Sia $f:(a,b)\to\mathbb{R}$, $x_0 \in (a,b)$ e $\exists f^j(x_0)$ $\forall j \leq n$

Polinomio di Taylor centrato in x_0 di grado n

$$P(x,x_0) = \frac{f(0)}{0!}\cdot(x-x_0)^0 + \frac{f'(x_0)}{1!}(x-x_0)^1 + \frac{f''}{2!}(x-x_0)^2 + \frac{f'''}{3!}(x-x_0)^3 + \dots + \frac{f^n}{n!}(x-x_0)^n$$

$$P(x,x_0) = \sum_{j=0}^{n} \frac{f^j(x_0)}{j!}\cdot(x-x_0)$$

Polinomio di Mac Laurin (centrato in 0)

$$P(x,0) = \sum_{j=0}^{n} \frac{f^j(x_0)}{j!}(x)$$

Errore $\to$ $f(x) - P_n(x,x_0) = o\,(x-x_0)^n_{\,x\to x_0}$ resto nella forma di Peano

$f:(a,b) \to \mathbb{R}$ derivabile almeno n volte in (a,b), $x_0 \in (a,b)$

allora $\exists\, \bar{x} \in (x_0, x)$ t.c.

$$f(x) = f(0) + f(\bar{x}) + f'(x_0)(x - x_0) + \frac{f''}{2}(x-x_0)^2 + \dots + \frac{f^{n-1}}{(n-1)!}(x-x_0)^{n-1} + \frac{f^{n}(\bar{x})}{n!}(x-x_0)^{n}$$

resto nella forma mentale

Polinomio di Mac Laurin noti

- $f(x) = e^x$ $\quad f^j(x) = e^x \quad f^j(0) = 1 \quad \forall j \in \mathbb{N}$

$$P_n(x;0) = \sum_{j=0}^{n} \frac{x^j}{j!}$$

- $f(x) = \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots (-1)^{n+1} \cdot \frac{x^n}{n} = \sum_{j=1}^{n} (-1)^{j+1} \frac{x^j}{j}$

solo quelli dispari

- $f(x) = \sin x = 0 + 1 \cdot x + 0 \cdot x^2 - 1 \cdot \frac{x^3}{3!} + \dots + (-1)^n \cdot \frac{x^{2n+1}}{(2n+1)!} = \sum_{j=0}^{n} (-1)^j \frac{x^{2j+1}}{(2j+1)!}$

- $f(x) = \cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \frac{(-1)^n \cdot x^{2n}}{(2n)!} =$

$$= \sum_{j=0}^{n} \frac{(-1)^j}{(2j)!} x^{2j}$$