rette in $\mathbb{R}^2$

forma parametrica, traslato spazo di dim 1

$$\ell = \underset{\langle \underline{v} \rangle}{\underset{\shortparallel}{\ell_0}} + \underline{p} \qquad \underline{p} \in \ell \qquad \underline{v} \neq \underline{0} \qquad \begin{array}{l} \underline{v} = (v_1, v_2) \\ \underline{p} = (p_1, p_2) \end{array}$$

$$\ell = \{ \lambda \underline{v} + \underline{p} : \lambda \in \mathbb{R} \} = \{ (\lambda v_1 + p_1, \lambda v_2 + p_2) . \lambda \in \mathbb{R} \}$$

$$= \begin{cases} x = \lambda v_1 + p_1 \\ y = \lambda v_2 + p_2 \end{cases} \quad \lambda \in \mathbb{R}$$

in coordinate

$$\ell = \{ (x, y) \in \mathbb{R}^2 \quad ax + by + c = 0$$
$$(a, b) \neq (0, 0)$$

rette in $\mathbb{R}^3$

parametrica come prima

coordinate

$$\ell = \left\{ (x, y, z) \in \mathbb{R}^3 : \begin{cases} a_1 x + b_1 y + c_1 z + d_1 = 0 \\ a_2 x + b_2 y + c_2 z + d_2 = 0 \end{cases} \right\}$$

con rango $\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{pmatrix} = 2$

piani in $\mathbb{R}^3$

parametrica. $H = H_0 + \underline{p}$ $\qquad \underline{v}, \underline{w}, \underline{p} \in \mathbb{R}^3$

$H_0 = \langle \underline{v}, \underline{w} \rangle$ componente omogenea

$$H = \{ \alpha \underline{v} + \beta \underline{w} + \underline{p} \cdot \alpha, \beta \in \mathbb{R} \}$$

coordinate

$$H = \{ (x, y, z) \in \mathbb{R}^3 : ax + by + cz + d = 0 \}$$
$$(a, b, c) \neq 0$$

é ortogonale al piano

8.1

1) $P_1 = \langle \underline{v}, \underline{w} \rangle + \underline{p}$ piano in forma parametrica

$\underline{v} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ $\underline{w} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ $\underline{p} = (1, 1, 1)$

scrivi in coordinate

$$P_1 = \left\{ \alpha \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} : \alpha, \beta \in \mathbb{R} \right\} = \left\{ \begin{pmatrix} 0 \\ \alpha \\ \alpha \end{pmatrix} + \begin{pmatrix} \beta \\ \beta \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} : \alpha, \beta \in \mathbb{R} \right\} = \left\{ \begin{pmatrix} \beta + 1 \\ \alpha + \beta + 1 \\ \alpha + 1 \end{pmatrix} : \alpha, \beta \in \mathbb{R} \right\}$$

$\begin{cases} x = \beta + 1 \\ y = \alpha + \beta + 1 \\ z = \alpha + 1 \end{cases}$ $\beta = x - 1$ sostituisco $y = z - 1 + x - 1 + 1 \rightarrow x - y + z - 1 = 0$ forma cartesiana

$\alpha = z - 1$

$$P_1 = \{ x - y + z - 1 = 0 : x, y, z \in \mathbb{R} \}$$

$L_1 = \langle \underline{z} \rangle + \underline{q}$ $\underline{z} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ $\underline{q} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$

$L_1 = \left\{ \begin{pmatrix} -\lambda + 2 \\ \lambda + 1 \\ -1 \end{pmatrix} \right\}$ $\begin{cases} x = -\lambda + 2 \\ y = \lambda + 1 \\ z = -1 \end{cases}$ $x = -y + 1 + 2 \rightarrow x = -y + 3$

$\lambda = y - 1$

$$L_1 = \left\{ \begin{cases} x + y - 3 = 0 \\ z + 1 = 0 \end{cases} x, y, z \in \mathbb{R} \right\}$$

2)

2) $L_2 = \left\{ (x,y,z) \in \mathbb{R}^3 : \begin{matrix} x+y-z=0 \\ -x+2y+z+2=0 \end{matrix} \right\}$ retta in $\mathbb{R}^3$ in coordinate

↓ devo vedere dim

$$rg\begin{pmatrix} 1 & 1 & -1 \\ -1 & 2 & 1 \end{pmatrix} = 2$$

$\begin{cases} z = x+y \\ z = x-2y-2 \end{cases}$ $\quad x+y = x-2y-2 \quad 3y=-2 \quad y = -\frac{2}{3}$

quindi $L_2 = \left\{ (x,y,z) \in \mathbb{R}^3 : y = -\frac{2}{3},\ z = x - \frac{2}{3} \right\}$

$L_2 = \left\{ \begin{pmatrix} x \\ -2/3 \\ x-2/3 \end{pmatrix} : x \in \mathbb{R} \right\}$ ← forma parametrica

$L_2 = \left\langle \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\rangle + \begin{pmatrix} 0 \\ -2/3 \\ -2/3 \end{pmatrix}$

8.2)

1) $A = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\quad \overline{AB} = r = \{ x-2y+3=0 \}$ $\qquad y = \frac{-x-3}{-2} = \frac{x}{2} + \frac{3}{2}$

$B = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$P = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ $\quad$ retta ortogonale passante per P $\qquad y+2 = -2(x-3)$

$y+2 = -2x+6$

$y = -2x+4$

proiezione $\begin{cases} y = -2x+4 \\ y = \frac{x}{2} + \frac{3}{2} \end{cases}$ $\quad -2x+4 = \frac{x}{2} + 3/2$

$V = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ ← proiezione ort di P su r $\qquad \frac{x}{2} + 2x = 4 - 3/2$

$\frac{5}{2}x = 5/2 \qquad x = 1 \Longrightarrow y = 2$

$d(P,V) = \sqrt{(3-1)^2 + (-2-2)^2}$

$\sqrt{4+16} = \sqrt{20}$

2) $S = \begin{cases} x = t-2 \\ y = -t \\ z = 2t+1 \end{cases}$ retta in $\mathbb{R}^3$ in forma parametrica

trovare dist tra s e $p = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

trovare piano $N \perp s$ e passante per P

$s = \left\langle \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \right\rangle + \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$

La comp. omogenea di N avrà eq. $N_0 = \{ \langle (1,-1,-2), (x,y,z) \rangle = 0 \}$ quindi: $\{ x - y + 2z = 0 \}$

traslo il piano per farlo passare per p

$N_d = \{ (x,y,z) \in \mathbb{R}^3 : x - y + 2z + d = 0 \}$

devo determinare d

$1 - 1 + 2 + d = 0 \qquad d = -2$

$N = \{ (x,y,z) \in \mathbb{R}^3 : x - y + 2z - 2 = 0 \}$

$\text{dist}(S, P) = \text{dist}(Q, p)$ dove Q è l'intersezione tra N e S

$S = \begin{cases} x = t-2 \\ y = -t \\ z = 2t+1 \end{cases}$

$N = \{ (x,y,z) \in \mathbb{R}^3 : x - y + 2z - 2 = 0 \}$

$Q = \begin{cases} x = t-2 \\ y = -t \\ z = 2t+1 \\ x - y + 2z - 2 = 0 \end{cases}$

$t - 2 + t + 4t + \cancel{2} - \cancel{2} = 0 \quad 6t - 2 = 0 \quad t = 1/3$

$x = 1/3 - 2 = -5/3$

$y = -1/3$

$z = 2/3 + 1 = 5/3$

$Q = \left(-\frac{5}{3}, -\frac{1}{3}, \frac{5}{3} \right)$

la $\text{dist}(Q, P) = \sqrt{(-5/3 - 1)^2 + (-1/3 - 1)^2 + (5/3 - 1)^2} =$

$\sqrt{\left(-\frac{8}{3}\right)^2 + \left(-\frac{4}{3}\right)^2 + \left(\frac{2}{3}\right)^2} =$

$\sqrt{\frac{64}{3} + \frac{16}{9} + 4/9} = \sqrt{\frac{84}{9}} = \frac{\sqrt{84}}{3}$

8.4) determinare dist(S, H) (S: retta, H: piano)

$$S = < \underset{\underline{z}}{\begin{pmatrix}1\\0\\-1\end{pmatrix}} > + \underset{\underline{p}}{\begin{pmatrix}1\\1\\0\end{pmatrix}} \quad H = < \underset{\underline{v}}{\begin{pmatrix}1\\1\\1\end{pmatrix}}, \underset{\underline{w}}{\begin{pmatrix}2\\1\\0\end{pmatrix}} > + \underset{\underline{q}}{\begin{pmatrix}0\\-1\\1\end{pmatrix}}$$

sono paralleli? se no dist = 0

controllo generatori, $\underline{z} \in < \underline{v}, \underline{w} >$?

$$\det\begin{pmatrix}1 & 0 & -1\\1 & 1 & 1\\2 & 1 & 0\end{pmatrix} \overset{?}{\neq} 0 \quad -1-1(1-2) = -1+1 = 0$$

sono paralleli

dobbiamo trovare retta $N \perp H$ che interseca S

trovo vett $\perp$ a H facendo il prodotto vettoriale dei generatori

$$\underline{u} = \underline{v} \wedge \underline{w} = \begin{pmatrix}e_1 & e_2 & e_3\\1 & 1 & 1\\2 & 1 & 0\end{pmatrix} = \begin{pmatrix}-1\\2\\-1\end{pmatrix}$$

$$N = <\begin{pmatrix}-1\\2\\-1\end{pmatrix}> + \underline{p} = \text{t.c. intersechi } S$$

↑ così interseca sicuramente S

$$= <\begin{pmatrix}-1\\2\\-1\end{pmatrix}> + \begin{pmatrix}1\\1\\0\end{pmatrix}$$

devo trovare P punto di intersezione tra N e S

e Q intersezione tra N e H

Infine dist(S, H) = dist(P, Q)

$$P = \begin{cases} x = -t+1 \\ y = 2t+1 \\ z = -t \\ x = t+1 \\ y = 1 \\ z = -t \end{cases} \qquad t+1 = -t+1 \qquad t = 0 \Downarrow \begin{matrix} x = 1 \\ y = 1 \\ z = 0 \end{matrix}$$

$P = \begin{pmatrix}1\\1\\0\end{pmatrix}$ notare che per come abbiamo fatto è uguale a $\underline{p}$ perché abbiamo fatto passare N per $\underline{p}$

$$Q = \begin{cases} x = -t+1 \\ y = 2t+1 \\ z = -t \\ y = \frac{z}{2} + \frac{x}{2} - 3/2 \end{cases}$$

$$H = \left\langle \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \right\rangle + \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

$x = \lambda + 2\beta$

$y = \lambda + \beta - 1$

$z = \lambda + 1$

$\beta = \frac{x-\lambda}{2}$

$y = z - 1 + \frac{x-\lambda}{2} - 1 = z + \frac{x}{2} - \frac{(z-1)}{2} - 2 = \frac{z}{2} + \frac{x}{2} - 3/2$

$\lambda = z - 1$

$$\frac{z}{2} + \frac{x}{2} - 3/2 = 2t+1$$

$$-\frac{t}{2} - \frac{t}{2} + \frac{1}{2} - 3/2 = 2t+1$$

$$-t - 1 = 2t+1$$

$$3t = -2 \quad t = -2/3$$

$$\begin{cases} x = +2/3 + 1 \\ y = 2 \cdot -2/3 + 1 \\ z = +2/3 \end{cases} \qquad Q = \begin{pmatrix} 5/3 \\ -1/3 \\ 2/3 \end{pmatrix}$$

$$\text{dist}(P,Q) = \| Q - P \| =$$

$$\left\| \begin{pmatrix} 5/3 \\ -1/3 \\ 2/3 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right\| = \left\| \begin{pmatrix} 2/3 \\ -4/3 \\ 2/3 \end{pmatrix} \right\| = \sqrt{\left(\frac{2}{3}\right)^2 + \left(-\frac{4}{3}\right)^2 + \left(\frac{2}{3}\right)^2} =$$

$$= \sqrt{\frac{4}{9} + \frac{16}{9} + \frac{4}{9}} = \sqrt{\frac{24}{9}} = \frac{2\sqrt{6}}{3}$$

8.5) Due rette sono sghembe se non sono parallele e non hanno intersezioni

$\det(\underline{v}|\underline{w}|\underline{b}-\underline{a}) \neq 0$ $\quad l = \langle v \rangle + \underline{a} \quad S = \langle \underline{w} \rangle + \underline{b}$

l e S non sono $//$ SSE $\underline{w}$ non $// \underline{v}$ quindi $\underline{v} \neq \lambda \underline{w}$ (indipendenti)

se l e S intersecano, poiché $l = \{\alpha \underline{v} + \underline{a} : \alpha \in \mathbb{R}\}$ e $S = \{\beta \underline{w} + \underline{b} : \beta \in \mathbb{R}\}$

allora $\exists\ \hat{\alpha}, \tilde{\beta}$ t.c. $\hat{\alpha}\underline{v} + \underline{a} = \tilde{\beta}\underline{w} + \underline{b} \iff \underline{b} - \underline{a} = \hat{\alpha}\underline{v} - \tilde{\beta}\underline{w} \in \langle \underline{v}, \underline{w} \rangle$

in pratica $\underline{b}-\underline{a}$ comb l. di $\underline{v}$ e $\underline{w}$

combinando con la condizione precedente

$\underline{v}, \underline{w}, \underline{b}-\underline{a}$ indipendenti

8.3) $S = \left\langle \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \right\rangle + \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

$\underline{z} \qquad \underline{q}$

$l = (x, y, z) \in \mathbb{R}^3 : \left.\begin{matrix} x + y - z = 0 \\ -x + 2y + z + 2 = 0 \end{matrix}\right\}$

$\Downarrow$ da es 8.1.2

$\left\langle \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\rangle + \begin{pmatrix} 0 \\ -2/3 \\ -2/3 \end{pmatrix}$

$\underline{v} \qquad \underline{p}$

$$\det \begin{pmatrix} 2 & 1 & 0 \\ 1 & 0 & -2/3 - 1 \\ 2 & 1 & -2/3 + 1 \end{pmatrix} \neq 0 \quad \text{per essere sghembe}$$

$$\det \begin{pmatrix} 2 & 1 & 0 \\ 1 & 0 & -5/3 \\ 2 & 1 & 1/3 \end{pmatrix} = 2\det \begin{pmatrix} 0 & -5/3 \\ 1 & 1/3 \end{pmatrix} - \det \begin{pmatrix} 1 & -5/3 \\ 2 & 1/3 \end{pmatrix}$$

$$\frac{10}{3} - \left(\frac{1}{3} + \frac{10}{3} \right) = -\frac{1}{3} \quad \text{sono sghembe}$$

2) trovare retta R sghemba a S e l

$R = \langle \underline{w} \rangle + \underline{u}$

$\underline{w}$ non deve essere parallelo né a $\underline{z}$ né a $\underline{v}$ (devono essere lin. ind.)

$\det(\underline{w} | \underline{z} | \underline{v}) \neq \det \left(\underline{w} \left| \begin{matrix} 2 \\ 1 \\ 1 \end{matrix} \right| \begin{matrix} 1 \\ 0 \\ 1 \end{matrix} \right) \neq 0$ ad es. $w = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

siccome per essere sghembe $\Rightarrow \det\left(\underline{w} \mid \underline{z} \mid \underline{q}-\underline{u}\right) \neq 0$

$$\det\begin{pmatrix} 1 & 2 & 0-\alpha \\ 0 & 1 & 1-\beta \\ 0 & 1 & -1-\gamma \end{pmatrix} \qquad \Rightarrow \det\left(\underline{w} \mid \underline{v} \mid \underline{p}-\underline{u}\right) \neq 0$$

$$\det\begin{pmatrix} 1 & 2 & -\alpha \\ 0 & 1 & 1-\beta \\ 0 & 1 & -1-\gamma \end{pmatrix} = -1-\gamma-1+\beta \neq 0$$

$$\beta-\gamma-2 \neq 0$$

$$\gamma \neq \beta-2$$

$$\det\begin{pmatrix} 1 & 1 & 0-\alpha \\ 0 & 0 & -2/3-\beta \\ 0 & 1 & -2/3-\gamma \end{pmatrix} = \det\begin{pmatrix} 0 & -2/3-\beta \\ 1 & -2/3-\gamma \end{pmatrix} = \frac{2}{3}+\beta \neq 0 \quad \beta \neq -\frac{2}{3}$$

va bene un qualsiasi $\underline{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ t.c. $y \neq -\frac{2}{3}$ e $z \neq y-2$

ad es $\underline{v} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$