Limiti notevoli

$e = \lim_{n \to +\infty} \left(1 + \frac{1}{n}\right)^n$ $\quad \forall \{a_n\},\ a_n \to +\infty \quad e = \lim_{n \to +\infty} \left(1 + \frac{1}{a_n}\right)^{a_n}$

$\forall \{a_n\},\ a_n \to 0 \quad e = \lim_{n \to +\infty} (1 + a_n)^{\frac{1}{a_n}}$

$a_n \neq 0$

si deduce che $\lim_{x \to +\infty} \left(1 + \frac{1}{x}\right)^x = e$ $\quad \frac{1}{x} = y \quad x \to +\infty \quad y \to 0^+$

$$\lim_{y \to 0^\pm} (1 + y)^{\frac{1}{y}} = e$$

1) $\lim_{x \to 0} \frac{\ln(1+x)}{x} = 1 \left[\frac{0}{0}\right]$ F.I.

$$\lim_{x \to 0} (1+x)^{\frac{1}{x}} = e$$

$$\frac{\ln(1+x)}{x} = \frac{1}{x} \ln(1+x) \quad \lim_{x \to 0} \ln\Big(\underbrace{(1+x)^{\frac{1}{x}}}_{t}\Big) = \lim_{t \to e} \ln(t) = 1$$

$x \to 0 \quad t \to e$

2) $\lim_{x \to 0} \frac{e^x - 1}{x} = 1 \quad \left[\frac{0}{0}\right]$ F.I.

$$\lim_{x \to 0} \frac{\ln(1+\boxed{x})}{x} = 1 \qquad x = e^t - 1$$

$$x \to 0 \quad t \to 0$$

$$\lim_{t \to 0} \frac{\ln(1 + e^t - 1)}{e^t - 1} = \lim_{t \to 0} \frac{t}{e^t - 1} = 1$$

3) $$\lim_{x\to 0} \frac{(1+x)^{\alpha}-1}{x} = \alpha \in \mathbb{R}$$

$$e^{x} = (1+t)^{\alpha} \qquad x = \ln(1+t)^{\alpha}$$

$$x \to 0 \qquad t \to 0$$

$$(1=) \lim_{x\to 0} \frac{e^{x}-1}{x} = \lim_{t\to 0} \frac{(1+t)^{\alpha}-1}{a\ln(1+t)} =$$

$$= \lim_{t\to 0} \frac{(1+t)^{\alpha}-1}{t} \cdot \frac{t}{a\ln(1+t} = \lim_{t\to 0} \frac{(1+t)^{\alpha}-1}{t} \cdot \underbrace{\lim_{t\to 0} \frac{t}{a\ln(1+t)}}_{=\frac{1}{a}}$$

$$\frac{t}{\ln(1+t)} \to 1$$

$$1 = \overbrace{\lim_{x\to 0} \frac{(1+t)^{\alpha}-1}{t}}^{\alpha} \cdot \frac{1}{\alpha}$$

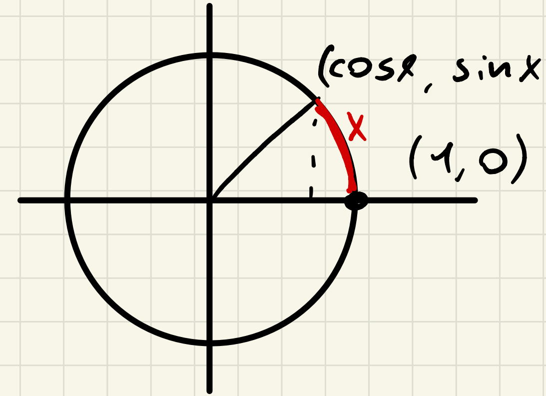
es. $\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x} = \frac{1}{2}$

<u>oss.</u> Se $f(x) \to 0$ $x \to x_0$, allora $\lim_{\boxed{x \to x_0}} \frac{\ln(1+f(x))}{f(x)} = 1$

$$\lim_{x \to x_0} \frac{e^{f(x)} - 1}{f(x)} = 1$$

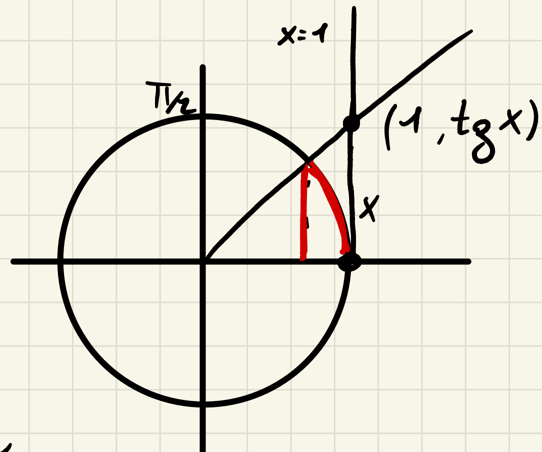
$$\lim_{x \to x_0} \frac{(1+f(x))^{\alpha} - 1}{f(x)} = \alpha$$

4) $\lim_{x \to 0} \underbrace{\frac{\sin x}{x}}_{\text{pari}}$ $\left[\frac{0}{0}\right]$ F.I.



$x > 0$

$$\lim_{x \to 0^+} \frac{\sin x}{x}$$



$$0 < x < \frac{\pi}{2}$$

$$\sin x < x < tg\,x = \frac{\sin x}{\cos x}$$

$$\Updownarrow$$

moltiplico sinx

$$\frac{\cos x}{\sin x} < \frac{1}{x} < \frac{1}{\sin x} \Longleftrightarrow \cos x < \frac{\sin x}{x} < 1$$

costante

applico il teorema dei carabinieri

$$\lim_{x \to 0} \cos x = 1$$

quindi

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

Ricavo:

$$\lim_{x \to 0} \frac{1-\cos x}{x^2}$$

$$\cos^2 x + \sin^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$$= \frac{(1-\cos x)(1+\cos x)}{x^2(1+\cos x)} = \frac{\sin^2 x}{x^2} \cdot \frac{1}{1+\cos x}$$

$$\lim_{x \to 0} \underbrace{\left(\frac{\sin x}{x}\right)^2}_{\downarrow\, 1} \cdot \underbrace{\frac{1}{1+\cos x}}_{\downarrow\, 1/2} = \frac{1}{2}$$

- $\lim\limits_{x\to 0} \dfrac{tg\,x}{x} = \lim\limits_{x\to 0} \underbrace{\dfrac{\sin x}{x}}_{\to 1} \cdot \underbrace{\dfrac{1}{\cos x}}_{\to 1} = 1$

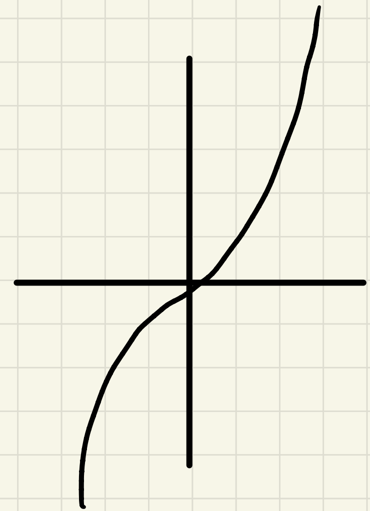
- $\lim\limits_{x\to 0} \dfrac{arctg\,x}{x}$ $\qquad x = tg\,t$

  $\qquad x \to 0 \quad t \to 0$

$$\| $$

$$\lim_{t\to 0} \frac{arctg(tg\,t)}{tg\,t} = \lim_{t\to 0} \frac{t}{tg\,t} = 1$$

è il reciproco quindi
è uguale



Osservazione

se $\lim_{x \to x_0} f(x) = 0$ allora $\lim_{x \to x_0} \frac{\sin f(x)}{f(x)} = 1$

$x_0 \in \mathbb{R}$

$$\lim_{x \to x_0} \frac{1 - \cos f(x)}{f(x)^2} = \frac{1}{2}$$

$$\lim_{x \to x_0} \frac{\operatorname{tg} f(x)}{f(x)} = 1$$

$$\lim_{x \to x_0} \frac{\operatorname{arctg} f(x)}{f(x)} = 1$$

es.

$$\lim_{x \to \frac{1}{2}} \frac{\sin \underbrace{\sqrt[3]{2x-1}}_{\to 0}}{\underbrace{\sqrt[5]{2x-1}}_{\to 0}} \quad \left[\frac{0}{0}\right] \text{ F.I.}$$

$$\frac{\sin \sqrt[3]{2x-1}}{\sqrt[3]{2x-1}} \to 1$$

$$\lim_{x \to \frac{1}{2}} \underbrace{\boxed{\frac{\sin \sqrt[3]{2x-1}}{(2x-1)^{\frac{1}{3}}}}}_{\to 1} \cdot \frac{(2x-1)^{\frac{1}{3}}}{(2x-1)^{\frac{1}{5}}} = \lim_{x \to \frac{1}{2}} (2x-1)^{\frac{1}{3} - \frac{1}{5}} \to 0 = 0$$