)(\nu_i^{post} - \vartheta)$

Assume that weights are zero at the beginning;
Each pattern is presented (enforced) during 0.5 sec (One after the other).
note that $p_j^{\mu} = \pm 1$ *but* $\nu_j \geq 0$

b) Compare with: $\frac{d}{dt} w_{ij} = a_0 + a_1^{pre} \nu_j^{pre} + a_1^{post} \nu_i^{post} + a_2^{corr} \nu_j^{pre} \nu_i^{post} + ..$

Associative Recall

Hierarchical organization of
Associative memory



animals

birds fish

Name as fast as possible
an example of a bird

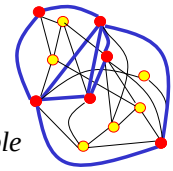
swan (or goose or raven or …)

Write down first letter: *s* for *swan* or *r* for *raven* …

Associative Recall

Nommez au plus vite possible
un exemple d'un /d'une

name as fast as possible
an example of a



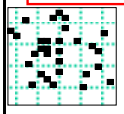
outil	**tool**
couleur	**color**
fruit	**fruit**
instrument de musique	**music instrument**

Lecture 5 – Network of neurons
and associative memory

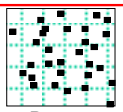
- -Introduction
- -Classification by similarity
- -Detour: magnetic materials
- -Associative Memory
- -Hopfield model
- -How many patterns?

Memory Capacity

learning of prototypes



Prototype $\vec{p}^1$



Prototype $\vec{p}^2$

interactions

(1) $w_{ij} = \frac{1}{N} \sum_{\mu} p_i^{\mu} p_j^{\mu}$

Sum over all prototypes

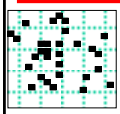
Q; How many prototypes can be stored?

dynamics $S_i(t+1) = \mathrm{sgn}\left(\sum_j w_{ij} S_j \right)$

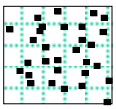
Sum over all interactions with i

Q; How many prototypes can be stored?

blackboard



Prototype $\vec{p}^1$



Prototype $\vec{p}^2$

Random patterns

Interactions (1) $w_{ij} = \sum_{\mu} p_i^{\mu} p_j^{\mu}$

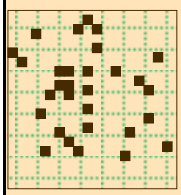
Dynamics (2) $S_i(t+1) = \mathrm{sgn}\left(\sum_j w_{ij} S_j \right)$

Minimal condition: pattern is fixed point of dynamics

-Assume we start directly in one pattern
-Pattern stays

Attention: Retrieval requires more (pattern completion)

Exercise 3 (homework)



Prototype $\vec{p}^1$

$w_{ij} = \frac{1}{N} \sum_{\mu} p_i^{\mu} p_j^{\mu}$

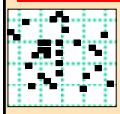
$S_i(t+1) = \mathrm{sgn}\left(\sum_j w_{ij} S_j \right)$

Sum over all interactions with i

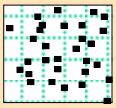
Assume 4 patterns. At time t=0, overlap with Pattern 3, no overlap with other patterns.
discuss temporal evolution
(assume that patterns are orthogonal)

Exercise 4 now: Associative memory

Q: How many prototypes can be stored?



Prototype $\vec{p}^1$



Prototype $\vec{p}^2$

End of lecture, exercise+ Computer exercise : 12:00

Random patterns

Interactions (1) $w_{ij} = \sum_{\mu} p_i^{\mu} p_j^{\mu}$

Dynamics (2) $S_i(t+1) = \mathrm{sgn}\left(\sum_j w_{ij} S_j \right)$

Random patterns → random walk

a) show relation to erf function: importance of p/N
b) network of 1000 neurons – allow at most 1 wrong pixel?
c) network of N neurons – at most 1 promille wrong pixels?

The end