Rolle

$$f(x) = \begin{cases} x^2 - 2x + 1 & 0 \le x < 3/2 \\ x - 5/4 & 3/2 \le x \le 9/4 \end{cases}$$

- continua $[0, 9/4]$ ✓
- der. $(0, 9/4)$
- $f(0) = f(9/4)$ ✓

$f(0) = 1 \quad f(9/4) = 1$

$$\lim_{x \to 3/2^-} x^2 - 2x + 1 = \frac{9}{4} - 3 + 1 = \frac{9}{4} - 2 = \frac{1}{4}$$

$$\lim_{x \to 3/2^+} x - \frac{5}{4} = \frac{3}{2} - \frac{5}{4} = \frac{1}{4}$$

$$f'(x) = \begin{cases} 2x - 2 & 0 < x < \frac{3}{2} \\ 1 & \frac{3}{2} < x < 9/4 \end{cases}$$

$\lim_{x \to 3/2^-} f'(x) = 3 - 2 = 1 \qquad \lim_{x \to 3/2^+} f'(x) = 1$

$\boxed{\text{derivabile in } 3/2}$

$\exists\, f'(c) \in (0, \frac{9}{4})$ t.c. $f'(c) = 0$

$f'(x) = 0$ $\quad (0, \frac{3}{2})$ $\quad 2c - 2 = 0$ $\quad c = 1$ $\quad f'(1) = 0$

$[\frac{3}{2}, \frac{9}{4})$ $\quad 1 = 0$ imp.

$$f(x) = \begin{cases} \operatorname{arctg} x & -1 \leq x < 0 \\ 2x^2 + 6x & 0 \leq x \leq 1 \end{cases}$$

$[-1, 1]$

CASA

Lagrange

$$f(x)=\begin{cases}\sqrt{1-x^2} & -1\leq x\leq 0\\ a x^3+1 & 0<x\leq 1\\ a(3x-2)+1 & 1<x\leq 2\end{cases}\qquad [-1,2]$$

$$f(0)=\lim_{x\to 0^-} f(x)=1 = \lim_{x\to 0^+} f(x)=1$$ continua

$$f(1)=\lim_{x\to 1^-} f(x)=a+1 \quad \lim_{x\to 1^+} f(x)=a+1$$ $\forall a\in\mathbb{R}$ f é continua

$$f'(x)=\begin{cases}\dfrac{-2x}{2\sqrt{1-x^2}} & -1<x<0\\ a3x^2 & 0<x<1\\ a(3) & 1<x\leq 2\end{cases}$$

$$\lim_{x\to 0^-} f'(x)=0 \quad \lim_{x\to 0^+} f'(x)=0$$

$$\lim_{x\to 1^-} f'(x)=3a \quad \lim_{x\to 1^+} f'(x)=3a$$

$\forall a\in\mathbb{R}$ f é derivabile

soddisfa sempre Lagrange

$$f(x) = |e^x - 1| + 1 \quad [-1, 0]$$

$$f(x) = \begin{cases} x-1 & 0 \leq x \leq 1 \\ \log x & 1 < x \leq 2 \end{cases} \quad [0, 2]$$

Lagrange e trovare $c = \dfrac{f(b) - f(a)}{b - a}$

Monotonia

$f(x) = 1 - e^{x^2}$

$\mathrm{Dom}(f) = \mathbb{R}$

$f'(x) = -e^{x^2} \quad 2x$

$f'(x) \geq 0 \qquad -2x\underbrace{e^{x^2}}_{\text{sempre } \geq 0} \geq 0$

$-2x \geq 0$

$x \leq 0$

	0	
+	|	−
↗	|	↘

f è crescente in $(-\infty, 0)$

decrescente in $(0, +\infty)$

pto di massimo assoluto $x = 0$

$f(x) = \log(x^2+2x+3)$

$D: \quad x^2+2x+3>0 \quad \forall x \in \mathbb{R}$

$\Delta = 4-4(3) < 0 \qquad \underline{\cup}$

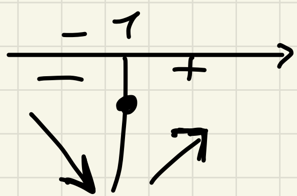
$D: \mathbb{R}$

$f'(x) = \dfrac{2x+2}{x^2+2x+3}$

$f'(x) \geq 0 \qquad \dfrac{2x+2}{x^2+2x+3} \geq 0$ sempre pos.

$2x+2 \geq 0$

$x \geq -1$



$(-\infty, -1)$ dec.

$(-1, +\infty)$ cres.

$x = 1$ p.to di min ass.

$f(x) = e^{3x} - 2e^{x} + 1$

Taylor e McLaurin

McLaurin di $f(x) = x\log(x^2+2)$ ordine 2

$\hookrightarrow x_0 = 0$

$f(0) = 0$

$$f'(x) = \log(x^2+2) + x \cdot \frac{2x}{x^2+2} = \log(x^2+2) + \frac{2x^2}{x^2+2}$$

$f'(0) = \log(2)$

$$f''(x) = \frac{2x}{x^2+2} + \frac{4x(x^2+2) - 2x^2(2x)}{(x^2+2)^2}$$

$f''(0) = 0$

$$P(x) = \cancel{f(0)} + f'(0)x + \cancel{\frac{f''(0)}{2}}x^2 + o(x^2) = x\log 2 + o(x^2)$$

Taylor

$$f(x) = \frac{x-3}{x^2+1} \qquad x_0 = 1 \qquad or = 2$$

$$f(1) = \frac{-2}{2} = \boxed{-1}$$

$$f'(x) = \frac{1(x^2+1)-(x-3)\cdot 2x}{(x^2+1)^2} = \frac{x^2+1-2x^2+6x}{(x^2+1)^2} = \frac{-x^2+6x+1}{(x^2+1)^2}$$

$$f'(1) = \frac{-1+6+1}{2^2} = \frac{6}{4} = \boxed{\frac{3}{2}}$$

$$f''(x) = \frac{(-2x+6)\,(x^2+1)^{\cancel{2}\,1} - (-x^2+6x+1)\cdot 2\cancel{(x^2+1)}\cdot 2x}{(x^2+1)^{\cancel{4}\,3}} =$$

$$\frac{(-2x+6)(x^2+1)+(x^2-6x-1)\,4x}{(x^2+1)^3}$$

$$f''(1) = -2$$

$$P(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2}(x - x_0)^2 + o((x - x_0)^2)$$

$$-1 + \frac{3}{2}(x-1) + \frac{-2}{2}(x-1)^2 + o((x-1)^2) = -x^2 + \frac{7}{2}x - \frac{7}{2} + o((x-1)^2)$$

$f(x) = x\sin$ Mclawin ordine 4

Studio di f.ne

$f(x) = x\, e^{\frac{1}{x-2}}$

1) Dominio $\quad D: (-\infty ; 2) \cup (2 ; +\infty)$

2) $\lim\limits_{x \to -\infty} x e^{\frac{1}{x-2}} = -\infty$

$\lim\limits_{x \to 2^-} x e^{\frac{1}{x-2}} = 2 e^{\frac{1}{0^-}} = 2 e^{-\infty} = 0$

$\lim\limits_{x \to 2^+} x e^{\frac{1}{x-2}} = 2 e^{\frac{1}{0^+}} = 2 e^{+\infty} = +\infty$

$\lim\limits_{x \to +\infty} x e^{\frac{1}{x-2}} = +\infty$

3) asintoti

A. V. $\to$ $x = 2^+$ A. Or. $\to$ NO

A. obl. $\to m = \lim\limits_{x \to \infty} \frac{f(x)}{x} = \lim\limits_{x \to \infty} \frac{\cancel{x} e^{\frac{1}{x-2}}}{\cancel{x}} = \boxed{1}$

$\exists$ finito $\neq 0$

$q = \lim\limits_{x \to \infty} f(x) - mx = \lim\limits_{x \to \infty} x e^{\frac{1}{x-2}} - x =$

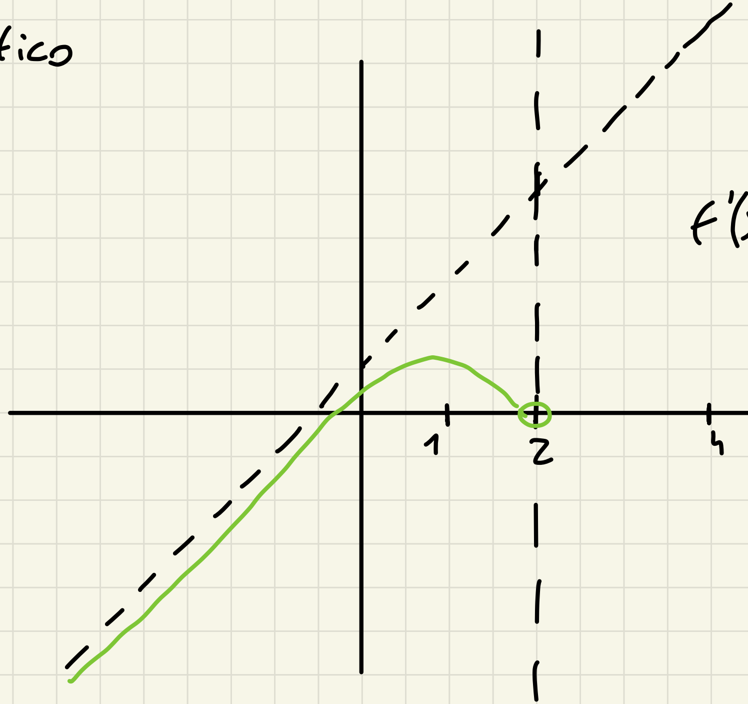
$= \lim\limits_{x \to \infty} x \left(e^{\frac{1}{x-2} \to 0} - 1 \right) = \lim\limits_{x \to \infty} \frac{x}{x-2} = 1$

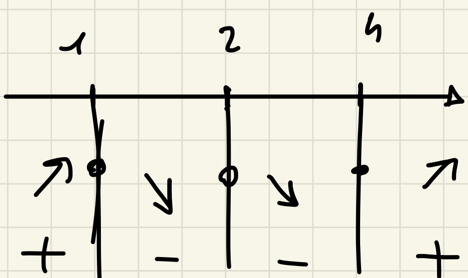
$x \left(e^{\frac{1}{x-2} \to 0} - 1 \right) \sim x \cdot \frac{1}{x-2} =$

$= \frac{x}{x-2}$

$\boxed{y = x + 1}$

grafico





$$f(x) = x e^{\frac{1}{x-2}}$$

$$f'(x) = e^{\frac{1}{x-2}} + x e^{\frac{1}{x-2}} \cdot \frac{-1}{(x-2)^2} \cdot 1$$

$$e^{\frac{1}{x-2}} \left(1 - \frac{1}{(x-2)^2} \right) =$$

$$e^{\frac{1}{x-2}} \left(\frac{x^2 - 5x + 4}{x^2 - 4x + 4} \right)$$

$$e^{\frac{1}{x-2}} \quad \frac{(x-1)(x-4)}{(x-2)^2}$$

$\downarrow$

sempre pos

$$f'(x) \geq 0 \iff (x-1)(x-4) \geq 0$$

$$x \leq 1 \lor x \geq 4$$