Swinging It

Team 2-08: Ben Grant and Berwin Lan

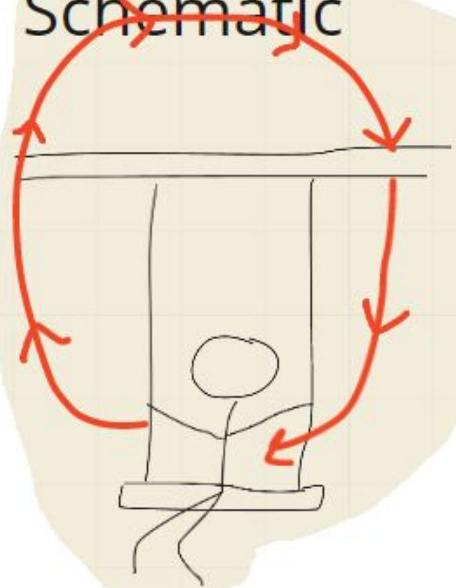
Question

What is the minimum speed you (as a kindergartener) need to be swinging at in order to make a complete revolution around the fulcrum?

Our question is primarily aimed at prediction, although it also has design implications for playground safety.

Model

Schematic



FBD(s)

ODEs

$$F = ma$$

$$-mg\sin\theta = ma = mr\alpha$$

$$\alpha = \frac{-mg\sin\theta}{mr}$$

$$\alpha = \frac{-g}{r}\sin\theta$$

2nd order:

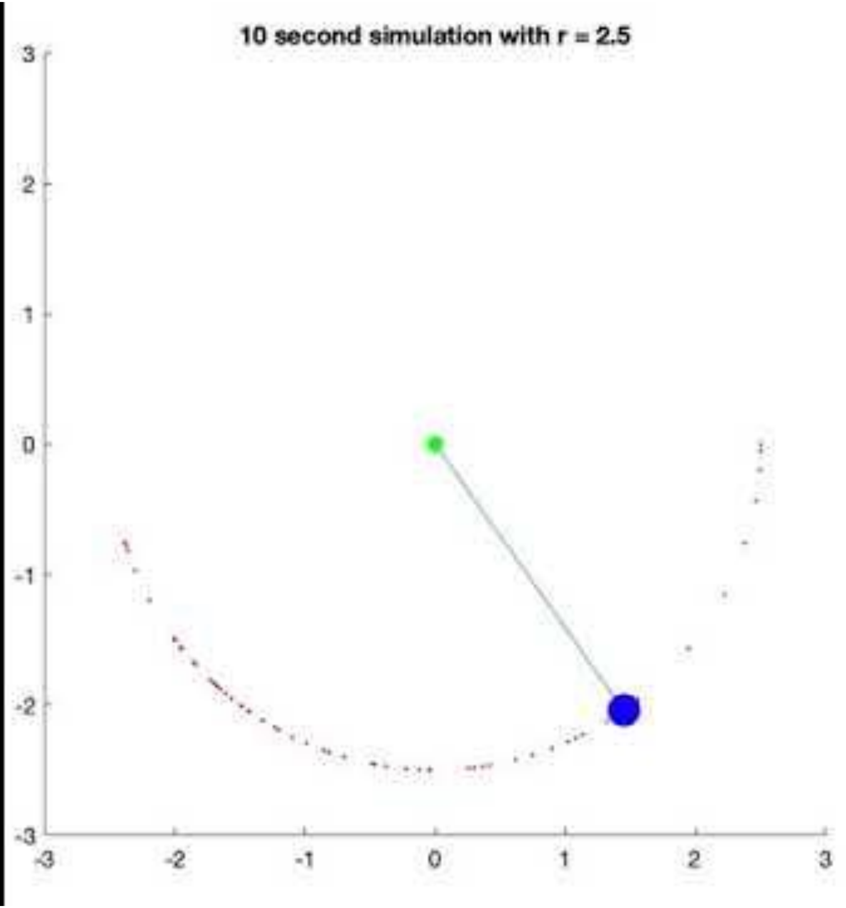
$$\frac{d^2\theta}{dt^2} = \frac{-g}{r}\sin\theta$$

1st orders:

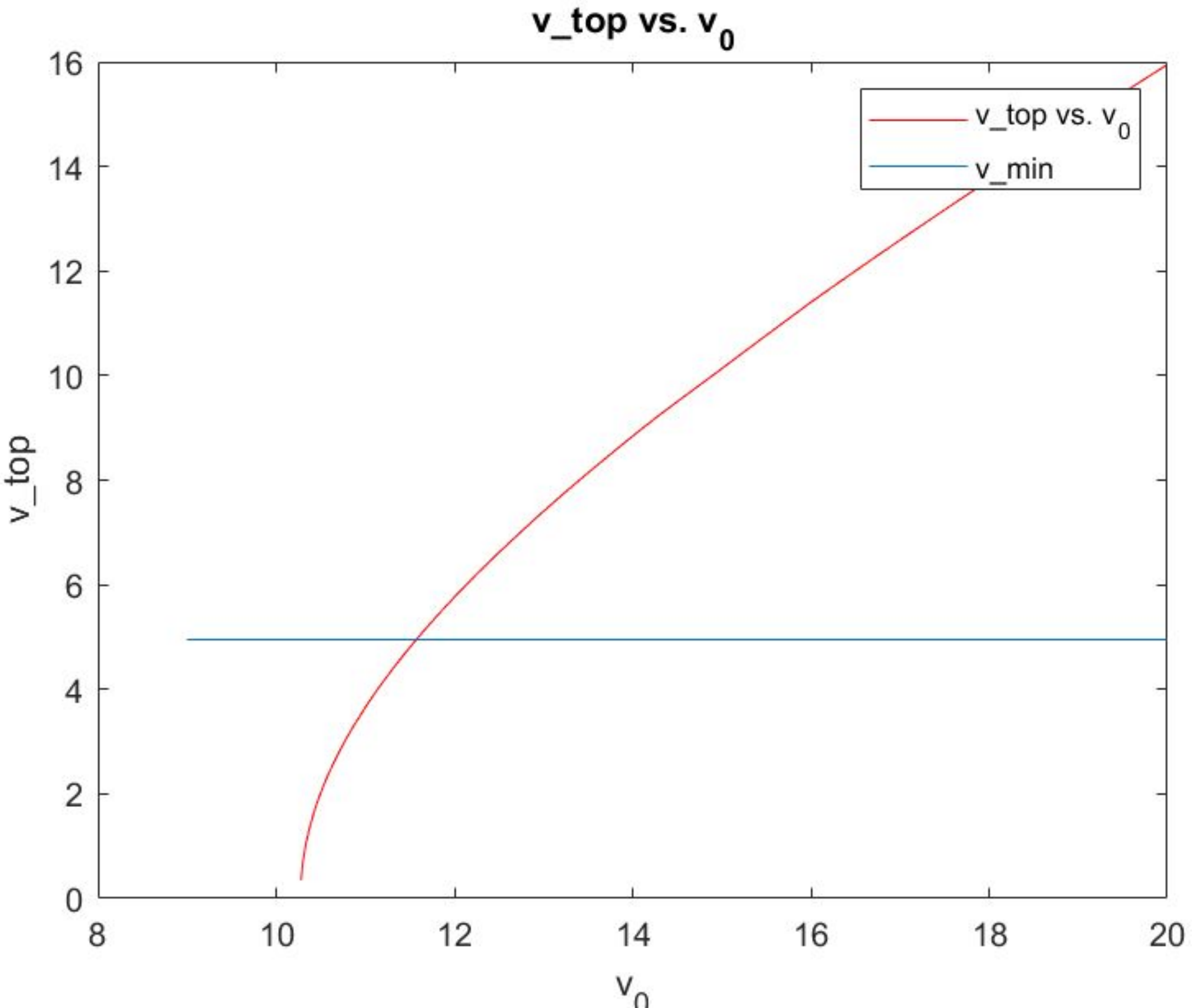
$$\frac{d\omega}{dt} = \frac{-g}{r}\sin(\theta)$$

$$\frac{d\theta}{dt} = \omega$$

Model



Results



v_top vs. v0
v_top vs. v0
v_min
v_top
v0
16
14
12
10
8
6
4
2
0
8
10
12
14
16
18
20

Interpretation

- We can conclude that at initial speeds greater than 11.58 m/s starting at an initial angle of 0, with length = 2.5 m and mass = 21 kg, kindergartener-you would be able to make it over the swing. That's almost 26 mph!
- We used the omnicalculator pendulum period calculator to validate our first cut model. Beyond that, we graphed our outputs as angle v. time with each iteration to verify our models and ensure that they made logical sense.

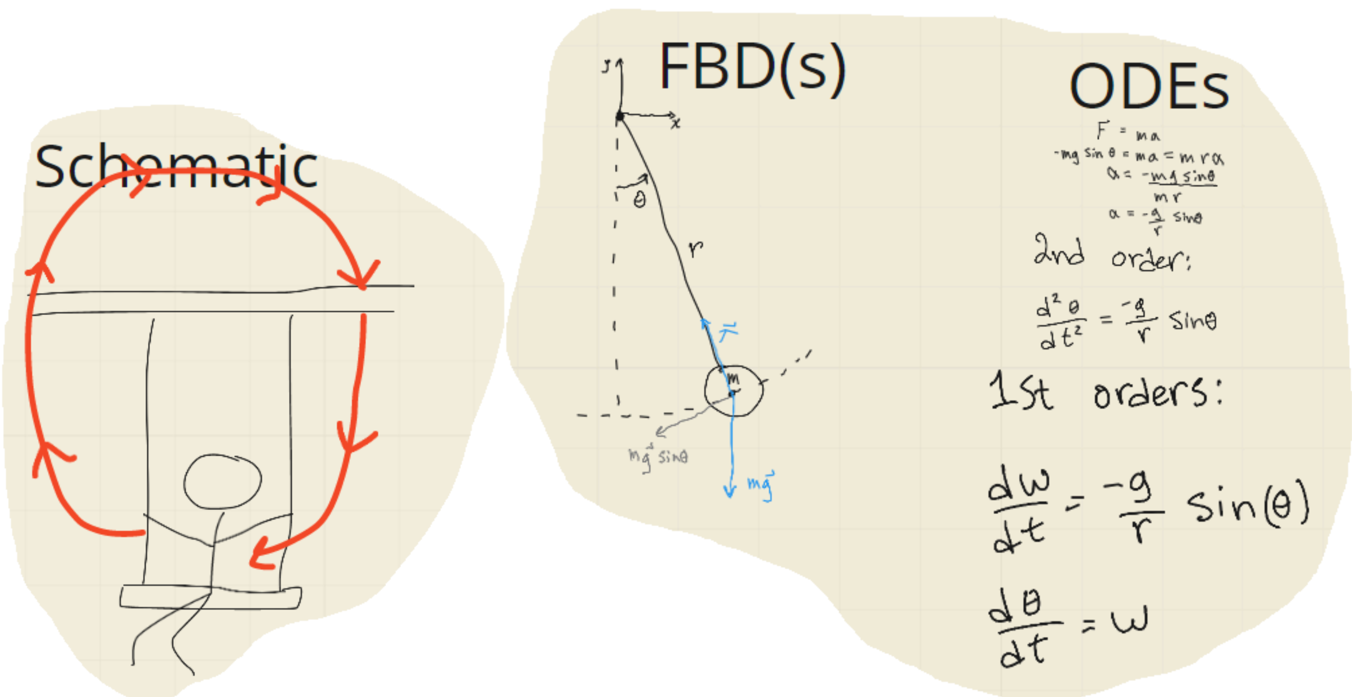
Resources

- Simple Pendulum Calculator
- The everyday effects of wind drag on people - McIlveen - 2002 - Weather

Swinging it - Ben Grant and Berwin Lan

Modeling Question

What is the minimum speed you need to be swinging at in order to make a complete revolution around the fulcrum?



ODEs

Second order:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{r}\sin(\theta),$$

where θ is angular displacement, t is time, g is gravitational const., r is length of rope.

First orders:

$$\frac{d\omega}{dt} = -\frac{g}{r}\sin(\theta) \quad \text{and} \quad \frac{d\theta}{dt} = \omega,$$

where ω is angular velocity.

Our first cut model uses a rigid massless rope with a point mass on the end in a vacuum. Many assumptions were made for this initial model, which was the first in our planned chain of iterations.

Complicating factors:	**Chain of iterations:**
Variation in length and mass	• Simple pendulum under gravity
Air resistance	• Rope length variation
Length of pendulum	• Air resistance
Friction at hinge	• Friction at hinge
Mass of rope	• Rope mass (physical pendulum?)

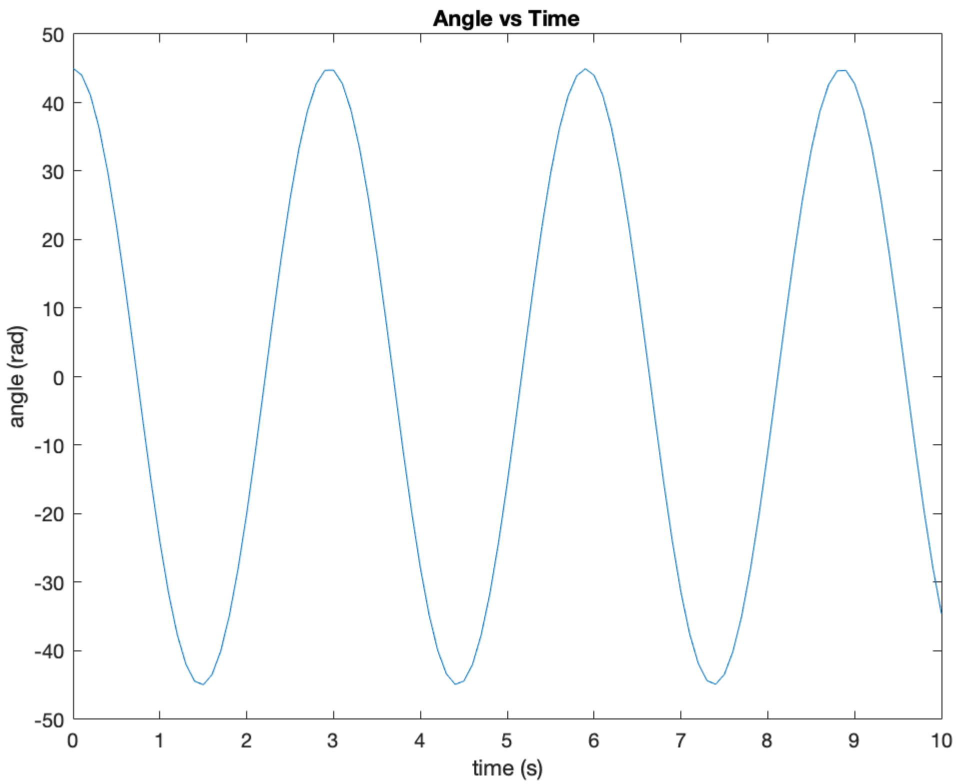
We used the omnicalculator pendulum period calculator to validate this model.

T = 2π√(L/g)

Acceleration of gravity (g)	1 g
Pendulum length (L)	2 m
Pendulum period (T)	2.8375 sec
Pendulum frequency (f)	0.3524 Hz

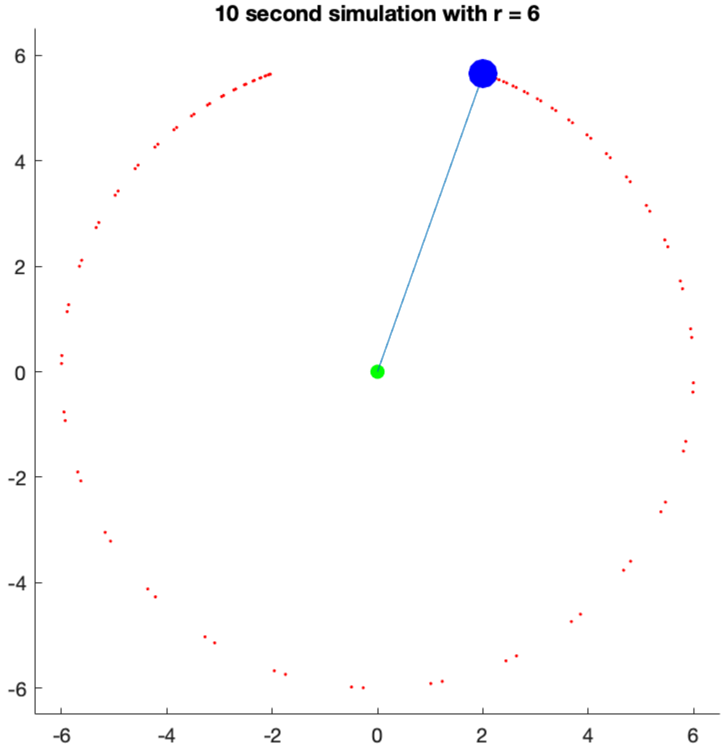
```
[T, M] = swing_it1(45, 0);
% convert to degrees
M(:,1) = M(:,1) .* (360 / 2 / pi);

figure;
plot(T, M(:, 1));
xlabel("time (s)");
ylabel("angle (rad)");
title("Angle vs Time");
```



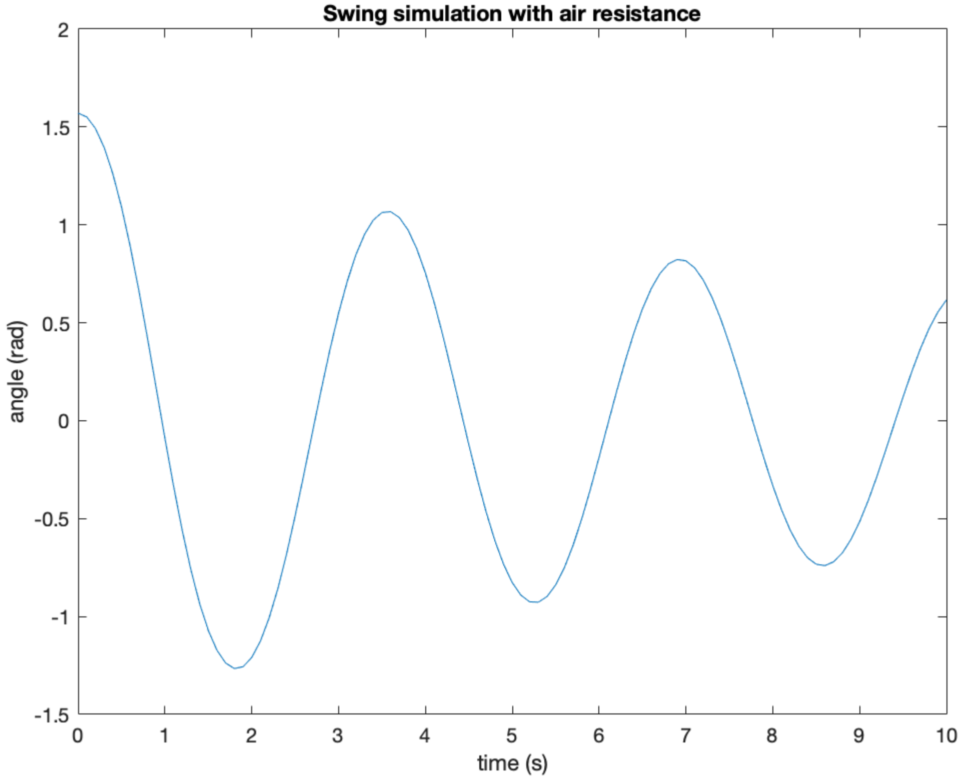
The second iteration, swing_it2, took out some hard coded values in favor of passing parameters in. The time_span parameter was added in order to use animations in our final presentation, and we also added in the ability to pass in the length of the rope.

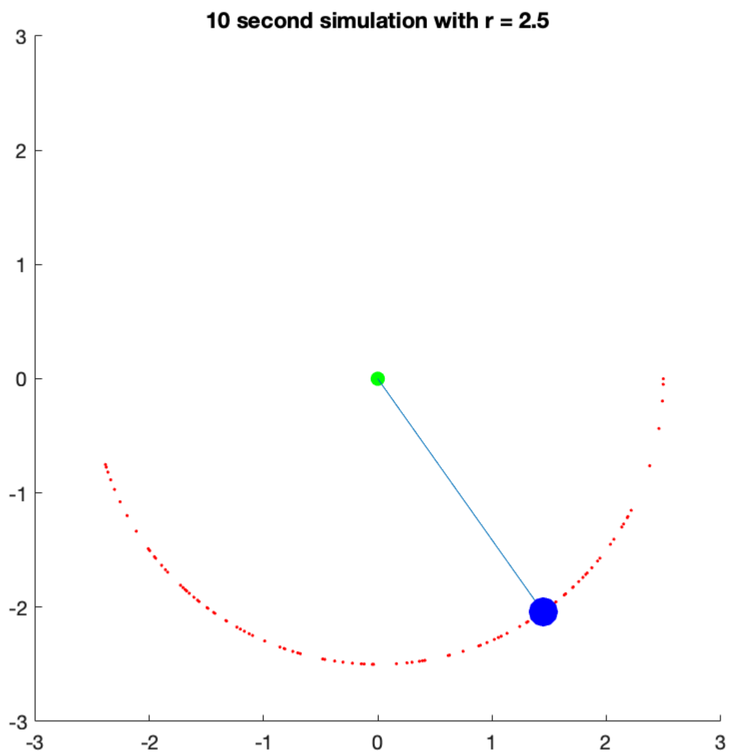
```
animation_test();
```



Air resistance was added in swing_it3, and a mass parameter was also added since it was needed for our calculations. For a pendulum in a vacuum, mass has no impact on the simulation; however, for a model in air, the mass will have an impact due to the dependence of the acceleration of air resistance on inertia and weight. Our numbers for the cross-sectional area and drag coefficients were pulled from this source.

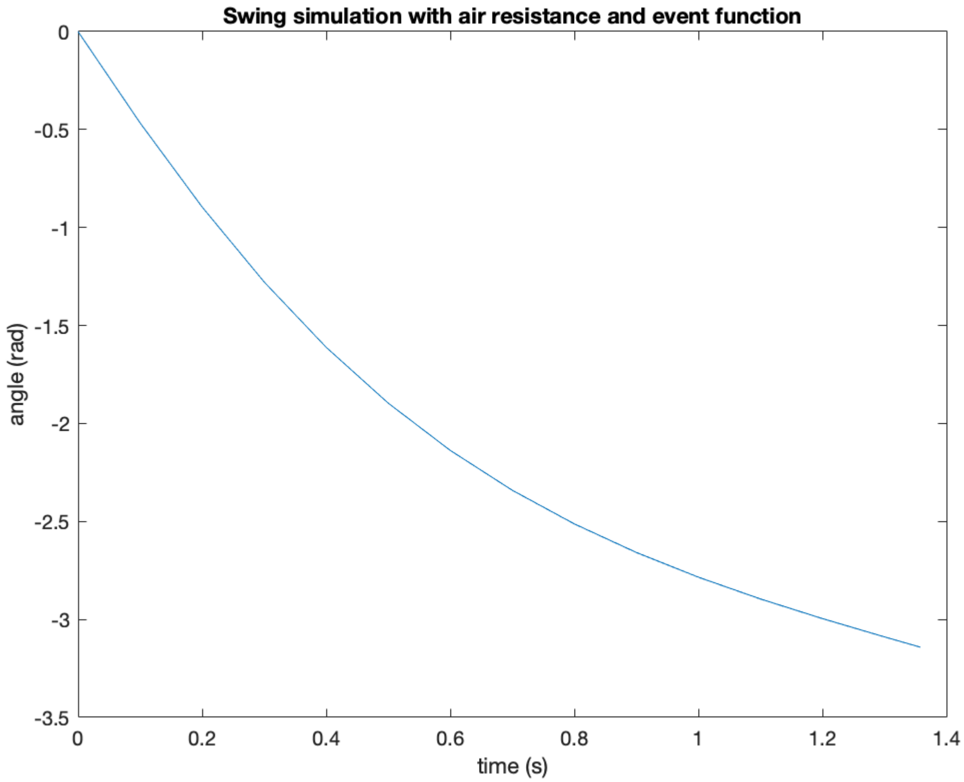
```
animation_test_drag();
```

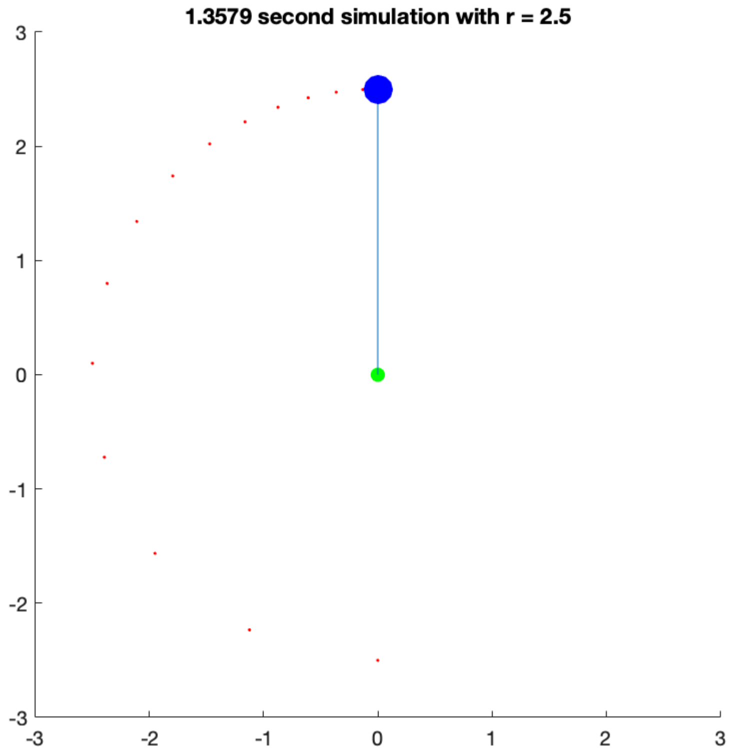




An event function for the swing going over the top of the fulcrum was added in swing_it4: the simulation stops once the swing reaches a vertical position above the fulcrum. The event function itself does not address the minimum velocity required for the rope to be approximated as a rigid body, which we addressed during the sweep.

```
animation_test_event();
```



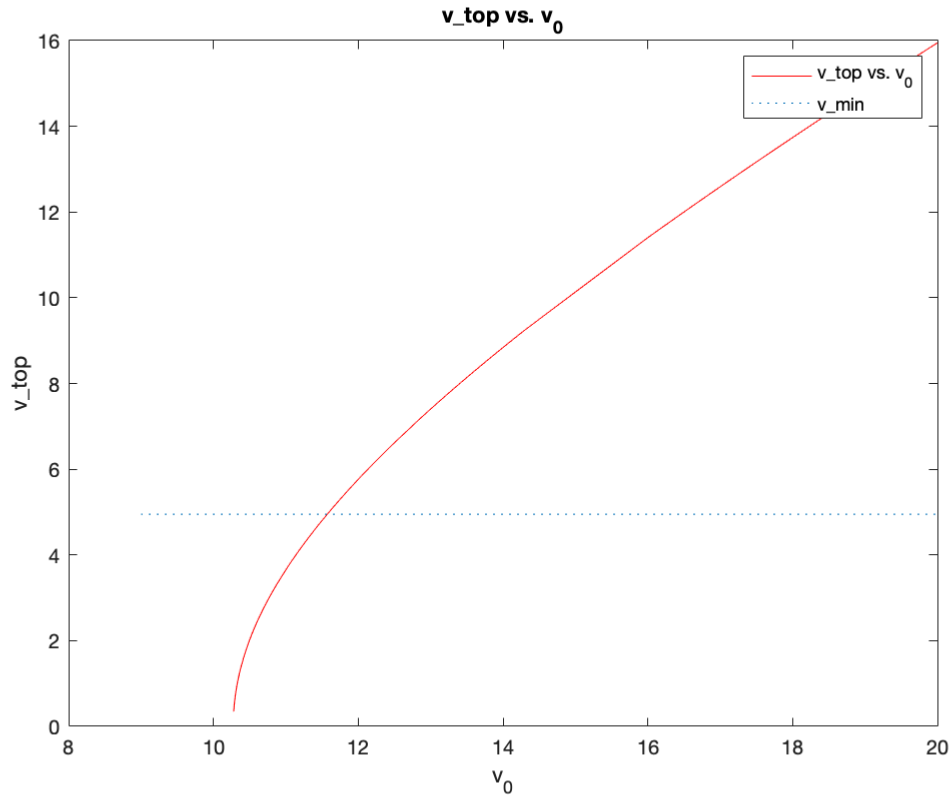


Manually playing around with the parameters shows that with an initial angle of 0 degrees, the initial velocity needs to be between 11 and 12 m/s in order to make it over the fulcrum with a rigid rope.

In our sweep, we swept the initial velocity from an initial angle of 0 (at the swing's lowest point), keeping all other parameters constant. The minimum critical velocity is the point at which a flexible, non-rigid rope can be approximated as a rigid body, as our model does. This point is where the gravitational acceleration is equal to the centripetal acceleration, represented by v_min on the graph.

The graph v_top vs. v_0 plots the initial velocities that successfully make it over the top of the swing with a rigid rope. The y-axis shows the velocities at the top of the swing, with the blue line being the critical velocity that must be exceeded in order for our assumption of a rigid rope to be valid. **Thus, we can conclude that at initial speeds greater than 11.58 m/s starting at an initial angle of 0, with length = 2.5 m and mass = 21 kg, kindergartener-you would be able to make it over the swing.**

```
animation_sweeping();
```



v_top vs. v0
v_top vs. v0
v_min
v_top
v0
16
14
12
10
8
6
4
2
0
8
10
12
14
16
18
20