1) what is the number of code words with weight 3 in Ham(r,2)?

$\binom{r}{3}$

2) Describe parameters of Ham (r,2) and prove them

$[2^r-1, 2^r-r-1, 3]$

by definition the length of the code words is 2^r-1 since it's the number of columns of the parity check matrix.

3 narrow sense BCH code over $\mathbb{F}_3$ 1 error correcting

α solution to primitive polynomial x^3+x+1, $q=2$ $m=3$ t 1

so have to take α, α^2 and the lcm of their minimal polynomials

$g(x) = x^3+x+1$

Is it perfect?

$d=3$ because gen poly. of 3 elements

$\sum_{0}^{1} \binom{n}{i}(q-1)^i$ $\qquad n = 2^3-1 = 7$

$1 + \frac{7!}{1!(7-1)!}(2-1)^1 = 1+7 = 8 = 2^3 \neq 2^{n-m} = 2^4$ not perfect codes

decode $r = [1,0,1,0,1,0,1]$

$\alpha^6 = (\alpha^3)^2 = (\alpha+1)^2 = \alpha^2+1$

$\alpha^4 = \alpha^3 \cdot \alpha = (\alpha+1)\alpha = \alpha^2+\alpha$

$S_1 = r(\alpha) = \cancel{1} + \cancel{\alpha^2} + \underset{\overset{\|}{\cancel{\alpha^2}+\alpha}}{\alpha^4} + \underset{\overset{\|}{\alpha^2+\cancel{1}}}{\alpha^6} = \alpha^2+\alpha$

$S_2 = r(\alpha^2) = \cancel{1} + \underset{\overset{\|}{\cancel{\alpha^2}+\cancel{\alpha}}}{\alpha^4} + \underset{\overset{\|}{\alpha}}{\alpha^8} + \underset{\overset{\|}{\underset{\overset{\|}{\cancel{\alpha^2}+\alpha+\cancel{1}}}{\alpha^5}}}{\alpha^{12}} = \alpha$

$\alpha^8 = (\alpha^2+\alpha)(\alpha^2+\alpha) = \alpha^4+\alpha^2 = \cancel{\alpha^2}+\alpha+\cancel{\alpha^2}$

$\alpha^5 = \alpha^3 \cdot \alpha^2 = (\alpha+1)(\alpha)^2 = \alpha^3+\alpha^2 = \alpha^3+\alpha^2 = \alpha+1+\alpha^2$

$X_1 Y_1 = \alpha^2+\alpha$

$X_1^2 Y_1 = \alpha$

$X_1 = \frac{X_1^2 Y_1}{X_1 Y_1} = = \frac{\alpha}{\alpha^2+\alpha} = \frac{\alpha}{\alpha^4} = \alpha^{-3} = \alpha^4$ codeword $\rightarrow (1,0,1,0,0,0,1)$

$1+x^2+x^6$

to decode divide by generator

$$\begin{array}{l|l} x^6+x^2+1 & x^3+x+1 \\ \hline x^6+x^4+x^3 & x^3+x+1 \\ \hline \cancel{x^4}+x^3+\cancel{x^2}+1 & \\ \cancel{x^4}+\cancel{x^2}+x & \\ \hline x^3+x+1 & \\ x^3+x+1 & \\ \hline 0 & \end{array}$$

$\rightarrow$ original message $(1,1,0,1)$

ⓐ

$$G := \mathrm{Id}_{12} B$$

$$\begin{array}{c}
\begin{array}{cccccccccccccccccccccccc} & & & & & & & & & & & & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \end{array}\\
\left(\begin{array}{cccccccccccc|cccccccccccc}
1&0&1&0&1&1&1&0&0&0&1&0&1&0&0&1&0&0&0&0&0&0&0&1\\
1&0&0&0&0&0&0&0&0&0&0&0&1&1&0&1&1&1&0&0&0&1&0&1\\
0&1&0&0&0&0&0&0&0&0&0&0&1&0&1&1&1&0&0&0&1&0&1&1\\
0&0&1&0&0&0&0&0&0&0&0&0&0&1&1&1&0&0&0&1&0&1&1&1\\
0&0&0&1&0&0&0&0&0&0&0&0&1&1&1&0&0&0&1&0&1&1&0&1\\
0&0&0&0&1&0&0&0&0&0&0&0&1&1&0&0&0&1&0&1&1&0&1&1\\
0&0&0&0&0&1&0&0&0&0&0&0&1&0&0&0&1&0&1&1&0&1&1&1\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&1&0&1&1&0&1&1&1&1\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&1&0&1&1&0&1&1&1&0&1\\
0&0&0&0&0&0&0&0&1&0&0&0&0&1&0&1&1&0&1&1&1&0&0&1\\
0&0&0&0&0&0&0&0&0&1&0&0&1&0&1&1&0&1&1&1&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&1&0&0&1&1&0&1&1&1&0&0&0&1&1\\
0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1&1&1&1&1&1&1&1&0
\end{array}\right)
\end{array}
\begin{array}{c}\text{recieved}\\ \begin{pmatrix}1\\0\\1\\0\\1\\1\\1\\0\\0\\0\\1\\0\\1\\0\\0\\1\\0\\0\\0\\0\\0\\0\\0\\0\\1\end{pmatrix}\end{array}
=
\begin{array}{c|c}
1 & 0\\ 2 & 1\\ 3 & 1\\ 4 & 0\\ 5 & 1\\ 6 & 1\\ 7 & 1\\ 8 & 1\\ 9 & 0\\ 10 & 1\\ 11 & 0\\ 12 & 0
\end{array}
\leftarrow \text{syndrome} = s$$

8 times 1, have to add 1

$$t = (0,0,0,0,1,0,1,1,0,0,1,1)$$

$$\begin{array}{cccccccccccc}
 & & & & & & & & & & & \\
1&0&0&0&1&0&1&1&0&1&1&1\\
\hline
1&0&0&0&0&0&0&0&0&1&0&0
\end{array}$$

ⓑ $w(s) = 7$

ⓒ

$i = 6$

$e : [00000100000001000000000100]$

RS-code over $\mathbb{F}_8 = \{0, 1, \alpha, \alpha^2, \dots \alpha^6\}$

α zero of the minimal polynomial $\pi(x) = x^3 + x + 1$ work mod 2

create code with $t=2$

generator poly of degree $2t=4$

$g(x) = (x+1)(x+\alpha)(x+\alpha^2)(x+\alpha^3) = (x^2+\alpha^3 x+\alpha)(x^2+\alpha^5 x+\alpha^5) = x^4+\alpha^2x^3+\alpha^5x^2+\alpha^5x+\alpha^6$

codewords length 7, gen poly. degree 4, dimension is 3

$[7,3,5]$ code

we recieve $[\alpha, \alpha^6, \alpha^2, \alpha^4, \alpha^2, \alpha^6, \alpha]$

$\Downarrow$

$r(x) = \alpha x^6 + \alpha^6 x^5 + \alpha^2 x^4 + \alpha^4 x^3 + \alpha^2 x^2 + \alpha^6 x + \alpha$

- calculate 2t syndromes and determine syndrome polynomial

$S_0 = r(1) = \alpha + \alpha^6 + \alpha^2 + \alpha^4 + \alpha^2 + \alpha^6 + \alpha = \alpha^{21} = \alpha^4$

$S_1 = r(\alpha) = \alpha^7 + \alpha^{11} + \alpha^6 + \alpha^7 + \alpha^4 + \alpha^7 + \alpha = \alpha^{21+11+6+4+1} = \alpha^{43} = \alpha^4$

$S_2 = r(\alpha^2) = \alpha^{13} + \alpha^{16} + \alpha^{10} + \alpha^{10} + \alpha^6 + \alpha^8 + \alpha = \alpha^2$

$S_3 = r(\alpha^3) = \alpha^{19} + \alpha^{21} + \alpha^{14} + \alpha^{13} + \alpha^8 + \alpha^8 + \alpha = \alpha^2 + \alpha = \alpha^4$

syndrome poly.

$S(x) = \alpha^4 x^3 + \alpha^2 x^2 + \alpha^4 x + \alpha^4$

apply Euclid's algorithm to x^4 and $s(x)$