

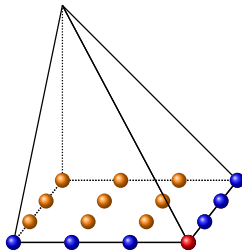
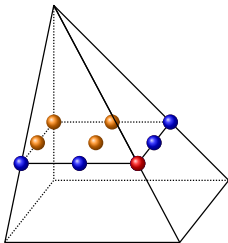
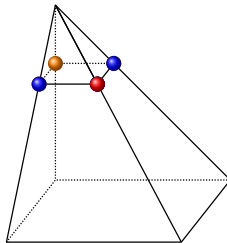
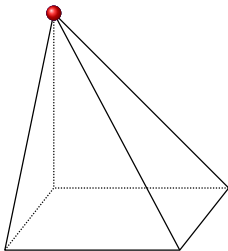
# Recursive Lexicographical Search: Finding all Markov Perfect Equilibria in Directional Dynamic Games

Fedor Iskhakov, University of New South Wales  
John Rust, Georgetown University  
Bertel Schjerning, University of Copenhagen

Australian National University  
Research School of Economics  
February 15, 2016

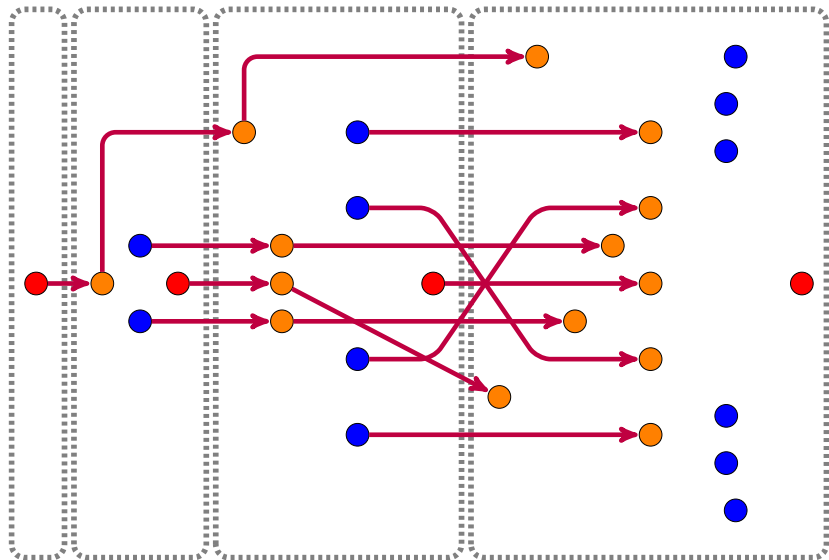
# Transitions due to technological progress

As  $c$  decreases, the game falls through the layers of the pyramid



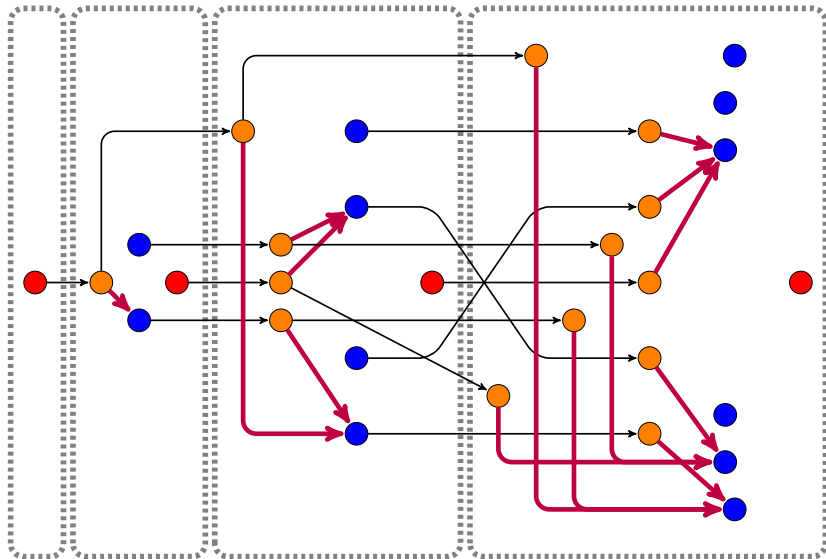
# Transitions due to technological progress

As  $c$  decreases, the game falls through the layers of the pyramid



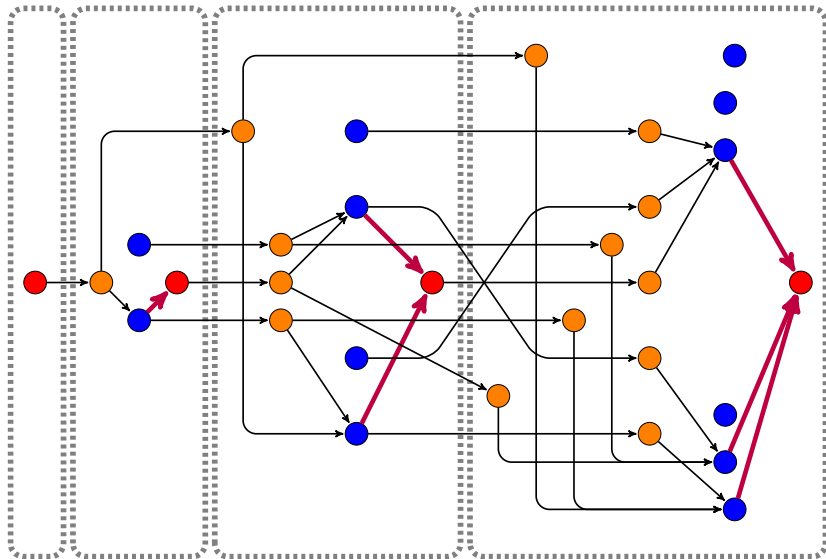
# Strategy-specific partial order on $S$

Strategy  $\sigma_1$  of firm 1: invest at all interior points



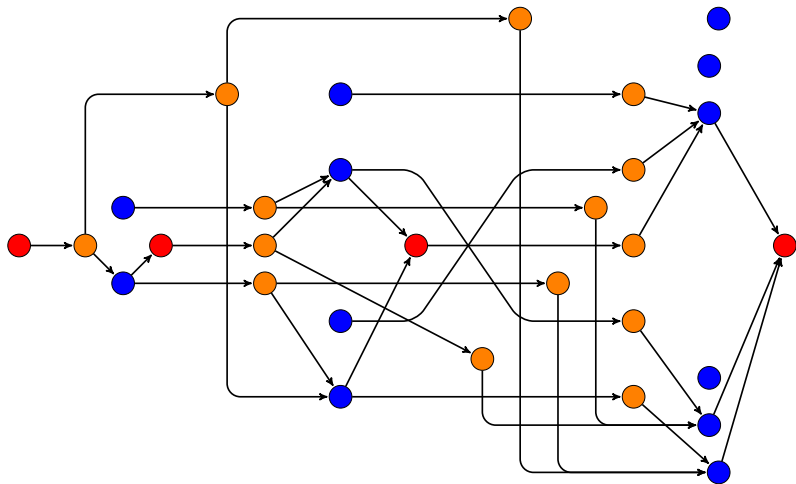
## Strategy-specific partial order on $S$

Strategy  $\sigma_2$  of firm 2: invest at all edge points



# Strategy-specific partial order on $S$

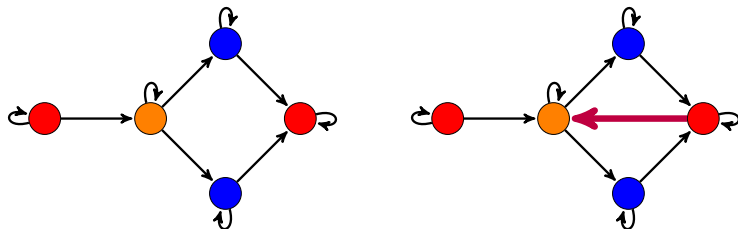
Strategy  $\sigma = (\sigma_1, \sigma_2)$  of both firms



# No loop (anti-cycling) condition

Hypothetical strategy profile inducing cycles

**Self-loops** appear when the game remains in the same state for two or more consecutive periods of time

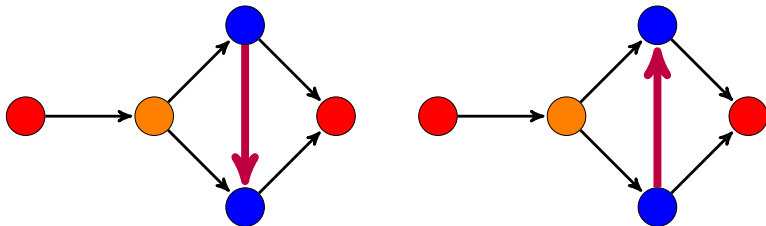


But loops between different states are not allowed

# Consistency of strategy specific partial orders

Two hypothetical inconsistent strategies

Two strategies that induce **opposite** transitions are **inconsistent**



Note that in both cases the no-loop condition is satisfied



# Definition of the Dynamic Directional Games

## Definition (Dynamic Directional Games, DDG)

Finite state Markovian stochastic game is a DDG if it holds:

- 1 Every feasible Markovian strategy  $\sigma$  satisfies the no loop condition.
- 2 Every pair of feasible Markovian strategies  $\sigma$  and  $\sigma'$  induce consistent partial orders on the state space.

# Next: Stages, stage games and state recursion

- ① Strategy **independent** partial order over  $S$
- ② DAG to represent the directionality of the game
- ③ Recursion on the game DAG to form partition of totally ordered subsets of  $S$
- ④ Stages on the state space and induced subgames of DDG
- ⑤ Continuation strategies and stage games

# Strategy independent partial order on $S$

Coarsest common refinement of partial orders induced by all strategies

