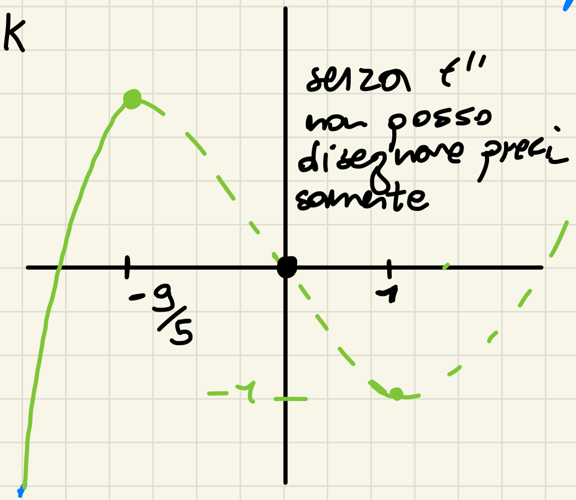
<u>es</u>. Determinare, al variare del parametro $K \in \mathbb{R}$, il numero di soluzioni dell'equazione $x^5 + x^4 - 3x^3 = K$

$\Downarrow$

studio fne $f(x) = x^5 + x^4 - 3x^3$



senza f'' non posso disegnare precisamente

$D = \mathbb{R}$

$$\lim_{x \to \mp\infty} f(x) = \begin{matrix} -\infty \\ +\infty \end{matrix}$$

no asintoti obliqui quindi neanche orizz.

$$f'(x) = 5x^4 + 4x^3 - 9x^2 = x^2 \underbrace{(5x^2 + 4x - 9)}$$

$x = 0$
$x = -\frac{9}{5}$
$x = 1$
$\Big\}$ p.ti staz.

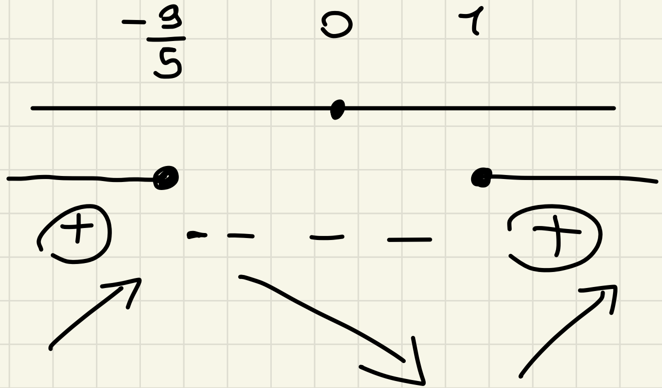
$$5x^2 + 4x - 9 = 0$$

$$x = \frac{-2 \pm \sqrt{49}}{5} = \frac{2 - \pm 7}{5} \begin{cases} -\frac{9}{5} \\ 1 \end{cases}$$

$f'(x) > 0$

$\Updownarrow$

$x < -\frac{9}{3} \vee x > 1$



$x = -\frac{9}{5}$ p.to di max relativo

$x = 1$ p.to di min relativo

$$f\left(-\frac{9}{5}\right) = \left(-\frac{9}{5}\right)^3 \left(\left(-\frac{9}{5}\right)^2 + \left(-\frac{9}{5}\right) - 3\right) = M_{rel}$$

$$f(1) = 1 + 1 - 3 = -1 = m_{rel}$$

$$\begin{cases} 1 & K < -2 \\ 2 & K = -1 \\ 3 & -1 < K < M_{rel} \\ 2 & K = M_{rel} \\ 1 & K > M_{rel} \end{cases}$$

continua anche se sarebbe finito

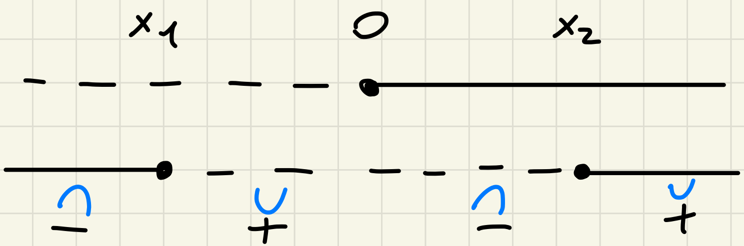
$$f''(x) = 20x^3 + 12x^2 - 18x = 2x\,(10x^2 + 6x - 9)$$

$10x^2 + 6x - 9 = 0 \qquad x = \dfrac{3 \pm \sqrt{99}}{10} = \dfrac{3 \pm 3\sqrt{11}}{10}$

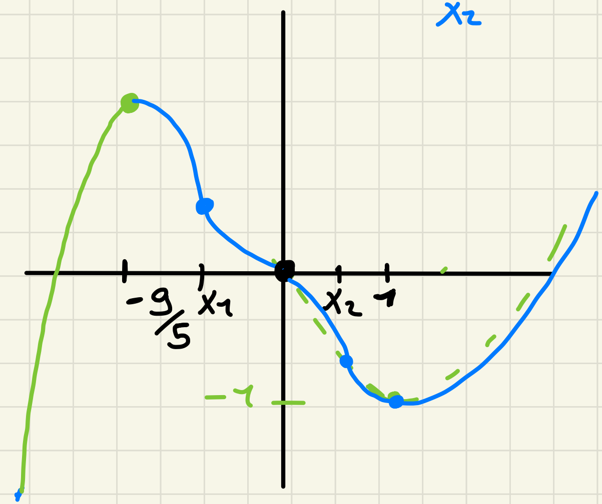
$$x_1: \quad -\frac{3}{2} < \boxed{\frac{3 - 3\sqrt{11}}{10}} < -1$$

$$x_2: \quad \frac{6}{10} < \boxed{\frac{3 + 3\sqrt{11}}{10}} < \frac{9}{10}$$

segno f''

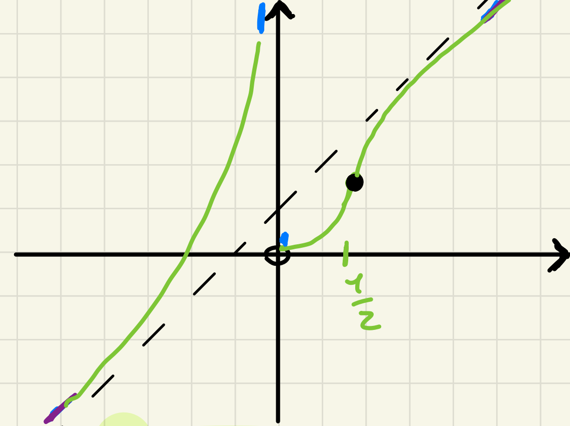


tutti p.ti di flesso



ES $f(x) = x + e^{-1/x}$ $f(x) = K$ $K \in \mathbb{R}$

$D: \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, +\infty)$



$\lim_{x \to \mp\infty} f(x) = \mp\infty$ $\lim_{x \to 0^-} f(x) = +\infty$ $\lim_{x \to 0^+} f(x) = 0^+$

• $x=0$ as. verticale

•• i) $\frac{f(x)}{x} \xrightarrow[x \to \mp\infty]{} 1$ = "m" ii) $f(x) - x = e^{-1/x} \xrightarrow[x \to \pm\infty]{} 1$ $\Rightarrow$ $y = x+1$ è as. obliquo di f

per $x \to -\infty$ e $x \to +\infty$

f può essere prolungata per continuità da Destra in $x=0$ ponendo $f(0)=0$

$x + e^{-1/x} > \overbrace{x+1}^{\text{as.}}$

$$\boxed{\begin{aligned} e^{-1/x} > 1 &\iff x < 0 \\ e^{-1/x} < 1 &\iff x > 0 \end{aligned}}$$

$$f'(x) = 1 + e^{-1/x} \cdot \frac{+1}{x^2} > 0 \quad \forall x \in D$$

quindi f strettamente crescente in

$I_1 = (-\infty, 0)$ e $I_2 (0, +\infty)$

$$f''(x) = e^{-1/x} \cdot \frac{1}{x^2} \cdot \frac{1}{x^2} + e^{-1/x} \cdot \frac{-2}{x^3}$$

$$= e^{-1/x}\left[\left(\frac{1}{x^2}\right)^2 + \frac{-2}{x^3}\right] = e^{-1/x}\left(\frac{1-2x}{x^4}\right) > 0 \iff x < \frac{1}{2}$$

$x \neq 0$

connessa in $(-\infty, 0)$ e

$\left(0, \frac{1}{2}\right)$

$x = \frac{1}{2}$ p.to di flesso

$$f(x)=\begin{cases} ax+b & x\leq 0 \\ \dfrac{\ln x}{\ln x-2} & x>0,\ x\neq e^2 \end{cases}$$

- per quali a e b é continua in $x=0$
- .. " " derivabile in $x=0$
- ... studiare f per $x>0$

i) $\lim_{x\to 0^-} f(x)=b \qquad f(0)=b \qquad \lim_{x\to 0^+} f(x)=1 \qquad \boxed{b=1 \quad \forall a}$

ii) $\begin{cases} ax+1 & x\leq 0 \\ \dfrac{\ln x}{\ln x-2} & x>0 \end{cases} \qquad f'(x)\begin{cases} a & x\leq 0 \\ \dfrac{\frac{1}{x}(\ln x-2)-\ln x\,\frac{1}{x}}{(\ln x-2)^2} = \dfrac{-\frac{2}{x}}{(\ln x-2)^2} & x>0 \end{cases} \quad \xrightarrow[x\to+\infty]{} -\infty$

Non é derivabile per nessun a

iii) $\frac{\ln x}{\ln x - 2}$