

2D reduction

Thursday, November 4, 2021

15:35

Input

problem size

$m \times n$

Hardware sources

① #SM (how many blocks can run concurrently)

$\# \max T$ / $\# \max W \rightarrow$ upper bound for threads/warps / block

② $\# \min T$ / $\# \min W \rightarrow$ lower bound for threads/warps / block to later FGM

e.g. $\#SM = 16 \rightarrow \# \max T = 1024$ / $\# \max W = 32$

$\# \min T = 512$ / $\# \min W = 16$

③ RF / shared mem per sm

- check when generating Algorithm

Concern

① efficiency or performance

e.g.1 $16 \times 128 \rightarrow 16 \times 1$

- performance: 16 blocks on 16 sm

each block has 128 threads

e.g.2 $8 \times 16 \times 128 \rightarrow 8 \times 16 \times 1$

- efficiency: for each 16×128

we use 2 blocks

each block has 1024 threads

② we assume each block has similar run time

so it's best to have $k \times \#SM$ blocks

If $n < 32$

only consider efficiency scheduling

① $n = 2, 4, 8, 16 \rightarrow$ assign $\frac{32}{n}$ rows to 1 warp

Algo: one warp reduce and get results for $\frac{32}{n}$ rows

② $n \neq 2, 4, 8, 16 \rightarrow$ assign 32 rows to 1 warp

Algo: one warp first read $n \times 32$ elements to shared mem and each thread add one row

$TW =$ Total number of warps needed

$$\leq K \times \frac{(\#SM \times \# \max W)}{512}$$

e.g. $\sqrt{768} \leq 2 \times 512$

option1: one kernel launch $\rightarrow 32$ blocks

each block has $\frac{768}{32} = 24$ warps

option2: kernel launch $\rightarrow 16$ blocks with 32 warps

kernel launch $\rightarrow 16$ blocks with 16 warps

scheduling performance

If $m < \#SM$

- we can assign one or more sm to one row

- one sm only need to compute one row

block formation

$$\#brn \text{ (blocks per row upper bound)} = \left\lfloor \frac{\#SM}{m} \right\rfloor$$

e.g. $\#SM = 16$, $m = 15$, $\#brn = 1$

$$\#br = \min\{brn, \max\{1, \left\lfloor \frac{n}{\# \min T} \right\rfloor\}\}$$

e.g. $brn = 2$, $n = 200$, $\# \min T = 512 \rightarrow \#br = 1$

$$\#wb = \min\{\# \max W, \left\lfloor \frac{n}{32 \times \#br} \right\rfloor\}$$

Algo detail



- intra row \rightarrow block-wide loop

- inter-blocks \rightarrow atomic add to global mem

- inter-warp \rightarrow atomic add to global mem
atomic add to shared mem
non-atomic write to shared mem

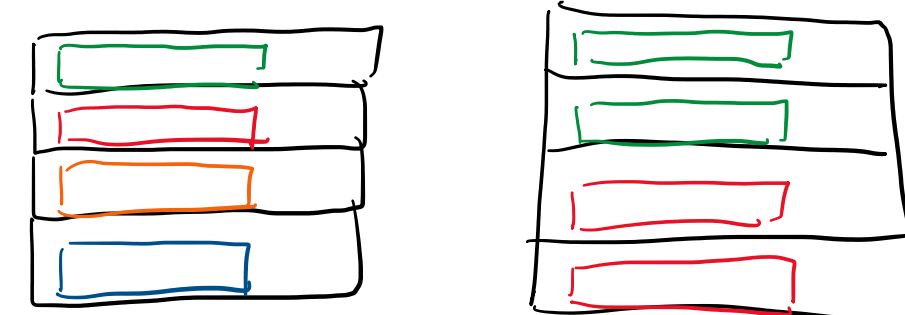
If $m > \#SM$

$$\#wrn \text{ (warps per row upper bound)} = \left\lfloor \frac{\# \max W \times \#SM}{m} \right\rfloor$$

$$\#wr = \min\{\#wrn, \left\lfloor \frac{n}{32} \right\rfloor\}$$

\rightarrow If $\#wr > 1$

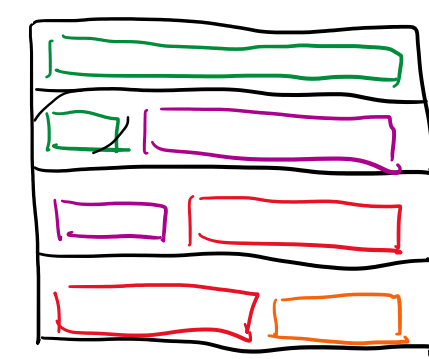
option1: 1 or k rows per block



- no sync between blocks

- simple control flow

option2: allow 1 row belong to multiple blocks



- $wr > \# \min T$, $\frac{\# \max W}{2} \rightarrow$ 1 row per block

- $\# \min T < wr < \frac{\# \max W}{2} \rightarrow k$ rows per block ($k \geq 1$)

- $wr < \# \min T$, $\frac{\# \max W}{2} \rightarrow k$ rows per block ($k > 1$)

e.g.1 $m = 25$, $wr = 20 \rightarrow$ option 2

16 blocks, each block has 32 warps

e.g.2 $m = 31$, $wr = 16 \rightarrow$ option 1

32 blocks, each block has 16 warps

If $wr < 1$

$$\#wrn = \left\lfloor \frac{k \times \# \max W \times \#SM}{m} \right\rfloor$$

smallest k for $\#wrn > 1$

scheduling efficiency

$$wr = \left\lfloor \frac{n}{32} \right\rfloor$$

$\# \max W$ warps per block

$$\frac{m \times wr}{\# \max W} \text{ blocks}$$