1

$$\begin{cases} (t+4)x + y + w = 1 \\ tx + (2t-2)y + tz - 10w = t \\ -x - y - z - w = -1 \end{cases}$$

$$\left(\begin{array}{cccc|c} (t+4) & 1 & 0 & 1 & 1 \\ t & 2t-2 & t & -10 & t \\ -1 & -1 & -1 & -1 & -1 \end{array}\right) \xrightarrow{r_1 \to r_1 - r_2} \left(\begin{array}{cccc|c} 4 & -2t-1 & -t & +11 & 1-t \\ t & 2t-2 & t & -10 & t \\ -1 & -1 & -1 & -1 & -1 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 4 & -2t-1 & -t & +11 & 1-t \\ t-1 & 2t-3 & t-1 & -11 & t-1 \\ -1 & -1 & -1 & -1 & -1 \end{array}\right) \to \left(\begin{array}{cccc|c} t+3 & -4 & -1 & 0 & 0 \\ t-1 & 2t-3 & t-1 & -11 & t-1 \\ -1 & -1 & -1 & -1 & -1 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} t+3 & -4 & -1 & 0 & 0 \\ t & 2t-2 & t & -10 & t \\ -1 & -1 & -1 & -1 & -1 \end{array}\right) \to \left(\begin{array}{cccc|c} -1 & -1 & -1 & -1 & -1 \\ t+3 & -4 & -1 & 0 & 0 \\ t & 2t-2 & t & -10 & t \end{array}\right)$$

$$\left(\begin{array}{cccc|c} -1 & -1 & -1 & -1 & -1 \\ 0 & -4 & -1 & 0 & t+3 \\ t & 2t-2 & t & -10 & t \end{array}\right) \to \left(\begin{array}{cccc|c} -1 & -1 & -1 & -1 & -1 \\ 0 & -1 & -4 & 0 & t+3 \\ t & t & 2t-2 & -10 & t \end{array}\right)$$

$$\left(\begin{array}{cccc|c} -1 & -1 & -1 & -1 & -1 \\ 0 & -1 & -4 & 0 & t+3 \\ 1 & 1 & 2-\frac{2}{t} & -\frac{10}{t} & 1 \end{array}\right) \to \left(\begin{array}{cccc|c} -1 & -1 & -1 & -1 & -1 \\ 0 & -1 & -4 & 0 & t+3 \\ 0 & 0 & 1-\frac{2}{t} & -\frac{10}{t}-1 & 0 \end{array}\right)$$

ho ∞ sol. tranne per $t=0$

A $n \times n$ invertibile $\iff \det(A) \neq 0$ possibile trovare U invertibile t.c.

UA ha una riga di soli 0? No perché il prodotto tra due m. invertibili é invertibile, ma se UA ha una riga di tutti $0 \iff \det(UA) = 0$ ma una matrice per essere invertibile deve avere $\det \neq 0$

2

$$A=\begin{pmatrix} 2 & 1 & 1 \\ -1 & 1 & 1 \\ -1 & 0 & 2 \end{pmatrix} \qquad \begin{matrix} 2 & 1 & 1 & 2 & 1 \\ -1 & 1 & 1 & -1 & 1 \\ -1 & 0 & 2 & -1 & 0 \end{matrix}$$

3

invertibile

$$(4-1)-(-1-2)=3+3=6$$

$$\left(\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 & 1 & 0 \\ -1 & 0 & 2 & 0 & 0 & 1 \end{array}\right) \xrightarrow{r_3 \to r_3 - r_2} \left(\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & -1 & 1 \end{array}\right) \xrightarrow{r_2 \to r_2 + \frac{1}{2} r_1} \left(\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 3/2 & 3/2 & 1/2 & 1 & 0 \\ 0 & -1 & 1 & 0 & -1 & 1 \end{array}\right)$$

$$\xrightarrow{r_3 \to r_3 + \frac{2}{3} r_2} \left(\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 3/2 & 3/2 & 1/2 & 1 & 0 \\ 0 & 0 & 2 & \frac{1}{3} & -\frac{1}{3} & 1 \end{array}\right) \xrightarrow{r_1 \to r_1 - \frac{2}{3} r_2} \left(\begin{array}{ccc|ccc} 2 & 0 & 0 & \frac{2}{3} & -2/3 & 0 \\ 0 & 3/2 & 3/2 & 1/2 & 1 & 0 \\ 0 & 0 & 2 & 1/3 & -\frac{1}{3} & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 2 & 0 & 0 & 2/3 & -\frac{2}{3} & 0 \\ 0 & 3/2 & 0 & 1/4 & 5/4 & 3/4 \\ 0 & 0 & 2 & 1/3 & -1/3 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/3 & -\frac{1}{3} & 0 \\ 0 & 1 & 0 & 2/12 & 10/8 & 1/2 \\ 0 & 0 & 1 & 1/6 & -1/6 & 1/2 \end{array}\right)$$

$M = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \quad A_1 = \left\{ \begin{pmatrix} \lambda & 2\lambda \\ 1-\lambda & \lambda \end{pmatrix}, \lambda \in \mathbb{R} \right\}$

4

$A_2 = \left\{ \begin{pmatrix} \lambda & 2\lambda \\ 0 & \lambda \end{pmatrix} + \begin{pmatrix} 2 & 4 \\ 0 & 2 \end{pmatrix}, \lambda \in \mathbb{R} \right\}$

$A_3 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

deve contenere $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ per essere $< M_{2x2}$

$$\begin{pmatrix} \lambda+2 & 2\lambda+4 \\ 0 & \lambda+2 \end{pmatrix} \quad \begin{cases} \lambda = -2 \\ 2\lambda = -4 \\ \lambda = -2 \end{cases} \quad \lambda = -2$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \in A_2$$

$M \cup A_2 < M_{2x2}$

$$\left\{ \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} \lambda+2 & 2\lambda+4 \\ 0 & \lambda+2 \end{pmatrix} \right\} \quad \begin{cases} \lambda+2=0 & \lambda=-2 \\ 2\lambda+4=0 & \lambda=-2 \\ \lambda+2=0 & \lambda=-2 \end{cases}$$

c'è l'origine
é sottospazio
di $M(2x2)$

5 $t=0$

⑥ $A=\begin{pmatrix} 1 & -1 & 2 \\ 3 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix}$ $B=\begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ -2 & 1 & 0 \end{pmatrix}$

$$A^2=\begin{pmatrix} -2 & 1 & 5 \\ 3 & -4 & 5 \\ 3 & 1 & 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ -2 & 1 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 3 & 2 & 2 & 0 & 1 \end{array}\right) \quad \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 3 & 2 & 2 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 5 & 2 & 3 & 1 \end{array}\right) \quad \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 1 & 0 \\ 0 & -1 & 0 & -\frac{2}{5} & \frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 5 & 2 & 3 & 1 \end{array}\right) \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{5} & -\frac{1}{5} & -\frac{2}{5} \\ 0 & -1 & 0 & -\frac{2}{5} & \frac{2}{5} & -\frac{1}{5} \\ 0 & 0 & 5 & 2 & 3 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/5 & -1/5 & -2/5 \\ 0 & 1 & 0 & 2/5 & -2/5 & 1/5 \\ 0 & 0 & 1 & 2/5 & 3/5 & 1/5 \end{array}\right)$$

$\det(A^2 \cdot B^{-1}) = \det(A^2) \cdot \det(B^{-1})$ per teorema di Binet

$$\det\begin{pmatrix} -2 & 1 & 5 \\ 3 & -4 & 5 \\ 3 & 1 & 0 \end{pmatrix} \quad \begin{matrix} -2 & 1 & 5 & -2 & 1 \\ 3 & -4 & 5 & 3 & -4 \\ 3 & 1 & 0 & 3 & 1 \end{matrix}$$

$15+15-(-60-10)$

$30+70=100$

$$\det\begin{pmatrix} 1/5 & -1/5 & -2/5 \\ 2/5 & -2/5 & 1/5 \\ 2/5 & 3/5 & 1/5 \end{pmatrix} \quad \begin{matrix} 1/5 & -1/5 & -2/5 & 1/5 & -1/5 \\ 2/5 & -2/5 & 1/5 & 2/5 & -2/5 \\ 2/5 & 3/5 & 1/5 & 2/5 & 3/5 \end{matrix}$$

$$\left(\frac{-2-2-12}{125}\right)-\left(\frac{8+3-2}{125}\right)=\frac{-16-9}{125}=\frac{-25}{125}$$

$$=-\frac{1}{5}$$

$\det(A^2 \cdot B^{-1}) =$
$-\frac{1}{5} \cdot 100 = 20$

⑦ $\begin{cases} 2x-2y=a \\ ax-y=0 \\ -x+y=1 \end{cases}$

$a=3$ $\left(\begin{array}{cc|c} 2 & -2 & 3 \\ 3 & -1 & 0 \\ -1 & 1 & 1 \end{array}\right) \to \left(\begin{array}{cc|c} 0 & -4 & 5 \\ 3 & -1 & 0 \\ -1 & 1 & 1 \end{array}\right) \to \left(\begin{array}{cc|c} 0 & -4 & 5 \\ 0 & 2 & 3 \\ -1 & 1 & 1 \end{array}\right)$

$\left(\begin{array}{cc|c} 0 & 0 & 11 \\ 0 & 2 & 3 \\ -1 & 1 & 1 \end{array}\right) \to \left(\begin{array}{cc|c} -1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 11 \end{array}\right)$ $rg(A) \neq rg(A|\underline{b})$
no sol.

$a=2$ $\left(\begin{array}{cc|c} 2 & -2 & 2 \\ 2 & -1 & 0 \\ -1 & 1 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 0 & 0 & 3 \\ 0 & 1 & 2 \\ -1 & 1 & 1 \end{array}\right)$ $rg(A) \neq rg(A|\underline{b})$ no sol

$a=-2$ $\left(\begin{array}{cc|c} 2 & -2 & -2 \\ -2 & -1 & 0 \\ -1 & 1 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 0 & 0 & 0 \\ -2 & -1 & 0 \\ -1 & 1 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & -3 & -2 \\ -1 & 1 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} -1 & 1 & 1 \\ 0 & -3 & -2 \\ 0 & 0 & 0 \end{array}\right)$

$rg(A) = rg(A|\underline{b})$ ho soluzione, rg max$=2$ quindi una sola soluzione

8) $(1,0,2)$ $(3,1,-1)$ $(-1,1,0)$ $(0,-2,1)$ ind

ind ind

$$\begin{cases} \lambda + 3\alpha - \beta = 0 \\ \alpha + \beta = 0 \\ 2\lambda - \alpha = 0 \end{cases}$$

$-\frac{\beta}{2} - 3\beta - \beta = 0$ $\quad$ $-\frac{9\beta}{2} = 0 \Rightarrow \beta = 0 \Rightarrow \alpha = 0$

$\Rightarrow \lambda = 0$

$\alpha = -\beta$

$\lambda = \frac{\alpha}{2} = \frac{-\beta}{2}$

$$\begin{cases} 3\lambda - \alpha = 0 \\ \lambda + \alpha - 2\beta = 0 \\ -\lambda + \beta = 0 \end{cases} \qquad \begin{cases} 3\beta - \beta = 0 \\ \beta + \alpha - 2\beta = 0 \\ \beta = \lambda \end{cases}$$

$3\beta = \beta \Rightarrow \beta = 0 \Rightarrow \alpha = 0 \Rightarrow \lambda = 0$

$\alpha = \beta$

i primi 3 ind anche ultimi 3 ind

(11) $\begin{cases} 3x-2y+z=0 \\ kx+y+z=0 \\ x+ky-z=0 \end{cases}$

$k=\pi$

$\begin{cases} 3x-2y+z=0 \\ \pi x+y=-z \\ x+\pi y-z=0 \end{cases}$

$3x+2x+z=0 \quad x=-\frac{z}{5}$

$x+\pi y+\pi x+y=0$

con $k=\pi$
no sol. solo con
$x=y=z=0$

$x(1+\pi)+y(1+\pi)=0$

$x=-y$

$-y=-\frac{z}{5}$

$y=\frac{\pi x+y}{5}$

$\pi x-4y=0$

$-\pi y-4y=0$

$-y(\pi+4)=0 \quad y=0$

(12) $\begin{cases} 3x-2y+z=0 \\ kx+y+z=1 \\ x+ky-z=-1 \end{cases}$

voglio vedere ho una soluzione sola, quindi $\det(A)\neq 0$

$$\begin{array}{ccccc} 3 & -2 & 1 & 3 & -2 \\ k & 1 & 1 & k & 1 \\ 1 & k & -1 & 1 & k \end{array}$$

$$(-3-2+k^2)-(1+3k+2k)$$

$-6+k^2-5k=0 \quad k^2-5k-6=0$

$\Delta = 25+24=49$

$k_{1,2}=\dfrac{5\pm 7}{2}$ $\begin{cases} 6 \Rightarrow \text{ho } \det=0 \\ -1 \Rightarrow \text{ho } \det=0 \end{cases}$ non ho 1 sola sol.

$$\left(\begin{array}{ccc|c} 3 & -2 & 1 & 0 \\ -1 & 1 & 1 & 1 \\ 1 & -1 & -1 & -1 \end{array}\right) \left(\begin{array}{ccc|c} 0 & 1 & 4 & 3 \\ -1 & 1 & 1 & 1 \\ 1 & -1 & -1 & -1 \end{array}\right) \left(\begin{array}{ccc|c} 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & -1 \end{array}\right)$$

ho infinite soluzioni per $k=-1$

(13)

$V_1 = (1, -1, \alpha, -1)$

$V_2 = (1, \alpha - 2, \frac{\alpha-1}{2}, 0) \subset \mathbb{R}^4$

$V_3 = (-1, -1, 1, -1)$

$$\begin{pmatrix} 1 \\ -1 \\ \alpha \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ \alpha-2 \\ \frac{\alpha-1}{2} \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$$

$$\begin{cases} x + y - z = 0 \\ -x + (\alpha-2)y - z = 0 \\ \alpha x + \left(\frac{\alpha-1}{2}\right)y + z = 0 \\ -x - z = 0 \end{cases}$$

$x = z - y$ $\quad x = z - 2z = -z$

$z + (\alpha-2)2z - z = 0$

$-\alpha z + \left(\frac{\alpha-1}{2}\right)2z + z = 0$

$-z + y - z = 0$ $\quad y = 2z$

$z + (\alpha-2)2z - z = 0 \quad (\alpha-2)2z = 0 \quad \alpha = 2 \vee z = 0$

$-\alpha z + \left(\frac{\alpha-1}{2}\right)2z + z = 0$

$z(1-\alpha) + \left(\frac{\alpha-1}{2}\right)2z = 0$

$z\left(1 - \alpha + \left(\frac{\alpha-1}{\cancel{2}}\right) \cdot \cancel{2}\right) = 0$

$z \cdot 0 = 0 \quad \forall z \in \mathbb{R}$ lin dip.

per qualche α sono dipendeti

(14) $\begin{cases} a(x,y,z,w) = A \\ b(x,y,z,w) = B \\ c(x,y,z,w) = C \\ d(x,y,z,w) = D \end{cases}$

∞^2 sol.

$\infty^{n-rg(A)}$

devo avere $rg(A) = 2$

solo se ho 2 eq. lin ind.

(15)

$p(x) = x^2 + 2x^3 + 1$

$q(x) = -x + 1$

BOH

(16) $V = \langle (1,0,-1), (2,2,0) \rangle$

$V_\lambda = V \cup \{(\lambda, 1-\lambda, 0)\} < \mathbb{R}^3$?

no perché non ho l'origine

(21) $I = \{ (x, y, z, w) \in \mathbb{R}^4 : z = x - w + y \}$

$$\begin{pmatrix} x \\ y \\ x-w+y \\ w \end{pmatrix}$$

(22) $\begin{cases} -x - y - z = b_1 \\ 3x - 9y - 6z = b_2 \\ 5x - 7y - 4z = b_3 \end{cases}$

$$\left(\begin{array}{ccc|c} -1 & -1 & -1 & b_1 \\ 3 & -9 & -6 & b_2 \\ 5 & -7 & -4 & b_3 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -1 & -1 & -1 & b_1 \\ 0 & -12 & -9 & b_2+3b_1 \\ 5 & -7 & -4 & b_3 \end{array}\right) \quad \left(\begin{array}{ccc|c} -1 & -1 & -1 & b_1 \\ 0 & -12 & -9 & b_2+3b_1 \\ 0 & -12 & -9 & b_3+5b_1 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -1 & -1 & -1 & b_1 \\ 0 & -12 & -9 & b_2+3b_1 \\ 0 & 0 & 0 & b_3+2b_1-b_2 \end{array}\right)$$

$b_3 + 2b_1 - b_2 = 0$

ho soluzione se

$b_2 = b_3 + 2b_1$

(23) Binet $\det(A\cdot B) = \det(A)\cdot\det(B)$

se AM è invertibile ($\det \neq 0$, rg max, tutte colonne indipendenti)

allora $\det(AM) \neq 0$ questo vuol dire che $\det(A) \neq 0$

(26)

v_3 lin dip da v_1 e v_2

non genero tutto $\mathbb{R}^3$ perché ho solo due vettori l.i.

$$\begin{pmatrix}1\\0\\2\end{pmatrix}, \begin{pmatrix}1\\1\\4\end{pmatrix}, \begin{pmatrix}-4\\1\\0\end{pmatrix}$$

$$\left\{\begin{pmatrix}-1\\6\\10\end{pmatrix}, \begin{pmatrix}0\\1\\2\end{pmatrix}\right\}$$ sono due vett lin ind. quindi posso generare uno spazio di dim 2

(27) $A\underline{x} = \underline{b}$ più incognite che equazioni

se $rg(A)$ è max ho soluzione

(31) $\begin{cases} x = 2 \\ y = 1+t \\ z = -1-t \end{cases}$

$$\begin{pmatrix}2\\1+t\\-1-t\end{pmatrix} = K\begin{pmatrix}1+t\\2\\-1-t\end{pmatrix}$$ sono paralleli

$2 = K + Kt$ sì per $K = 1$

$2 = 1+t$

$t = t$

(33) $A\underline{x}=b$ 3 eq. , infinite sol.

4 incognite $rg(A)=rg(A|\underline{b})<4$

$$\left(\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & b_1 \\ 0 & 0 & x_3 & x_4 & b_2 \\ 0 & 0 & 0 & x_4 & b_3 \end{array}\right)$$

Anche se A ha rg max ho infinite (∞^1) soluzioni

$rg(A)=3$ $n=4$ $sol=\infty^{n-rg(A)}$

(36)

A $n \times m$ $A\underline{x}=\underline{b}+\lambda\underline{c}$

b e c lin dip.

$\forall\lambda$ il sistema non ha soluzione