1) $f(x) = x^3 + x + 1$ $\qquad g'(3) = ?$ $\qquad g'(y_0) = \dfrac{1}{f'(x_0)}$

monotona strettamente, quindi invertibile

A

$$x^3 + x + 1 = 3$$

$$x^3 + x - 2 = 0$$

$$\begin{array}{c|ccc|c}
 & 1 & 0 & 1 & -2 \\
1 & & 1 & 1 & 2 \\
\hline
 & 1 & 1 & 2 & 0
\end{array}$$

$$(x^2 + x + 2)(x - 1) = 0$$

$$x = 1$$

$$f'(x) = 3x^2 + 1$$

$$g'(3) = \frac{1}{f'(1)} = \frac{1}{4}$$

B

2) $f(x) = xe^x - 3e^x$

$$f'(x) = e^x + xe^x - 3e^x = e^x(x-2)$$

$f'(x)=0 \quad x=2$

$f'(x)>0 \quad \overbrace{e^x}^{>0\ \forall x}(x-2)$

$x>2$

$x = 2$ p.to minimo globale

C

3) $f(x) = x^2 + 2\ln x$

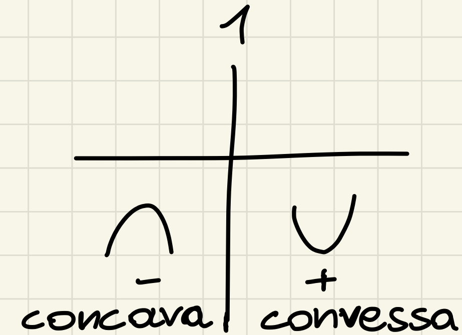
$f'(x) = 2x + \frac{2}{x}$

$f''(x) = 2 - \frac{2}{x^2}$

$f''(x) > 0 \qquad 2 - \frac{2}{x^2} > 0$

$\frac{2}{x^2} < 2 \qquad x > 1$

$x \in [1, +\infty)$



$$4)\ \int \sqrt[3]{x+3}\,dx = \int_{-3}^{5} \underbrace{1}_{f'} \cdot \sqrt[3]{\underbrace{x+3}_{f}}\,dx = \frac{(x+3)^{4/3}}{4/3} + c = \quad C$$

$$= \frac{3}{4}\sqrt[3]{(x+3)^4} + c = \frac{3x+9}{4}\sqrt[3]{(x+3)} + c$$

$$\int_{-2}^{5} \sqrt[3]{x+3}\,dx = \frac{3(5)+9}{4}\sqrt[3]{8} - \left(\frac{-6+9}{4}\sqrt[3]{1}\right) = \frac{24\cdot 2}{4} - \frac{3}{4}$$

$$\frac{48-3}{4} = \frac{45}{4}$$

ES. 2

$$f(x) = x(\sin x - \sin(x^2))$$

$$\int x(\sin x - \sin(x^2))\,dx$$

$$\int x\sin x - x\sin(x^2)\,dx = \int x\sin x\,dx - \int x\sin(x^2)\,dx =$$

$$= \int x\sin x\,dx - \frac{1}{2}\int 2x\sin(x^2)\,dx = \int x\sin x\,dx + \frac{1}{2}\cos(x^2)dx =$$

$$= -x\cos x + \int \cos x\,dx + \frac{1}{2}\cos(x^2)dx =$$

$g(x) = x \quad g'(x) = 1$

$f'(x) = \sin x \quad f(x) = -\cos x$

$$-x\cos x + \sin x + \frac{1}{2}\cos(x^2) + C$$

b) $\lim_{x\to 0} -\cancel{x\cos x} + \cancel{\sin x}^{0} + \frac{1}{2}\cos(x^2) + c = 0$

$\to 1$

$c = -\frac{1}{2}$

c) $F'(x) = -\cancel{\cos x} + x\sin x + \cancel{\cos x} + \frac{1}{\cancel{2}}\sin(x^2)\cancel{2}x = x(\sin x - \sin(x^2))$

$F''(x) = \sin x - \sin(x^2) + x(\cos x - \cos(x^2)\cdot 2x)$

$\sin x - \sin(x^2) + x\cos x - 2x^2\cos(x^2)$

$F'''(x) = \cos x - \cos(x^2)\cdot 2x + \cos x - x\sin x + 4x\cos(x^2) + 4x^3\sin(x^2)$

$\cancel{2x\cos(x^2)} + 2\cos x \cancel{+4x^3\sin(x^2)} - \cancel{x\sin x}$

$F'''(0) = 2$

$F^{IV}(x) = \{[2\cos(x^2)]_{1} - [\cancel{2x\sin(x^2)\cdot 2x}]_{0}\} - [\cancel{2\sin x}]_{0} + \{[\cancel{12x^2\sin(x^2)}]_{0} + [\cancel{-4x^3\cos(x^2)\cdot 2x}]_{1}\}$

$-\cancel{\sin x} - \cancel{x\cos x}$ $\boxed{F^{IV}(0) = 2}$ RIGUARDA

$$F(x) = -x\cos x + \sin x + \frac{\cos(x^2)}{2} - \frac{1}{2} \qquad P_4(x,0)$$

$$\cos t = 1 - \frac{t^2}{2} + \frac{t^4}{4!} + o(t^4)$$

$$\sin t = t - \frac{t^3}{3!} + o(t^4)$$

x rientra qua

$$F(x) = -x\left(1 - \frac{x^2}{2} + \frac{x^4}{4!} + o(x^4)\right) + x - \frac{x^3}{3!} + o(x^4) + \cancel{\frac{1}{2}}\left(1 - \frac{x^4}{2} + o(x^4)\right) - \cancel{\frac{1}{2}}$$

$$= -x + \frac{x^3}{2} + o(x^4) + x - \frac{x^3}{6} - \frac{x^4}{4}$$