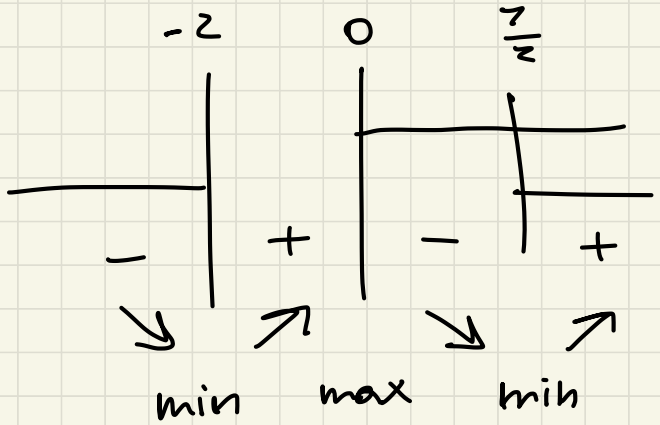
4) $f(x) = x^4 + 2x^3 - 2x^2$

$f'(x) = 4x^3 + 6x^2 - 4x = 0$

$x(4x + 6x - 4) = 0 \qquad x = 0$

$\Delta = 36 + 64 = 100$

$x_{1,2} = \frac{-6 \pm 10}{8} \begin{cases} -2 \\ \frac{1}{2} \end{cases}$



2) $f(x)=\ln^2 x - \ln \ln x \qquad D: (0, +\infty)$

$$f'(x) = 2\ln x \cdot \frac{1}{x} - \left(\frac{1}{\ln x} \cdot \frac{1}{x}\right) = \frac{2\ln x}{x} - \frac{1}{x\ln x}$$

$$f'(x)=0 \quad \frac{2\ln x}{x} - \frac{1}{x\ln x} = 0 \quad \frac{2\ln x}{x} = \frac{1}{x\ln x} \quad 2\ln x = \frac{1}{\ln x} \quad \ln x = \frac{1}{2\ln x}$$

$$\ln x = \sqrt{\frac{1}{2}} \qquad x = e^{\frac{1}{\sqrt{2}}}$$

3) $f(x) = x^2 - \ln x \qquad x = 1$

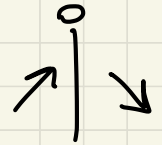
$f'(x) = 2x - \frac{1}{x} \qquad f'(1) = 2 - 1 = 1$

$f'(x_0) + X + f(x_0) - f'(x_0) \cdot x_0) \qquad f'(x_0)(x - x_0) + f(x_0)$

$1(x - 1) + 1 = x$

4)

$$f(x) = \begin{cases} x - e^x & x \leq 0 \\ x^3 + x & x > 0 \end{cases}$$



$$f'(x) = \begin{cases} 1 - e^x & x \leq 0 \\ 3x^2 + 1 & x > 0 \end{cases}$$

$1 - e^x = 0 \quad x = 0 \qquad 1 - e^x > 0$

$e^x < 1 \quad x < 0$

$3x^2 + 1 > 0$

$x^2 > -\frac{1}{3} \qquad \forall x \in \mathbb{R}$

5) $f(x) = \begin{cases} a\sin x + b & -1 \le x \le 0 \\ 1 - 2^x & 0 < x \le 2 \end{cases}$

$\lim_{x \to 0^-} a\sin x + b = \lim_{x \to 0^+} 1 - 2^x = 0 \quad b = 0 \quad \forall a \in \mathbb{R}$

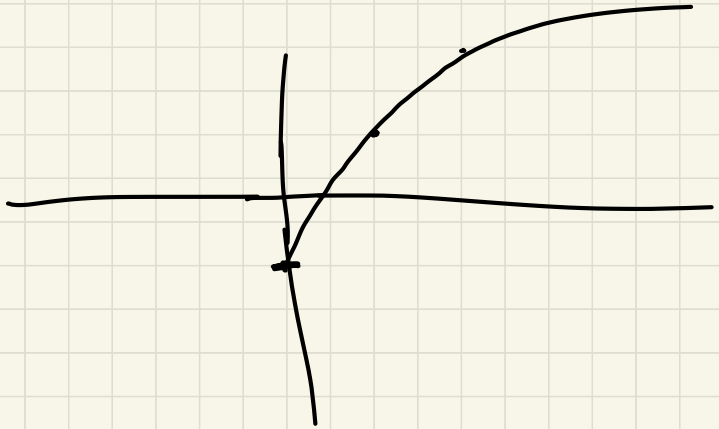
6) $f_k(x) = x^3 - 3kx^2 + 3x + 1$

$f_k'(x) = 3x^2 - 6kx + 3 > 0$

$\Delta = 36k^2 - 36 < 0 \quad k^2 < 1 \quad -1 \le k \le 1$

7) 1) continua

2) derivabile



8) $f: \mathbb{R} \to \mathbb{R}$ $\quad f(1) = 1$

$f'(1) = 2$

$g(x) = e^{f(x)}$ $\quad g'(1)$

$g'(x) = e^{f(x)} \cdot f'(x)$ $\quad g'(1) = e^{1} \cdot 2 = 2e$