$$\begin{cases} x_1 + x_2 + x_3 = 1 \\ -x_1 + x_3 = 1 \\ x_1 + x_2 + 2x_3 = 0 \\ x_3 = 0 \\ x_1 + x_3 = 0 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ -1 & 0 & 1 & 1 \\ 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right) \xrightarrow[\substack{r_3 \to r_3 + r_1 \\ r_5 \to r_5 - r_1}]{r_2 \to r_2 + r_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 \end{array}\right)$$

$$\xrightarrow{r_5 \to r_5 + r_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{array}\right) \xrightarrow{\substack{r_4 \to r_4 - r_3 \\ r_5 \to r_5 - 2r_3}} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 3 \end{array}\right) \xrightarrow{r_5 \to r_5 - 3r_4}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$rg(A) = 3$

$rg(A|\underline{b}) = 4$

no soluzioni

Discutere risolubilitá sistema al variare di $k \in \mathbb{R}$

$$\begin{cases} 4x-6y=-2 \\ -2x+ky=1 \\ 6x-9y=-k \end{cases} \Rightarrow \left(\begin{array}{cc|c} 4 & -6 & -2 \\ -2 & k & 1 \\ 6 & -9 & -k \end{array}\right) \xrightarrow{r_2 \to r_1} \left(\begin{array}{cc|c} -2 & k & 1 \\ 4 & -6 & -2 \\ 6 & -9 & -k \end{array}\right)$$

$r_2 \to r_2 + 2r_1$
$r_3 \to r_3 + 3r_1$

$$\left(\begin{array}{cc|c} -2 & k & 1 \\ 0 & -6+2k & 0 \\ 0 & -9+3k & -k+3 \end{array}\right) \xrightarrow{r_3 \to r_3 - \frac{3}{2} r_2} \left(\begin{array}{cc|c} -2 & k & 1 \\ 0 & -6+2k & 0 \\ 0 & 0 & -k+3 \end{array}\right)$$

$k=3$ ammette soluzioni perché ho:

$$\left(\begin{array}{cc|c} -2 & 3 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right) \qquad \infty^{2-1} = \infty^1$$

$$-2x+3y=1$$
$$y = \frac{1}{3} + \frac{2}{3}x$$

$$S_{k=3} = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \;\middle|\; y = \frac{1}{3} + \frac{2}{3}x \right\} = \left(\begin{matrix} x \\ \frac{1}{3} + \frac{2}{3}x \end{matrix} \right) \;\middle|\; x \in \mathbb{R} \Big\}$$

$$\begin{pmatrix} 0 \\ 1/3 \end{pmatrix} + \left\langle \begin{matrix} 1 \\ 2/3 \end{matrix} \right\rangle$$

$k \neq 3$?

$rg(A) = 2$
$\to$ no sol.
$rg(A|\underline{b}) = 3$

$$\begin{pmatrix} x \\ \frac{2}{3}x \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{3} \end{pmatrix} = x\begin{pmatrix} 1 \\ \frac{2}{3} \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{3} \end{pmatrix}$$

OP. su matrici

$+ : M_{m,n}(\mathbb{R}) \times M_{m,n}(\mathbb{R}) \to M_{m,n}(\mathbb{R})$

$(A, B) \to C$

$c_{ij} = a_{ij} + b_{ij}$ $\quad i \in [1, m]$, $j \in [1, n]$

prodotto scalare $K \times M_{m,n}(K) \to M_{m,n}(K)$

$(\lambda, A) \to \lambda A$

$(\lambda, a_{ij}) \to \lambda a_{ij}$

prodotto righe per colonne

$M_{m,n}(K) \times M_{n,p}(K) \to M_{m,p}(K)$

$A \times B \to AB$

$$c_{ij} = \sum_{k=1}^{n} a_{nk} b_{kj}$$

$A = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ $\quad B = \begin{pmatrix} -1 & 1 & 1 \end{pmatrix}$ $\quad C = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$ $\quad D = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ $\quad E = \begin{pmatrix} -1 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix}$

$A + B$ non si puó

$C + D = \begin{pmatrix} -1 & 2 \\ 0 & 1 \end{pmatrix}$

$A \cdot B = \begin{pmatrix} -2 & 2 & 2 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$ $\qquad B \cdot A$ non si puó

$$C \cdot D = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 \cdot 0 + 1 \cdot (-1) & -1\ 1 + 1\ 0 \\ 1 \cdot 0 + 1 \cdot (-1) & 1\ 1 + 1 \cdot 0 \end{pmatrix} =$$

$$= \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}$$

$C \cdot E$ no se puede

$$EC = \begin{pmatrix} -1 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1+1 & -1+1 \\ -1+0 & 1+0 \\ -1+1 & 1+1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ -1 & 1 \\ 0 & 2 \end{pmatrix}$$

$$B \cdot E = \begin{pmatrix} -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1+1+1 \,, & -1+1 \end{pmatrix} = (3, 0)$$

Determinante

$\det : Mat_n(K) \to K$

matrice M a cui si toglie riga i e colonna j

$$\det M = \sum_{j=1}^{n} (-1)^{i+j} m_{ij} \det(M_{ij}) \quad \forall\, i \in [1, n]$$

proprietà

- det é lineare nelle righe (colonne) $M = \begin{pmatrix} R_1 \\ R_{i1} + R_{i2} \\ R_n \end{pmatrix}$

$$\det M = \det\begin{pmatrix} R_1 \\ R_{i1} \\ \vdots \\ R_n \end{pmatrix} + \det\begin{pmatrix} R_1 \\ R_{i2} \\ \vdots \\ R_n \end{pmatrix}$$

- $\det(\lambda M) = \lambda^n \det(M)$

- $\det(AB) = \det A \cdot \det B$ (teorema di Binet)