STAC50: Assignment 3

Deadline to hand in: Nov. 26th in class

Q. 1 For healthy individuals the level of prothrombin in the blood is approximately normally distributed with mean 20 mg/100 mL and standard deviation 4 mg/100 mL. Low levels indicate low clotting ability. In studying the effect of gallstones on prothrombin, the level of each patient in a sample is measured to see if there is a deficiency. Let μ be the true average level of prothrombin for gallstone patient.

(a) What are the appropriate null and alternative hypotheses?

(b) Let $\bar{X}$ denote the sample average level of prothrombin in a sample of $n = 20$ randomly selected gallstone patients. Consider the test procedure with test statistic $\bar{X}$ and rejection region $\bar{x} \leq 17.92$. What is the probability distribution of the test statistic when H_0 is true? What is the probability of type I error for the test procedure?

(c) What is the probability distribution of the test statistic when $\mu = 16.7$? Using the test procedure of part (b), what is the probability that gallstone patients will be judged not deficient in prothrombin, when in fact $\mu = 16.7$ (a type II error)?

(d) How would you change the test procedure of part (b) to obtain the test with significance level 0.05? What impact would this change have on the error probability of part (c)?

Q. 2 The diameter of a ball bearing was measured by 12 inspectors, each using two different kinds of calipers. Download caliper.txt from the class Blackboard.

(a) Identify the treatments and experimental units. Identify the type of design.

(b) Is there a significant difference between the means of the population of measurements represented by the two samples? Perform a t test at 5% level. Write down the null and alternative hypotheses.

(c) Answer the question in (a) by performing ANOVA and F test at 5% level.

(d) Construct a 95% confidence interval on the difference in the mean diameter measurements for the two types of calipers.

Q. 3 A scientist randomly and equally allocated $N = 65$ chicks to five diets (a control and four different diets). After a month each chick's calcium content (mg) in 2 cm length of bone is measured resulting in the following:

	Control	1	2	3	4
Mean	11.54	11.00	11.42	11.44	11.28
St.Dev	.27	.47	.31	.42	.31

Construct the ANOVA table (i.e. compute the between and within SS) and test if there appears to be any differences in means (use $\alpha = 0.01$).

Q. 4 Four different designs for a digital computer circuit are being studied to compare the amount of defects. Download defect.txt from the class Blackboard.

(a) Is the amount of defects present the same for all four designs? Use $\alpha = 0.05$.

(b) Check the model assumptions. In particular, how do you think about the normality assumption?

(c) Use the Kruskal-Wallis test for the data and compare the results with (a).

Q.5 An experiment is conducted to study the impact of hormone on the liver of rat. Two types of hormones (I, II) each with two levels are involved. We consider the following four treatments: (A) Hormone I at high level; (a) Hormone I at low level; (B) Hormone II at high level; (b) Hormone II at low level. Each treatment is applied to six randomly selected rats. The response is the amount of glycogen (in mg) in the liver of a rat after a certain period of time.

Treatment			Responses			
A	106	101	120	86	132	97
a	51	98	85	50	111	72
B	103	84	100	83	110	91
b	50	66	61	72	85	60

Suppose we are interested in the following three contrasts:

Comparison	A	a	B	b
Hormone I vs Hormone II	1	1	-1	-1
Low Level vs High level	1	-1	1	-1
Equivalence of Level	1	-1	-1	1

(a) Use ANOVA to check if there exist differences between the treatments (α =5%).

(b) If there is a difference among the four treatments, perform pairwise comparisons using Tukey's procedure.

(c) Show that the contrasts are orthogonal to each other.

(d) Use contrast sum of squares to test if the contrasts are significant($\alpha = 5\%$). Interpret the results.