Def campo

$\mathbb{K} \neq \emptyset$ munito di 2 operazioni $+$ e $\cdot$

- $(\mathbb{K}, +)$ è un gruppo abeliano con el. neutro $+ : \mathbb{K} \to \mathbb{K}$
  - $(a+b)+c = a(b+c)$
  - $a+0 = a = 0+a$
  - $\forall a \in \mathbb{K}\ \exists -a \in \mathbb{K}$ t.c. $a+(-a) = 0$
  - $a+b = b+a$
- $(\mathbb{K} \setminus \{0\}, \cdot)$ $\cdot : \mathbb{K} \to \mathbb{K}$ gruppo abeliano
  - $(ab)c = a(bc)$
  - $a \cdot 1 = a$
  - $\forall a \in \mathbb{K} \setminus \{0\}\ \exists a^{-1} \in \mathbb{K} \setminus \{0\}$ t.c. $a \cdot a^{-1} = 1$
  - $ab = ba$

prop distr.

$a(b+c) = ab + ac = (b+c)a$

Def anello

insieme $A \neq \emptyset$ con $+, \cdot$

- $(A,+)$ gruppo abeliano el. neutro 0
- $(A,\cdot)$ $(ab)c = a(bc)$ $\forall a,b,c \in A$

l' anello è unitario se $\exists 1 \in A$

" " abeliano se $ab = ba$ $\forall a,b \in A$

es. di anello $\to \mathbb{Z}$

Def. $R_n = \{ (a,b) \in \mathbb{Z}^2 \mid n \mid a-b \}$ classe di equivalenza resto
$n \in \mathbb{Z}$

- riflessiva
- simmetrica
- transitiva

$\forall a \in \mathbb{Z}$ $(a,a) \in R_n$? $n \mid a-a$ $a = n0$

$\forall a,b \in \mathbb{Z}$ t.c. $(a,b) \in R_n$ $\Rightarrow (b,a)$

$\Updownarrow$

$\exists k \in \mathbb{Z}$ $-a+b = -k(n)$

$b - a = -kn$

$\forall a,b,c$ t.c. $(a,b) \in R_n$ $\Rightarrow a-b = nq_1$ $q_1 \in \mathbb{Z}$

$(b,c) \in R_n$ $b-c = nq_2$ $q_2 \in \mathbb{Z}$

$a-b+b-c = nq_1 + nq_2$

$a-c = n(\underset{\in \mathbb{Z}}{q_1+q}) \quad (a,c) \in R_n$

$$\mathbb{Z}/_{R_n} = \{ [a]_m \mid a \in \mathbb{Z} \}$$

$$[a]_n = \{ b \in \mathbb{Z} \mid (a,b) \in R_n \} = \{ b \in \mathbb{Z} \mid b - a = n\,q \ \exists q \in \mathbb{Z} \}$$

$$= \{ b \in \mathbb{Z} \mid b = a + nq,\ q \in \mathbb{Z} \} =$$

$$= \{ a + nq \mid q \in \mathbb{Z} \}$$

$$[3]_6 = \{ b \in \mathbb{Z} \mid (3,b) \in R_6 \}$$

$$\Updownarrow$$

$$6 \mid b-3 \Leftrightarrow \exists q \in \mathbb{Z} \quad b-3 = 6q$$

$$b = 3 + 6q$$

$q = 0 \rightarrow b = 3$

$q = 1 \rightarrow b = 9$

$q = 2 \rightarrow b = 15$

infiniti

$[15]_6 = [3]_6$

$$\{ [0]_6 , [1]_6 \dots [5]_6 \} = \mathbb{Z}/_{6\mathbb{Z}}$$

$[6]_6 = [0]_6$

$+ : \mathbb{Z}/_{n}\mathbb{Z} \times \mathbb{Z}/_{n}\mathbb{Z} \to \mathbb{Z}/_{n}\mathbb{Z}$

- $([a]_n, [b]_n) \to [a+b]_n =: [a]_n + [b]_n$

$[a_1]_n = [a_2]_n$
$[b_1]_n = [b_2]_n$ $\Longrightarrow [a_1]_n + [b_1]_n = [a_2]_n + [b_2]_n$

$\to n \mid a_2 - a_1 \Rightarrow a_2 = a_1 + qn \qquad s, q \in \mathbb{Z}$

$\to n \mid b_2 - b_1 \Rightarrow b_2 = b_1 + sn$

$[a_2]_n + [b_2]_n = [a_2 + b_2]_n = [a_1 + qn + b_1 + sn]_n =$

$= [a_1 + b_1 + n(q+s)]_n = [a_1 + b_1] = [a_1]_n + [b_1]_n$

$\forall a, b, c \in \mathbb{Z} \ (([a]_n + [b]_n) + [c]_n = [a+b]_n + [c]_n =$

$[(a+b) + c]_n = [a + (b+c)]_n = [a_n] + [b+c]_n =$

$[a]_n + ([b]_n + [c]_n)$

- $[0]_n + [a]_n = [a]_n$

Abeliano

- $\forall a \in \mathbb{Z} \ [-a]_n + [a]_n = [0]_n$

$(\mathbb{Z}/n\mathbb{Z}, +, \cdot)$ anello

non c'é inverso $\to$ non gruppo

$[3]_4 [3]_4 = [9]_4 = [1]_4$

$n=4 \quad [2]_4 [0] = [0]$

$[2]_4 [3]_4 = [6]_4 = [2]_4$ no inv.

se P primo $\mathbb{Z}/p\mathbb{Z}$ campo

$(\mathbb{Z}/n\mathbb{Z}, +, \cdot)$ anello unitario commutativo

$\mathbb{Z}/5\mathbb{Z} = \{[0]_5 \dots [4]_5\}$

$[1]_5 [1]_5$

$[2]_5 [3]_5 = [6]_5 = [1]_5$

$[3]_5 [2]_5 = [6]_5 = [1]_5$

hanno tutti inverso

$[4]_5 [4]_5 = [16]_5 = [1]_5$

Spazi vettoriali:

$V \neq \emptyset$ str. di s.V. su $\mathbb{K}$ $\quad + : V \times V \to V$
rispetto alla quale V é un gruppo abeliano
É definita $\mathbb{K} \times V \to V$
$a, b \in \mathbb{K}$, $v, w \in V$

- $(a+b)v = av + bv$
- $a(v+w) = av + aw$
- $1_K v = v$
- $ab(v) = a(bv)$

Sottospazio

$W \neq \emptyset$ $\quad W \subseteq V$ $\quad W$ ē sottospazio vett. di V se ē a sua volta uno spazio vettoriale.

Criterio:

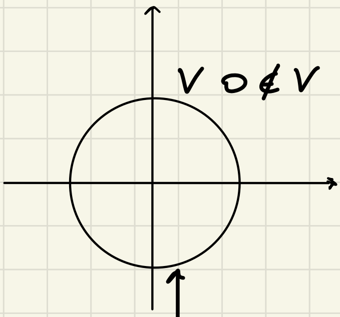
SSE $\forall \alpha, \beta \in \mathbb{K}, \forall v, w \in W \quad \alpha v + \beta w \in W$

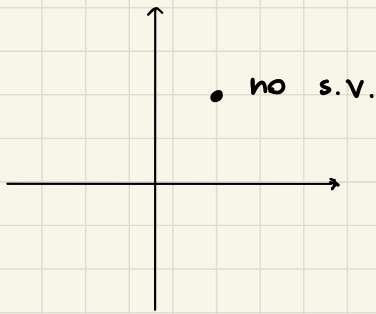
chiuso rispetto alle op. lineari

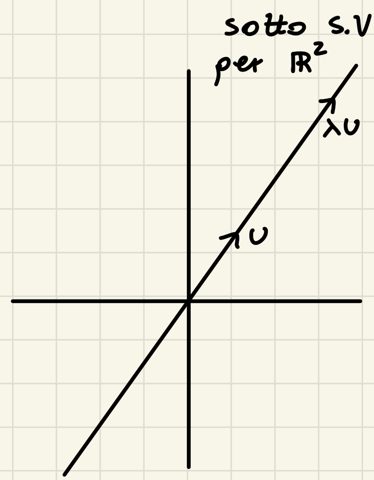
Def.

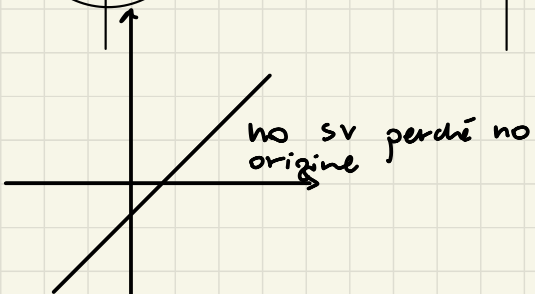
$\{v_1, \dots v_n\} \subseteq V$ é di generatori per V se

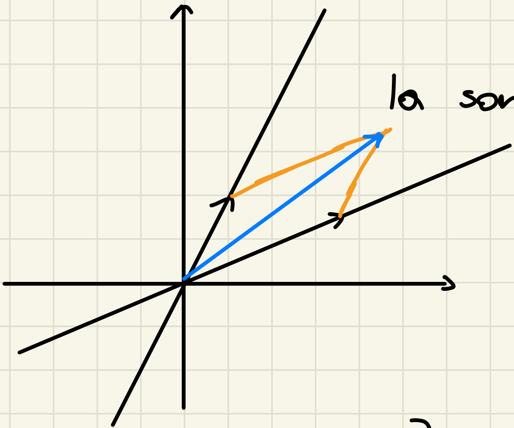
$\forall v \in V, \exists\ \lambda_1 \dots \lambda_n \in \mathbb{K} \quad v = \sum_{i=1}^{n} \lambda_i v_i$











la somma non è contenuta

$$V_1=\{(x\,y\,z)\in\mathbb{R}^3 \mid (x\,y\,z)=1\,0\,0\} = \{1\,0\,0\} \neq \{(0\,0\,0)\} \text{ no S.V.}$$

$$V_2=\{(x\,y\,z)\in\mathbb{R}^3 \mid y=z=0\} = \{(x,0,0) \mid x\in\mathbb{R}\} \quad \text{SPAN}_{\mathbb{R}}\{(1,0,0)\}$$

↑ S.V.

$$=\left\{x\begin{pmatrix}1\\0\\0\end{pmatrix} \,\middle|\, x\in\mathbb{R}\right\} \qquad \langle(1\,0\,0)\rangle$$

$$V_3=\{(x\,y\,z)\in\mathbb{R}^3 \mid \underset{\displaystyle z=-2x+y}{\underset{\downarrow}{-2x+y-z=0}}\} = \left\{\begin{pmatrix}x\\y\\-2x+y\end{pmatrix} \,\middle|\, x,y\in\mathbb{R}\right\}$$

riusciamo a scriverlo come SPAN

$$=\left\{\begin{pmatrix}x\\0\\-2x\end{pmatrix}+\begin{pmatrix}0\\y\\y\end{pmatrix} \,\middle|\, x,y\in\mathbb{R}\right\} = \left\{x\begin{pmatrix}1\\0\\-2\end{pmatrix}+y\begin{pmatrix}0\\1\\1\end{pmatrix} \,\middle|\, \dots\right\} =$$

es

$$=\left\langle\begin{pmatrix}1\\0\\-2\end{pmatrix},\begin{pmatrix}0\\1\\1\end{pmatrix}\right\rangle$$

$$V=\{\dots \mid -2x+2y+z+2=0\} \text{ no S.V. } (0,0,0)\notin V$$

$$r=\{(x\,y\,z)\in\mathbb{R}^3 \mid 3x-z=0\}$$

$$V_z=3x \quad \left\{\begin{pmatrix}x\\y\\3x\end{pmatrix}\dots\right\} \quad \begin{pmatrix}x\\0\\3x\end{pmatrix}+\begin{pmatrix}0\\y\\0\end{pmatrix} \quad \left\langle\begin{pmatrix}1\\0\\3\end{pmatrix},\begin{pmatrix}0\\1\\0\end{pmatrix}\right\rangle$$

$$V_6 = \{ a_3 x^3 + a_2 x^2 + a_1 x + a_0 \in \mathbb{R}[3]_3 \mid a_2 = 0 \}$$

$$= \{ a_3 x^3 + a_1 x + a_0 \cdot 1 \mid a_3, a_1 + a_0 \in \mathbb{R} \} = \langle 1, x, x^3 \rangle$$

$$V_8 = \{ \text{" "} \mid a_1 + a_1 = 1 \}$$

a