

COMPILING MATHEMATICAL FUNCTIONS IN BIOCHEMICAL REACTION NETWORKS

Internship INRIA Lifeware - Julien Bienvenu

Soutenance de stage juillet 2022

Context – Practical view point

M. Patrick Amar & Sys2Diag

- Synthetic vesicles
- Multiple usages (cheap medical testing)

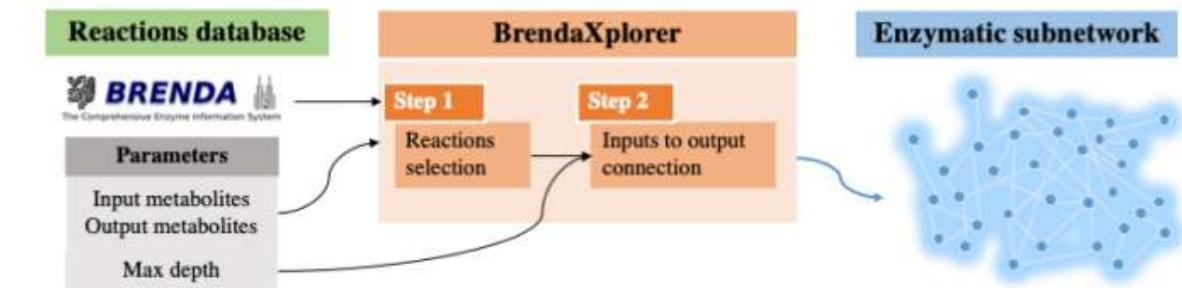


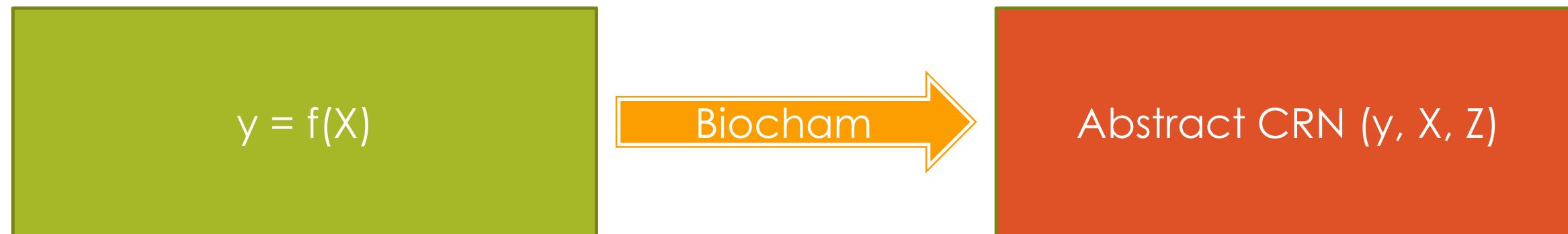
Fig. 2 Database pipeline implemented in [BrendaXplorer](#) software to extract concrete chemical reactions, currently used with [SiliCell Maker](#) software for implementing logical circuits.

But : Need to figure out real-life CRNs

- Brenda database & BrendaXplorer

Context – Theoretical viewpoint

- Lifeware & BioCham
- Able through polynomialization to perform the following pipeline :



Context – Theoretical viewpoint

Input: $A = f(\text{time})$ or $A = f(X)$

Formal Derivation

$$\begin{aligned} A &= f(T) \\ T &= t \end{aligned}$$

$\cdots \Rightarrow$

$$\begin{aligned} \frac{dA}{dt} &= f'(T) \\ \frac{dT}{dt} &= 1 \\ A(0) &= f(0) \\ T(0) &= 0 \end{aligned}$$

Polynomialization

$$\begin{aligned} \frac{dA}{dt} &= P(A, \mathbf{B}, T) \\ \frac{dB}{dt} &= \mathbf{P}(A, \mathbf{B}, T) \\ \frac{dT}{dt} &= 1 \\ A(0) &= f(0) \\ \mathbf{B}(0) &= \mathbf{IC} \\ T(0) &= 0 \end{aligned}$$

Quadratization

CRN Generation

Quadratic PIVP

$\forall t, A(t) = f(t)$

Output CRN

Halting time with X

$$\begin{aligned} \frac{dA}{dt} &= P(A, \mathbf{B}, T)X \\ \frac{dB}{dt} &= \mathbf{P}(A, \mathbf{B}, T)X \\ \frac{dT}{dt} &= X \\ \frac{dX}{dt} &= -X \\ A(0) &= f(0) \\ \mathbf{B}(0) &= \mathbf{IC} \\ T(0) &= 0 \\ X(0) &= \text{input} \end{aligned}$$

Quadratic PIVP

$\lim_{t \rightarrow \infty} A(t) = f(X(0))$

Fig. 1. Symbolic compilation pipeline implemented in [BIOCHAM v4](#) to transform any elementary mathematical function f into an abstract CRN [13,17,26], either a function of time (plain arrows) or an input/output function (dashed arrows): P and \mathbf{P} are polynomials, \mathbf{B} denotes the set of abstract auxiliary species introduced by polynomialization given with initial conditions \mathbf{IC} .

Internship's objective(s)

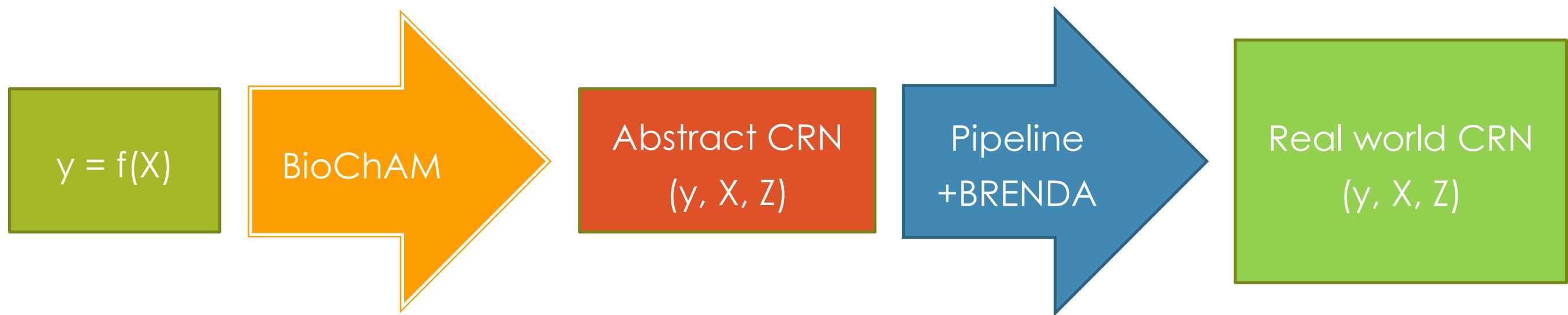
Compile general elementary mathematical functions in concrete CRN, by constraining our compilation pipeline to use the ODE terms of a limited set of well-characterized real enzymatic reactions

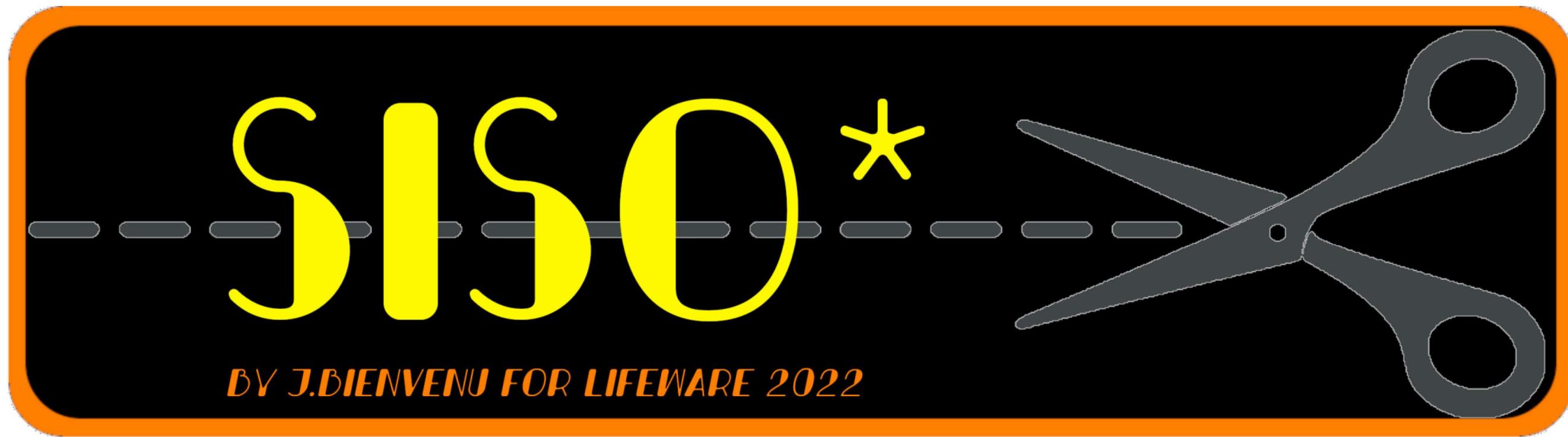
Static mapping from the abstract CRN to a concrete CRN (using Brenda as a backend module to Biocham)

Bonus objective (*if there's time*):

Symbolic computation theory of our CRN compilation pipeline in order to constrain it to the limited resources of a reaction catalogue in the early ODE transformation phases

Internship's objective(s)





My approach – Dividing in three

Biocham:
Figuring out how to use
and exploit the
abstract CRNs that it
returns

Brenda(Xplorer):
Figuring out how to use
it and exploit the
database of reactions
it returns

Processing :
Where the magic
happens

Pre-Processing Bricks

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What do we have in Brenda ?

EC code	Enzyme Name	Organism	Reactants	Products	Reversibility	KM	Kcat	pH	...
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SOAP API

- Brenda has a smart API allowing us to get an ID number for every different molecule
- Ex : for ID = 1 :
['OH-', 'H2o', 'h2O', 'H2O[side 2]', 'H3O+', '1alpha-hydroxy-vitamin D3', 'HO-', '1alpha-hydroxyvitamin D3', 'H2O']
- Limited to 10 requests/second in theory (in reality ~100/mn)
- >100 000 requests leads to a long time of computing
- Some metabolites' name are still not recognized

Some statistics

304 848 reactions

116 215 different metabolite name

6475 different enzyme names

6502 different ECs

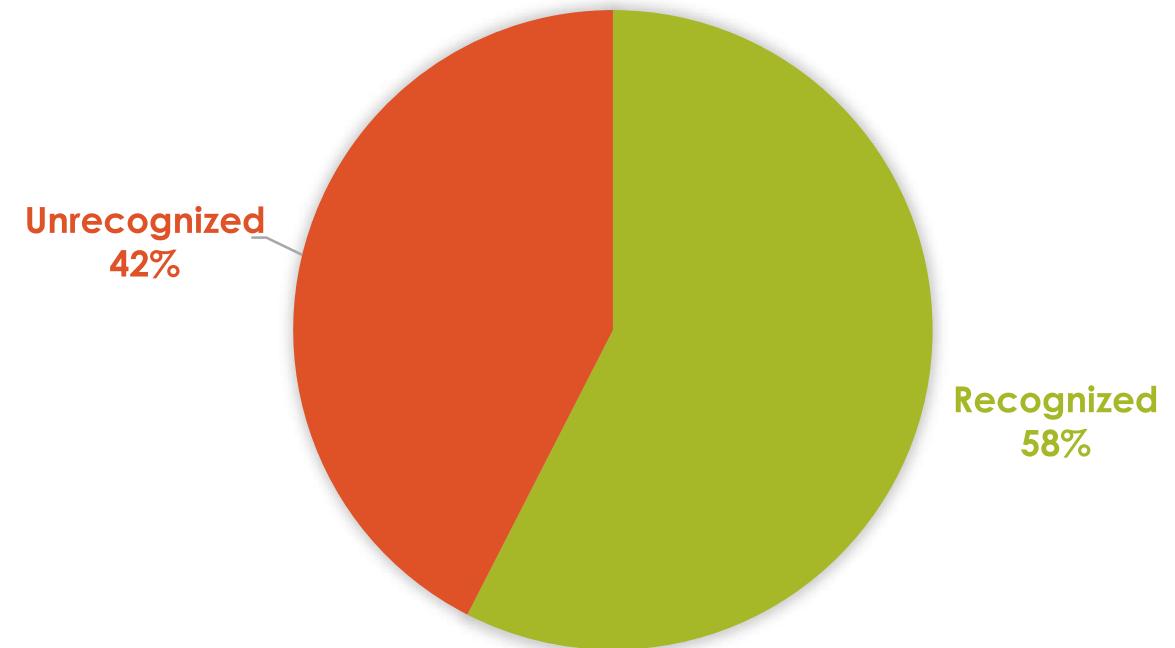
Among the 116 215, there are :

35 755 names not recognized by the API

53 162 molecules considered as different by the API

Among the 304 848 reactions of the data-base, there are 129 482 of them with at least one unrecognized metabolite.

REACTIONS



Weird ECs

'1.14.14.143', '(+)-menthofuran synthase')	'1.14.14.120', 'dammarenediol 12-hydroxylase')
'1.14.13.104', '(+)-menthofuran synthase')	'1.14.13.183', 'dammarenediol 12-hydroxylase')
'1.14.19.7', '(S)-2-hydroxypropylphosphonic acid epoxidase')	'4.1.3.2', 'malate synthase')
'1.11.1.23', '(S)-2-hydroxypropylphosphonic acid epoxidase')	'2.3.3.9', 'malate synthase')
'1.14.21.2', '(S)-cheilanthifoline synthase')	'1.10.3.12', 'menaquinol oxidase (H ⁺ -transporting)')
'1.14.19.65', '(S)-cheilanthifoline synthase')	'7.1.1.5', 'menaquinol oxidase (H ⁺ -transporting)')
'1.14.14.99', '(S)-limonene 3-monoxygenase')	'1.14.13.202', 'methyl farnesoate epoxidase')
'1.14.13.47', '(S)-limonene 3-monoxygenase')	'1.14.14.127', 'methyl farnesoate epoxidase')
'1.14.21.1', '(S)-stylopine synthase')	'1.14.19.70', 'mycocyclosin synthase')
'1.14.19.64', '(S)-stylopine synthase')	'1.14.21.9', 'mycocyclosin synthase')
'1.14.14.133', '1,8-cineole 2-endo-monoxygenase')	'4.3.1.5', 'phenylalanine ammonia-lyase')
'1.14.13.156', '1,8-cineole 2-endo-monoxygenase')	'4.3.1.24', 'phenylalanine ammonia-lyase')
'1.14.13.53', '4'-methoxyisoflavone 2'-hydroxylase")	'1.14.13.185', 'pikromycin synthase')
'1.14.14.89', '4'-methoxyisoflavone 2'-hydroxylase")	'1.14.15.33', 'pikromycin synthase')
'5.3.2.8', '4-oxalomesaconate tautomerase')	'1.14.13.121', 'premnaspirodiene oxygenase')
'5.3.3.16', '4-oxalomesaconate tautomerase')	'1.14.14.151', 'premnaspirodiene oxygenase')
'5.6.2.1', 'DNA topoisomerase')	'1.14.14.121', 'protopanaxadiol 6-hydroxylase')
'5.99.1.2', 'DNA topoisomerase')	'1.14.13.184', 'protopanaxadiol 6-hydroxylase')
'5.99.1.3', 'DNA topoisomerase (ATP-hydrolysing)')	'1.14.21.4', 'salutaridine synthase')
'5.6.2.2', 'DNA topoisomerase (ATP-hydrolysing)')	'1.14.19.67', 'salutaridine synthase')
'1.14.13.71', "N-methylcoclaurine 3'-monoxygenase")	'1.14.19.62', 'secologanin synthase')
'1.14.14.102', "N-methylcoclaurine 3'-monoxygenase")	'1.3.3.9', 'secologanin synthase')
'3.1.11.7', "adenosine-5'-diphospho-5'-[DNA] diphosphatase")	'1.14.13.70', 'sterol 14alpha-demethylase')
'3.6.1.71', "adenosine-5'-diphospho-5'-[DNA] diphosphatase")	'1.14.14.154', 'sterol 14alpha-demethylase')
'1.14.13.158', 'amorpha-4,11-diene 12-monoxygenase')	'1.14.13.75', 'vinorine hydroxylase')
'1.14.14.114', 'amorpha-4,11-diene 12-monoxygenase')	'1.14.14.104', 'vinorine hydroxylase')
'1.14.13.120', 'costunolide synthase')	
'1.14.14.150', 'costunolide synthase')	

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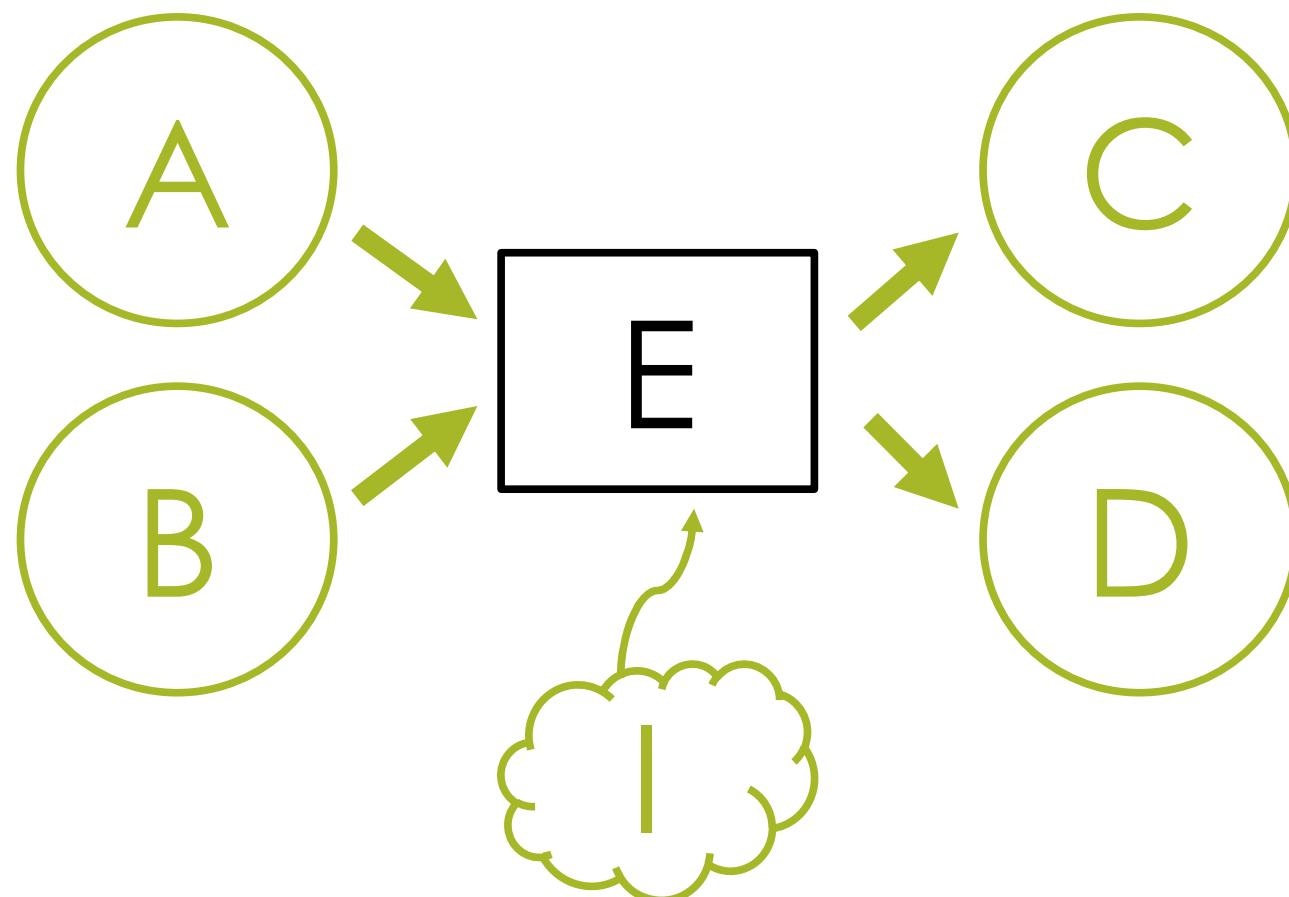
Processing :
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Pre-Processing Bricks

My approach – General ideas

- Various objects and classes :
 - Metabolite
 - Reaction
 - Kinetics
 - Environment...
- Loose concept of « matching »
- Abstract CRNs as graphs, with a pre-traversal algorithm
- Recursive programming, with the previous heuristic to find matching reactions

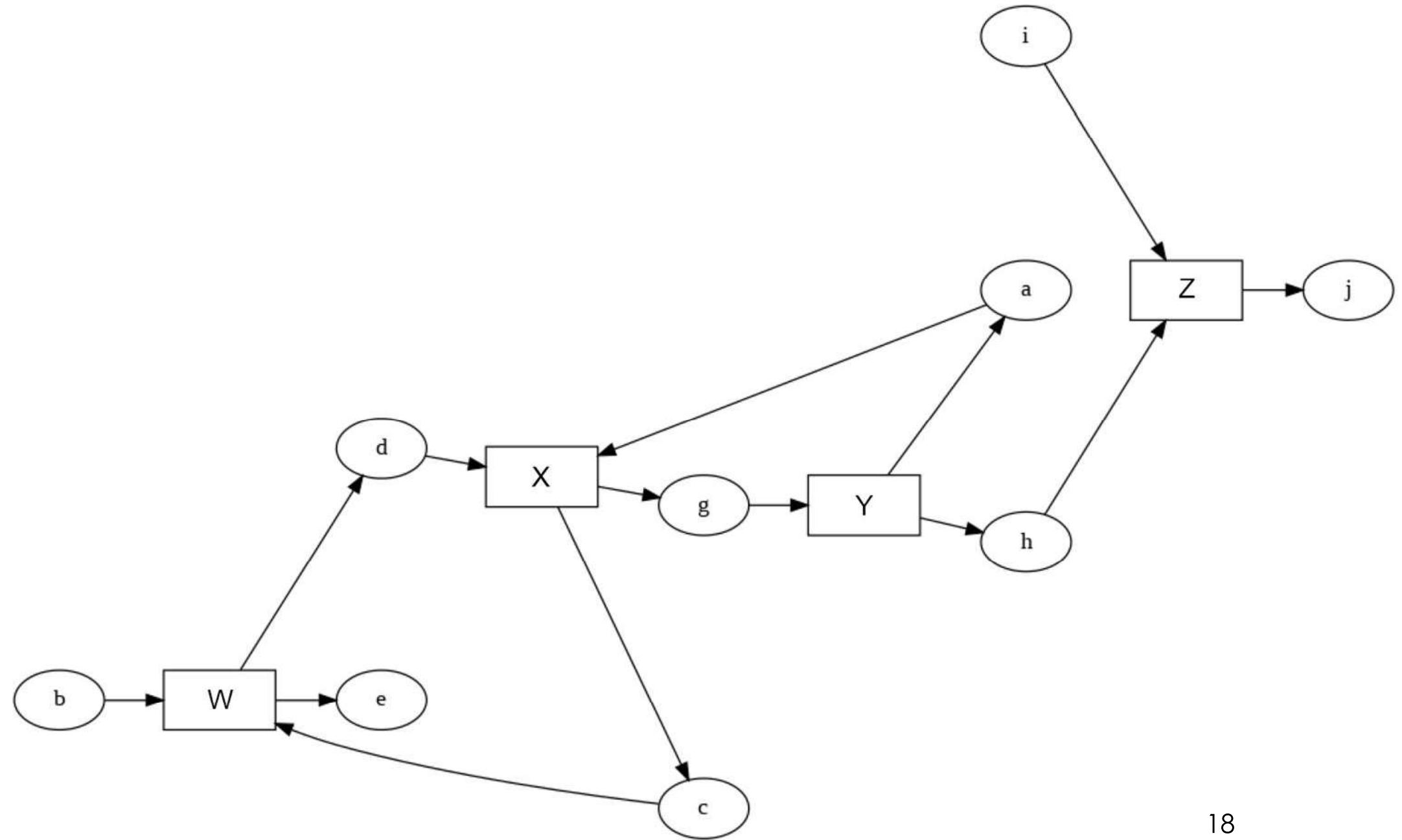
Some description of aCRNs



A	E	in
B		in
C		out
D		out
I		inhib

Matrix of an aCRN

	W	X	Y	Z
A	in	out		
B	in			
C	in	out		
D	out	in		
E	out			
G	out	in		
H		out	in	
I			in	
J			out	

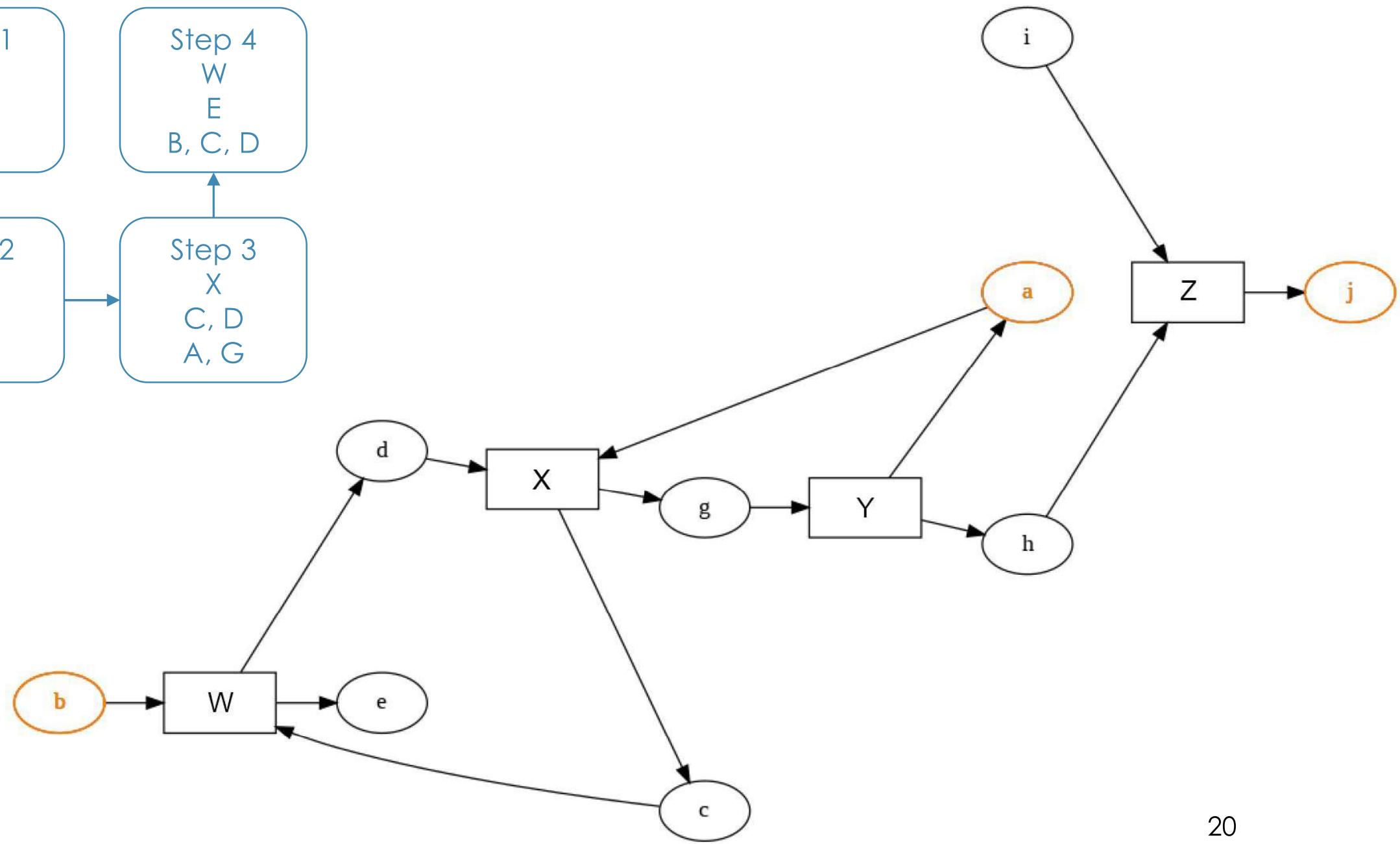
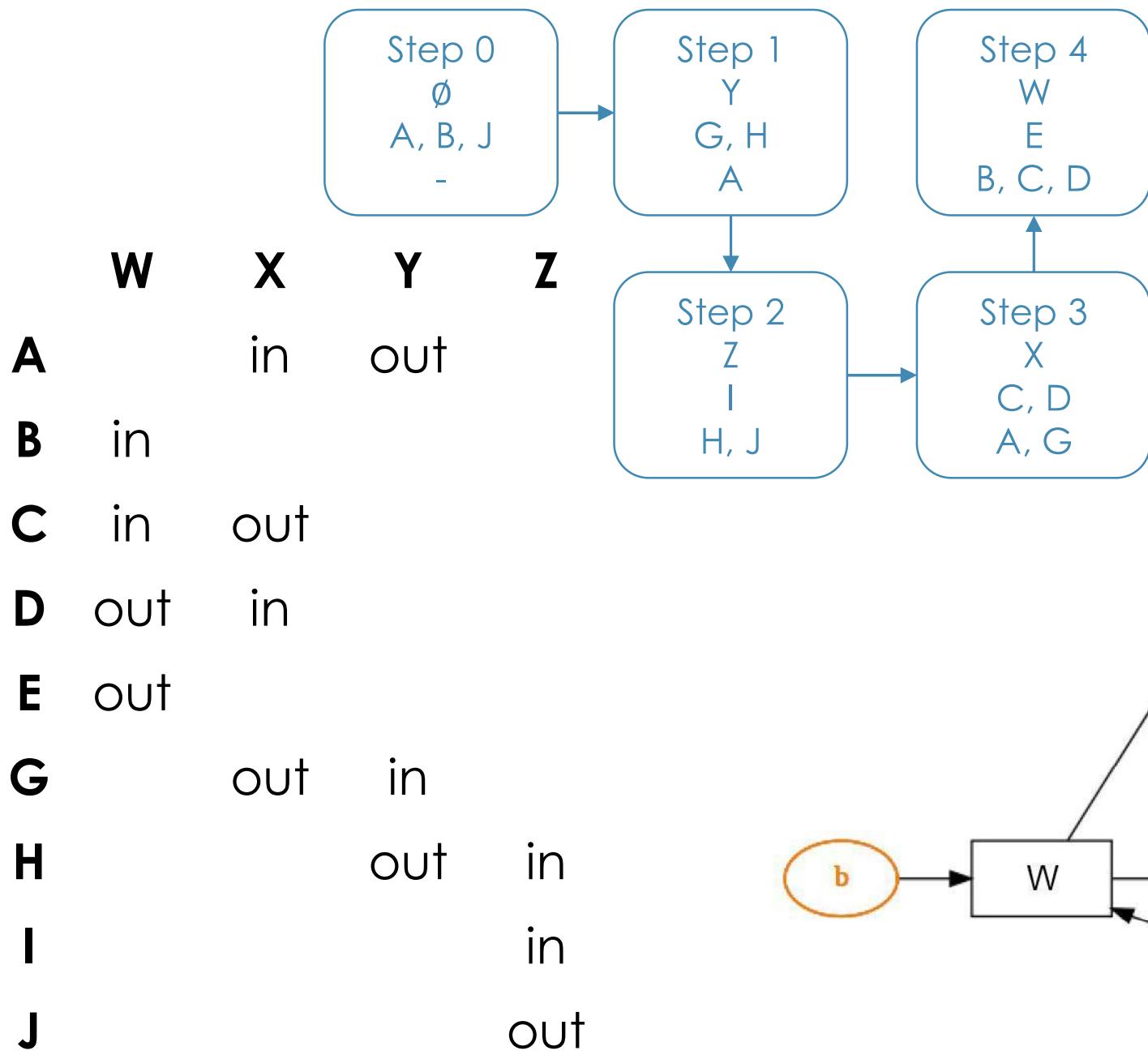


Pre-traversal of aCRN

- Step 0 : Consider and color as green the predefined metabolites
- Step 1 : Select the enzyme with the most green metabolites and the least red
- Step 2 : Note the enzyme and the new filled metabolites
- Step 3 : Green-light these metabolites for all of their enzymes
- Step 4 : Go back to step 1...

Real-time demo on whiteboard !

Pre-traversal an aCRN



Notations

- Abstract metabolites/enzymes will be denoted with capital letters (sometimes referred to as lettered-metabolites or lettered-enzymes orally)
- A step of the pre-traversal is described as follows :
 - An enzyme E to fill
 - A series of metabolites (A_i) to fill
 - A series of already filled metabolites (X_j) which are involved in E

Final pipeline

- Pre-traversal optimises the pipeline : aims to show incompatibilities ASAP
- Pre-selecting enzyme candidates for every abstract one in advance, matching with it's abstract kinetic
- Recursive programming

When do abstract and concrete reactions match ?

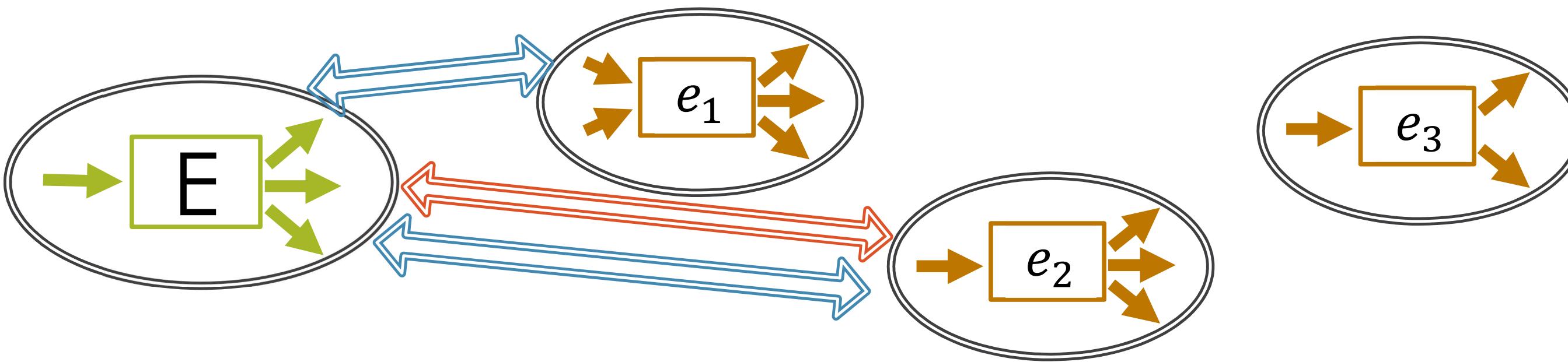
« hard-match »

exactly same number of ins and outs

« soft-match »

abstract has less ins and less outs than concrete
→ Grey-listing

- « kinetics-match » : takes kinetics into account as well
- Consequences on the graphs considered !



Recursive algorithm

For the n -th step proceed as follows :

- If $n \geq$ number of enzymes, (D_E, D_M, G_M) is a solution
- Else, let E, A, X be a description of the n -th step and K set of candidates for E
 - o For $c \in K$, if $c \in B_E$ or $x \notin c.in \cup c.out$, c is discarded
 - o For every remaining c of K :
 - Let $e_{in} = c.in (x.in \cup G_M)$ and $e_{out} = c.out (x.out \cup G_M)$.
 - If $|e_{in}| < |A.in|$ or $|e_{out}| < |A.out|$, c is discarded
 - Else : $D_E[E] \leftarrow c$ and are considered every $|A.in|$ -permutation ai of e_{in} and $|A.out|$ -permutation ao of e_{out}
 - For every couple $(ai, ao) : D_M[A.in] \leftarrow ai$, $D_M[A.out] \leftarrow ao$, and the step $n + 1$ is initiated with :
 - o $B_M \leftarrow B_M \cup ai \cup ao$
 - o $B_E \leftarrow B_E \cup c$
 - o $G_M \leftarrow G_M \cup (e_{in} \setminus ai) \cup (e_{out} \setminus ao)$.

Case Study – D-glucose Λ Acetone → Resorufin

- Same aCRN as this one, but with letters
- First goal : find the same solution as this one
- Secondary goal : acquire new solutions
- Resazurin → Resorufin still not found
- Added end step
- 2-propanone == acetone
- Some complete results !

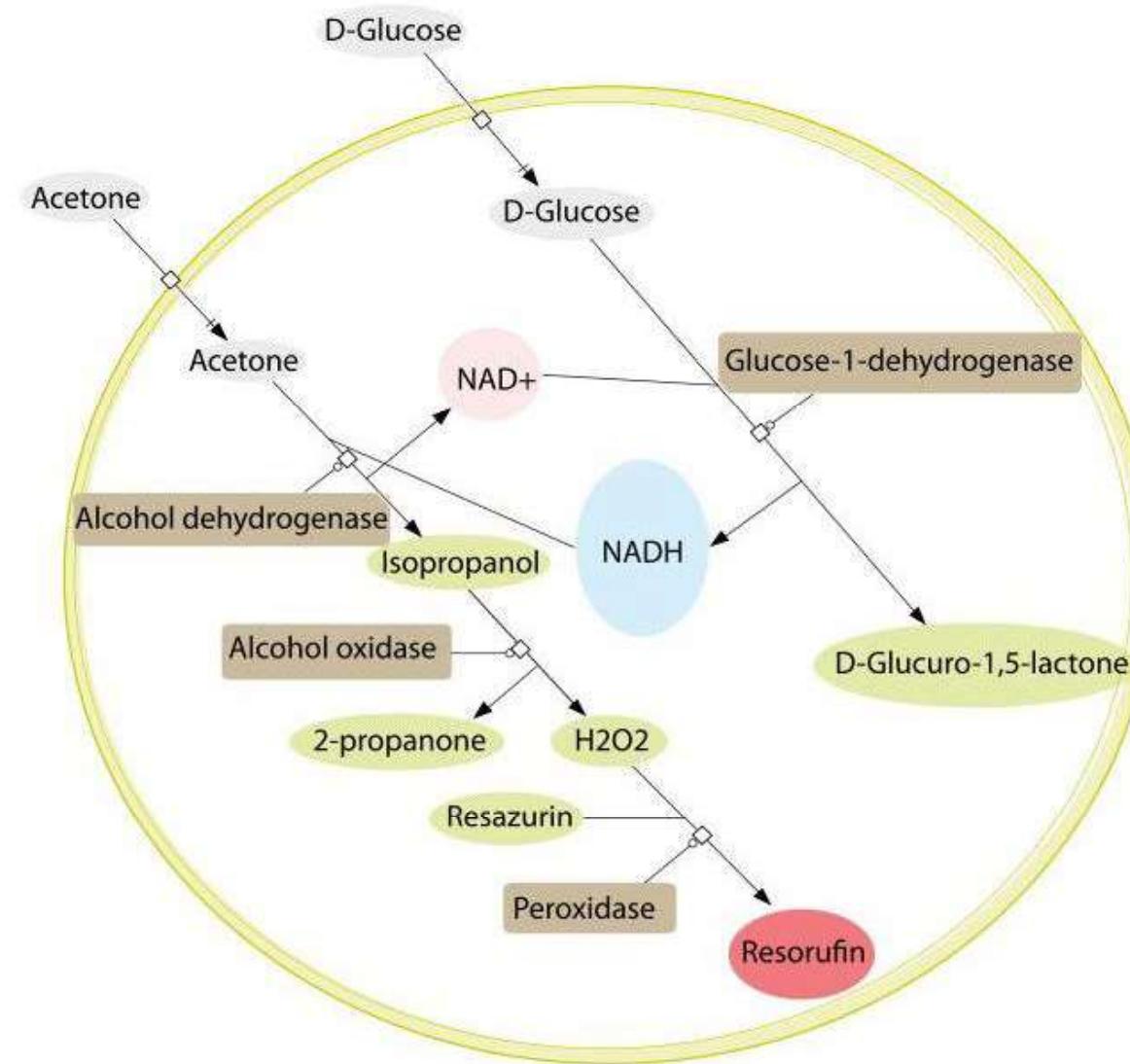


Figure 2.

Many new results

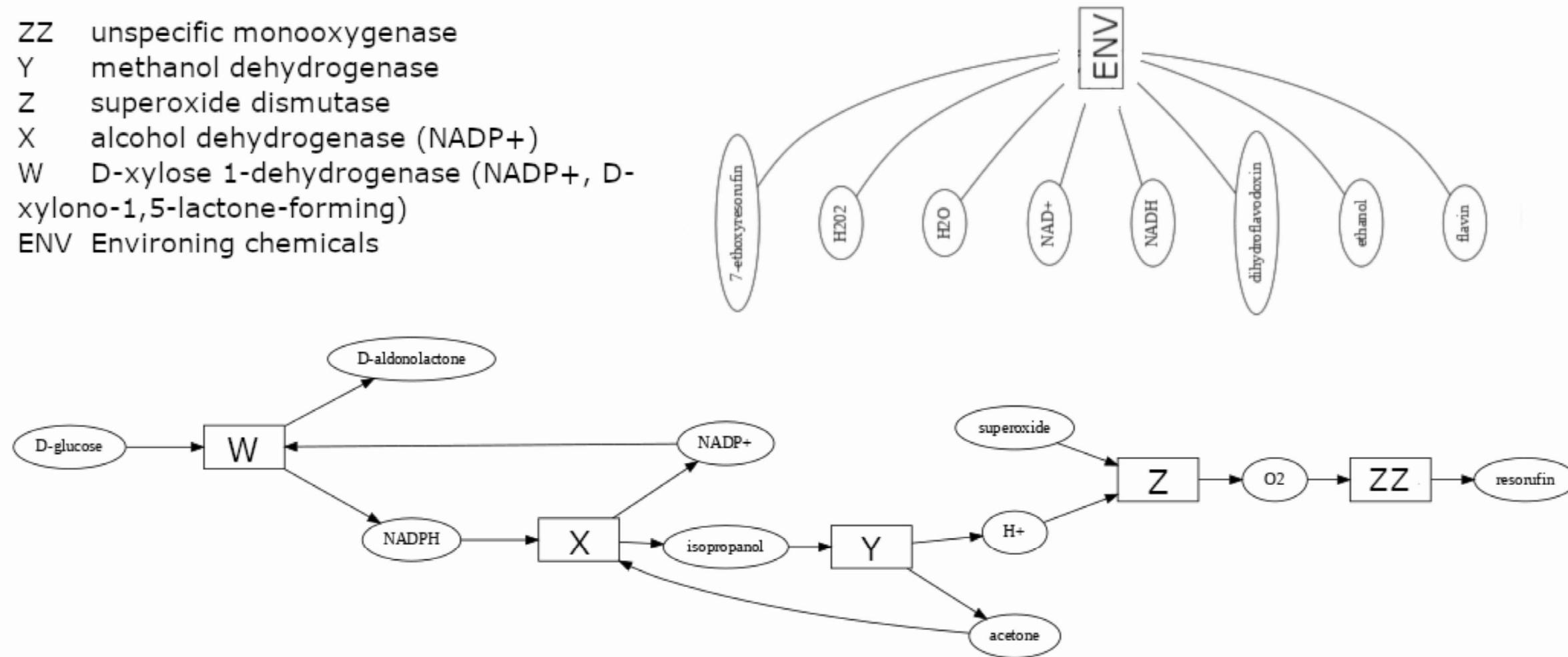
- ~600 with distinct enzymes for 9h computation time on a regular computer
- Not kinetic dependent
- Quite convincing
- But really time-costly to get every solution) (doesn't seem usable with a more SEPI-like search)

Some of the solutions

- {'Y': 'alcohol oxidase', 'X': 'alcohol dehydrogenase', 'W': 'aldose 1-dehydrogenase (NAD+)'}
• {'Y': 'alcohol oxidase', 'X': 'alcohol dehydrogenase', 'W': 'D-xylose 1-dehydrogenase'}
• {'Y': 'alcohol oxidase', 'X': 'alcohol dehydrogenase', 'W': 'inositol 2-dehydrogenase'}
• {'Y': 'alcohol oxidase', 'X': 'alcohol dehydrogenase', 'W': 'glucose/galactose 1-dehydrogenase'}
• {'Y': 'alcohol oxidase', 'X': 'alcohol dehydrogenase', 'W': 'glucose-6-phosphate 3-dehydrogenase'}
• {'Y': 'alcohol oxidase', 'X': 'alcohol dehydrogenase', 'W': 'aldehyde dehydrogenase (NAD+)'}
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• {'Y': 'alcohol oxidase', 'X': 'carbonyl reductase (NADPH)', 'W': 'alcohol dehydrogenase (NADP+)'}
• {'Y': 'alcohol oxidase', 'X': 'carbonyl reductase (NADPH)', 'W': 'glucose/galactose 1-dehydrogenase'}
• {'Y': 'alcohol oxidase', 'X': 'carbonyl reductase (NADPH)', 'W': 'alcohol dehydrogenase [NAD(P)+]'}
• {'Y': 'alcohol oxidase', 'X': 'carbonyl reductase (NADPH)', 'W': '(R)-aminopropanol dehydrogenase'}
• {'Y': 'alcohol oxidase', 'X': 'alcohol dehydrogenase (NADP+)', 'W': 'glucose/galactose 1-dehydrogenase'}
• {'Y': 'alcohol oxidase', 'X': 'alcohol dehydrogenase (NADP+)', 'W': 'alcohol dehydrogenase [NAD(P)+]'}
• {'Y': 'alcohol oxidase', 'X': 'alcohol dehydrogenase (NADP+)', 'W': '(R)-aminopropanol dehydrogenase'}
• {'Y': 'secondary-alcohol oxidase', 'X': 'alcohol dehydrogenase', 'W': 'D-xylose 1-dehydrogenase'}
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• ...

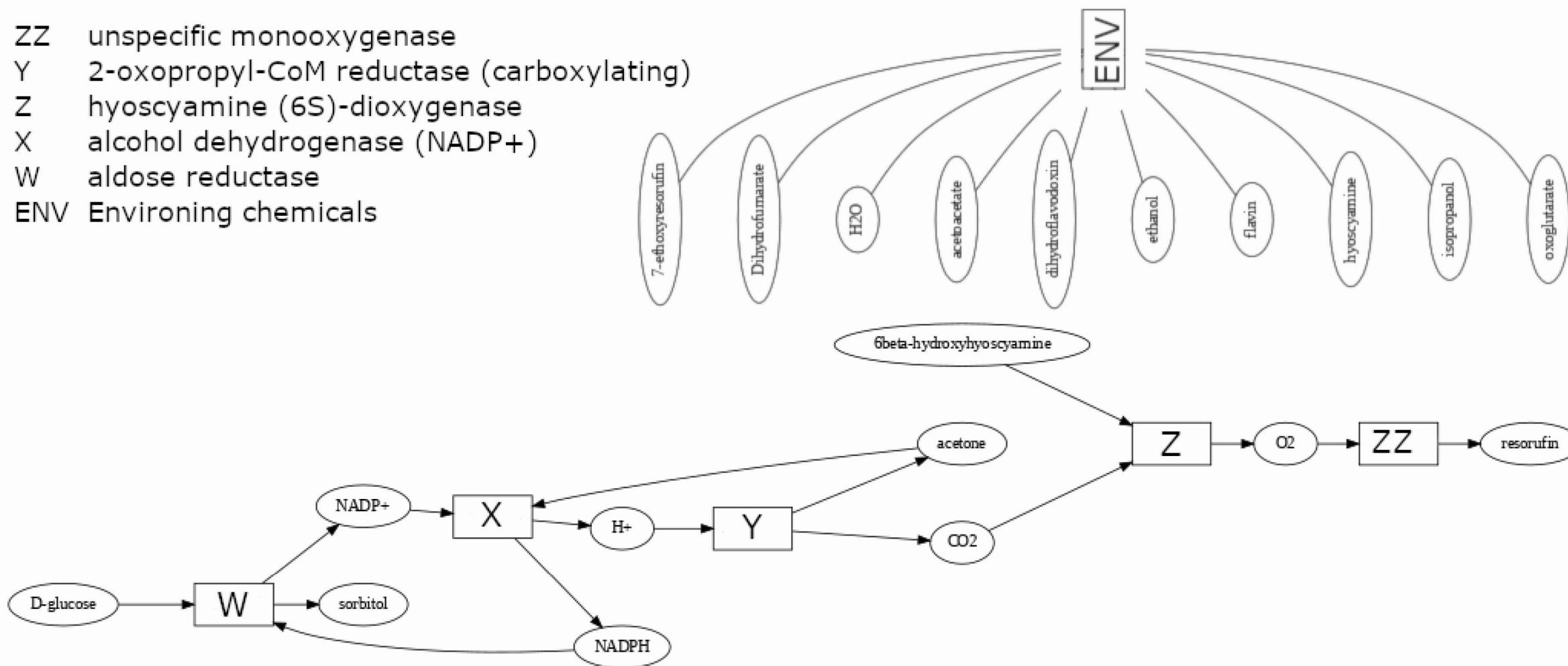
Case Study – a couple of solutions

ZZ unspecific monooxygenase
Y methanol dehydrogenase
Z superoxide dismutase
X alcohol dehydrogenase (NADP+)
W D-xylose 1-dehydrogenase (NADP+, D-xylonolactone-forming)
ENV Environing chemicals

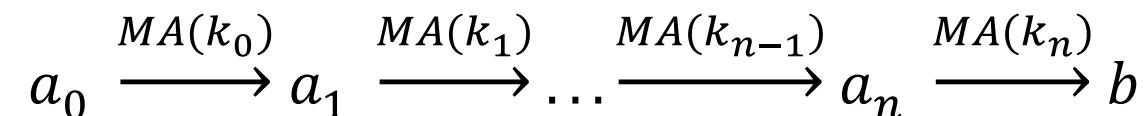


Case Study – a couple of solutions

ZZ unspecific monooxygenase
Y 2-oxopropyl-CoM reductase (carboxylating)
Z hyoscyamine (6S)-dioxygenase
X alcohol dehydrogenase (NADP⁺)
W aldose reductase
ENV Environing chemicals



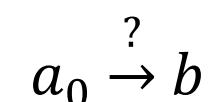
Kinetics – Chain contraction



With $D : u \mapsto \frac{du}{dt}$ and $Q_{(k_j)_{0 \leq j \leq n}} = \frac{X}{\prod_{j=0}^n k_j} \prod_{j=1}^n (X + k_j)$

b and a_0 can be linked by $a_0 = Q_{(k_j)_{0 \leq j \leq n}} \circ D(b)$

What can be deduced from it on a kinetics standpoint ?



Kinetics – Chain contraction proof

$$a_0 \xrightarrow{MA(k_0)} a_1 \xrightarrow{MA(k_1)} \dots \xrightarrow{MA(k_{n-1})} a_n \xrightarrow{MA(k_n)} b$$

- $\frac{db}{dt} = k_n a_n \quad \frac{da_0}{dt} = -k_0 a_0 \quad \frac{da_j}{dt} = k_{j-1} a_{j-1} - k_j a_j$
- $n = 0 :$ $\frac{db}{dt} = k_0 a_0 \quad \text{ie } Q_{k_0} = \frac{X}{k_0}$

- $n \mapsto n + 1 :$
 $\frac{da_1}{dt} = k_0 a_0 - k_1 a_1 \text{ ie } \frac{1}{k_0} \left(\frac{d}{dt} + k_1 \right) (a_1) = a_0$
so $Q_{(k_j)_{0 \leq j \leq n+1}} = \frac{1}{k_0} (X + k_1) Q_{(k_j)_{1 \leq j \leq n+1}}$ ■

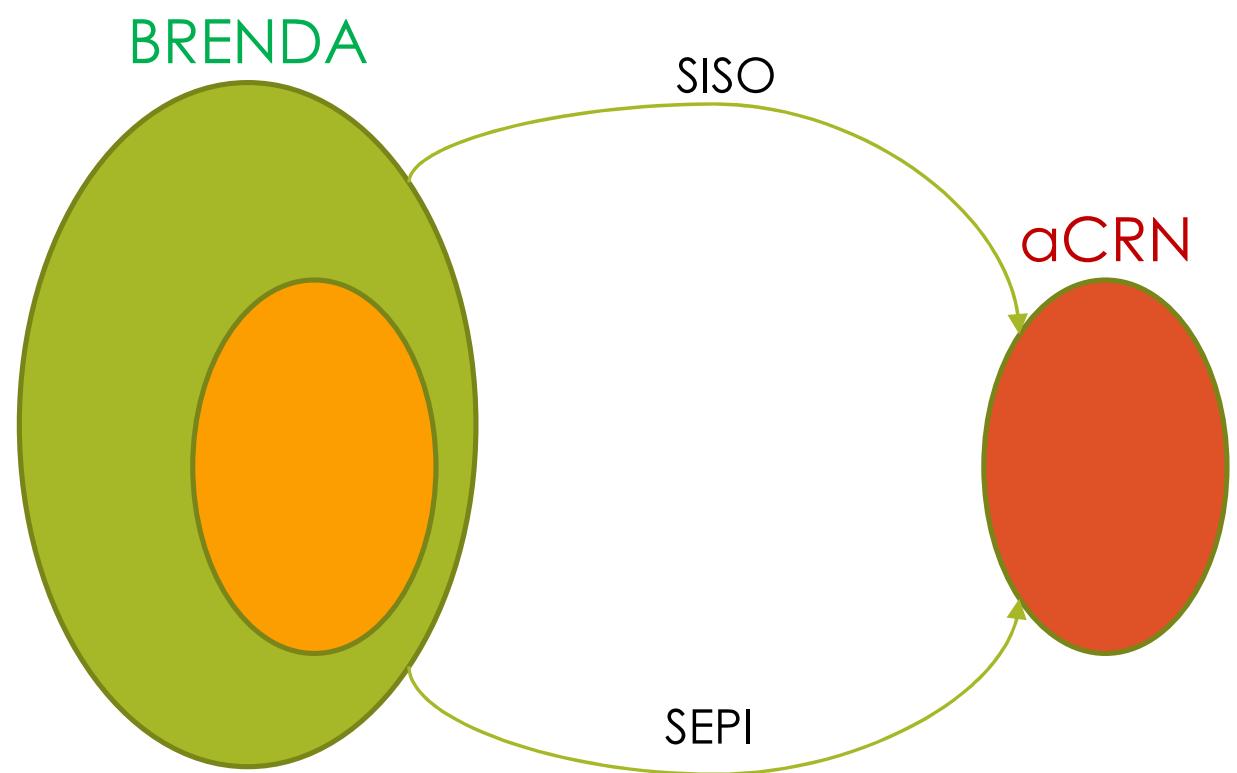
Kinetics - Independence

Hypothesis for the following reductions :

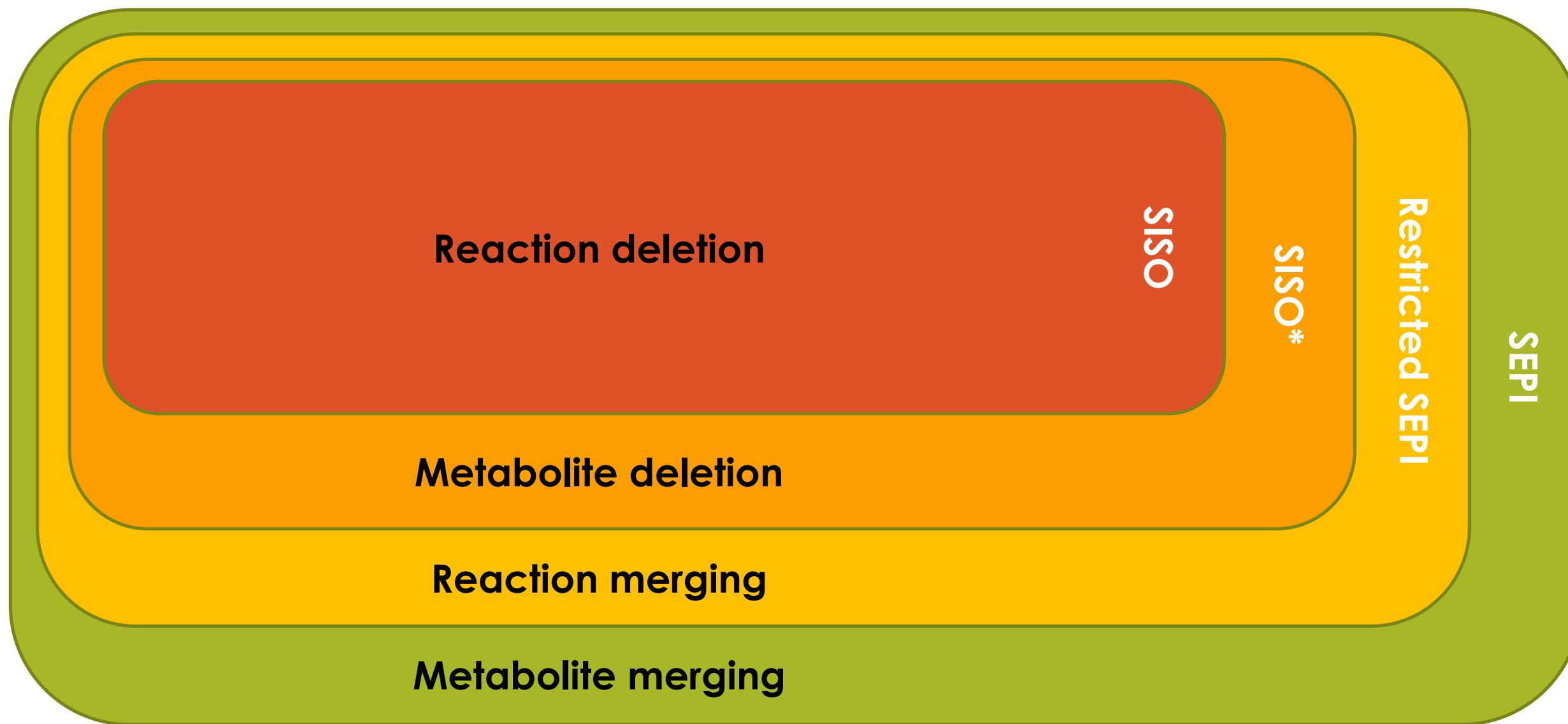
The aCRN considered are « morally » rate-independent.

SEPI, SISO...

- SEPI : subgraph epimorphism
- SISO : subgraph isomorphism
- We will describe them by the operations they permit



SEPI, SISO... – Nuances



SEPI, SISO – Implementations

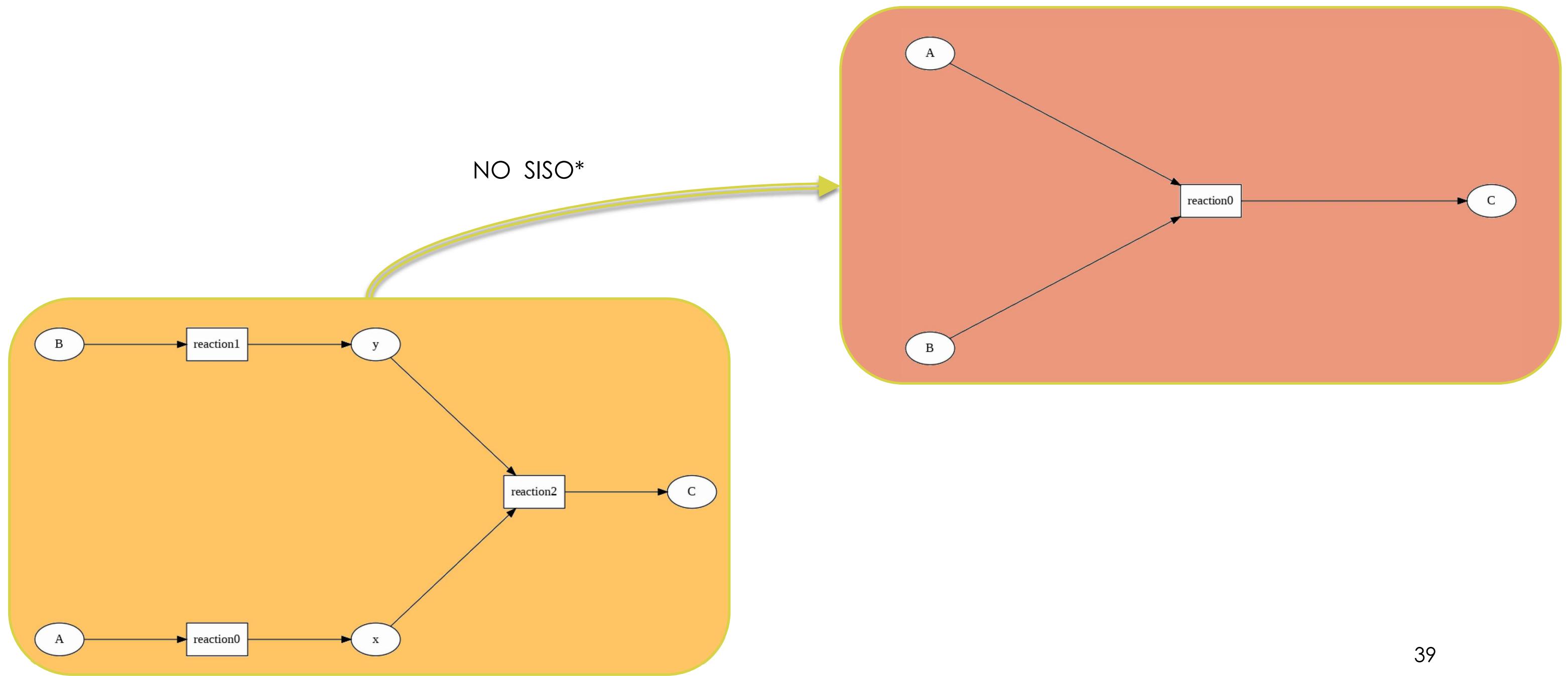
BIOCHAM

- SEPI
- Restricted SEPI
- But only up to ~100 reactions

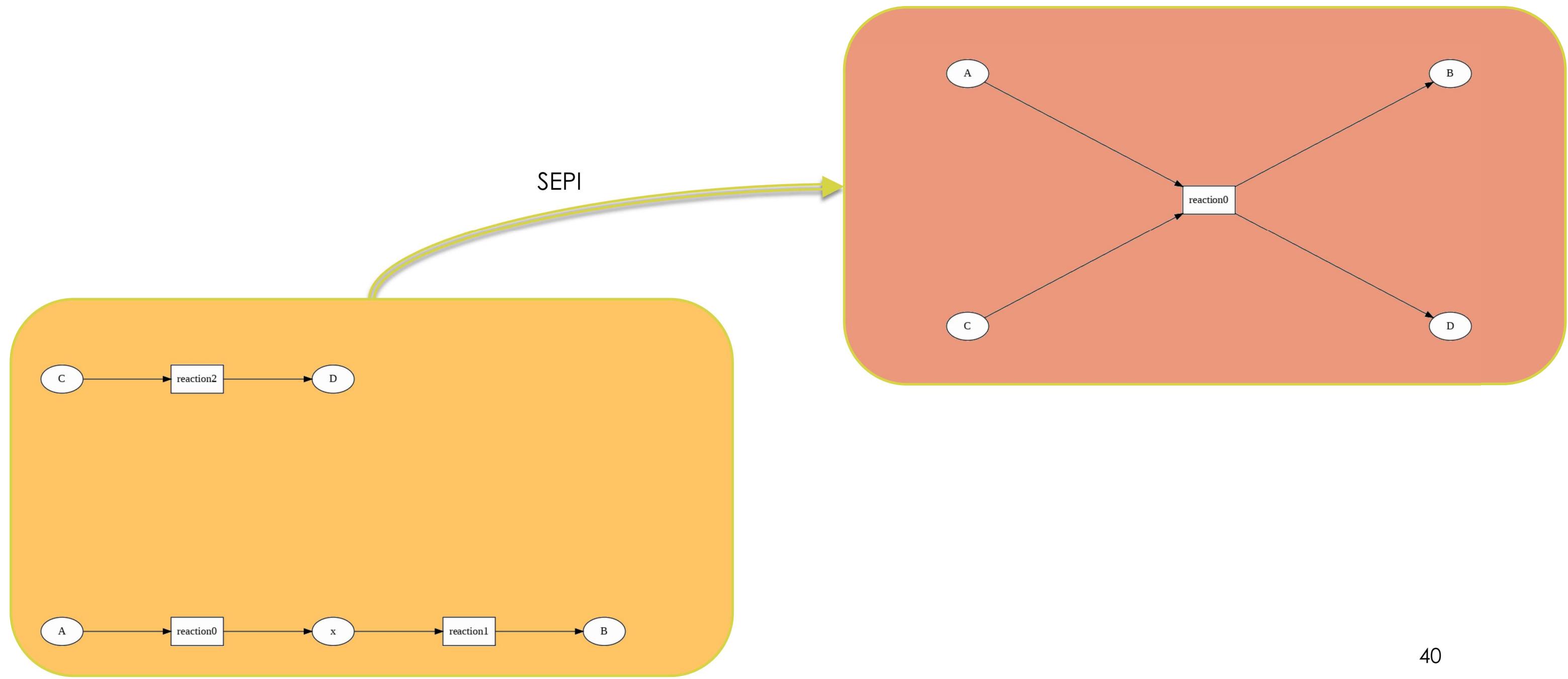
My Pipeline

- SISO
- SISO*
- Can handle BRENDA

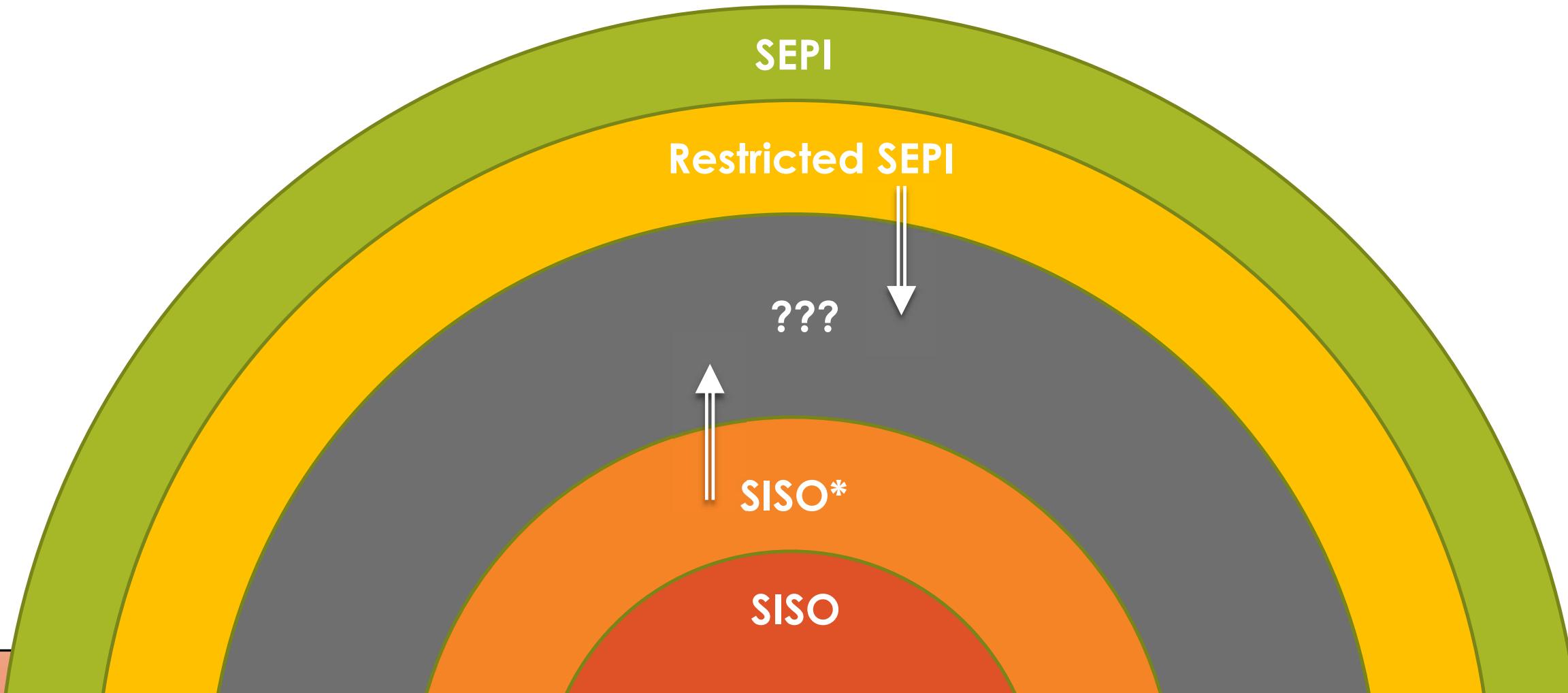
SISO* – Too restrictive



Restricted SEPI – Too lenient



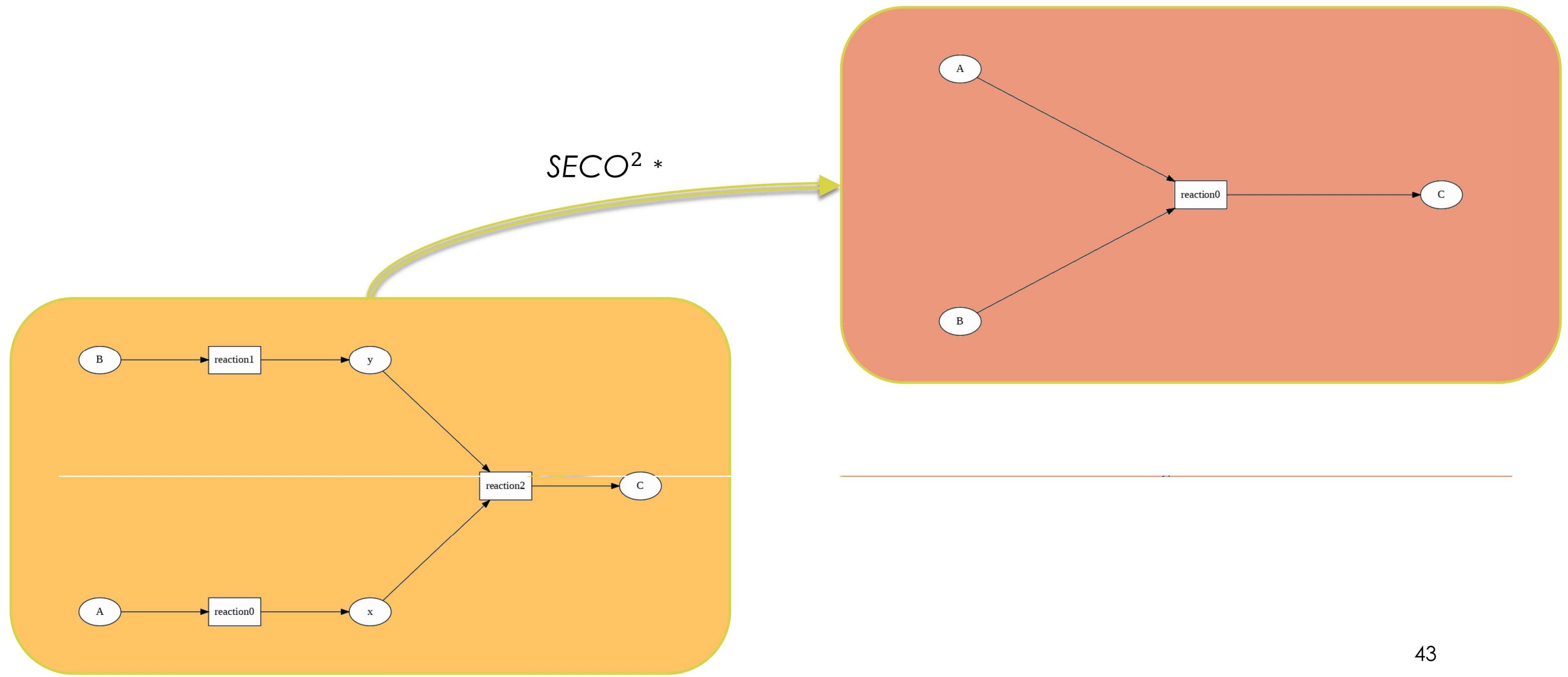
SEPI, SISO – Somewhere in between



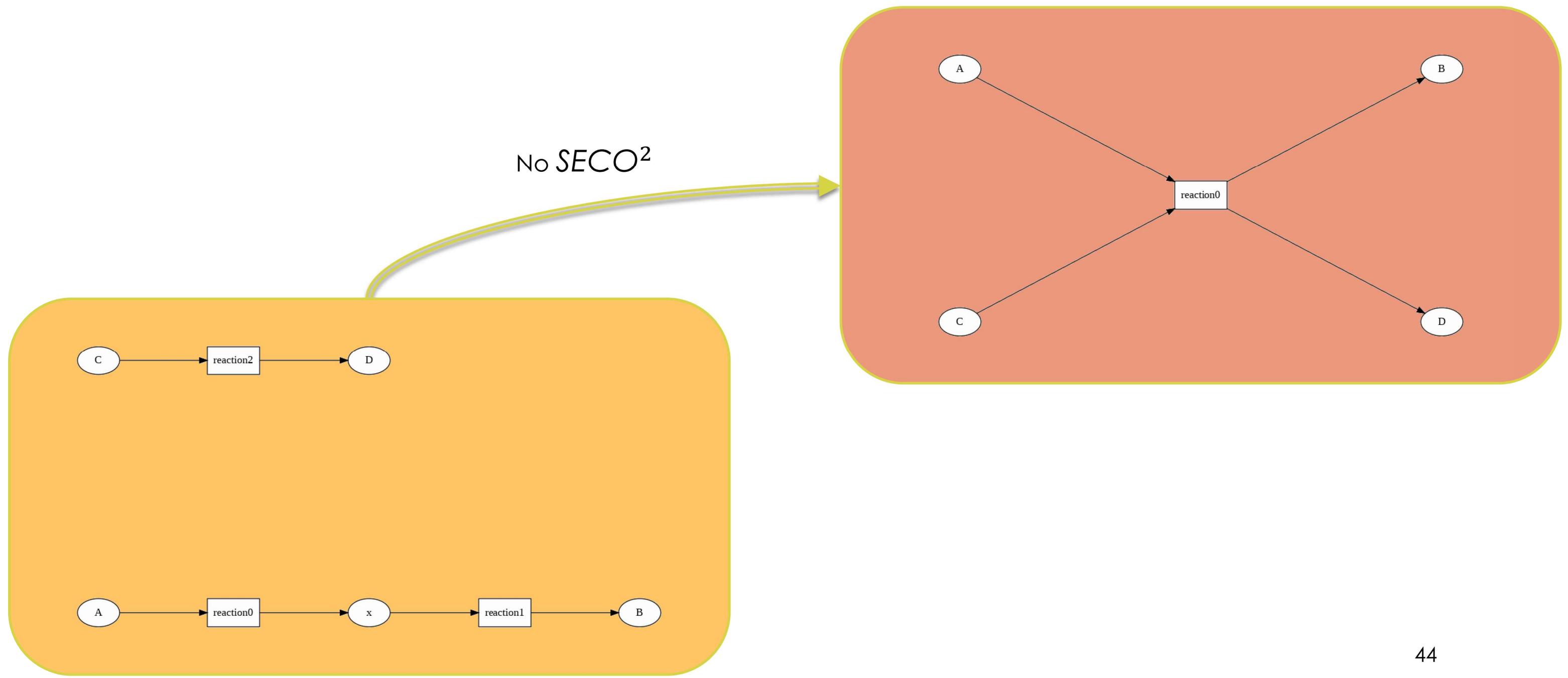
SEPI, SISO – Solution ?

- The concretization morphism is somewhere between SISO* and restricted SEPI
- No definitive solution as of now but some ideas have emerged the past week, including :
- *SECO*² (« subgraph epimorphism connexity-conserving »), merges reactions only if the resulting connexity is weaker than the original.

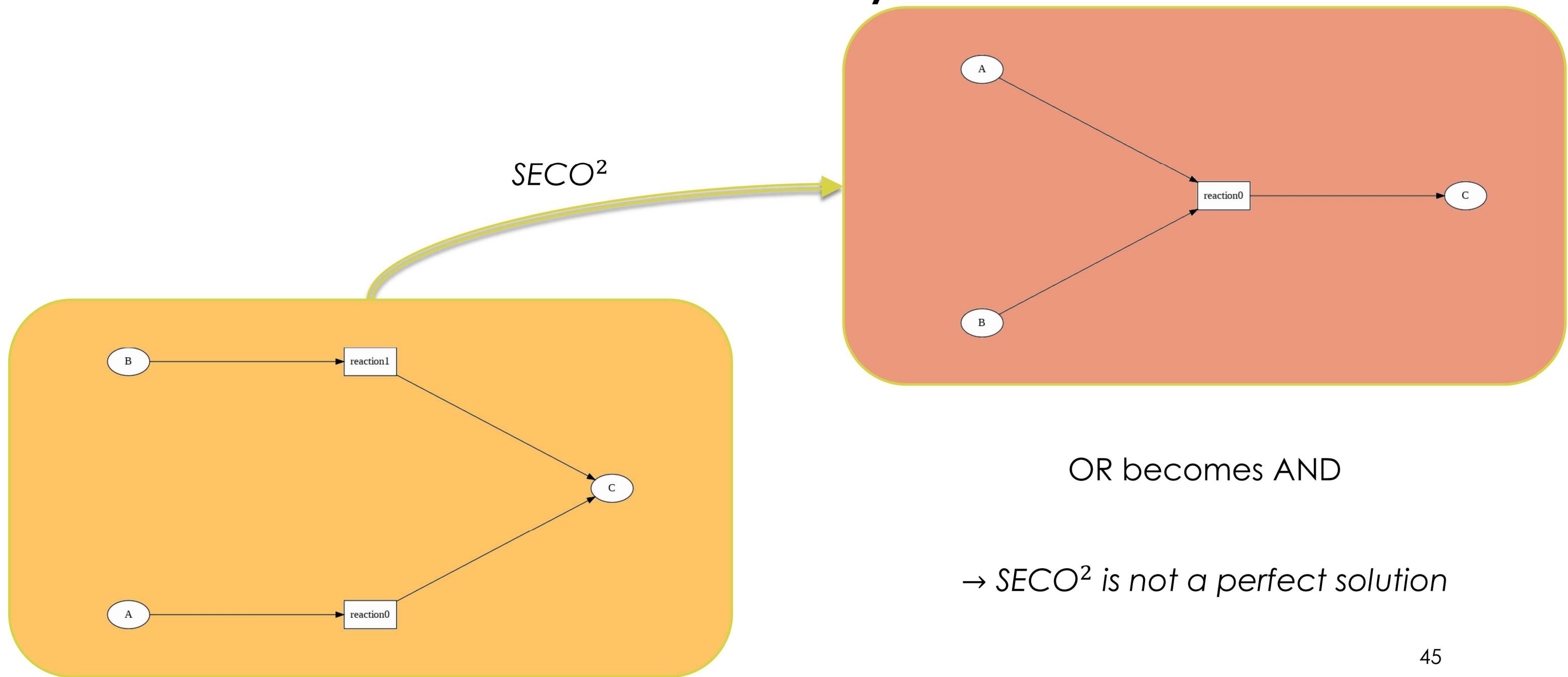
$SECO^2$ – Too restrictive ?



SECO² – Too lenient ?



$SECO^2$ – Solves many issues but...



What about now ?

- Kinetics – Adding them to $SISO \star$, how contractions affects them...
- Inhibitors not yet incorporated
- Between $SECO^2$ and $SISO \star$, find the morally correct description of what we are looking for
 - $SECO^2 \rightarrow$ more restrictions to the fusions ?
 - $SISO \star \rightarrow$ automate insertions ?