$f(x) = \sqrt{\dfrac{x^3-1}{x}}$ C.E. $\dfrac{x^3-1}{x} \geq 0$ $N \geq 0$ $x^3 \geq 1$ $\boxed{x \geq 1}$

$D > 0$ $x > 0$

		0		1	
N	$-$		$-$	$\bullet$	$+$
D	$-$		$+$		$+$
	$+$		$-$		$+$

1) $D: (-\infty, 0) \cup [1, +\infty)$

2) $\lim\limits_{x \to -\infty} \sqrt{\dfrac{x^3-1}{x}} = +\infty$ $\lim\limits_{x \to +\infty} \sqrt{\dfrac{x^3-1}{x}} = +\infty$

$\lim\limits_{x \to 0^-} \sqrt{\dfrac{x^3-1}{x}} = \sqrt{\dfrac{0-1}{0^-}} = +\infty$

$f(1) = \sqrt{\dfrac{1-1}{1}} = 0$

↑ perchè qua é definita, coincide con il limite

3) A.V. → $x=0$ sinistro

A Or. → null

A. Ob $m = \lim\limits_{x \to \infty} \dfrac{f(x)}{x} = \lim\limits_{x \to \infty} \dfrac{1}{x}\sqrt{\dfrac{x^3-1}{x}}$ $\dfrac{1}{x}\sqrt{\dfrac{x^3-1}{x}} \sim \dfrac{1}{x}\sqrt{\dfrac{x^3}{x}} = \dfrac{1}{x}\sqrt{x^2} = \dfrac{1}{x}|x|$

$\lim\limits_{x \to -\infty} \dfrac{-x}{x} = -1$ $\lim\limits_{x \to +\infty} \dfrac{x}{x} = 1$

$$g = \lim_{x \to -\infty} f(x) - mx$$

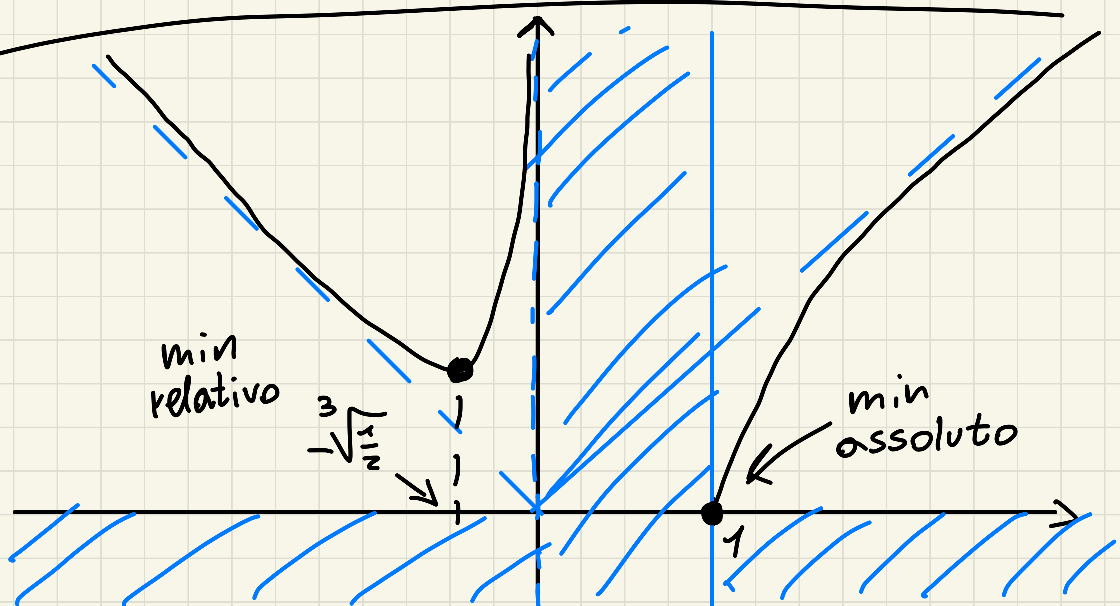
$$g = \lim_{x \to +\infty} f(x) - mx \quad \text{é uguale}$$

$$\lim_{x \to -\infty} \left(\sqrt{\frac{x^3-1}{x}} + x \right) \cdot \frac{\sqrt{\frac{x^3-1}{x}} - x}{\sqrt{\frac{x^3-1}{x}} - x}$$

$$\lim_{x \to -\infty} \frac{\frac{x^3-1}{x} - x^2}{\sqrt{\frac{x^3-1}{x}} - x} = \lim_{x \to -\infty} \frac{-\frac{1}{x}}{\sqrt{\frac{x^3-1}{x}} - x} = \frac{0}{+\infty} = 0$$

sx $y = -x$

dx $y = x$

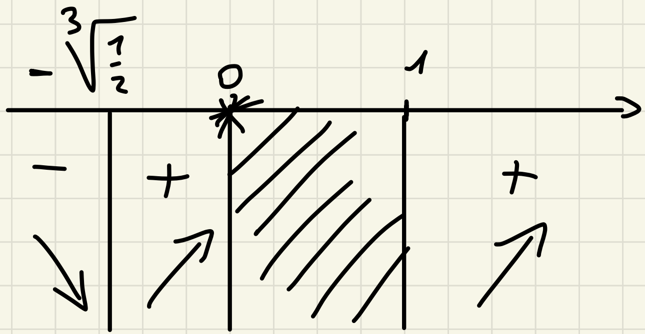


9) $y = \sqrt{\frac{x^3-1}{x}}$

$$y' = \frac{1}{2\sqrt{\frac{x^3-1}{x}}} \cdot \left(\frac{3x^2(x) - (x^3-1)}{x^2} \right) = \underbrace{\frac{1}{2}}_{\text{pos}} \underbrace{\sqrt{\frac{x^3-1}{x}}}_{\text{pos}} \cdot \frac{2x^3+1}{\underbrace{x^2}_{\text{pos}}}$$

unica cosa da studiare

$y' \geq 0 \qquad 2x^3+1 \geq 0 \qquad x \geq -\sqrt[3]{\frac{1}{2}}$



$x = -\sqrt[3]{\frac{1}{2}}$ p.to di minimo relativo

5) $f(x) = K$

$$\begin{cases} y = f(x) \\ y = K \end{cases}$$

$K < 0 \Rightarrow$ imp

$0 \leq K < f\left(-\sqrt[3]{\frac{1}{2}}\right) \Rightarrow 1$ soluzione

$K = f\left(-\sqrt[3]{\frac{1}{2}}\right) \Rightarrow 3$ soluzioni di cui 2 coincidenti

$K > f\left(-\sqrt[3]{\frac{1}{2}}\right) \Rightarrow 3$ sol. distinte

Integrali

quasi immediati

- $\int (2x-4)^3 \, dx$

$g(f(x)) \longrightarrow g'(f(x)) \cdot f'(x)$

mi manca derivata

$$\frac{1}{2} \int (2x-4)^3 \cdot 2 \, dx = \frac{1}{2} \frac{(2x-4)^4}{4} = \boxed{\frac{(2x-4)^4}{8}} + C$$

- $\int x^2 \cos(2x^3-7) \, dx$

$$\frac{1}{6} \int 6x^2 \cdot \cos(2x^3-7) \, dx = \boxed{\frac{1}{6} \cdot \sin(2x^3-7)} + C$$

$$\bullet \int \frac{4}{5x+8}\,dx$$

$$\frac{4}{5}\int \frac{1 \cdot 5}{5x+8}\,dx = \frac{4}{5}\ln|5x+8| + c$$

$$\bullet \int \frac{x+1}{x^2+2x}\,dx = \frac{1}{2}\int \frac{2x+2}{x^2+2x}\,dx = \frac{1}{2}\ln|x^2+2x| + c$$

$$\bullet \int \frac{e^{1/x}}{x^2}\,dx = -\int -\frac{1}{x^2} \cdot e^{1/x}\,dx = -e^{1/x} + c$$

$$\bullet \int \frac{1}{1+4x^2}\,dx = \qquad \int \frac{1}{1+x^2}\,dx = \operatorname{arctg} x + c$$

$\hookrightarrow 8x$

$$\frac{1}{1+(\ )^2}$$

$$= \int \frac{1}{1+(2x)^2}\,dx =$$

$$= \frac{1}{2}\int \frac{2}{1+(2x)^2}\,dx = \frac{1}{2}\operatorname{arctg}(2x) + c$$

- $$\int \frac{8}{(7x-4)^3}\,dx = \int 8\,(7x-4)^{-3}\,dx = \frac{8}{7}\int 7(7x-4)^{-3}\,dx =$$

$$= \frac{8}{7}\cdot\frac{(7x-4)^{-2}}{-2} + c = \frac{-\cancel{8}^{\,4}}{7\cdot\cancel{14}(7x-4)^2} + c$$

$\hookrightarrow 3(7x-4)^2 \cdot 7$ escludendo log e arctg

Integrali per parti

$$\int x e^x\,dx$$

$f(x) = x$ $\quad f'(x) = 1$ $\quad x \xrightarrow{d} 1$, $\quad x \xrightarrow{i} \frac{x^2}{2}$

$g'(x) = e^x$ $\quad \int g(x)\,dx = e^x$ $\quad e^x \xrightarrow{d} e^x$, $\quad e^x \xrightarrow{i} e^x$

$$\int f(x)\,g'(x)\,dx = f(x)g(x) - \int f'(x)\,g(x)\,dx$$

$$xe^x - \int e^x\,dx = xe^x - e^x + c$$

- $\int x \log x \, dx$

$x \xrightarrow{d} 1 \qquad x \xrightarrow{\int} \frac{x^2}{2}$

$\log x \xrightarrow{d} \frac{1}{x} \qquad \log x \xrightarrow{\int} ???$

$f(x) = \log x \qquad f'(x) = \frac{1}{x}$

$g'(x) = x \qquad g(x) = \frac{x^2}{2}$

$$= \frac{x^2}{2} \log x - \int \frac{1}{\cancel{x}} \frac{x^{\cancel{2}}}{2} dx = \frac{x^2}{2} \log x - \frac{1}{2} \int x \, dx =$$

$$\frac{x^2}{2} \log x - \frac{x^2}{4} + c$$

$\int 1 \cdot \log x$

$f(x) = \log x \qquad f'(x) = \frac{1}{x}$

$g'(x) = 1 \qquad g(x) = x$

$$= x \log x - \int \frac{1}{x} \cdot x \, dx =$$

$$= x \log x - \int 1 \, dx = x \log x - x + c$$

$\int \operatorname{arctg} x \, dx$ A casa

Sostituzione

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx \qquad \sqrt{x} = t \qquad x = t^2$$

$$1 dx = 2t dt$$

$$dx = 2t dt$$

$$\int \frac{e^t}{\cancel{t}} \cdot 2\cancel{t} dt =$$

$$= 2\int e^t dt = 2e^t + c =$$

$$= 2e^{\sqrt{x}} + c$$