1 balza

2 $f: \mathbb{R} \to \mathbb{R}$ $\quad f'(0)=0 \quad f''(x)=\ln(e+x) \quad f$ in $x=0$?

$\int \ln(e+x)$ $\quad t = x+e \quad x = t-e$

$dx = 1\,dt$

$\int 1 \cdot \ln(t)\,dt$

$f(x) = \ln(t) \quad f'(x) = \frac{1}{t} \qquad t\ln t - \int 1\,dt = t\ln t - t + c$

$g'(x) = 1 \qquad g(x) = t$

$t\ln t - t = (x+e)\cdot\ln(x+e) - x - e = f'(x)$

$(x+e)\cdot\ln(x+e) > x+e \qquad \overset{CE}{x > -e}$

$$\ln(x+e) > \ln e$$

$$x > 0$$

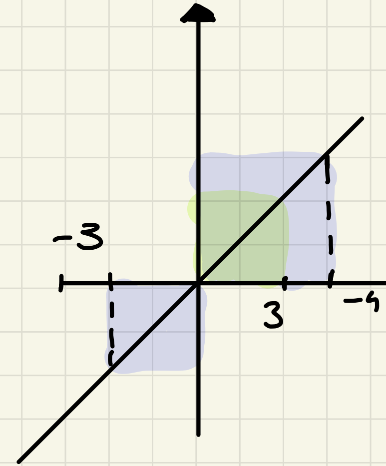
0

$\searrow \; | \; \nearrow$

p.to di min

(3) $f: \mathbb{R} \to \mathbb{R}$

$$\int_{-3}^{4} f(x)\,dx = \int_{3}^{4} f(x)\,dx$$



(4) $f(x) = \sqrt[3]{\frac{x^3}{2}+1}$

$$f'(x) = \frac{1}{3\sqrt[3]{\left(\frac{x^3}{2}+1\right)^2}} \cdot \frac{3}{2}x^2 = \frac{x^2}{2\sqrt[3]{\left(\frac{x^3}{2}+1\right)^2}}$$

(5) BALZA

⑥ $f(x) = \begin{cases} a\sin x - b^2 & -2 \leq x \leq 0 \\ 1 - e^x & 0 < x \leq 3 \end{cases}$

$$\lim_{x \to 0^-} a\sin x - b^2 = \lim_{x \to 0^+} 1 - e^x = 0$$

$$\lim_{x \to 0^-} a\sin x - b^2 \qquad -b^2 = 0 \qquad \boxed{b = 0}$$

0

$f'(x) = \begin{cases} a\cos x & -2 \leq x \leq 0 \\ -e^x & 0 < x \leq 3 \end{cases}$

1

$$\lim_{x \to 0^-} a\cos x = \lim_{x \to 0^+} -e^x = -1$$

$\boxed{a = 1}$

7 BALZA

8 $f(x) = \ln x$, $g(x) = x^3$ $h(x) = 2 - x$

$(h \circ g \circ f)(x) = 2 - (\ln x)^3$

ESERCIZI

$f: \mathbb{R} \to \mathbb{R} \qquad f(x) = (x^2 - 2x)e^{-x}$

1) $D: \mathbb{R}$

2) segno $\quad (x^2 - 2x) > 0 \quad x(x-2) > 0$

$x > 0 \quad x > 2$

3) lim

$$\lim_{x \to -\infty} (\overset{+\infty}{x^2} - 2x)\,\overset{+\infty}{e^{-x}} = +\infty$$

$$\lim_{x \to +\infty} (x^2 - 2x)\,e^{-x} =$$

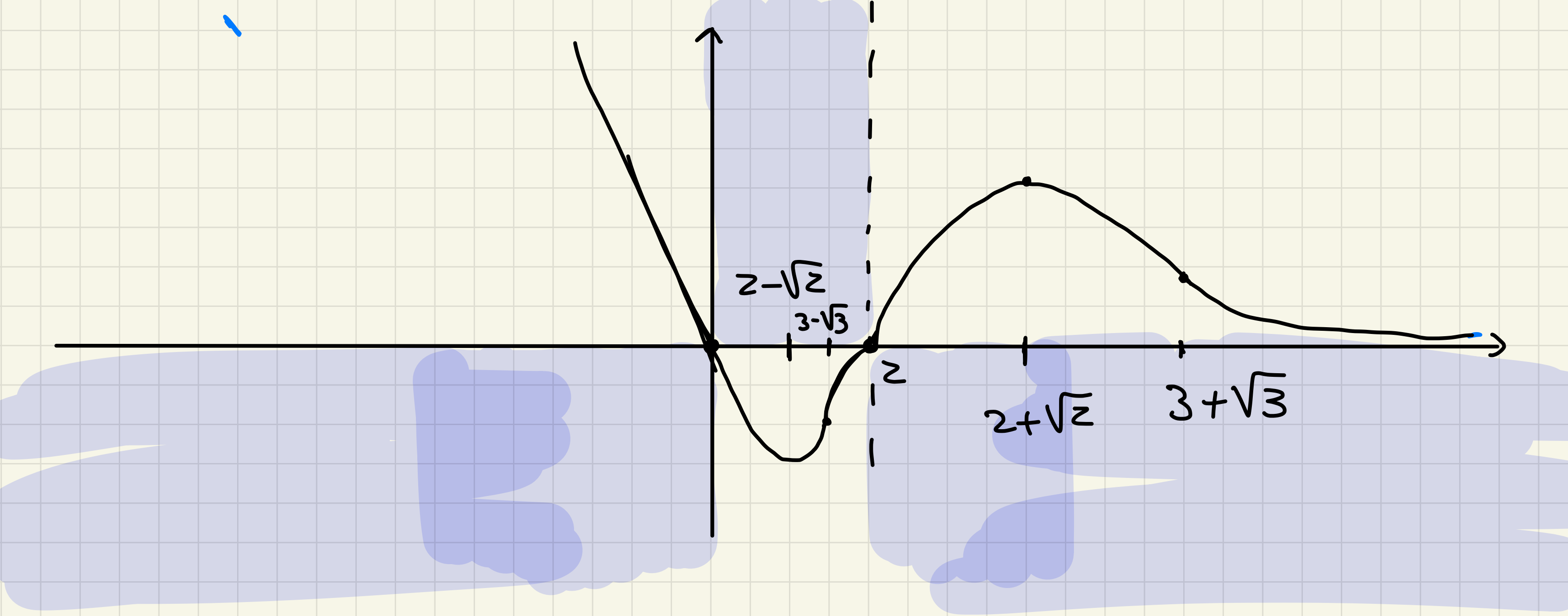
$$\lim_{x \to +\infty} \frac{x^2 - 2x}{e^x} \sim \frac{x^2}{e^x} \xrightarrow{x \to +\infty} 0$$

(

$f(x) > 0$

$x < 0 \ \vee \ x > 0$

	0		2	
$-$		$+$		$+$
$-$		$-$		$+$
$+$		$-$		$+$


2-√2
3-√3
2
2+√2
3+√3

4) $\lim_{x \to -\infty} \frac{f(x)}{x} = \lim_{x \to -\infty} \frac{x^{\cancel{2}} - 2\cancel{x}}{\cancel{x} e^x} = \lim_{x \to -\infty} \frac{x-2}{e^x} = 0$

no as. obliquo

5) $f'(x) = \frac{(2x-2)}{e^x} - \frac{(x^2-2x)}{e^x} = \frac{-x^2+4x-2}{e^x} = \frac{-(x^2-4x+2)}{e^x}$

$D_{f'}$: $\mathbb{R}$

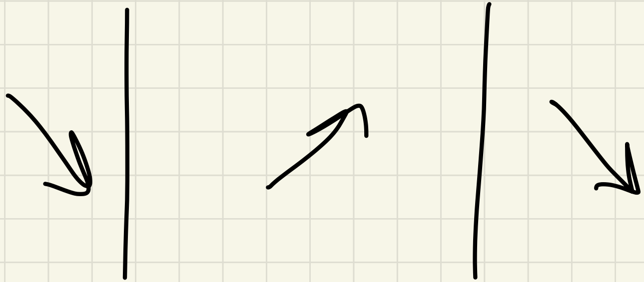
segno $\quad D>0 \ x \in \mathbb{R}$

$-x^2+4x-2 > 0$

$\Delta = 16 - 8 = 8$

$x_{1,2} = \frac{-4 \pm 2\sqrt{2}}{-2} = \begin{cases} \frac{4-2\sqrt{2}}{2} = 2-\sqrt{2} \\ \frac{4+2\sqrt{2}}{2} = 2+\sqrt{2} \end{cases}$

$$2-\sqrt{2} < x < 2+\sqrt{2}$$



min max

$$f'(x) = \frac{-(x^2-4x+2)}{e^x}$$

$$f''(x) = \frac{(-2x+4)\cancel{e^x} + (x^2-4x+2)\cancel{e^x}}{(e^x)^{\cancel{2}}} = \frac{x^2-6x+6}{e^x}$$

$f''(x) > 0 \qquad \dfrac{x^2 - 6x + 6}{e^x} > 0$

$x^2 - 6x + 6 > 0$

$\Delta = 36 - 24 = 12$

$x_{1/2} = \dfrac{6 \pm \sqrt{12}}{2} = \dfrac{6 \pm 2\sqrt{3}}{2} = 3 \pm \sqrt{3}$

	$3-\sqrt{3}$		$3+\sqrt{3}$	
$+$		$-$		$+$
$\cup$		$\cap$		$\cup$

Mac Lavin

$f(0) + f'(0)x + \dfrac{f''(0)}{2!}x^2$

$$\boxed{0 - 2x + 3x^2 + o(x^2)}$$

$$f(x) = \frac{1}{x\ln^2 x} \quad : (1, +\infty) \longrightarrow \mathbb{R}$$

$$\int \frac{1}{x} (\ln x)^{-2} dx = -(\ln x)^{-1} + c = \frac{-1}{\ln x} + c$$

$x = e \qquad g(x) = \frac{e}{x}$

$$\frac{-1}{\ln e} + c = \frac{e}{e} \qquad \frac{-1}{\ln e} + c = 1 \qquad -1 + c = 1 \qquad c = 2$$

$$\boxed{\frac{-1}{\ln x} + 2}$$

$$\int_e^{e^3} \frac{1}{x\ln^2 x} dx \qquad \frac{\left(\frac{-1}{\ln e^3} + 2\right) - \left(\frac{-1}{\ln e} + 2\right)}{e^3 - e} = \frac{\frac{-1}{3} + 2 + 1 - 2}{e^3 - e} =$$

$$= \frac{-\frac{1}{3} + 1}{e^3 - e} = \frac{\frac{2}{3}}{e^3 - e} = \frac{2}{3(e^3 - e)}$$