① $\int \underset{f}{x}\,\underset{g}{\ln^2 x}\,dx = \frac{x^2}{2}\ln^2 x - \int \frac{x^{\cancel{2}}}{\cancel{2}}\cdot\frac{\cancel{2}\ln x}{\cancel{x}}\,dx =$

$\frac{x^2\ln^2 x}{2} - \int \underset{f}{x}\,\underset{g}{\ln x}\,dx = \frac{x^2\ln^2 x}{2} - \frac{x^2}{2}\log x + \int \frac{x^{\cancel{2}}}{2}\,\frac{1}{\cancel{x}}\,dx =$

$\frac{x^2\ln^2 x}{2} - \frac{x^2}{2}\ln x - \frac{x^2}{4} + c = \frac{x^2}{2}\left(\ln^2 x - \ln x - \frac{1}{2}\right) + c$

• trovare primitiva t.c. $\lim_{x\to 0^+} F(x) = 0$

$\lim_{x\to 0^+} \overset{\to 0}{\boxed{\frac{x^2}{4}}}\underbrace{\left(2\ln^2 x - 2\ln x - 1\right)}_{+\infty \quad \text{FI}} + c = 0$ se $c=0$

vince $\frac{x^2}{4}$ per gerarchia $\quad F(x) = \frac{x^2}{4}\left(2\ln^2 x - 2\ln x - 1\right)$

studiare $F(x)$ e tracciarne il grafico

$$F(x) = \frac{x^2}{4}\left(2\ln^2 x - 2\ln x - 1\right)$$

- $D: (0, +\infty)$
- $\lim_{x \to 0^+} F(x) = 0$ (dal p.to precedente)

$$\lim_{x \to +\infty} \frac{x^2}{4}\left(2\ln^2 x - 2\ln x - 1\right) = \lim_{x \to +\infty} \frac{x^2}{4}\left[\overbrace{\ln^2 x\left(2 - \underbrace{\frac{2}{\ln x}}_{\to 0} \underbrace{- \frac{1}{\ln^2 x}}_{\to 0}\right)}^{\to +\infty}\right] =$$

$[+\infty(+\infty - \infty + 1)]$ F.I.

$$= +\infty$$

- Segno di $F(x)$

$t = \log x$

$$\underbrace{\frac{x^2}{4}}_{\text{pos}}\left(2\ln^2 x - 2\ln x - 1\right) \geq 0$$

$$2\ln^2 x - 2\ln x - 1 \geq 0$$

$$2t^2 - 2t - 1 \geq 0 \quad \forall t \in \mathbb{R} \Rightarrow 2\ln^2 x - 2\ln x - 1 > 0 \quad \forall x \in (0, +\infty)$$

- derivabile perché ottenuta da operazioni algebriche tra polinomi e $\log x$.

- derivata $F'(x) = x\log^2 x$ per costruzione

$x > 0 \quad \forall x \in (0, +\infty)$

$\log^2 x \quad \forall x \in (0, +\infty), x \neq 1$

$f'(x) > 0 \quad \forall x \in (0,1) \cup (1, +\infty)$

$f(1) = 0$

	1	
$+$		$+$
$\nearrow$		$\nearrow$

flesso a tg. orizzontale ascendente

$$\lim_{x \to 0^+} F'(x) = \lim_{x \to 0^+} x\log^2 x = 0$$

• derivata seconda e convessità

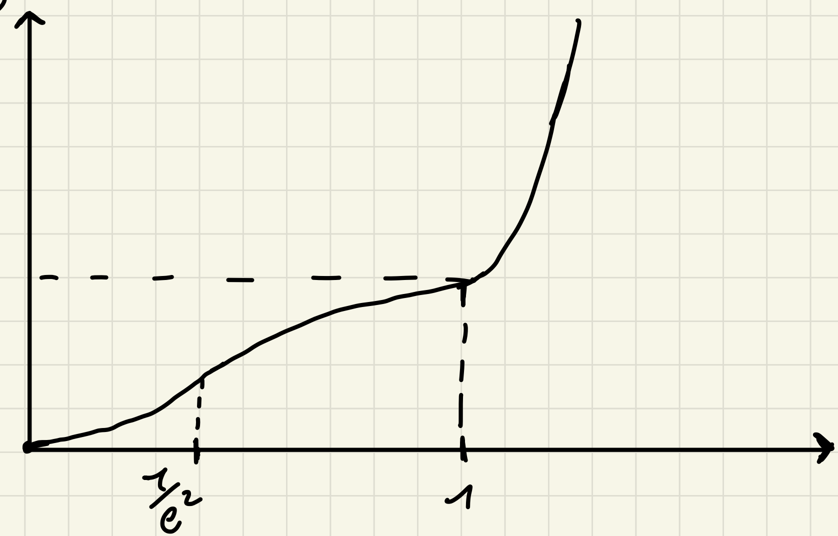
$$f''(x) = \log^2 x + \cancel{x}\, 2\log x \,\frac{1}{\cancel{x}} = \log x\,(\log x + 2) > 0 \iff$$

$\log x \leq -2$ oppure $\log x \geq 0$

$x \leq \frac{1}{e^2}$ o $x \geq 1$

$\frac{1}{e^2}$ $\quad$ 1 $\quad$ flessi a tg. orizzontale

$+$	$\frac{1}{e^2}$	$-$	1	$+$
$\cup$	•	$\cap$	•	$\cup$



Calcolare al variare di $\alpha \in \mathbb{R}$, il limite della succ. $n^{\alpha} \cdot F\left(\frac{1}{n}\right)$

$$\lim_{n \to +\infty} n^{\alpha} F\left(\frac{1}{n}\right) = \lim_{n \to +\infty} \left(\frac{1}{n}\right)^{-\alpha} F\left(\frac{1}{n}\right)$$

$$\lim_{x \to 0^+} x^{-\alpha} F(x) = \lim_{x \to 0^+} x^{-\alpha} \left(2\log^2 x - 2\log x + 1\right) =$$

$$\lim_{x \to 0^+} \boxed{\frac{x^{2-\alpha}}{\cancel{4}\,2} \cdot \cancel{2}\log^2 x - \frac{\cancel{2}}{\cancel{4}\,2}\, x^{2-\alpha}} \log x + \boxed{\frac{x^{2-\alpha}}{4}} = \begin{cases} 0 & \text{se } \alpha < 2 \\ +\infty & \text{se } \alpha \geq 2 \end{cases}$$

0 $\alpha < 2$ 1 $\alpha = 2$ $+\infty$ $\alpha > 2$

$-t = \log x$

$$x = \frac{1}{e^{t}} \qquad \frac{1}{2} \frac{1}{e^{t(2-\alpha)}} \cdot t^2 - \frac{1}{e^{t(2-\alpha)}} \cdot t = \begin{cases} 0 & \alpha < 2 \\ +\infty & \alpha = 2 \\ +\infty & \alpha > 2 \end{cases}$$

$$\lim_{n \to +\infty} n^{\beta} F(n)$$

$$\lim_{x \to +\infty} x^{\beta} F(x) = \lim_{x \to +\infty} \frac{1}{4}\left(x^{2+\beta}\left(2\log^2 x - 2\log x + 1\right)\right) =$$

$$= \begin{cases} +\infty & \beta \geq -2 \\ 0 & \beta < -2 \end{cases}$$

[difficile] $\sum F\left(\frac{1}{n}\right)$ cosa fa?

so che $\dfrac{F\left(\frac{1}{n}\right)}{\frac{1}{n^\alpha}} \to 0$ se $n<2$

$\Rightarrow F\left(\frac{1}{n}\right) < \frac{1}{n^\alpha}$ definitivamente

$\downarrow$

studio dif

so che è pos $\forall n \in \mathbb{N} \setminus \{0\}$

$$f(x) = \begin{cases} \dfrac{xe^{\frac{x}{2}} + \log(1-x)}{x^3} & x > 0 \\ a - \dfrac{17}{48}\,x & x \leq 0 \end{cases}$$

a per cui f é continua e derivabile in 0

$$\lim_{x \to 0^+} \frac{xe^{\frac{x}{2}} + \log(1-x)}{x^3} \qquad \lim_{x \to 0^-} f(x) = a = f(0)$$

faccio taylor al IV in $x=0$ di $xe^{\frac{x}{2}} + \log(1-x)$

$$e^t = 1 + t + \frac{t^2}{2} + \frac{t^3}{3!} + o(t^3)$$

$$e^{x/2} = 1 + x + \frac{x^2}{8} + \frac{x^3}{48} + o(x^3)$$

$$xe^{\frac{x}{2}} = x + \frac{x^2}{2} + \frac{x^3}{8} + \frac{x^4}{48} + o(x^4)$$

$$\log(1+t) = t - \frac{t^2}{2} + \frac{t^3}{3} - \frac{t^4}{4} + o(t^4)$$

$$\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + o(x^4)$$

$$xe^x + \log(1-x) = \cancel{x} + \cancel{\frac{x^2}{2}} + \frac{x^3}{8} + \frac{x^4}{48} \cancel{-x} - \cancel{\frac{x^2}{2}} - \frac{x^3}{3} - \frac{x^4}{4} + o(x^4) =$$

$$= \frac{-5}{24}x^3 - \frac{11}{48}x^4 + o(x^4)$$

$$\lim_{x\to 0^+} \frac{xe^x + \log(1-x)}{x^3} = \lim_{x\to +\infty} \frac{\frac{-5}{24}\cancel{x^3} - \frac{11}{48}x^{\cancel{4}} + \overset{0}{o(x^4)}}{\cancel{x^3}} = \frac{-5}{24}$$

$$a = \frac{-5}{24}$$

$f(x)$ derivabile

$$\lim_{h\to 0} \frac{f(h)-f(0)}{h} = \lim_{h\to 0^-} \frac{-\frac{5}{24} - \frac{11}{48}h + \frac{5}{24}}{h} = -\frac{11}{48}$$

$f(x)$ é derivabile in x_0 SSE $\lim_{h\to 0^+} \frac{f(h)-f(0)}{h} = -\frac{11}{48}$

$$\lim_{h\to 0^+} \frac{\frac{he^{\frac{h}{2}} + \ln(1-h) + \frac{5}{24}}{h^3}}{h} = \lim_{x\to 0^+} \frac{he^{h/2} + \ln(1-h) + \frac{5}{24}h^3}{h^4}$$

AtSA

$$f(x) = \begin{cases} e^{-\frac{1+x}{x^2}} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

- continua e derivabile in $x=0$
- grafico e immagine