1) var 1)

$f'(x) = (2x \cdot e^{1+x}) + (x^2 \cdot e^{1+x})$

A

$f'(x_0) = f'(-1) = -2 \cdot 1 + 1 = -1 = m$

$f(x_0) = f(-1) = 1 \cdot 1 = 1$

$g(x) = f(x_0) + f'(x_0)\ (x - x_0) = 1 - 1(x+1) = 1 - x - 1 = -x$

var 2) $f(x) = -x^2 e^{1-x} \quad x_0 = 1$

$f'(x) = -2x \cdot e^{1-x} + (-x^2 e^{1-x} \cdot (-1)) = -2x e^{1-x} + x^2 e^{1-x}$

$f'(x_0) = -2 e^0 + e^0 = -1$

$f(x_0) = -1 e^0 = -1$

$g(x) = f(x_0) + f'(x_0)(x - x_0) = -1 + (-1)(x-1) = -1 - x + 1 = -x$

2) var 1)

$f(x) = x \ln x \qquad f(x_0) = 0$

$f'(x) = \ln x + 1 \qquad f'(x_0) = 1$

$f''(x, \quad \frac{1}{x} \qquad f''(x_0) = 1$

A

$P_2(x, 1) = 0 + 1(x-1) + \frac{1}{2!}(x-1)^2 =$

$= x - 1 + \frac{1}{2}(x^2 - 2x + 1) = x - 1 + \frac{x^2}{2} - x + \frac{1}{2} = \frac{x^2}{2} - \frac{1}{2}$

var 2)

$f(x) = (x+1)\ln(x+1) \qquad f(0) = 1 \ln 1 = 0$

$f'(x) = 1(\ln(x+1)) + \left(\frac{1}{\cancel{x+1}} + \cancel{x+1}\right) = \qquad f'(0) = 1$

$= \ln(x+1) + 1$

$f''(x) = \frac{1}{x+1}(x+1) = 1 \qquad f''(0) = 1$

$P_2(x, 0) = 0 + 1(x) + \frac{1}{2}x^2 = \frac{x^2}{2} + x$

3) var c) $f(x) = e^{2x} - 3e^{x} + 1$

$f'(x) = 2e^{2x} - 3e^{x}$

$f'(x) > 0$ $\quad 2e^{2x} - 3e^{x} > 0$ $\quad e^{x}(2e^{x} - 3)$ $\quad e^{x} > 0 \ \forall x \in \mathbb{R}$

$2e^{2x} > 3e^{x}$ $\quad e^{x} > \frac{3}{2}$

A

$\ln(2e^{2x}) > \ln(3e^{x})$ $\quad x > \ln \frac{3}{2}$

$\ln 2 + \ln e^{2x} > \ln 3 + \ln e^{x}$

$\ln 2 + 2x > \ln 3 + x$

$x > \ln 3 - \ln 2$ $\quad x > \ln \frac{3}{2}$

4) var 1) $f(x) = \sqrt{1+x^2}$ $\quad f'(1) = ?$

$$f'(x) = \frac{1}{2\sqrt{1+x^2}} \cdot 2x = \frac{x}{\sqrt{1+x^2}} \qquad f'(1) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

var 2)

$f(x) = \sqrt{1+x^2}$ $\quad f'(-1)$

SONO SBAGIATI

$$f'(x) = \frac{x}{\sqrt{1+x^2}} \qquad f'(-1) = \frac{-1}{\sqrt{2}}$$

5) $f(x) = x^3 - 3x + 14$

$f'(x) = 3x^2 - 3$

$f'(x) = 0 \qquad 3x^2 - 3 = 0$

$3(x^2 - 1) = 0$

$x = \pm 1$ possibili p ti estremanti

$f'(x) > 0 \qquad x < -1 \quad \vee \quad x > 1$

$x = -1$

A

$-1 \qquad +1$

↗ | ↘ | ↗

p.to max — p.to min

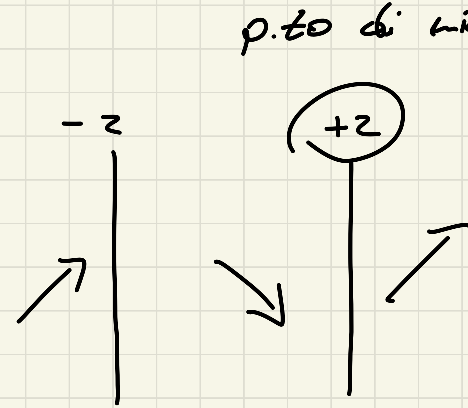
var 2) min locale di $f(x) = x^3 - 12x + 14$ A

$f'(x) = 3x^2 - 12$

potenziali p.ti estremanti

$f'(x) = 0 \quad 3(x^2 - 4) = 0 \quad x = \pm 2$

$f'(x) > 0 \quad x < -2 \lor x > 2$



6) var. 1) $f(x) = \sqrt[3]{x+1}$ $[-1, 7]$

BALZA

NO FATTO

esercizi

e) $f'(x) \underset{(<)}{>} 0 \quad \forall x \in I$

sempre pos

$f:(2,+\infty)\to\mathbb{R} \qquad f(x) = xe^{\frac{1}{x-2}}$

1) $f'(x) = e^{\frac{1}{x-2}} + x\left(e^{\frac{1}{x-2}}\cdot\left(\frac{-1}{(x-2)^2}\right)\right) = e^{\frac{1}{x-2}} - \frac{xe^{\frac{1}{x-2}}}{(x-2)^2}$

sempre pos.

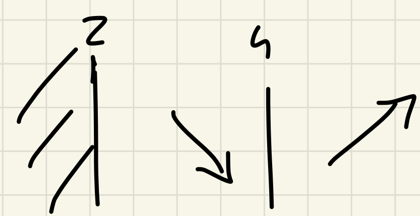
$f'(x_0)>0$

$e^{\frac{1}{x-2}} - \frac{xe^{\frac{1}{x-2}}}{(x-2)^2} > 0 \qquad e^{\frac{1}{x-2}}\left(1 - \frac{x}{(x-2)^2}\right) > 0$

$1 - \frac{x}{(x-2)^2} > 0 \qquad \frac{(x-2)^2 - x}{(x-2)^2} > 0 \qquad \frac{x^2-5x+4}{(x-2)^2} > 0$

$\Delta = 25-16 = 9 \qquad x_{1,2} = \frac{5\pm 3}{2} \begin{cases} 4 \\ -1 \end{cases}$

$x<-1 \qquad x>4$



$2<x<4$ monotona dec.

$x>4$ mon. cresc.

ii) $f\left(\frac{5}{2}\right) = \frac{5}{2} e^{\frac{1}{5/2 - 2}}$

$\frac{5}{2} e^{\frac{1}{\frac{1}{2}}} = \frac{5}{2} e^{2}$

$f(4) = 4 e^{\frac{1}{2}} = 4\sqrt{e}$

$f\left(\frac{5}{2}, 4\right) = \left[4\sqrt{e}, \frac{5}{2} e^{2}\right]$

iii) as. vert

$\lim_{x \to 2^+} x e^{\frac{1}{x-2}} \to 2^+ e^{\frac{1}{0^+}} \to +\infty$ $x=2$

as. or. $\lim_{x \to +\infty} x e^{\frac{1}{x-2}} = +\infty$ no

as. obl $m = \lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} e^{\frac{1}{x-2}} = 1$

$q = \lim_{x \to +\infty} f(x) - mx = \lim_{x \to +\infty} x e^{\frac{1}{x-2}} - x$

$$\lim_{x\to+\infty} xe^{\frac{1}{x-2}} - x = \lim_{x\to+\infty} x\left(e^{\frac{1}{x-2}} - 1\right) = \lim_{x\to+\infty} \frac{e^{\frac{1}{x-2}} - 1}{1/x} =$$

$$e^{\frac{1}{x-2}} - 1 \xrightarrow{f'} e^{\frac{1}{x-2}} \cdot \frac{-1}{(x-2)^2} \qquad = \lim_{x\to+\infty} \frac{\frac{+e^{\frac{1}{x-2}}}{(x-2)^2}}{+1/x^2} = \lim_{x\to+\infty} \frac{e^{\frac{1}{x-2}} \cdot x^2}{(x-2)^2} =$$

$$1/x \xrightarrow{f'} -x^{-2} = \frac{-1}{x^2}$$

$$= \lim_{x\to+\infty} \underbrace{e^{\underbrace{\frac{1}{x-2}}_{\to 0}}}_{\to 1} \cdot \underbrace{\lim_{x\to+\infty} \frac{x^2}{(x-2)^2}}_{\to 1} = 1 \cdot 1 = 1$$

$y = x+1$ as obl.

2) $f: I \to \mathbb{R}$

F é una primitiva di f se $F' = f$

due primitive differiscono per una costante perchè la derivata di una f.ne costante é 0

F' é idetica per ogni $F + c$ con c costante

$$\int \ln(1+x)\, dx$$

$$f(x) = \ln(1+x) \qquad f'(x) = \frac{1}{x+1}$$

$$g'(x) = 1 \qquad g(x) = x$$

$$\int 1 \cdot \ln(1+x)\, dx$$

$$= \frac{\ln(1+x)}{x+1} - \int \frac{x}{x+1}\, dx$$

$$f'(x) = x \qquad f(x) = \frac{x^2}{2}$$

$$g(x) = x+1 \qquad g'(x) = 1$$

$$\frac{\ln(1+x)}{x+1} \cdot \frac{x^2}{2} - \int \frac{x^2}{2} = \frac{\ln(1+x)}{x+1} \cdot \frac{x^2}{2} - \frac{1}{2}\left(\frac{x^3}{3}\right) + c$$