IV. Deep Neural Networks

1) Efficient Training Methods

1.1) Implementing Backpropagation(Mathematica)
1.2) DNNs with TensorFlow/Keras
1.3) Data pre-processing

1.1)

Implementing backpropagation

1) $$a_i^{(1)} = x_i$$

2) $$z_i^{(l)} = \sum_{j=1}^{n_{l-1}} w_{ij}^{(l)} a_j^{(l-1)} + b_i^{(l)} \qquad a_i^{(l)} = \sigma(z_i^{(l)})$$

3) $$\Delta_i^{(L)} = \frac{\partial E}{\partial a_i^{(l)}} \sigma'(z_i^{(L)})$$

4) $$\Delta_i^{(l)} = \sigma'(z_i^{(l)}) \sum_{j=1}^{n_{l+1}} \Delta_j^{(l+1)} w_{ji}^{(l+1)}$$

5) $$\frac{\partial E}{\partial b_i^{(l)}} = \Delta_i^{(l)} \qquad \frac{\partial E}{\partial w_{ij}^{(l)}} = \Delta_i^{(l)} a_j^{(l-1)}$$

1.2)

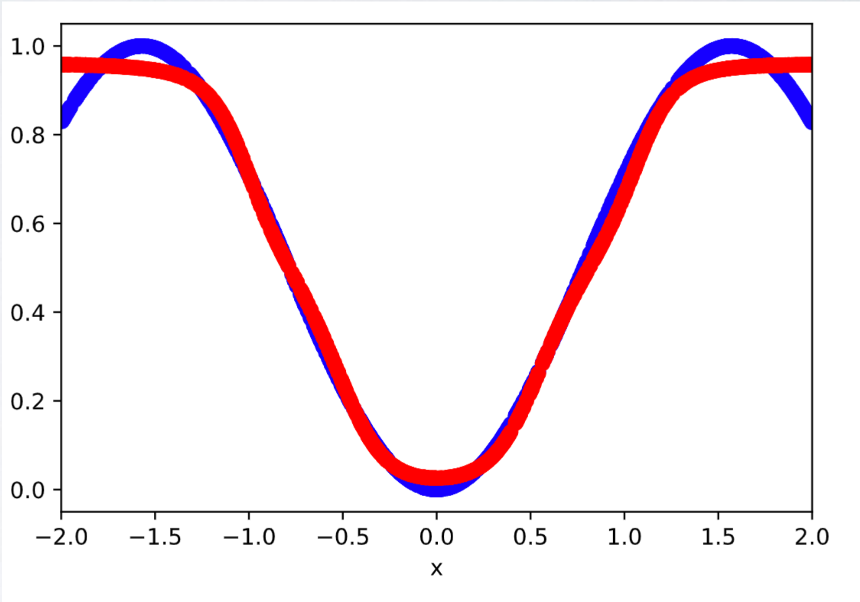
DNNs with Tensorflow/Keras

```
import numpy as np
import matplotlib.pyplot as plt

import tensorflow as tf
from tensorflow.keras import activations

from keras.models import Sequential
from keras.layers import Dense, Activation
from keras.optimizers import Adam
# for 2nd attempt
from keras.initializers import glorot_uniform
from keras.callbacks import EarlyStopping

import keras.backend as K
```



see example notebook `NNbasic.ipynb`

1.2)

Efficient Training Methods (Regularisation)

Parameter initialisation e.g. Glorot-Bengio initialisation

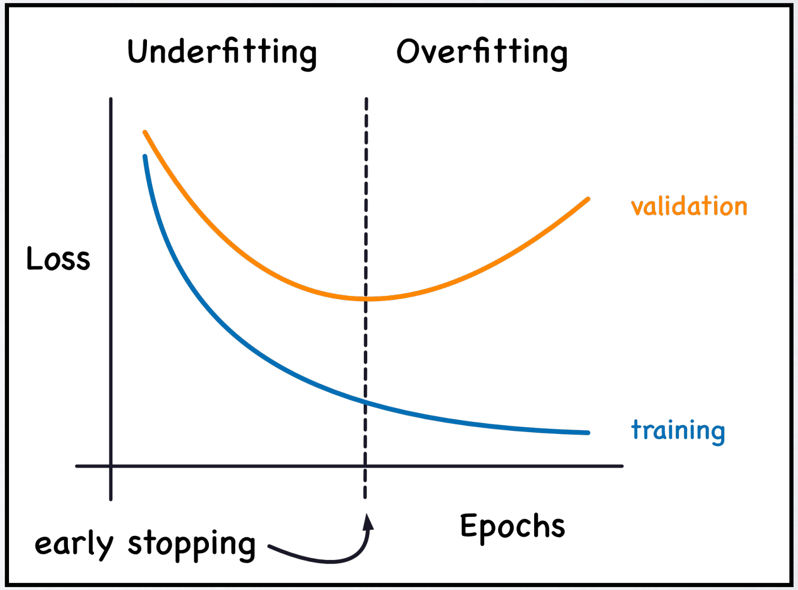
We initialized the biases to be 0 and the weights W_{ij} at each layer with the following commonly used heuristic:

$$W_{ij} \sim U\left[-\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}\right], \tag{1}$$

where $U[-a, a]$ is the uniform distribution in the interval $(-a, a)$ and n is the size of the previous layer (the number of columns of W).

http://proceedings.mlr.press/v9/glorot10a/glorot10a.pdf

Cross validation of loss function on testing and training sets: **Early stopping**



1.2)

Efficient Training Methods (Regularisation)

Dropout

see https://arxiv.org/pdf/1803.08823.pdf

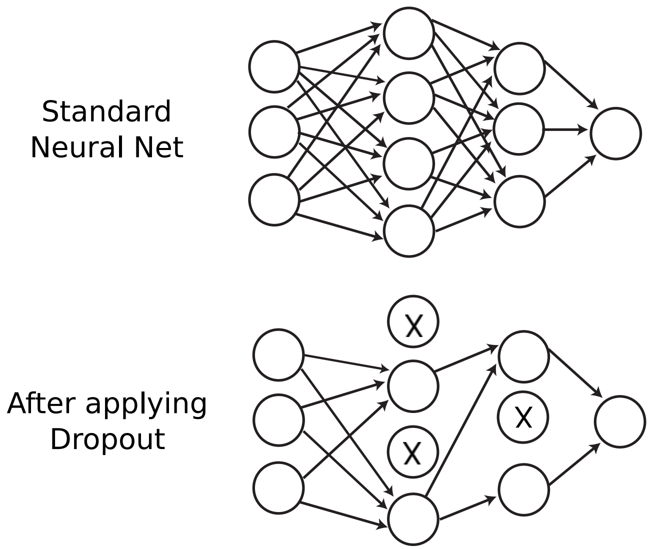


FIG. 39 **Dropout** During the training procedure neurons are randomly "dropped out" of the neural network with some probability p giving rise to a thinned network. This prevents overfitting by reducing correlations among neurons and reducing the variance in a method similar in spirit to ensemble methods.

1.2)

Efficient Training Methods (Regularisation)

mini-batch: update model parameters
using small subsets of the data

controlled split of test training datasets:
e.g. `sklearn.model_selection.train_test_split`

Hyperparameter tuning: learning rate, layers and depths,…
see KerasTuner: https://keras.io/keras_tuner/

1.3)

Data pre-processing

So far we have made sure to choose functions that lie between 0 and 1 so the sigmoid (logistic) activation function was appropriate

In general we should orgainse our data to make sure the choice of network architecture is appropriate

$$x'(x, \langle x \rangle, \sigma_x) = \frac{x - \langle x \rangle}{\sigma_x}$$ 'standardise'

$$f'(f(x), f_{\mathrm{min}}, f_{\mathrm{max}}) = \frac{f(x) - f_{\mathrm{min}}}{f_{\mathrm{max}} - f_{\mathrm{min}}}$$ 'normalise'

After calling the trained network, de-standardisation/de-normalisation should be applied e.g.

$$f(x) = f'(x') \times (f_{\mathrm{max}} - f_{\mathrm{min}}) + f_{\mathrm{min}}$$