

Quadrato $L \Rightarrow \text{Area} = A = L^2 \Rightarrow A \sim \exp\left(\frac{1}{2}\right)$

① $E[A]$ e $P(A \leq 4)$

$$E[A] = \frac{1}{\lambda} = \frac{1}{\left(\frac{1}{2}\right)} = 4$$

$$P(A \leq 4) = F_A(4) = 1 - e^{-\frac{1}{2} \cdot 4} = 1 - e^{-1} = 1 - \frac{1}{e}$$

② L lato del quadrato, sapendo che la v.a. L ha $E[L] = \sqrt{\pi}$ quanto vale la sua varianza

$$\begin{aligned} \text{VAR}(L) &= E[L^2] - E[L]^2 \\ &\quad \hookrightarrow \text{perché } L^2 = A \\ &= E[A] - E[L]^2 = 4 - (\sqrt{\pi})^2 = 4 - \pi \end{aligned}$$

③ determinare $F_L(l)$ e $f_L(l)$

$$F_A(a) = \begin{cases} 0 & \text{se } a \leq 0 \\ 1 - e^{-\frac{1}{2}a} & \text{se } a > 0 \end{cases}$$

$$F_L(l) = P(L \leq l) = P(\sqrt{A} \leq l) = P(A \leq l^2) = 1 - e^{-\frac{l^2}{2}}$$

$$\text{quindi } F_L(l) = \begin{cases} 0 & \text{se } l < 0 \\ 1 - e^{-\frac{1}{2}l^2} & \text{se } l \geq 0 \end{cases}$$

$F_L(l)$ è assolutamente continua essendo continua e derivabile a tratti allora

$$f_L(l) = F_L'(l) = \begin{cases} 0 & \text{se } l < 0 \\ -\left(-\frac{1}{2} \cdot 2l\right) \cdot e^{-\frac{1}{2}l^2} = \frac{1}{2} l e^{-\frac{1}{2}l^2} & \text{se } l \geq 0 \end{cases}$$