I

$$A=\begin{pmatrix}1\\0\\1\end{pmatrix} \quad B=\begin{pmatrix}1\\1\\1\end{pmatrix} \quad C\begin{pmatrix}2\\1\\2\end{pmatrix}$$

$$x\begin{pmatrix}1\\0\\1\end{pmatrix}+y\begin{pmatrix}2\\1\\2\end{pmatrix}+z\begin{pmatrix}2\\1\\2\end{pmatrix}=0$$

$$\begin{cases} x+y+2z=0 \\ y+z=0 \\ x+y+2z=0 \end{cases} \quad \begin{cases} x-z+2z=0 \\ y=-z \end{cases} \quad \begin{cases} x=-z \\ y=-z \end{cases}$$

$$S=\left\{ (x\,y\,z)\in\mathbb{R}^3 \;\middle|\; x=-z \quad y=-z \right\} = \left\{ \begin{pmatrix}-z\\-z\\z\end{pmatrix} \;\middle|\; z\in\mathbb{R} \right\} = \left\langle \begin{pmatrix}-1\\-1\\1\end{pmatrix} \right\rangle$$

linearmente dipendenti

II

$\{A\} \neq \{\underline{0}\}$ L.I.

$\{A,B\}$ $\exists\, \lambda\in\mathbb{R}$ t.c. $A=\lambda B$ $\qquad \begin{pmatrix}1\\0\\1\end{pmatrix}=\lambda\begin{pmatrix}1\\1\\1\end{pmatrix} \Rightarrow$ L.I.

$\{A,B,C\}$ $\exists\, \lambda,\mu\in\mathbb{R}$ t.c. $C=\lambda A+\mu B$

$$\begin{cases} \lambda+\mu=2 \\ \mu=1 \\ \cancel{\lambda+\mu=2} \end{cases} \Rightarrow \begin{matrix}\lambda=1\\ \mu=1\end{matrix}$$

per essere L.I. dovrei avere 0

completare $\{A, B, C\}$ a una base di $\mathbb{R}^3$

base canonica $= \left\{ \underset{e_1}{\begin{pmatrix}1\\0\\0\end{pmatrix}}, \underset{e_2}{\begin{pmatrix}0\\1\\0\end{pmatrix}}, \underset{e_3}{\begin{pmatrix}0\\0\\1\end{pmatrix}} \right\}$

$e_1 = \lambda A + \mu B \quad \begin{cases} \lambda + \cancel{\mu} = 1 \\ \mu = 0 \\ \lambda + \cancel{\mu} \; 0 \end{cases}$ falso sono L.I.

$\left\{ \begin{pmatrix}1\\0\\0\end{pmatrix}, A, B \right\}$ base di $\mathbb{R}^3$

$U_2 = \{ (xyz) \in \mathbb{R}^3 \mid y = z = 0 \} = \left\langle \begin{pmatrix}1\\0\\0\end{pmatrix} \right\rangle \neq \underline{0}$ L.I. $\left\{ \begin{pmatrix}1\\0\\0\end{pmatrix} \right\}$ base di U_2

$\dim(U_2) = 1$

$U_3 = \{ \text{'' ''} \mid -2x + y - z = 0 \} = \left\langle \begin{pmatrix}1\\0\\-2\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix} \right\rangle$ $\left\{ \begin{pmatrix}1\\0\\-2\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix} \right\}$ base di U_3

$\dim(U_3) = 2$

$U_2 \cap U_3 = \{ (xyz) \in \mathbb{R}^3 \mid y = z = 0, \; 2x + y - z = 0 \} = \{ \underline{0} \}$

$\begin{cases} y = 0 \\ z = 0 \\ -2 + y - z = 0 \end{cases} \quad x = 0$

$\dim(U_2 \cap U_3) = 0$

$U_2 + U_3 = \left\langle \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}1\\0\\-2\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix} \right\rangle$ è una base di $U_2 + U_3$

$\dim(U_2 + 3) = 3$

$\begin{pmatrix}1\\0\\0\end{pmatrix} = \lambda \begin{pmatrix}1\\0\\-2\end{pmatrix} + \mu \begin{pmatrix}0\\1\\1\end{pmatrix} \quad \begin{cases} \lambda + \quad = 1 \\ 0 + \mu \; = 0 \\ 2\lambda + \cancel{\mu} = 0 \end{cases} \quad \begin{matrix} \lambda = 1 \\ \\ \lambda = 0 \end{matrix}$

falso, sono L.I.

$\dim(U_2 + U_3) = \dim(U_2) + \dim(U_3) - \dim(U_2 \cap U_3)$

$3 = 1 + 2 - 0$

vero

soddisfo formula Grassman

$U_4 = \{ (x, y, z) \in \mathbb{R}^3 \mid 3x - z = \}$ U_2 e U_4 provare

$U_g = \{ a_3x^3 + a_2x^2 + a_1x + a_0 \mid a_1 + a_2 = 0 \}$

$a_3x^3 - a_1x^2 + a_1x + a_0 \mid a_1, a_0, a_3 \in \mathbb{R}^3 \}$

$\langle x^3, -x^2+x, 1 \rangle$

$\mathbb{R}[x]_3$ dim=4 $\{1, x, x^2, x^3\}$

pol. max grado 3

$\{b_1, b_2, b_3, b_4\}$

BOH

Sistemi lineari

due sistemi sono equivalenti se hanno il medesimo insieme di soluzioni

3 op. che trasformano il sistema in un sist. equivalente

- scambio di righe
- moltiplicazione riga per $x \neq 0$
- somma due righe

Ridurre la scala

$$\begin{pmatrix} 0 & 1 & 1 & -1 & 0 \\ 1 & -1 & 2 & 1 & 1 \\ 1 & 0 & 3 & 0 & 1 \end{pmatrix} \xrightarrow{r_1 \leftarrow r_3} \begin{pmatrix} 1 & 0 & 3 & 0 & 1 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & 1 & 1 & -1 & 0 \end{pmatrix} \xrightarrow{r_2 \leftrightarrow r_2 - r_1} \begin{pmatrix} 1 & 0 & 3 & 0 & 1 \\ 0 & -1 & -1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 \end{pmatrix} \rightarrow$$

$$\xrightarrow{r_3 \leftrightarrow r_3 + r_2} \begin{pmatrix} 1 & 0 & 3 & 0 & 1 \\ 0 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

rango = 2

$$\begin{pmatrix} 1 & -1 & 0 & -1 \\ 1 & 0 & 1 & -1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 2 & -1 \end{pmatrix} \xrightarrow[r_4 \leftrightarrow r_4 - r_1]{r_2 \leftrightarrow r_2 - r_1} \begin{pmatrix} 1 & -1 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 2 & 0 \end{pmatrix} \xrightarrow[r_4 \leftrightarrow r_4 - 2r_2]{r_3 \leftrightarrow r_3 - r_2}$$

$$\rightarrow \begin{pmatrix} 1 & -1 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

rango = 2

$$\begin{cases} x_1 - x_2 \quad - x_4 = -2 \\ x_1 + x_3 - x_4 = -1 \\ x_2 + x_3 - 1 \\ x_1 + x_2 + 2x_3 - x_4 = 0 \end{cases}$$

$$\left(\begin{array}{cccc|c} \multicolumn{4}{c|}{A} & \underline{b} \\ 1 & -1 & 0 & -1 & -2 \\ 1 & 0 & 1 & -1 & -1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 2 & -1 & 0 \end{array}\right) \xrightarrow[r_4 \to r_4 - r_1]{r_2 \to r_2 - r_1} \left(\begin{array}{cccc|c} 1 & -1 & 0 & -1 & -2 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 2 & 2 & 0 & 2 \end{array}\right) \xrightarrow[r_4 \to r_4 - 2r_2]{r_3 \to r_3 - r_2} \overbrace{\left(\begin{array}{cccc|c} 1 & -1 & 0 & -1 & -2 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)}^{A|\underline{b}}$$

$rg(A) = 2$ per teorema di Rouche-capelli ha soluzione

$rg(A|\underline{b}) = 2$ perché sono uguali

incognite

$n-r$ → rango

ho ∞^{n-r} soluzioni → ho ∞^2 soluzioni (dipendono da due parametri

$$\begin{cases} x_1 - x_2 \quad - x_4 = -2 \\ x_2 + x_3 = 1 \end{cases} \to \quad x_2 = 1 - x_3$$

$x_1 - 1 + x_3 - x_4 = -2 \quad x_1 + x_3 - x_4 = -1 \qquad x_1 = -1 - x_3 + x_4$

$$S = \left\{ \begin{pmatrix} -1 - x_3 + x_4 \\ 1 - x_3 \\ x_3 \\ x_4 \end{pmatrix} \middle| \; x_3, x_4 \in \mathbb{R} \right\}$$

$$\begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \underbrace{\begin{pmatrix} -x_3 \\ -x_3 \\ x_3 \\ 0 \end{pmatrix} + \begin{pmatrix} x_4 \\ 0 \\ 0 \\ x_4 \end{pmatrix}}_{x_3 \begin{pmatrix} -1 \\ -1 \\ 1 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}} = \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \left\langle \begin{pmatrix} -1 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\rangle$$

$$\begin{cases} x_1 - x_2 - x_4 = 0 \\ x_1 + x_3 - x_4 = 0 \\ x_2 + x_3 = 0 \\ x_1 + x_2 + 2x_3 - x_4 = 1 \end{cases}$$