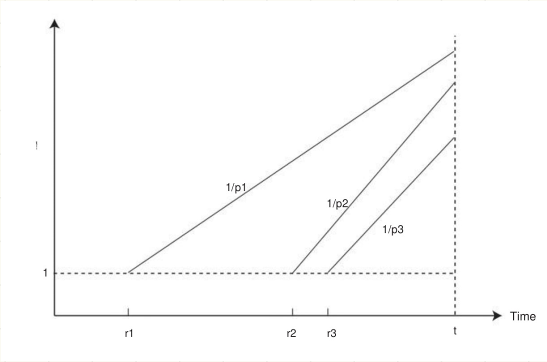
1) A variant of Highest response ratio next

called minimax response next, aims to minimize the maximum response ratio

Come up with a scheduling policy that meets this requirement
take inspiration from the figure



Suppose we have to make a scheduling decision at time t.
In what order should task t_1, t_2, t_3 be performed (from time t), so that the maximum response ratio $\max_s((w+p_s)/p_s)$ is as small as possible?

<u>Solution:</u>

the response ratio is determined when the process is finished
look at the time when all processes finished (this is $T = t + \sum_s p_s$)
choose the process for which $(T - r_s)/p_s$) is smallest as the last process to schedule
continue like this

<u>proof of optimality:</u>

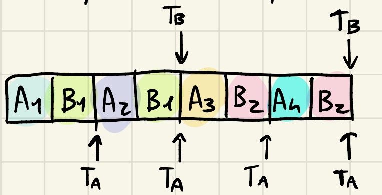
- call P the task with the highest response ratio, the time at which it ends is T_p
- suppose there is a better order, then either:
  - P in this order ends later. Since the response ratio increases if a process is placed later P would have a higher response ratio, and therefore also a higher maximum response ratio
    This is not possible
  - P ends earlier in this better order
    consequently, there must be at least one other job Q may have been placed later ending on or after T_p.
    But we already knew that P's response ratio was the smallest at this point
    Also not possible.

2) Rate monotonic scheduling

It can be proved that RMT will always respect the deadlines if $U = \sum_{j=1}^{n} \underbrace{p_j}_{\text{duration}} / \underbrace{T_j}_{\text{period}} \leq n(2^{1/n} - 1)$

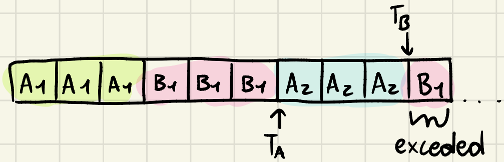
1) Give an example of (at least 2) jobs that meet their deadlines with RMS and $U=1$

$T_A = 2 \quad p_A = 1 \qquad T_B = 4 \quad p_B = 2 \qquad U = \frac{1}{2} + \frac{2}{4} = 1$



2) Give an example that doesn't work with U at most 0.95

$T_A = 6 \quad p_A = 3 \quad T_B = 9 \quad p_B = 4 \quad U = \frac{3}{6} + \frac{4}{9} = 0.9444$



3) same but U at most 0.84

to solve this we can generalize

$T_A = 2n \quad T_B = 3n \qquad U = \frac{n}{2n} + \frac{n+1}{3n} = \frac{1}{2} + \frac{1}{3} + \frac{1}{3n}$

$p_A = n \quad p_B = n+1$

$U \geq 0.84$

$\frac{1}{2} + \frac{1}{3} + \frac{1}{3n} \geq 0.84$

$\frac{1}{3n} \geq \frac{84}{100} - \frac{5}{6}$

$\frac{1}{3n} \geq \frac{84 \cdot 6 - 500}{600} = \frac{4}{600}$

$\frac{1}{3n} \geq \frac{1}{150} \qquad n \leq 50$ it works so $n > 50$