Polar

September 29, 2020

1 Imports

[1]:
```
import json
from functools import reduce
from os import listdir
from os.path import isfile, join
from pathlib import Path

import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
import statsmodels.api as sm
from pylab import rcParams
from scipy.stats import shapiro
from statsmodels.graphics.gofplots import qqplot
from statsmodels.stats.diagnostic import het_breuschpagan
```

2 Setup

[2]:
```
# Set figure size
rcParams['figure.figsize'] = (4, 4)

# Folder for images
Path('img').mkdir(parents=True, exist_ok=True)

# Nice float format
pd.options.display.float_format = "{:,.6f}".format
```

3 Data description

Last year I purchased a Polar watch that tracks my vitals during workouts. I used the Polar Flow website to obtain a copy of my data. For privacy reasons I shall not be sharing the dataset.

[3]:
```
path = './data/'
```

First, we create a list of files in the download.

[4]:
```
files = [f for f in listdir(path) if isfile(join(path, f))]
```

We shall only consider files containing the string `'training-session'`.

[5]:
```
files = [f for f in files if 'training-session' in f]
```

The number of files under consideration is:

[6]:
```
len(files)
```

[6]:
```
284
```

We loop over each of the files and them to a list.

[7]:
```
data = []

for f in files:
    with open(join(path, f)) as f:
        d = json.load(f)
        data.append(d)
```

We extract the relevant information from the items in the list.

[8]:
```
workouts = []

for d in data:
    workouts.append(d['exercises'][0])
```

Finally we create a dataframe containing the workout information.

[9]:
```
df = pd.DataFrame(workouts)
```

4 Data structure

We find the following columns in the dataframe.

[10]:
```
df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 284 entries, 0 to 283
Data columns (total 17 columns):
 #   Column          Non-Null Count  Dtype
---  ------          --------------  -----
 0   startTime       284 non-null    object
 1   stopTime        284 non-null    object
 2   timezoneOffset  284 non-null    int64
 3   duration        284 non-null    object
 4   sport           284 non-null    object
 5   kiloCalories    283 non-null    float64
```

```
 6   heartRate       283 non-null    object
 7   zones           284 non-null    object
 8   samples         284 non-null    object
 9   distance        130 non-null    float64
 10  latitude        130 non-null    float64
 11  longitude       130 non-null    float64
 12  ascent          120 non-null    float64
 13  descent         121 non-null    float64
 14  speed           130 non-null    object
 15  autoLaps        102 non-null    object
 16  laps            2 non-null      object
dtypes: float64(6), int64(1), object(10)
memory usage: 37.8+ KB
```

We remove columns that containt data from features I do not use in my training.

Due to privacy concerns I shan't be extracting longitudinal and latitudinal data.

[11]:
```
df = df.drop(['zones', 'samples', 'autoLaps',
              'laps', 'latitude', 'longitude'], axis=1)
```

[12]:
```
df.head()
```

[12]:
```
                 startTime                 stopTime  timezoneOffset  \
0  2019-05-24T13:18:14.000  2019-05-24T14:58:44.125             120
1  2019-05-04T12:03:34.000  2019-05-04T13:21:38.500             120
2  2019-04-12T12:48:57.000  2019-04-12T12:59:10.750             120
3  2019-06-12T13:13:09.000  2019-06-12T13:23:15.500             120
4  2019-05-24T14:59:06.000  2019-05-24T15:29:08.750             120

      duration             sport  kiloCalories  \
0  PT6030.125S  STRENGTH_TRAINING    658.000000
1  PT4684.500S  STRENGTH_TRAINING    373.000000
2   PT613.750S  TREADMILL_RUNNING     62.000000
3   PT606.500S  TREADMILL_RUNNING     71.000000
4  PT1802.750S  TREADMILL_RUNNING    416.000000

                               heartRate  distance  ascent  descent speed
0  {'min': 72, 'avg': 105, 'max': 136}       nan     nan      nan   NaN
1   {'min': 71, 'avg': 99, 'max': 138}       nan     nan      nan   NaN
2   {'min': 71, 'avg': 97, 'max': 107}       nan     nan      nan   NaN
3  {'min': 67, 'avg': 105, 'max': 121}       nan     nan      nan   NaN
4  {'min': 84, 'avg': 144, 'max': 170}       nan     nan      nan   NaN
```

5 Missing Values

The watch tracks different information for different workouts. For example when walking it tracks location but when walking on a treadmill it doesn't, hence there is quite a lot of missing data.

```
[13]: missing = (df.isna().sum() / df.shape[0] * 100)
missing.name = 'Missing %'
missing = missing.to_frame()
missing = missing.sort_values('Missing %', ascending=False)
missing = missing[missing['Missing %'] > 0]
np.round(missing, 2)
```

```
[13]:              Missing %
ascent         57.750000
descent        57.390000
distance       54.230000
speed          54.230000
kiloCalories    0.350000
heartRate       0.350000
```

6 Transforms

We apply certain transforms to make the data easier to work with. First we convert strings to datetimes.

```
[14]: df['startTime'] = pd.to_datetime(df['startTime'])
df['stopTime'] = pd.to_datetime(df['stopTime'])
```

We calculate the total duration of each individual workout in minutes.

```
[15]: df['totalTime'] = (df['stopTime'] - df['startTime'])
df['totalTime'] = df['totalTime'].apply(lambda x: round(x.seconds / 60, 2))
df.drop('duration', axis=1, inplace=True)
```

We split the datetime columns in to date and time.

```
[16]: df['startDate'] = pd.to_datetime(df['startTime']).dt.date
df['stopDate'] = pd.to_datetime(df['stopTime']).dt.date
df['startTime'] = pd.to_datetime(df['startTime']).dt.time
df['stopTime'] = pd.to_datetime(df['stopTime']).dt.time
```

We extract maximum, average and minimum heart rate values from the `heartRate` column.

```
[17]: df['heartRateMax'] = df['heartRate'].apply(lambda x: x['max'] if isinstance(x,␣
 ↪dict) else np.nan)
df['heartRateAvg'] = df['heartRate'].apply(lambda x: x['avg'] if isinstance(x,␣
 ↪dict) else np.nan)
df['heartRateMin'] = df['heartRate'].apply(lambda x: x['min'] if isinstance(x,␣
 ↪dict) else np.nan)
df.drop('heartRate', axis=1, inplace=True)
```

In a similar manner we extract the maximum, average and minimum values form the `speed` column.

[18]:
```
df['speedAvg'] = df['speed'].apply(lambda x: x['avg'] if isinstance(x, dict)␣
 ↪else np.nan)
df['speedMax'] = df['speed'].apply(lambda x: x['max'] if isinstance(x, dict)␣
 ↪else np.nan)
df.drop('speed', axis=1, inplace=True)
```

We reorder the data as follows.

[19]:
```
order = ['startDate',
         'stopDate',
         'startTime',
         'stopTime',
         'timezoneOffset',
         'totalTime',
         'sport',
         'kiloCalories',
         'heartRateMax',
         'heartRateAvg',
         'heartRateMin']
```

[20]:
```
df = df[order]
```

We check if there are any more `NaN`'s in the data.

[21]:
```
df.isna().sum()
```

[21]:
```
startDate         0
stopDate          0
startTime         0
stopTime          0
timezoneOffset    0
totalTime         0
sport             0
kiloCalories      1
heartRateMax      1
heartRateAvg      1
heartRateMin      1
dtype: int64
```

There is one row with `NaN`'s. This might due to my watch having little battery left to make the measurements.

[22]:
```
df = df.dropna()
```

Finally we proceed to sort the data with the latest workouts at the top of the dataframe.

[23]:
```
sort_cols = ['startDate','startTime']
df = df.sort_values(sort_cols, ascending=False)
```

```python
df = df.reset_index(drop=True)
```

[24]:
```python
df.head()
```

[24]:
```
    startDate    stopDate startTime         stopTime  timezoneOffset  \
0  2020-03-29  2020-03-29  21:50:21  22:23:41.750000             120
1  2020-03-27  2020-03-27  20:38:32  21:25:03.750000              60
2  2020-03-26  2020-03-26  21:07:46  21:52:55.625000              60
3  2020-03-25  2020-03-25  19:22:38  20:10:17.875000              60
4  2020-03-24  2020-03-24  13:09:06  13:48:46.750000              60

   totalTime    sport  kiloCalories  heartRateMax  heartRateAvg  heartRateMin
0  33.330000  WALKING    245.000000    116.000000    102.000000     69.000000
1  46.520000  WALKING    401.000000    132.000000    104.000000     70.000000
2  45.150000  WALKING    336.000000    122.000000    103.000000     87.000000
3  47.650000  WALKING    380.000000    125.000000    108.000000     87.000000
4  39.670000  WALKING    358.000000    141.000000    117.000000     90.000000
```

[25]:
```python
df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 283 entries, 0 to 282
Data columns (total 11 columns):
 #   Column          Non-Null Count  Dtype
---  ------          --------------  -----
 0   startDate       283 non-null    object
 1   stopDate        283 non-null    object
 2   startTime       283 non-null    object
 3   stopTime        283 non-null    object
 4   timezoneOffset  283 non-null    int64
 5   totalTime       283 non-null    float64
 6   sport           283 non-null    object
 7   kiloCalories    283 non-null    float64
 8   heartRateMax    283 non-null    float64
 9   heartRateAvg    283 non-null    float64
 10  heartRateMin    283 non-null    float64
dtypes: float64(5), int64(1), object(5)
memory usage: 24.4+ KB
```

7 Analysis

Given that we have produced a clean dataset we can proceed to analyse a few aspects.

7.1 Time span

The date of the first workout is:

[26]:
```
str(df['startDate'].min())
```

[26]: '2019-02-20'

The date of the last workout is:

[27]:
```
str(df['startDate'].max())
```

[27]: '2020-03-29'

Workouts measured:

[28]:
```
len(df)
```

[28]: 283

7.2 Descriptive statistics

[29]:
```
df.drop('timezoneOffset', axis=1).describe()
```

[29]:

	totalTime	kiloCalories	heartRateMax	heartRateAvg	heartRateMin
count	283.000000	283.000000	283.000000	283.000000	283.000000
mean	42.831060	315.975265	128.335689	105.194346	76.738516
std	29.647283	218.745576	18.254807	11.866621	8.987117
min	5.000000	29.000000	93.000000	82.000000	53.000000
25%	15.920000	121.500000	115.000000	96.000000	70.000000
50%	36.450000	277.000000	125.000000	103.000000	77.000000
75%	65.290000	441.500000	138.500000	111.000000	83.000000
max	172.730000	1,067.000000	178.000000	148.000000	99.000000

7.3 Kilocalories burned in total

First we count the total kilocalories I burned during the period in question.

[30]:
```
total_calories = df['kiloCalories'].sum()
print(total_calories)
```

89421.0

We convert this number to kilograms of body fat. According to this article it equates to

[31]:
```
total_calories / 7700
```

[31]: 11.613116883116883

7.4 Kilocalories burned by sport

[32]:
```
by_sport = df[['kiloCalories', 'sport']].groupby('sport', as_index=False)
by_sport = by_sport.sum()
by_sport['sport'] = by_sport['sport'].apply(lambda x: x.lower())
by_sport['kiloCalories'] = by_sport['kiloCalories'].astype(int)
by_sport = by_sport.rename(columns={'kiloCalories': 'total kilocalories'})
by_sport = by_sport.sort_values('total kilocalories', ascending=False)
by_sport
```

[32]:

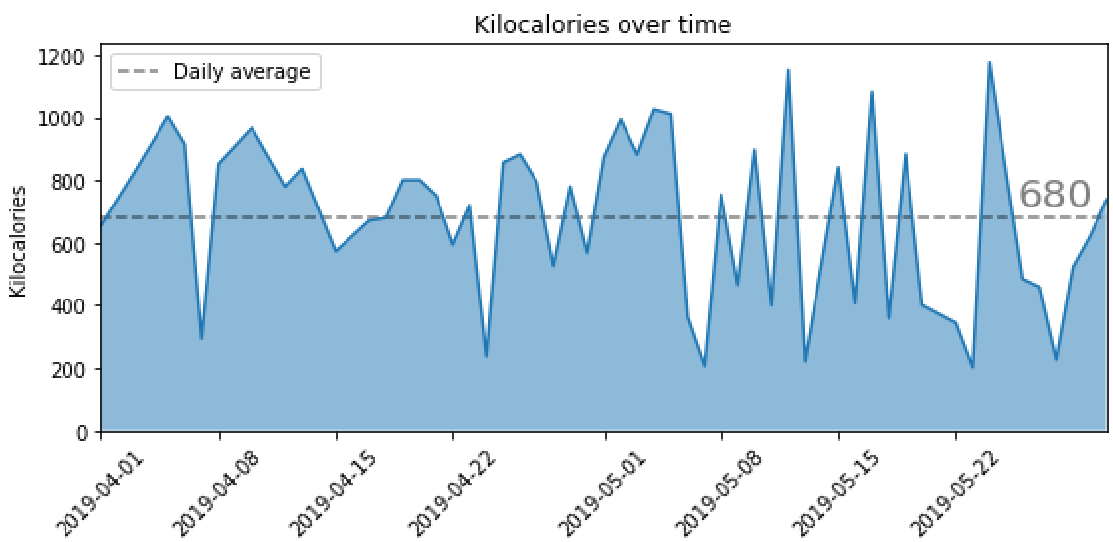
	sport	total kilocalories
4	walking	33080
2	strength_training	31547
3	treadmill_running	19825
0	cycling	4029
1	running	940

7.5 Kilocalories burned over time

Next we produce a plot of kilocalories burned over a two month period in 2019.

[33]:
```
width = 800
height = 400
dpi = 100

start = pd.to_datetime('2019-04-1')
stop = pd.to_datetime('2019-06-1')

daily = df[['startDate', 'kiloCalories']]
mask = (daily['startDate'] >= start) & (daily['startDate'] < stop)
daily = daily[mask]
daily = daily.groupby('startDate', as_index=False)
daily = daily.sum()
daily = daily.sort_values('startDate', ascending=False)
daily = daily.reset_index(drop=True)

plt.figure(figsize=(width/dpi, height/dpi))
plt.plot(daily['startDate'], daily['kiloCalories'])

plt.fill_between(x=daily['startDate'],
                 y1=0,
                 y2=daily['kiloCalories'],
                 alpha=1/2)

plt.hlines(xmin=daily['startDate'].min(),
           xmax=daily['startDate'].max(),
           y=daily['kiloCalories'].mean(),
           linestyle='dashed',
```

```
            label='Daily average',
            alpha=1/2)

plt.text(x=daily.loc[3, 'startDate'],
         y=daily['kiloCalories'].mean() + 75,
         s=round(daily['kiloCalories'].mean()),
         verticalalignment='center',
         horizontalalignment='center',
         alpha=1/2,
         fontsize=20)

plt.title('Kilocalories over time')
plt.xticks(rotation=45, horizontalalignment='center')
plt.xlim(daily['startDate'].min(), daily['startDate'].max())
plt.ylim(0, daily['kiloCalories'].max() * 1.05)
plt.ylabel('Kilocalories')
plt.legend(loc='best')
plt.tight_layout()
plt.savefig('./img/kilocalories_ts.png')
plt.show()
```



7.6 Workouts by sport

We check how many workouts I completed.

[34]:
```
stats = df[['sport', 'startTime']]
stats = stats.groupby(['sport'], as_index=False)
stats = stats.count()
stats = stats.rename(columns={'sport': 'Sport',
```

```
                                 'startTime': 'Count'})
stats = stats.sort_values('Count', ascending=False)
stats.head()
```

[34]:

	Sport	Count
4	WALKING	105
3	TREADMILL_RUNNING	90
2	STRENGTH_TRAINING	62
0	CYCLING	24
1	RUNNING	2

7.7 By hour of day

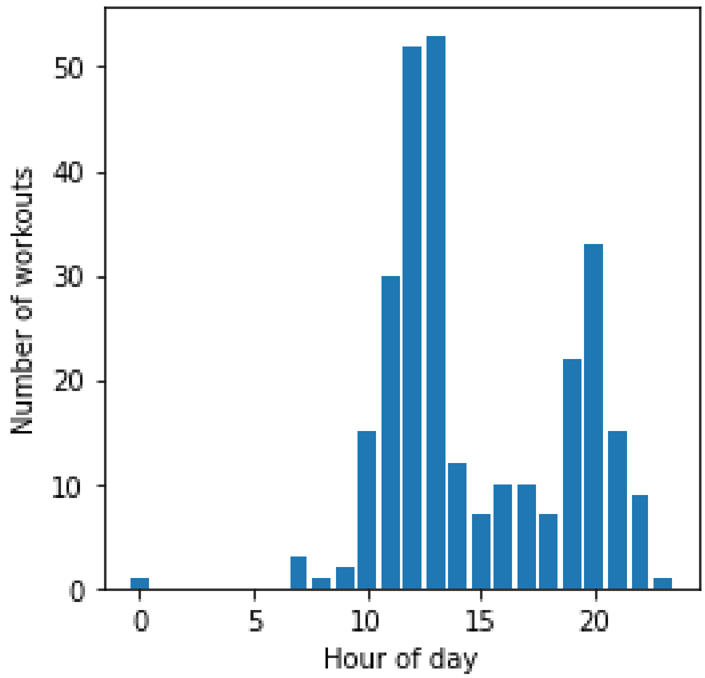
We count workouts by hour of day.

[35]:

```
by_hour = df[['startTime', 'sport']].copy()
by_hour['startTime'] = pd.to_datetime(by_hour['startTime'], format='%H:%M:%S')
by_hour['startHour'] = by_hour['startTime'].dt.hour
by_hour = by_hour.drop('startTime', axis=1)
by_hour = by_hour.groupby('startHour', as_index=False)
by_hour = by_hour.count()

all_hours = pd.DataFrame(range(0, 24), columns=['startHour'])

by_hour = pd.merge(all_hours, by_hour, how='left')
by_hour = by_hour.fillna(0)
by_hour = by_hour.sort_values('startHour')
by_hour = by_hour.rename(columns={'startHour': 'Hour of day',
                                  'sport': 'Total workouts'})
```

[36]:

```
plt.bar(by_hour['Hour of day'], by_hour['Total workouts'])
plt.ylabel('Number of workouts')
plt.xlabel('Hour of day')
plt.savefig('./img/workouts_by_hour_of_day.png')
plt.show()
```

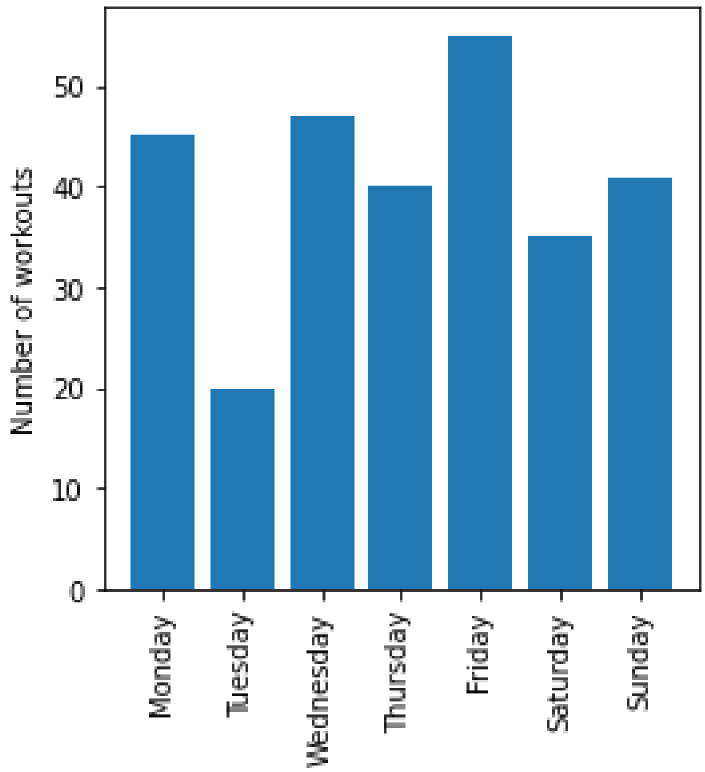


7.8 By day of week

We count workouts by day of week.

[37]:
```
by_day = df[['startDate', 'sport']].copy()
by_day['Day of week'] = pd.to_datetime(by_day['startDate']).dt.day_name()
by_day['Day number'] = pd.to_datetime(by_day['startDate']).dt.dayofweek
by_day = by_day.groupby(['Day of week', 'Day number'], as_index=False)
by_day = by_day.count()
by_day = by_day.drop('startDate', axis=1)
by_day = by_day.sort_values('Day number')
by_day = by_day.rename(columns={'sport': 'Total Workouts'})
```

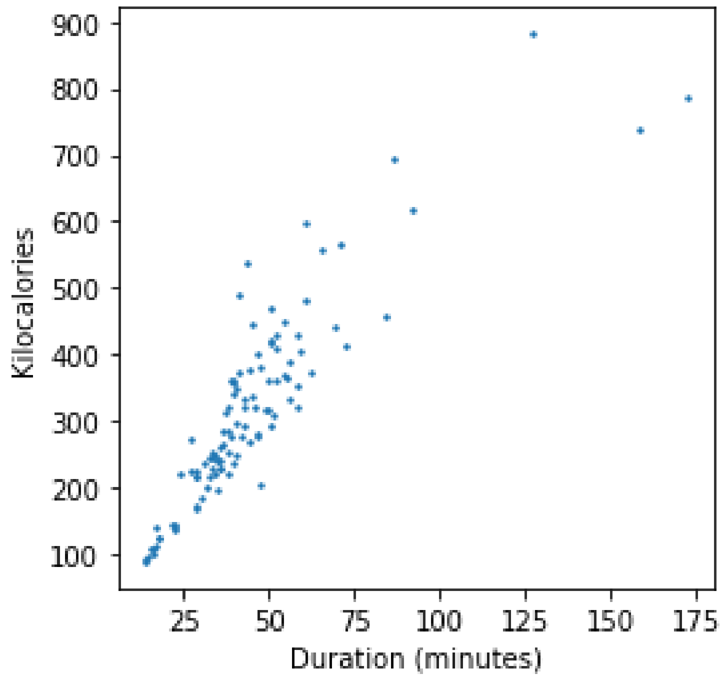
[38]:
```
plt.bar(by_day['Day of week'], by_day['Total Workouts'])
plt.xticks(rotation=90)
plt.ylabel('Number of workouts')
plt.savefig('./img/workouts_by_day_of_week.png')
plt.show()
```



7.9 Walks

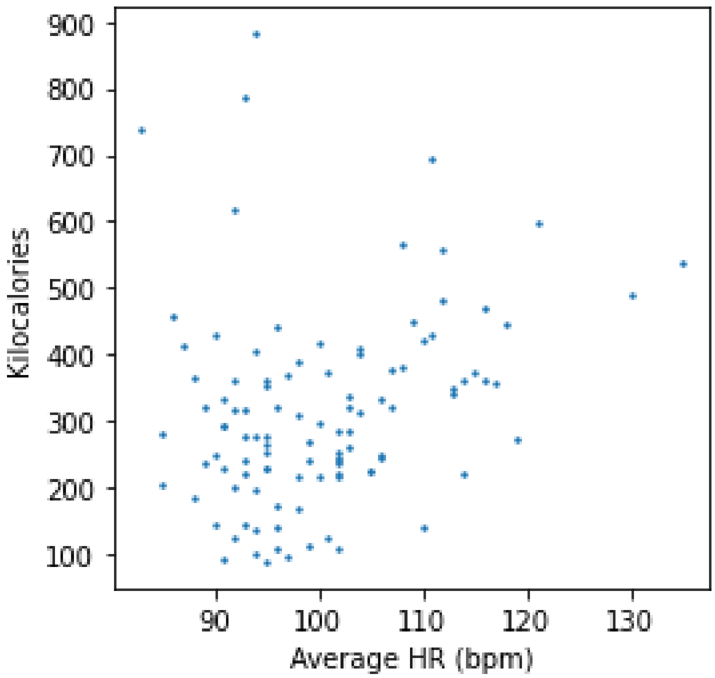
We plot `totalTime` versus `kiloCalories`. As can be seen their seems to exist a linear relationship between the two.

[39]:
```
walking = df[df['sport'] == 'WALKING']
plt.scatter(walking['totalTime'], walking['kiloCalories'], s=2)
plt.xlabel('Duration (minutes)')
plt.ylabel('Kilocalories')
plt.savefig('./img/kilocalories_vs_time.png')
plt.show()
```



We plot `heartRateAvg` against `kiloCalories`. Again we see a linear relationship although there are a couple of outliers

[40]:
```
walking = df[df['sport'] == 'WALKING']
plt.scatter(walking['heartRateAvg'], walking['kiloCalories'], s=2)
plt.ylabel('Kilocalories')
plt.xlabel('Average HR (bpm)')
plt.savefig('./img/kilocalories_vs_avg_hr.png')
plt.show()
```



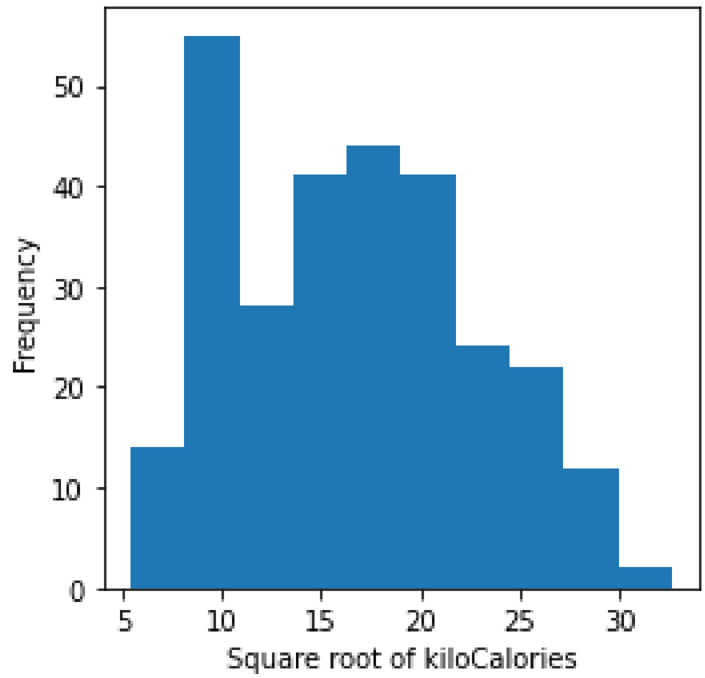
8 Regression

Now we proceed to build a regression model to predict `kiloCalories` burned during a workout based on the numeric features `totalTime` and `heartRateAvg` ie the total duration of the workout and the average heart rate.

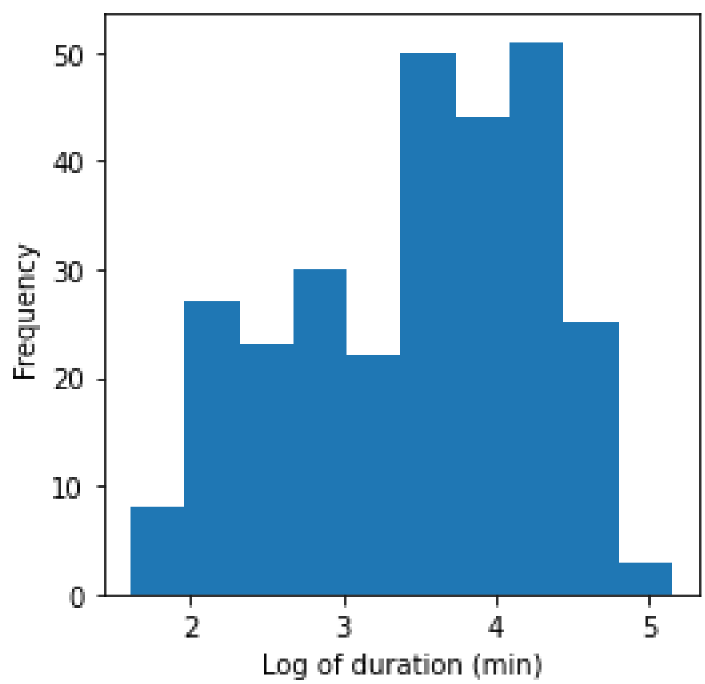
[41]:
```
X = df[['kiloCalories', 'totalTime', 'heartRateAvg']].copy()
```

First we visualize histograms of each of the variables.

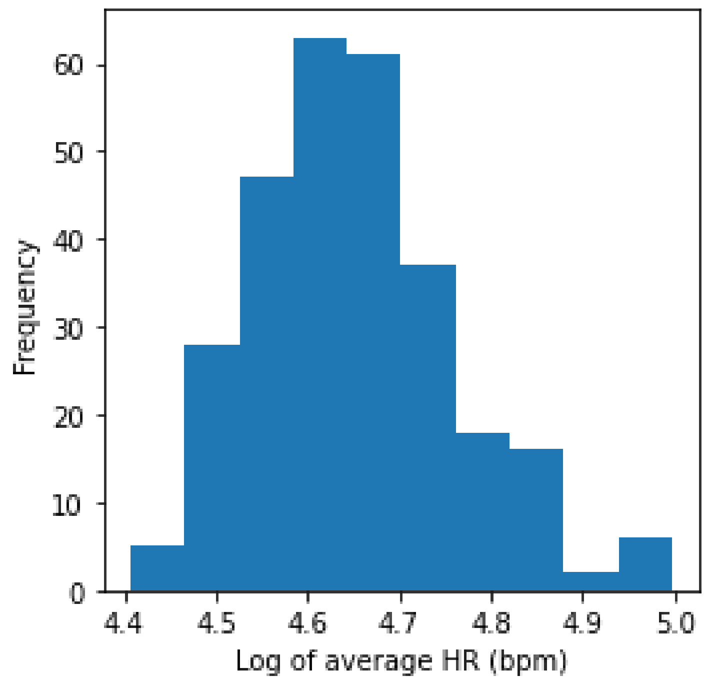
[42]:
```
plt.hist(np.sqrt(X['kiloCalories']))
plt.ylabel('Frequency')
plt.xlabel('Square root of kiloCalories')
plt.savefig('./img/kilocalories_histogram.png')
plt.show()
```



[43]:
```
plt.hist(np.log(X['totalTime']))
plt.ylabel('Frequency')
plt.xlabel('Log of duration (min)')
plt.savefig('./img/duration_histogram.png')
plt.show()
```



[44]:
```
plt.hist(np.log(X['heartRateAvg']))
plt.ylabel('Frequency')
plt.xlabel('Log of average HR (bpm)')
plt.savefig('./img/average_hr_histogram.png')
plt.show()
```



The model we shall fit is:

$\sqrt{c_i} = \beta_1 \ln t_i + \beta_2 \ln h_i + \varepsilon_i$

Where: * c_i - The i-th `kiloCalories` value. * t_i - The i-th `totalTime` value. * h_i - The i-th `heartRateAvg` value.

[45]:
```
X['kiloCalories'] = np.sqrt(X['kiloCalories'])
X['totalTime'] = np.sqrt(X['totalTime'])
X['heartRateAvg'] = np.log(X['heartRateAvg'])
```

We inspect the correlation matrix to check for multicollinearity. It should be noted that the correlation between `kiloCalories` and `totalTime` is quite high and this to be expected.

[46]:
```
C = X.corr(method='pearson')
C = C.style.background_gradient(cmap='coolwarm')
C = C.set_precision(2)
C = C.set_table_attributes('style="font-size: 15px"')
```

```
C
```

[46]: <pandas.io.formats.style.Styler at 0x7f4e6dd2f400>

Next we perform the regression.

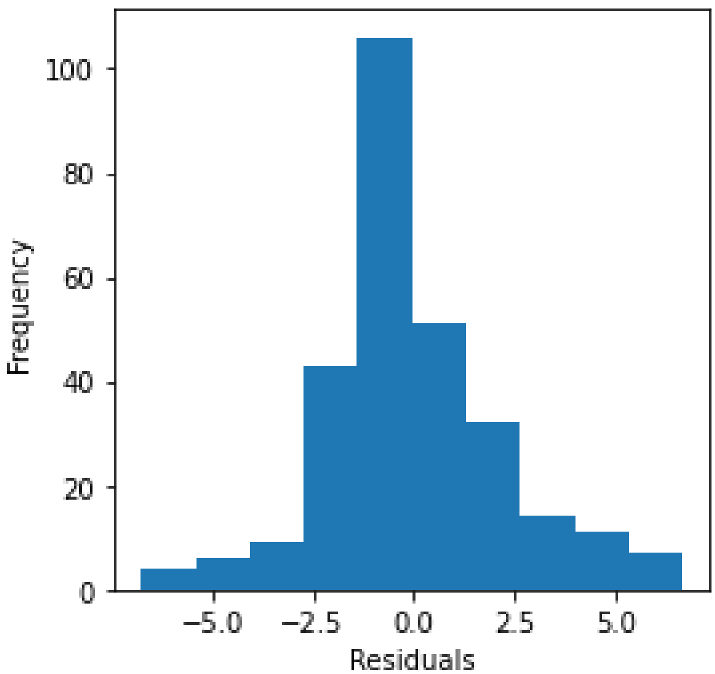
[47]:
```
y = X[['kiloCalories']]
X = X.drop('kiloCalories', axis=1)

mdl = sm.OLS(y, X)
res = mdl.fit()

residuals = res.resid

print(res.summary())
```

```
                                 OLS Regression Results
================================================================================
=======
Dep. Variable:            kiloCalories   R-squared (uncentered):
0.985
Model:                             OLS   Adj. R-squared (uncentered):
0.984
Method:                  Least Squares   F-statistic:
8966.
Date:                 Tue, 29 Sep 2020   Prob (F-statistic):
2.87e-255
Time:                         23:52:53   Log-Likelihood:
-625.71
No. Observations:                  283   AIC:
1255.
Df Residuals:                      281   BIC:
1263.
Df Model:                            2
Covariance Type:             nonrobust
================================================================================
                   coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------
totalTime        2.4806      0.058     43.039      0.000       2.367       2.594
heartRateAvg     0.3203      0.081      3.950      0.000       0.161       0.480
==============================================================================
Omnibus:                       25.323   Durbin-Watson:                   1.966
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               35.338
Skew:                           0.612   Prob(JB):                     2.12e-08
Kurtosis:                       4.224   Cond. No.                         5.90
==============================================================================

Warnings:
```

```
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

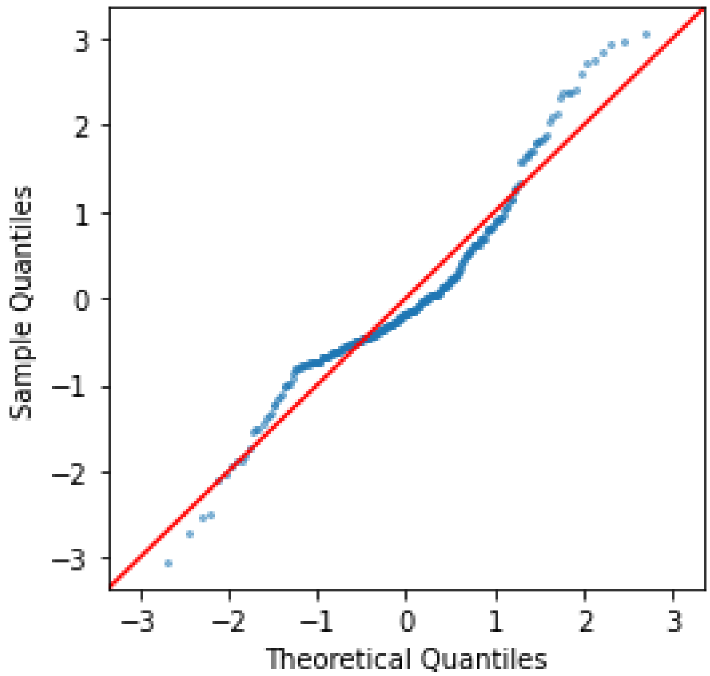
We note that the value of the Durbin-Watson test statistic is approximately 2, as it should be. We proceed to inspect the residuals of the model. First we view the histogram of the residuals.

[48]:
```
plt.hist(residuals)
plt.ylabel('Frequency')
plt.xlabel('Residuals')
plt.savefig('./img/mdl_residuals.png')
plt.show()
```



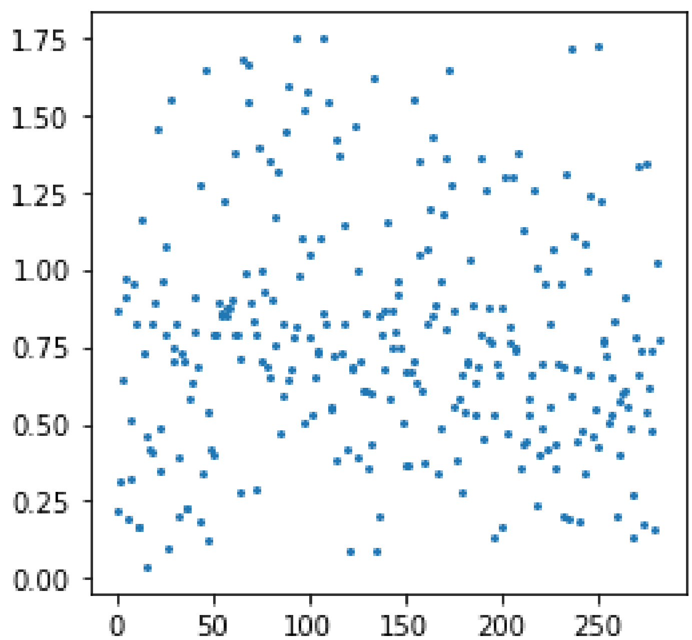
The next plot is a qqplot created to visually inspect the normality of the residuals.

[49]:
```
plt.figure()
ax = plt.gca()
qqplot(res.resid, ax=ax, color='#1f77b4', markersize=2, line='45', fit=True,␣
 ↪alpha=1/2)
plt.savefig('./img/mdl_qq.png')
plt.show()
```



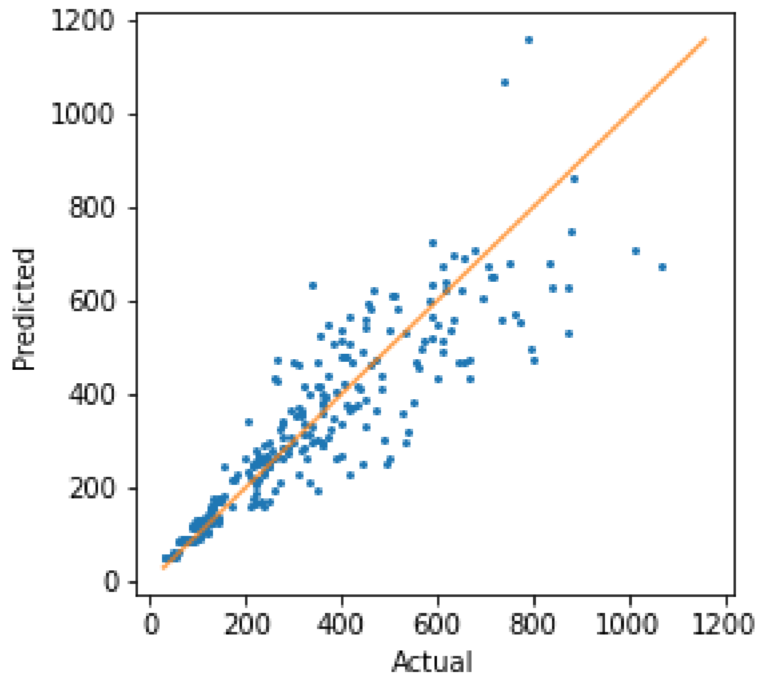
The 3 plot we make is a plot of the standardized residuals to check for homoskedasticity.

[50]:
```
residuals_std = np.sqrt(np.abs((residuals - np.mean(residuals)) / np.
 ↪std(residuals)))
plt.plot(residuals_std, 'o', markersize=2)
plt.savefig('./img/mdl_residuals_std.png')
plt.show()
```



Finally we compare the predicted `kiloCalories` with the actual values.

[51]:
```
y_pred = res.predict(X)
y_pred = y_pred.to_numpy().reshape(len(y_pred))
y_true = y.to_numpy().reshape(len(y),)

m = np.min(np.hstack([y_true, y_pred]))
M = np.max(np.hstack([y_true, y_pred]))

x = np.linspace(np.power(m, 2), np.power(M, 2), len(y))
plt.plot(np.power(y_true, 2), np.power(y_pred,2), 'o', markersize=2)
plt.plot(x,x, alpha=3/4)
plt.ylabel('Predicted')
plt.xlabel('Actual')
plt.savefig('./img/mdl_predicted_vs_actual.png')
plt.show()
```



[52]:
```
X['totalTime'] = np.power(X['totalTime'], 2)
X['heartRateAvg'] = np.exp(X['heartRateAvg'])
X['y_true'] = np.power(y, 2)
X['y_pred'] = np.power(y_pred, 2)
X['e'] = X['y_true'] - X['y_pred']
X = X.sort_values('e', ascending=False)
```

[53]:
```
X.head()
```

[53]:

	totalTime	heartRateAvg	y_true	y_pred	e
93	96.920000	128.000000	1,067.000000	674.706539	392.293461
237	75.150000	129.000000	870.000000	531.796222	338.203778
251	66.620000	130.000000	801.000000	475.501979	325.498021
154	102.000000	121.000000	1,013.000000	706.971271	306.028729
173	69.650000	129.000000	795.000000	495.456319	299.543681

We now move on to performing statistical tests for normality and homoskedasticity.

[54]:
```
def hypothesis_decision(x, alpha=0.05):
    if x < alpha:
        return 'Reject null hypothesis'
    else:
        return 'Fail to reject hypothesis'
```

First we carry out the Shapiro-Wilks test for normality. The hypotheses are:

H_0 : Data comes from a normal distribution.

H_1 : Data does not come from a normal distribution.

[55]:
```
_,shapiro_pval = shapiro(residuals)
```

The second test we perform is the Breusch-Pagan for homoscedasticity. The hypotheses are:

H_0 : Homoscedasticity.

H_1 : Lack of homoscedasticity / Heteroskedasticity.

[56]:
```
_,_,_,breusch_pval = het_breuschpagan(residuals, X)
```

We summarize the outcomes of the tests in a nice table.

[57]:
```
pvalues = [shapiro_pval, breusch_pval]
name = ['Shapiro-Wilks', 'Breusch-Pagan',]
null_hyp = ['Normality', 'Heteroskedasticity']

tests = pd.DataFrame([name, null_hyp, pvalues],
                     index=['Name', 'Null Hypothesis', 'P-value'])

tests = tests.transpose()
tests['Decision'] = tests['P-value'].apply(hypothesis_decision)
tests
```

[57]:

	Name	Null Hypothesis	P-value	Decision
0	Shapiro-Wilks	Normality	0.000000	Reject null hypothesis
1	Breusch-Pagan	Heteroskedasticity	0.000000	Reject null hypothesis

9 Model

Since we have succesfully obtained estimates for the β_i coefficients for `totalTime`:

[58]:
```
b1 = np.round(res.params['totalTime'], 4)
b1
```

[58]: 2.4806

and for `heartRateAvg`:

[59]:
```
b2 = np.round(res.params['heartRateAvg'], 4)
b2
```

[59]: 0.3203

It's now time for a bit of algebra.

$$\sqrt{c} = \beta_1\sqrt{t} + \beta_2 \ln h \tag{1}$$
$$c = \left(\beta_1\sqrt{t} + \beta_2 \ln h\right)^2 \tag{2}$$
$$c = \left(\beta_1\sqrt{t}\right)^2 + 2\beta_1\beta_2\sqrt{t}\ln(h) + \left(\beta_2 \ln h\right)^2 \tag{3}$$
$$c = \beta_1^2 t + 2\beta_1\beta_2\sqrt{t}\ln h + \beta_2^2 \ln^2 h \tag{4}$$

After these calculations we may state that `kiloCalories` c as a function of `totalTime` t (in minutes) and `heartRateAvg` h (in bpm) after substituting the β_i coefficients is given by:

$$c(t, h) = 6.1534t + 1.5890\sqrt{t}\ln h + 0.1026\ln^2 h \tag{5}$$

We restrict the functions domain to values observed in the data:

$$5 \leq \quad t \quad \leq 172 \tag{6}$$
$$82 \leq \quad h \quad \leq 148 \tag{7}$$
$$\tag{8}$$

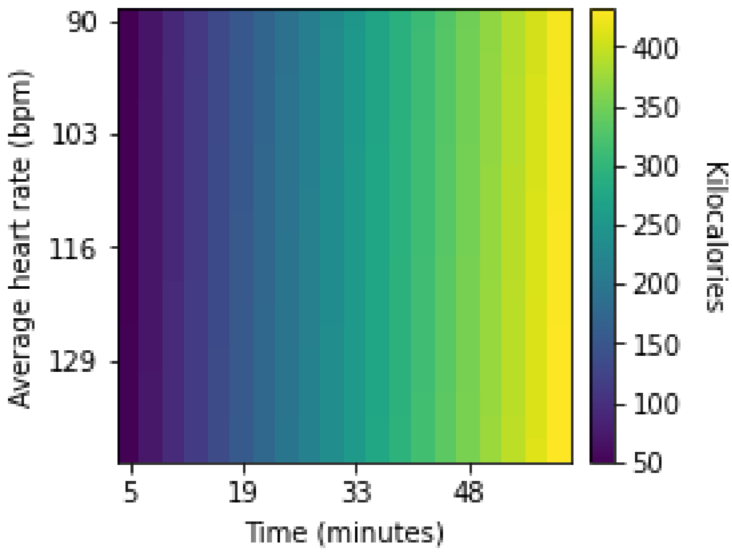
Next we define a function as described above to make a plot and see how does time and average heart rate impact the predictions.

[60]:
```python
def kilocalories(t, h):
    res = np.power(b1, 2) * t
    res += 2 * b1 * b2 * np.sqrt(t) * np.log(h)
    res += np.power(b2, 2) * np.power(np.log(h), 2)
    return np.round(res, 2)
```

As can be seen the total duration of the workout has an overwhelming influence on the predictions.

[61]:
```python
n = 5
t = np.linspace(5, 60, 20)
h = np.linspace(90, 140, 20)

cols = [str(int(x)) for x in list(t)]
rows = [str(int(x)) for x in list(h)]

t,h = np.meshgrid(t, h)

c = kilocalories(t, h)
c = pd.DataFrame(c, columns=cols, index=rows)

plt.imshow(c)
plt.ylabel('Average heart rate (bpm)')
plt.xlabel('Time (minutes)')
plt.xticks(ticks=range(len(cols))[::n], labels=cols[::n])
```

```
plt.yticks(ticks=range(len(rows))[::n], labels=rows[::n])
cbar = plt.colorbar(fraction=0.046, pad=0.04)
cbar.set_label('Kilocalories', rotation=270, labelpad=15)
plt.tight_layout()
plt.savefig('./img/mdl_grid.png')
plt.show()
```



10 Summary

- I downloaded data generated by my Polar watch that tracks heart rate and estimates burned calories during workouts.
- The data came in the form of ".json" files which were read, transformed and cleaned with `pandas`.
- The clean dataset contains 283 workouts over a nearly one year period.
- A linear regression model was built using `statsmodels` to predict the total `kilocalorie` expenditure based on the `duration` of the session and the `average heart rate`.
- After further transforming the data, the duration of a workout and kilocalories burned have 0.93 correlation.
- The estimated formula is: `calories ^ (1/2) = 2.4806 * time ^ (1/2) + 0.3203 * log( heart rate )`
- Both variables turned out to be statistically significant, although the impact of heart rate is small.
- The biggest errors made by the model were on high `duration` workouts.