$$\int_1^{e^3} \frac{\log x}{x}\, dx \qquad \int \underbrace{\frac{1}{x}}_{f'} \overset{g}{\left(\underbrace{\log x}_{f}\right)^{1}} dx = \frac{(\log x)^2}{2} + c$$

$$\int_1^{e^3} \log x \, \frac{1}{x}\, dx = \frac{\log^2 x}{2}\Big|_1^{e^3} = \frac{\log^2 e^3}{2} - \frac{\log^2 1}{2} = \frac{9}{2} - 0 = \frac{9}{2}$$

$$\int_0^1 \frac{\operatorname{arctg}^3 x}{1+x^2}\, dx = \int \operatorname{arctg}^3 x \cdot \frac{1}{1+x^2} = \frac{(\operatorname{arctg} x)^4}{4} + c$$

$$= \frac{\operatorname{arctg}^4 1}{4} - \frac{\operatorname{arctg}^4 0}{4} = \frac{\left(\frac{\pi}{4}\right)^4}{4} = \frac{\pi^4}{4^5}$$

$$\int_{-2}^{-1} \frac{x-1}{x^2-2x}\,dx \qquad \frac{1}{2}\int \frac{2x-2}{x^2-2x}\,dx = \frac{1}{2}\left(\ln|x^2-2x|\right)+c$$

$$\frac{1}{2}\left(\ln|3|\right) - \frac{1}{2}\left(\ln|8|\right) = \frac{1}{2}\left(\ln 3 - \ln 8\right) = \frac{1}{2}\ln\frac{3}{8}$$

$f(x) = x\sin x$

trova $F(x)$ t.c. $F\left(\frac{\pi}{2}\right)=0$

$$\int x\sin x\,dx \qquad -x\cos x + \int \cos x\,dx = -x\cos x + \sin x + c$$

$$-\frac{\pi}{2}\cdot\cos\left(\frac{\pi}{2}\right) + \sin\left(\frac{\pi}{2}\right) + c = 0$$

$$-\frac{\pi}{2}\cdot 0 + 1 + c = 0 \qquad c=-1 \qquad F(x) = -x\cos x + \sin x - 1$$

$f(x) = x^2 \cdot e^{2x}$

① $F(x)$ t.c. $F(0) = \frac{1}{4}$

② $\lim_{n \to +\infty} \frac{F(n)}{n^5}$

$\int x^2 \cdot e^{2x} \, dx =$ $\qquad f(x) = x^2 \quad g'(x) = e^{2x}$

$= \frac{e^{2x}}{2} \cdot x^2 - \int x e^{2x} \, dx \qquad f'(x) = 2x \quad g(x) = \frac{e^{2x}}{2}$

$f(x) = x \quad f'(x) = 1$

$\frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{1}{2} \int e^{2x} \, dx \qquad g'(x) = e^{2x} \quad g(x) = \frac{e^{2x}}{2}$

$\frac{1}{2} x^2 e^{2x} - \frac{1}{2} x e^{2x} + \frac{1}{4} e^{2x} + c =$

$= \frac{1}{2} e^{2x} \left(x^2 - x + \frac{1}{2} \right) + c$

$F(x) = \frac{1}{2} e^{2x} \left(x^2 - x + \frac{1}{2} \right)$

$F(0) = \frac{1}{4} \qquad \frac{1}{2} e^{2 \cdot 0} \left(+ \frac{1}{2} \right) + c = \frac{1}{4} + c = \frac{1}{4} \qquad c = 0$

$$\lim_{n\to+\infty} \frac{F(n)}{n^5} = \frac{\frac{1}{2}n^2e^{2n}}{n^5} = \frac{e^{2n}}{2n^3} \quad \lim_{n\to+\infty} \frac{e^{2n}}{2n^3} = +\infty$$

Taylor $\quad x_0 = 0 \quad$ ordine = 2

$$f(x) = 1 - \cos(4x)$$

$$\lim_{n \to +\infty} n^4 f\left(\frac{1}{n^2}\right)$$

$$1 - \cos(4x) = 1 - \Big(\overset{0}{f(0)} + \overset{0}{f'(x_0)(x - x_0)} + \overset{16/2\ x}{\frac{f''(x_0)}{2}}(x - x_0) + o(x^2)\Big)$$

$$P(x) = 8x^2 + o(x^2)$$

due modi

$f(x) = 1 - \cos(4x) \qquad f(0) = 0$

$f'(x) = 4 \operatorname{sen} 4x \qquad f'(0) = 0$

$f''(x) = 16 \cos(4x) \qquad f''(0) = 16$

$$\cos x = 1 - \frac{x^2}{2} + o(x^2)$$ ← metodo di merda

$$\cos(4x) = 1 - \frac{\overset{8}{\cancel{16}}x^2}{\cancel{2}} + o(x^2)$$

$$1 - \cos(4x) = 8x^2 + o(x^2)$$

$$\lim_{n\to+\infty} n^4 f\left(\frac{1}{n^2}\right) \qquad \lim_{n\to+\infty} n^4\left(1-\cos\left(\frac{4}{n^2}\right)\right)$$

$$\lim_{n\to+\infty} n^4 \cdot \left(8\cdot\frac{1}{n^4} + o\left(\frac{1}{n^4}\right)\right) = \lim_{n\to+\infty}\left(8+o(1)\right) = 8$$

$$f(x) = \begin{cases} \dfrac{e^x - e}{x-1} & x \neq 1 \\ \alpha & x = 1 \end{cases} \qquad \alpha \in \mathbb{R}$$

$$\lim_{x\to 1^-} \frac{e^x - e}{x-1} = \lim_{x\to 1^+} \frac{e^x - e}{x-1} = \alpha$$

De l' Hôpital

$$\lim_{x\to 1} \frac{e^x - e}{x-1} = \lim_{x\to 1} \frac{e^x}{1} = e$$

$$\lim_{x\to 1} \frac{e^x - e}{x-1} = \qquad \frac{e^x - 1}{x} \quad x \to 0$$

$$= \lim_{x\to 1} \frac{e\left(e^{x-1} - 1\right)}{x-1} = e \qquad \alpha = e$$

$\to 1$

$$f(x) = \begin{cases} \dfrac{e^x - e}{x-1} & x \neq 1 \\ e & x = 1 \end{cases}$$

$$f'(x) = \frac{e^x(x-1) - (e^x - e)}{(x-1)^2} \quad \boxed{x \neq 1}$$

$$\frac{xe^x - e^x - e^x + e}{(x-1)^2} = \frac{xe^x - 2e^x + e}{(x-1)^2}$$

$$\lim_{x\to 1} \frac{xe^x - 2e^x + e}{(x-1)^2} = \lim_{x\to 1} \frac{e^x + xe^x - 2e^x}{2(x-1)} =$$

$$= \lim_{x\to 1} \frac{e^x + e^x + xe^x - 2e^x}{2} = \frac{e}{2}$$

$f(x) = e^{x^2-7x+4}$

a) $(2,5)$ c) $(-1,10)$

b) $(4,8)$ d) $(-2,4)$

$f'(x) = e^{x^2-7x+4} \cdot (2x-7)$

$f'(x_0) \geq 0 \iff 2x-7>0$

$x > \frac{7}{2}$