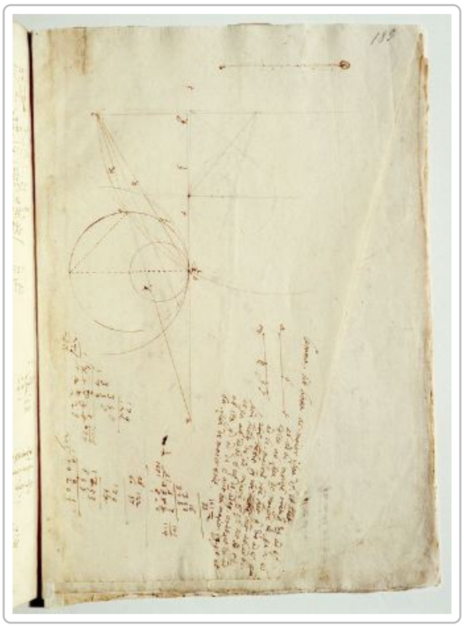
ChatGPT

Uniswap Tick Spacing and $\sqrt{1.0001}$

Uniswap v3 uses **ticks** spaced by 0.01% price changes: each tick moves the price by a factor of $\sqrt{1.0001}$ [1]. In other words, the ratio between adjacent ticks is $\sqrt{1.0001}$ ($\approx$1.00004999987). We found no historical reference explicitly linking the fraction **10202/101** to $\sqrt{1.0001}$. In fact, using that fraction requires an extra adjustment: ((10202/101)–1)/100 $\approx$1.00009901, which is about 0.005% above $\sqrt{1.0001}$. This slight error ($\approx$0.0049%) suggests 10202/101 is not an obvious simple approximation to $\sqrt{1.0001}$. We did not find any mathematical source or blockchain literature associating 10202/101 with Uniswap ticks.



Galileo's 1615 notebook shows an example of an early square-root approximation: he computes "179 * 57 = 10202" and then writes "$\sqrt{10202}$ = 101" [2]. In other words, Galileo effectively used 101 as the square root of 10202 (since 10202/101$\approx$101.0099). This yields an approximation error of about 0.005% for $\sqrt{10202}$. However, this is not related to $\sqrt{1.0001}$; it's simply an historical example of hand-computed square roots. There is no evidence Galileo (or others) used the ratio 10202/101 to approximate $\sqrt{1.0001}$ specifically. In summary, aside from such isolated computations (like Galileo's), we found no mention of 10202/101 being used as an approximation to $\sqrt{1.0001}$.

Rational Approximations of $\sqrt{1.0001}$

No classic text or table seems to record a special rational approximation for $\sqrt{1.0001}$. In general, $\sqrt{1+\varepsilon}$ can be approximated by binomial expansion or continued-fraction techniques. For very small ε=0.0001, the binomial series gives $\sqrt{1.0001} \approx 1 + 0.0001/2 - (0.0001)^2/8 + \ldots \approx 1.00004999987$. One could form rational approximations via continued fractions, but simple ratios like 10202/101 ($\approx$101.0099 as a multiplier) do not directly approximate 1.00005 unless one divides by 100 as above. We did not find any historical record of a simple fraction (such as p/q) being used for $\sqrt{1.0001}$ in mathematics literature. Modern computation (as in Uniswap's smart contracts) simply calculates $\sqrt{1.0001}$^tick using high-precision arithmetic [1], rather than relying on a fixed rational.

Historical Logarithm Bases Less Than 2

Throughout history, several logarithm systems used bases smaller than 2. Notable examples include:

- **Napier's original logarithms (1614):** Napier's "logarithms" were defined so that antilogarithms grow by a factor of e^{-1} per unit. Equivalently, Napier's tables can be viewed as using base $e^{-1} \approx 0.3679$ [3]. (He scaled values so that N = (e^{-1})^L, where L is Napier's logarithm.) Although e^{-1}<1, it illustrates an early non-integer base choice for logarithms.

- **Jost Bürgi's "progress tables" (1620):** Independently of Napier, Jost Bürgi tabulated powers of B = 1.0001 up to very high precision [4]. In his tables, each successive entry is 0.01% larger than the previous (multiply by 1.0001 each step). Thus Bürgi was effectively using **base 1.0001** for his computations. (Byrgi's tables reached the *"whole red number"* N ≈ 23027.0022, showing 1.0001^23027.0022 = 10 [5].)

- **John Wallis / Byrgius (Antilogarithms to base (1.0001)^(1/10)):** The 1911 *Encyclopædia Britannica* notes that "Byrgius gives antilogarithms to base (1.0001)^(1/10)" [3]. In other words, he tabulated values of (1.0001)^(n/10), i.e. antilogs to the base ≈1.000009999. This is another example of a very small-growth base (<2) used in early log tables.

- **Golden-ratio base (φ ≈ 1.618):** In the 20th century a "base-φ" (phi) positional system was studied, using the golden ratio φ as the base [6]. Although not a conventional logarithm in use today, base φ is an irrational base <2 used for numeral representation. Correspondingly, one can define log base φ, e.g. relating to Fibonacci scaling. (For example, in a phi-based system any real number can be expressed in "base φ" [6].)

Other common bases like 2, 10 or *e* are ≥2, so the above examples cover most historically noted cases of bases between 1 and 2. In summary, early log tables and numeral systems did experiment with unusual small bases – for instance, Napier's base e^{-1}, Bürgi's base *1.0001* [4], Byrgius's base *(1.0001)^(1/10)* [3], and the golden ratio φ (≈1.618) [6] – but we found **no reference** to $\sqrt{(1.0001)}$ specifically being linked to 10202/101 in any mathematical or historical source.

Sources: Authoritative documentation of Uniswap ticks [1]; 1911 *Encyclopædia Britannica* on Napier and Bürgi [3] [4]; galilean manuscript images [2]; modern descriptions of golden-ratio base [6].

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[1] TickMath | Uniswap
https://docs.uniswap.org/contracts/v3/reference/core/libraries/TickMath

[2] Overview of Folio Page 185 r
https://www.mpiwg-berlin.mpg.de/Galileo_Prototype/HTML/F185_R/F185_R.HTM

[3] [5] 1911 Encyclopædia Britannica/Logarithm - Wikisource, the free online library
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[4] sam.math.ethz.ch
https://www.sam.math.ethz.ch/sam_reports/reports_final/reports2012/2012-43_fp.pdf

[6] Golden ratio base - Wikipedia
https://en.wikipedia.org/wiki/Golden_ratio_base