## Primitive quasi immediate

$$\int f'(x) \cdot g'\big(f(x)\big)\, dx = g\big(f(x)\big) + c$$

## Integrazione per parti:

unico che ricompare

$$\int g'(x) \cdot f(x)\, dx = f(x)\, g(x) - \int g(x) \cdot f'(x)\, dx$$

si deve scegliere attentamente quale integrare e quale derivare per semplificare

## Integrazione per sostituzione

$$\int g'\big(f(x)\big)\, dx = \int g'(t) \cdot dx\, dt$$ = risolvo e sostituisco di nuovo $t$

$t = f(x)$

$dx = f'(x)\, dt$

# PRIMITIVE DI RAZIONALI FRATTE

$$\frac{P_n(x)}{Q_m(x)} \quad n, m \in \mathbb{N}$$

- $n \geq m$ divido i due polinomi

$$\frac{P_n(x)}{Q_m(x)} = R_{n-m} + \frac{\text{resto}}{Q_m(x)}$$

es. $\displaystyle\int \frac{x^2+3}{x-2}\,dx = \int x+2+\frac{7}{x-2}\,dx$

$$\begin{array}{rrr|l} x^2 & & +3 & x-2 \\ \hline -x^2 & +2x & & x+2 \\ \hline 0 & +2x & +3 & \\ & -2x & +4 & \\ \hline & & +7 & \end{array}$$

- $m > n$
  - $m = 1$ $\quad \dfrac{A}{Bx+C} \quad B \neq 0 \quad \displaystyle\int \frac{A}{Bx+C}\,dx = \frac{A}{B}\int \frac{1}{x+\frac{C}{B}} = \frac{A}{B}\ln\left|x+\frac{C}{B}\right| + \text{cost}$

• $m=2$ $\quad \dfrac{Ax+B}{ax^2+bx+c}$

i) $ax^2+bx+c$ ha due radici reali distinte $(\Delta>0)$

$x_1, x_2 \rightarrow$ radici

$$\int \frac{Ax+B}{ax^2+bx+c}\,dx = \int \frac{H(x+x_2)+K(x+x_1)}{(x+x_1)(x+x_2)}\,dx = \int \frac{H}{x+x_1}\,dx + \int \frac{K}{x+x_2}\,dx =$$

$$H(x+x_2)+K(x+x_1) = Ax+B \qquad = H\ln|x+x_1| + K\ln|x+x_2| + c$$

trovo $H$ e $K$

ii) $\Delta=0$ una sola radice $x_1$

$$\int \frac{Ax+b}{(ax+x_1)^2} = \frac{1}{a}\int \frac{A\left(\frac{t-x_1}{a}\right)+b}{(t)^2}\,dt$$ da qui si risolve normalmente

$ax+x_1=t$

$x=\frac{t}{a}-\frac{x_1}{a}$

$dx=\frac{1}{a}\,dt$

iii) $\Delta < 0$

modifico il denominatore

es. $\int \frac{3x+4}{4x^2+4x+2}dx = \int \frac{3x+4}{\underbrace{4x^2+4x+1}_{\text{quadrato}}+1}dx = \int \frac{3x+4}{(2x+1)^2+1}dx =$

$\Delta < 0$

$$= \frac{1}{2}\int \frac{3\left(\frac{t-1}{2}\right)+4}{t^2+1}dt =$$

$t = 2x+1$

$x = \frac{t-1}{2}$ $\qquad dx = \frac{1}{2}$

$$= \frac{1}{2}\int \frac{\frac{3}{2}t - \frac{3}{2}+4}{t^2+1}dt =$$

$$\frac{1}{2}\int \frac{\frac{3}{2}t + \frac{5}{2}}{t^2+1} d = \frac{1}{2}\left(\frac{1}{2}\cdot\frac{3}{2}\int\frac{2t}{t^2+1}dt + \frac{5}{2}\int\frac{1}{t^2+1}dt = \right.$$

$$= \frac{1}{2}\left(\frac{3}{4}\ln|t^2+1| + \frac{5}{2}\operatorname{arctg} t\right) + c$$