Student Solutions Manual

to accompany

Applied Linear Regression Models

Fourth Edition

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PREFACE

This Student Solutions Manual gives intermediate and final numerical results for all starred (*) end-of-chapter Problems with computational elements contained in *Applied Linear Regression Models*, 4th edition. No solutions are given for Exercises, Projects, or Case Studies.

In presenting calculational results we frequently show, for ease in checking, more digits than are significant for the original data. Students and other users may obtain slightly different answers than those presented here, because of different rounding procedures. When a problem requires a percentile (e.g. of the t or F distributions) not included in the Appendix B Tables, users may either interpolate in the table or employ an available computer program for finding the needed value. Again, slightly different values may be obtained than the ones shown here.

The data sets for all Problems, Exercises, Projects and Case Studies are contained in the compact disk provided with the text to facilitate data entry. It is expected that the student will use a computer or have access to computer output for all but the simplest data sets, where use of a basic calculator would be adequate. For most students, hands-on experience in obtaining the computations by computer will be an important part of the educational experience in the course.

While we have checked the solutions very carefully, it is possible that some errors are still present. We would be most grateful to have any errors called to our attention. Errata can be reported via the website for the book: http://www.mhhe.com/KutnerALRM4e.

We acknowledge with thanks the assistance of Lexin Li and Yingwen Dong in the checking of this manual. We, of course, are responsible for any errors or omissions that remain.

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1.20. a. $\hat{Y} = -0.5802 + 15.0352X$

(4) MSE = 66.8

LINEAR REGRESSION WITH ONE PREDICTOR VARIABLE

d.
$$\hat{Y}_h = 74.5958$$

1.21. a. $\hat{Y} = 10.20 + 4.00X$
b. $\hat{Y}_h = 14.2$
c. 4.0
d. $(\bar{X}, \bar{Y}) = (1, 14.2)$

1.24. a. $\frac{i: \quad 1 \quad 2 \quad \dots \quad 44 \quad 45}{e_i: \quad -9.4903 \quad 0.4392 \quad \dots \quad 1.4392 \quad 2.4039}$

$$\sum e_i^2 = 3416.377$$

$$\min Q = \sum e_i^2$$
b. $MSE = 79.45063, \sqrt{MSE} = 8.913508, \text{ minutes}$

1.25. a. $e_1 = 1.8000$
b. $\sum e_i^2 = 17.6000, MSE = 2.2000, \sigma^2$

1.27. a. $\hat{Y} = 156.35 - 1.19X$
b. $(1) b_1 = -1.19, (2) \hat{Y}_h = 84.95, (3) e_8 = 4.4433, \sigma^2$

INFERENCES IN REGRESSION AND CORRELATION ANALYSIS

- 2.5. a. $t(.95; 43) = 1.6811, 15.0352 \pm 1.6811(.4831), 14.2231 \le \beta_1 \le 15.8473$
 - b. H_0 : $\beta_1=0,\ H_a$: $\beta_1\neq 0.\ t^*=(15.0352-0)/.4831=31.122.$ If $|t^*|\leq 1.681$ conclude H_0 , otherwise H_a . Conclude H_a . P-value=0+
 - c. Yes
 - d. H_0 : $\beta_1 \le 14$, H_a : $\beta_1 > 14$. $t^* = (15.0352 14)/.4831 = 2.1428$. If $t^* \le 1.681$ conclude H_0 , otherwise H_a . Conclude H_a . P-value= .0189
- 2.6. a. $t(.975; 8) = 2.306, b_1 = 4.0, s\{b_1\} = .469, 4.0 \pm 2.306(.469),$ $2.918 \le \beta_1 \le 5.082$
 - b. H_0 : $\beta_1 = 0$, H_a : $\beta_1 \neq 0$. $t^* = (4.0 0)/.469 = 8.529$. If $|t^*| \leq 2.306$ conclude H_0 , otherwise H_a . Conclude H_a . P-value= .00003
 - c. $b_0 = 10.20, s\{b_0\} = .663, 10.20 \pm 2.306(.663), 8.671 \le \beta_0 \le 11.729$
 - d. H_0 : $\beta_0 \le 9$, H_a : $\beta_0 > 9$. $t^* = (10.20 9)/.663 = 1.810$. If $t^* \le 2.306$ conclude H_0 , otherwise H_a . Conclude H_0 . P-value= .053
 - e. H_0 : $\beta_1 = 0$: $\delta = |2 0|/.5 = 4$, power = .93 H_0 : $\beta_0 \le 9$: $\delta = |11 - 9|/.75 = 2.67$, power = .78
- 2.14. a. $\hat{Y}_h = 89.6313$, $s\{\hat{Y}_h\} = 1.3964$, t(.95; 43) = 1.6811, $89.6313 \pm 1.6811(1.3964)$, $87.2838 \leq E\{Y_h\} \leq 91.9788$
 - b. $s\{\text{pred}\} = 9.0222, 89.6313 \pm 1.6811(9.0222), 74.4641 \le Y_{h(\text{new})} \le 104.7985, \text{ yes}, \text{ yes}$
 - c. 87.2838/6 = 14.5473, 91.9788/6 = 15.3298, $14.5473 \le \text{Mean time per machine} \le 15.3298$
 - d. $W^2 = 2F(.90; 2, 43) = 2(2.4304) = 4.8608, W = 2.2047, 89.6313 \pm 2.2047(1.3964),$ $86.5527 \le \beta_0 + \beta_1 X_h \le 92.7099$, yes, yes
- 2.15. a. $X_h = 2$: $\hat{Y}_h = 18.2$, $s\{\hat{Y}_h\} = .663$, t(.995; 8) = 3.355, $18.2 \pm 3.355(.663)$, $15.976 \le E\{Y_h\} \le 20.424$

$$X_h = 4$$
: $\hat{Y}_h = 26.2$, $s\{\hat{Y}_h\} = 1.483$, $26.2 \pm 3.355(1.483)$, $21.225 \le E\{Y_h\} \le 31.175$

- b. $s\{\text{pred}\} = 1.625, 18.2 \pm 3.355(1.625), 12.748 \le Y_{h(\text{new})} \le 23.652$
- c. $s\{\text{predmean}\}=1.083,\ 18.2\pm3.355(1.083),\ 14.567\leq \bar{Y}_{h(\text{new})}\leq 21.833,\ 44=3(14.567)\leq \text{Total number of broken ampules}\leq 3(21.833)=65$

d.
$$W^2 = 2F(.99; 2, 8) = 2(8.649) = 17.298, W = 4.159$$

 $X_h = 2$: $18.2 \pm 4.159(.663), 15.443 \le \beta_0 + \beta_1 X_h \le 20.957$
 $X_h = 4$: $26.2 \pm 4.159(1.483), 20.032 \le \beta_0 + \beta_1 X_h \le 32.368$

yes, yes

2.24. a.

Source	SS	df	MS
Regression	76,960.4	1	76,960.4
Error	3,416.38	43	79.4506
Total	80,376.78	44	

Source	SS	df	MS
Regression	76,960.4	1	76,960.4
Error	3,416.38	43	79.4506
Total	80,376.78	44	
Correction for mean	261,747.2	1	
Total, uncorrected	342,124	45	

b. H_0 : $\beta_1 = 0$, H_a : $\beta_1 \neq 0$. $F^* = 76,960.4/79.4506 = 968.66$, F(.90;1,43) = 2.826. If $F^* \leq 2.826$ conclude H_0 , otherwise H_a . Conclude H_a .

- c. 95.75% or 0.9575, coefficient of determination
- d. +.9785
- e. R^2

2.25. a.

Source	SS	df	MS
Regression	160.00	1	160.00
Error	17.60	8	2.20
Total	177.60	9	

- b. H_0 : $\beta_1 = 0$, H_a : $\beta_1 \neq 0$. $F^* = 160.00/2.20 = 72.727$, F(.95; 1, 8) = 5.32. If $F^* \leq 5.32$ conclude H_0 , otherwise H_a . Conclude H_a .
- c. $t^* = (4.00 0)/.469 = 8.529, (t^*)^2 = (8.529)^2 = 72.7 = F^*$
- d. $R^2 = .9009, r = .9492, 90.09\%$
- 2.27. a. H_0 : $\beta_1 \geq 0$, H_a : $\beta_1 < 0$. $s\{b_1\} = 0.090197$, $t^* = (-1.19 0)/.090197 = -13.193, \ t(.05; 58) = -1.67155.$ If $t^* \geq -1.67155$ conclude H_0 , otherwise H_a . Conclude H_a . P-value= 0+
 - c. $t(.975; 58) = 2.00172, -1.19 \pm 2.00172(.090197), -1.3705 \le \beta_1 \le -1.0095$

- 2.28. a. $\hat{Y}_h = 84.9468, \ s\{\hat{Y}_h\} = 1.05515, \ t(.975;58) = 2.00172,$ $84.9468 \pm 2.00172(1.05515), \ 82.835 \le E\{Y_h\} \le 87.059$
 - b. $s\{Y_{h(\text{new})}\} = 8.24101, 84.9468 \pm 2.00172(8.24101), 68.451 \le Y_{h(\text{new})} \le 101.443$
 - c. $W^2 = 2F(.95; 2, 58) = 2(3.15593) = 6.31186, W = 2.512342,$ $84.9468 \pm 2.512342(1.05515), 82.296 \le \beta_0 + \beta_1 X_h \le 87.598, \text{ yes, yes}$
- 2.29. a.

i:	1	2	 59	60
$Y_i - \hat{Y}_i$:	0.823243	-1.55675	 -0.666887	8.09309
$\hat{Y}_i - \bar{Y}$:	20.2101	22.5901	 -14.2998	-19.0598

b.

Source	SS	df	MS
Regression	11,627.5	1	11,627.5
Error	3,874.45	58	66.8008
Total	15,501.95	59	

- c. H_0 : $\beta_1 = 0$, H_a : $\beta_1 \neq 0$. $F^* = 11,627.5/66.8008 = 174.0623$, F(.90; 1,58) = 2.79409. If $F^* \leq 2.79409$ conclude H_0 , otherwise H_a . Conclude H_a .
- d. 24.993% or .24993
- e. $R^2 = 0.750067, r = -0.866064$
- 2.42. b. .95285, ρ_{12}
 - c. $H_0: \rho_{12} = 0, H_a: \rho_{12} \neq 0.$ $t^* = (.95285\sqrt{13})/\sqrt{1 (.95285)^2} = 11.32194,$ t(.995; 13) = 3.012. If $|t^*| \leq 3.012$ conclude H_0 , otherwise H_a . Conclude H_a .
 - d. No
- 2.44. a. $H_0: \rho_{12} = 0, H_a: \rho_{12} \neq 0. \ t^* = (.87\sqrt{101})/\sqrt{1 (.87)^2} = 17.73321, \ t(.95; 101) = 1.663.$ If $|t^*| \leq 1.663$ conclude H_0 , otherwise H_a . Conclude H_a .
 - b. z' = 1.33308, $\sigma\{z'\} = .1$, z(.95) = 1.645, $1.33308 \pm 1.645(.1)$, $1.16858 \le \zeta \le 1.49758$, $.824 \le \rho_{12} \le .905$
 - c. $.679 \le \rho_{12}^2 \le .819$
- 2.47. a. -0.866064,
 - b. $H_0: \rho_{12}=0, H_a: \rho_{12}\neq 0.$ $t^*=(-0.866064\sqrt{58})/\sqrt{1-(-0.866064)^2}=-13.19326, t(.975;58)=2.00172.$ If $|t^*|\leq 2.00172$ conclude H_0 , otherwise H_a . Conclude H_a .
 - c. -0.8657217
 - d. H_0 : There is no association between X and Y

 H_a : There is an association between X and Y

$$t^* = \frac{-0.8657217\sqrt{58}}{\sqrt{1 - (-0.8657217)^2}} = -13.17243.$$
 $t(0.975, 58) = 2.001717.$ If $|t^*| \le 1.001717$

2.001717, conclude H_0 , otherwise, conclude H_a . Conclude H_a .

DIAGNOSTICS AND REMEDIAL MEASURES

3.4. c and d.

i:	1	2	 44	45
\hat{Y}_i :	29.49034	59.56084	 59.56084	74.59608
e_i :	-9.49034	0.43916	 1.43916	2.40392

e.

Ascending order:	1	2	 44	45
Ordered residual:	-22.77232	-19.70183	 14.40392	15.40392
Expected value:	-19.63272	-16.04643	 16.04643	19.63272

 H_0 : Normal, H_a : not normal. r = 0.9891. If $r \geq .9785$ conclude H_0 , otherwise H_a . Conclude H_0 .

g. $SSR^* = 15, 155, SSE = 3416.38, X_{BP}^2 = (15, 155/2) \div (3416.38/45)^2 = 1.314676,$ $\chi^2(.95; 1) = 3.84$. If $X_{BP}^2 \le 3.84$ conclude error variance constant, otherwise error variance not constant. Conclude error variance constant.

3.5. c.

e.

 H_0 : Normal, H_a : not normal. r = .961. If $r \ge .879$ conclude H_0 , otherwise H_a . Conclude H_0 .

g. $SSR^* = 6.4$, SSE = 17.6, $X_{BP}^2 = (6.4/2) \div (17.6/10)^2 = 1.03$, $\chi^2(.90; 1) = 2.71$. If $X_{BP}^2 \le 2.71$ conclude error variance constant, otherwise error variance not constant. Conclude error variance constant.

Yes.

3.7. b and c.

i:	1	2	 59	60
e_i :	0.82324	-1.55675	 -0.66689	8.09309
\hat{Y}_i :	105.17676	107.55675	 70.66689	65.90691

d.

Ascending order:	1	2	 59	60
Ordered residual:	-16.13683	-13.80686	 13.95312	23.47309
Expected value:	-18.90095	-15.75218	 15.75218	18.90095

 H_0 : Normal, H_a : not normal. r=0.9897. If $r\geq 0.984$ conclude H_0 , otherwise H_a . Conclude H_0 .

e. $SSR^* = 31,833.4, SSE = 3,874.45,$

 $X_{BP}^2=(31,833.4/2)\div(3,874.45/60)^2=3.817116,~\chi^2(.99;1)=6.63.$ If $X_{BP}^2\leq6.63$ conclude error variance constant, otherwise error variance not constant. Conclude error variance constant. Yes.

3.13. a. H_0 : $E\{Y\} = \beta_0 + \beta_1 X$, H_a : $E\{Y\} \neq \beta_0 + \beta_1 X$

b. $SSPE = 2797.66, SSLF = 618.719, F^* = (618.719/8) \div (2797.66/35) = 0.967557,$ F(.95; 8, 35) = 2.21668. If $F^* \leq 2.21668$ conclude H_0 , otherwise H_a . Conclude H_0 .

3.17. b.

	λ :	.3	.4	.5	.6	.7
\overline{S}	SE:	1099.7	967.9	916.4	942.4	1044.2

c. $\hat{Y}' = 10.26093 + 1.07629X$

e.

i:	1	2	3	4	5
· ·	36				.30
$\hat{Y}_{i}^{'}$:	10.26	11.34	12.41	13.49	14.57
Expected value:	24	.14	.36	14	.24
i:	6	7	8	9	10
e_i :	41	.10	47	.47	07
e_i :		.10	47	.47	07

f. $\hat{Y} = (10.26093 + 1.07629X)^2$

SIMULTANEOUS INFERENCES AND OTHER TOPICS IN REGRESSION ANALYSIS

- 4.3. a. Opposite directions, negative tilt
 - b. $B = t(.9875; 43) = 2.32262, b_0 = -0.580157, s\{b_0\} = 2.80394, b_1 = 15.0352, s\{b_1\} = 0.483087$

$$-0.580157 \pm 2.32262(2.80394)$$
 $-7.093 \le \beta_0 \le 5.932$ $15.0352 \pm 2.32262(0.483087)$ $13.913 \le \beta_1 \le 16.157$

- c. Yes
- 4.4. a. Opposite directions, negative tilt
 - b. $B = t(.9975; 8) = 3.833, b_0 = 10.2000, s\{b_0\} = .6633, b_1 = 4.0000, s\{b_1\} = .4690$ $10.2000 \pm 3.833(.6633)$ $7.658 \le \beta_0 \le 12.742$ $4.0000 \pm 3.833(.4690)$ $2.202 \le \beta_1 \le 5.798$
- 4.6. a. $B = t(.9975; 14) = 2.91839, b_0 = 156.347, s\{b_0\} = 5.51226, b_1 = -1.190, s\{b_1\} = 0.0901973$

$$156.347 \pm 2.91839(5.51226)$$
 $140.260 \le \beta_0 \le 172.434$ $-1.190 \pm 2.91839(0.0901973)$ $-1.453 \le \beta_1 \le -0.927$

- b. Opposite directions
- c. No
- 4.7. a. F(.90; 2, 43) = 2.43041, W = 2.204727

$$X_h = 3$$
: $44.5256 \pm 2.204727(1.67501)$ $40.833 \le E\{Y_h\} \le 48.219$

$$X_h = 5$$
: $74.5961 \pm 2.204727(1.32983)$ $71.664 \le E\{Y_h\} \le 77.528$

$$X_h = 7$$
: $104.667 \pm 2.204727(1.6119)$ $101.113 \le E\{Y_h\} \le 108.221$

- b. F(.90; 2, 43) = 2.43041, S = 2.204727; B = t(.975; 43) = 2.01669; Bonferroni
- c. $X_h = 4$: $59.5608 \pm 2.01669 (9.02797)$ $41.354 \le Y_{h(\text{new})} \le 77.767$

$$X_h = 7$$
: $104.667 \pm 2.01669(9.05808)$ $86.3997 \le Y_{h(\text{new})} \le 122.934$

4.8. a.
$$F(.95; 2, 8) = 4.46, W = 2.987$$

$$X_h = 0$$
: $10.2000 \pm 2.987(.6633)$ $8.219 \le E\{Y_h\} \le 12.181$

$$X_h = 1$$
: $14.2000 \pm 2.987(.4690)$ $12.799 \le E\{Y_h\} \le 15.601$

$$X_h = 2$$
: $18.2000 \pm 2.987(.6633)$ $16.219 \le E\{Y_h\} \le 20.181$

b.
$$B = t(.99167; 8) = 3.016$$
, yes

c.
$$F(.95; 3, 8) = 4.07, S = 3.494$$

$$X_h = 0$$
: $10.2000 \pm 3.494(1.6248)$ $4.523 \le Y_{h(\text{new})} \le 15.877$

$$X_h = 1$$
: $14.2000 \pm 3.494(1.5556)$ $8.765 \le Y_{h(\text{new})} \le 19.635$

$$X_h = 2$$
: $18.2000 \pm 3.494(1.6248)$ $12.523 \le Y_{h(\text{new})} \le 23.877$

d.
$$B = 3.016$$
, yes

4.10. a.
$$F(.95; 2, 58) = 3.15593, W = 2.512342$$

$$X_h = 45$$
: $102.797 \pm 2.512342(1.71458)$ $98.489 \le E\{Y_h\} \le 107.105$

$$X_h = 55$$
: $90.8968 \pm 2.512342(1.1469)$ $88.015 \le E\{Y_h\} \le 93.778$

$$X_h = 65$$
: $78.9969 \pm 2.512342(1.14808)$ $76.113 \le E\{Y_h\} \le 81.881$

b.
$$B = t(.99167; 58) = 2.46556$$
, no

c.
$$B = 2.46556$$

$$X_h = 48: 99.2268 \pm 2.46556(8.31158) \quad 78.734 \le Y_{h(\text{new})} \le 119.720$$

$$X_h = 59$$
: $86.1368 \pm 2.46556(8.24148)$ $65.817 \le Y_{h(\text{new})} \le 106.457$

$$X_h = 74$$
: $68.2869 \pm 2.46556(8.33742)$ $47.730 \le Y_{h(\text{new})} \le 88.843$

d. Yes, yes

4.16. a.
$$\hat{Y} = 14.9472X$$

b.
$$s\{b_1\} = 0.226424, \ t(.95; 44) = 1.68023, \ 14.9472 \pm 1.68023(0.226424), \ 14.567 \le \beta_1 \le 15.328$$

c.
$$\hat{Y}_h = 89.6834$$
, $s\{\text{pred}\} = 8.92008$, $89.6834 \pm 1.68023(8.92008)$, $74.696 \le Y_{h(\text{new})} \le 104.671$

4.17. b.

No

c. H_0 : $E\{Y\} = \beta_1 X$, H_a : $E\{Y\} \neq \beta_1 X$. SSLF = 622.12, SSPE = 2797.66, $F^* = (622.12/9) \div (2797.66/35) = 0.8647783$, F(.99; 9, 35) = 2.96301. If $F^* \leq 2.96301$ conclude H_0 , otherwise H_a . Conclude H_0 . P-value = 0.564

MATRIX APPROACH TO SIMPLE LINEAR REGRESSION ANALYSIS

5.4. (1) 503.77 (2)
$$\begin{bmatrix} 5 & 0 \\ 0 & 160 \end{bmatrix}$$
 (3) $\begin{bmatrix} 49.7 \\ -39.2 \end{bmatrix}$

5.6. (1) 2,194 (2)
$$\begin{bmatrix} 10 & 10 \\ 10 & 20 \end{bmatrix}$$
 (3) $\begin{bmatrix} 142 \\ 182 \end{bmatrix}$

$$5.12. \qquad \left[\begin{array}{cc} .2 & 0 \\ 0 & .00625 \end{array} \right]$$

5.14. a.
$$\begin{bmatrix} 4 & 7 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 25 \\ 12 \end{bmatrix}$$

b.
$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 4.5 \\ 1 \end{bmatrix}$$

$$5.18. \text{ a.} \qquad \left[\begin{array}{c} W_1 \\ W_2 \end{array}\right] = \left[\begin{array}{ccc} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{array}\right] \left[\begin{array}{c} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{array}\right]$$

b.
$$\mathbf{E}\left\{ \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} \right\} = \begin{bmatrix} \frac{1}{4}[E\{Y_1\} + E\{Y_2\} + E\{Y_3\} + E\{Y_4\}] \\ \frac{1}{2}[E\{Y_1\} + E\{Y_2\} - E\{Y_3\} - E\{Y_4\}] \end{bmatrix}$$

c.
$$\boldsymbol{\sigma}^{2}\{\mathbf{W}\} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} \sigma^{2}\{Y_{1}\} & \sigma\{Y_{1}, Y_{2}\} & \sigma\{Y_{1}, Y_{3}\} & \sigma\{Y_{1}, Y_{4}\} \\ \sigma\{Y_{2}, Y_{1}\} & \sigma^{2}\{Y_{2}\} & \sigma\{Y_{2}, Y_{3}\} & \sigma\{Y_{2}, Y_{4}\} \\ \sigma\{Y_{3}, Y_{1}\} & \sigma\{Y_{3}, Y_{2}\} & \sigma^{2}\{Y_{3}\} & \sigma\{Y_{3}, Y_{4}\} \\ \sigma\{Y_{4}, Y_{1}\} & \sigma\{Y_{4}, Y_{2}\} & \sigma\{Y_{4}, Y_{3}\} & \sigma^{2}\{Y_{4}\} \end{bmatrix}$$

$$\times \begin{bmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2} \\ \frac{1}{4} & -\frac{1}{2} \\ \frac{1}{4} & -\frac{1}{2} \end{bmatrix}$$

Using the notation σ_1^2 for $\sigma^2\{Y_1\}, \sigma_{12}$ for $\sigma\{Y_1, Y_2\}$, etc., we obtain:

$$\sigma^2\{W_1\} = \frac{1}{16}(\sigma_1^2 + +\sigma_2^2 + \sigma_3^2 + \sigma_4^2 + 2\sigma_{12} + 2\sigma_{13} + 2\sigma_{14} + 2\sigma_{23} + 2\sigma_{24} + 2\sigma_{34})$$

$$\sigma^{2}\{W_{2}\} = \frac{1}{4}(\sigma_{1}^{2} + \sigma_{2}^{2} + \sigma_{3}^{2} + \sigma_{4}^{2} + 2\sigma_{12} - 2\sigma_{13} - 2\sigma_{14} - 2\sigma_{23} - 2\sigma_{24} + 2\sigma_{34})$$

$$\sigma\{W_{1}, W_{2}\} = \frac{1}{8}(\sigma_{1}^{2} + \sigma_{2}^{2} - \sigma_{3}^{2} - \sigma_{4}^{2} + 2\sigma_{12} - 2\sigma_{34})$$

$$5.19. \qquad \left[\begin{array}{cc} 3 & 5 \\ 5 & 17 \end{array}\right]$$

5.21. $5Y_1^2 + 4Y_1Y_2 + Y_2^2$

5.23. a.
$$(1)\begin{bmatrix} 9.940 \\ -.245 \end{bmatrix}$$
 $(2)\begin{bmatrix} -.18 \\ .04 \\ .26 \\ .08 \\ -.20 \end{bmatrix}$ $(3) 9.604$ $(4) .148$ $(5)\begin{bmatrix} .00987 & 0 \\ 0 & .000308 \end{bmatrix}$ $(6) 11.41$ $(7) .02097$

$$(5) \begin{bmatrix} .00987 & 0 \\ 0 & .000308 \end{bmatrix} \qquad (6) 11.41 \qquad (7) .02097$$

c.
$$\begin{bmatrix} .6 & .4 & .2 & 0 & -.2 \\ .4 & .3 & .2 & .1 & 0 \\ .2 & .2 & .2 & .2 & .2 \\ 0 & .1 & .2 & .3 & .4 \\ -.2 & 0 & .2 & .4 & .6 \end{bmatrix}$$

d.
$$\begin{bmatrix} .01973 & -.01973 & -.00987 & .00000 & .00987 \\ -.01973 & .03453 & -.00987 & -.00493 & .00000 \\ -.00987 & -.00987 & .03947 & -.00987 & -.00987 \\ .00000 & -.00493 & -.00987 & .03453 & -.01973 \\ .00987 & .00000 & -.00987 & -.01973 & .01973 \end{bmatrix}$$

5.25. a. (1)
$$\begin{bmatrix} .2 & -.1 \\ -.1 & .1 \end{bmatrix}$$
 (2) $\begin{bmatrix} 10.2 \\ 4.0 \end{bmatrix}$ (3) $\begin{bmatrix} 1.8 \\ -1.2 \\ 1.8 \\ -.2 \\ -1.2 \\ -2.2 \\ .8 \\ .8 \\ .8 \end{bmatrix}$

(5)
$$17.60$$
 (6) $\begin{bmatrix} .44 & -.22 \\ -.22 & .22 \end{bmatrix}$ (7) 18.2 (8) $.44$

b.
$$(1) .22$$
 $(2) -.22$ $(3) .663$

MULTIPLE REGRESSION - I

6.9. c.
$$\begin{array}{c} Y \\ X_1 \\ X_2 \\ X_3 \end{array} \left[\begin{array}{cccccc} 1.0000 & .2077 & .0600 & .8106 \\ & 1.0000 & .0849 & .0457 \\ & & 1.0000 & .1134 \\ & & & & 1.0000 \end{array} \right]$$

6.10. a. $\hat{Y} = 4149.89 + 0.000787X_1 - 13.166X_2 + 623.554X_3$ b&c.

- e. $n_1 = 26$, $\bar{d}_1 = 145.0$, $n_2 = 26$, $\bar{d}_2 = 77.4$, s = 81.7, $t_{BF}^* = (145.0 77.4)/[81.7\sqrt{(1/26) + (1/26)}] = 2.99$, t(.995; 50) = 2.67779. If $|t_{BF}^*| \leq 2.67779$ conclude error variance constant, otherwise error variance not constant. Conclude error variance not constant.
- 6.11. a. H_0 : $\beta_1 = \beta_2 = \beta_3 = 0$, H_a : not all $\beta_k = 0$ (k = 1, 2,3). MSR = 725, 535, MSE = 20, 531.9, $F^* = 725, 535/20, 531.9 = 35.337$, F(.95; 3, 48) = 2.79806. If $F^* \leq 2.79806$ conclude H_0 , otherwise H_a . Conclude H_a . P-value = 0+.

b.
$$s\{b_1\} = .000365, s\{b_3\} = 62.6409, B = t(.9875; 48) = 2.3139$$

 $0.000787 \pm 2.3139(.000365) - .000058 \le \beta_1 \le 0.00163$
 $623.554 \pm 2.3139(62.6409)$ $478.6092 \le \beta_3 \le 768.4988$

- c. $SSR = 2,176,606, SSTO = 3,162,136, R^2 = .6883$
- 6.12. a. F(.95; 4, 48) = 2.56524, W = 3.2033; B = t(.995; 48) = 2.6822

X_{h1}	X_{h2}	X_{h3}		
302,000	7.2	0:	$4292.79 \pm 2.6822(21.3567)$	$4235.507 \le E\{Y_h\} \le 4350.073$
245,000	7.4	0:	$4245.29 \pm 2.6822(29.7021)$	$4165.623 \le E\{Y_h\} \le 4324.957$
280,000	6.9	0:	$4279.42 \pm 2.6822(24.4444)$	$4213.855 \le E\{Y_h\} \le 4344.985$
350,000	7.0	0:	$4333.20 \pm 2.6822(28.9293)$	$4255.606 \le E\{Y_h\} \le 4410.794$
295,000	6.7	1:	$4917.42 \pm 2.6822 (62.4998)$	$4749.783 \le E\{Y_h\} \le 5085.057$
b.Yes, no				

$$\begin{aligned} F(.95;4,48) &= 2.5652, \ S = 3.2033; \ B = t(.99375;48) = 2.5953 \\ \hline \frac{X_{h1} \quad X_{h2} \quad X_{h3}}{230,000 \quad 7.5 \quad 0:} \quad 4232.17 \pm 2.5953(147.288) \quad 3849.913 \leq Y_{h(\text{new})} \leq 4614.427 \\ 250,000 \quad 7.3 \quad 0: \quad 4250.55 \pm 2.5953(146.058) \quad 3871.486 \leq Y_{h(\text{new})} \leq 4629.614 \\ 280,000 \quad 7.1 \quad 0: \quad 4276.79 \pm 2.5953(145.134) \quad 3900.124 \leq Y_{h(\text{new})} \leq 4653.456 \\ 340,000 \quad 6.9 \quad 0: \quad 4326.65 \pm 2.5953(145.930) \quad 3947.918 \leq Y_{h(\text{new})} \leq 4705.382 \end{aligned}$$

- 6.14. a. $\hat{Y}_h = 4278.37$, $s\{\text{predmean}\} = 85.82262$, t(.975; 48) = 2.01063, $4278.37 \pm 2.01063(85.82262)$, $4105.812 \le \bar{Y}_{h(\text{new})} \le 4450.928$
 - b. $12317.44 \le \text{Total labor hours} \le 13352.78$

c. $\hat{Y} = 158.491 - 1.1416X_1 - 0.4420X_2 - 13.4702X_3$

d&e.

$$i:$$
 1 2 ... 45 46
 $e_i:$.1129 -9.0797 ... -5.5380 10.0524
Expected Val.: -0.8186 -8.1772 ... -5.4314 8.1772

- f. No
- g. $SSR^* = 21,355.5$, SSE = 4,248.8, $X_{BP}^2 = (21,355.5/2) \div (4,248.8/46)^2 = 1.2516$, $\chi^2(.99;3) = 11.3449$. If $X_{BP}^2 \le 11.3449$ conclude error variance constant, otherwise error variance not constant. Conclude error variance constant.
- 6.16. a. H_0 : $\beta_1 = \beta_2 = \beta_3 = 0$, H_a : not all $\beta_k = 0$ (k = 1, 2, 3). $MSR = 3,040.2, \ MSE = 101.2, \ F^* = 3,040.2/101.2 = 30.05, \ F(.90; 3, 42) = 2.2191$. If $F^* \leq 2.2191$ conclude H_0 , otherwise H_a . Conclude H_a . P-value = 0.4878

b.
$$s\{b_1\} = .2148, \ s\{b_2\} = .4920, \ s\{b_3\} = 7.0997, \ B = t(.9833; 42) = 2.1995$$

 $-1.1416 \pm 2.1995(.2148)$ $-1.6141 \le \beta_1 \le -0.6691$
 $-.4420 \pm 2.1995(.4920)$ $-1.5242 \le \beta_2 \le 0.6402$
 $-13.4702 \pm 2.1995(7.0997)$ $-29.0860 \le \beta_3 \le 2.1456$

- c. SSR = 9,120.46, SSTO = 13,369.3, R = .8260
- 6.17. a. $\hat{Y}_h = 69.0103, \ s\{\hat{Y}_h\} = 2.6646, \ t(.95; 42) = 1.6820, \ 69.0103 \pm 1.6820(2.6646), \ 64.5284 \leq E\{Y_h\} \leq 73.4922$
 - b. $s\{\text{pred}\} = 10.405, 69.0103 \pm 1.6820(10.405), 51.5091 \le Y_{h(\text{new})} \le 86.5115$

MULTIPLE REGRESSION – II

- 7.4. a. $SSR(X_1) = 136,366, SSR(X_3|X_1) = 2,033,566, SSR(X_2|X_1,X_3) = 6,674, SSE(X_1,X_2,X_3) = 985,530, df$: 1, 1, 1,48.
 - b. H_0 : $\beta_2 = 0$, H_a : $\beta_2 \neq 0$. $SSR(X_2|X_1, X_3) = 6,674$, $SSE(X_1, X_2, X_3) = 985,530$, $F^* = (6,674/1) \div (985,530/48) = 0.32491$, F(.95;1,17) = 4.04265. If $F^* \leq 4.04265$ conclude H_0 , otherwise H_a . Conclude H_0 . P-value = 0.5713.
 - c. Yes, $SSR(X_1) + SSR(X_2|X_1) = 136,366 + 5,726 = 142,092$, $SSR(X_2) + SSR(X_1|X_2) = 11,394.9 + 130,697.1 = 142,092$. Yes.
- 7.5. a. $SSR(X_2) = 4,860.26, SSR(X_1|X_2) = 3,896.04, SSR(X_3|X_2,X_1) = 364.16, SSE(X_1, X_2, X_3) = 4,248.84, df: 1, 1, 1, 42$
 - b. H_0 : $\beta_3 = 0$, H_a : $\beta_3 \neq 0$. $SSR(X_3|X_1, X_2) = 364.16$, $SSE(X_1, X_2, X_3) = 4,248.84$, $F^* = (364.16/1) \div (4,248.84/42) = 3.5997$, F(.975;1,42) = 5.4039. If $F^* \leq 5.4039$ conclude H_0 , otherwise H_a . Conclude H_0 . P-value = 0.065.
- 7.6. H_0 : $\beta_2 = \beta_3 = 0$, H_a : not both β_2 and $\beta_3 = 0$. $SSR(X_2, X_3 | X_1) = 845.07$, $SSE(X_1, X_2, X_3) = 4,248.84$, $F^* = (845.07/2) \div (4,248.84/42) = 4.1768$, F(.975; 2,42) = 4.0327. If $F^* \le 4.0327$ conclude H_0 , otherwise H_a . Conclude H_a . P-value = 0.022.
- 7.9. H_0 : $\beta_1 = -1.0$, $\beta_2 = 0$; H_a : not both equalities hold. Full model: $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \varepsilon_i$, reduced model: $Y_i + X_{i1} = \beta_0 + \beta_3 X_{i3} + \varepsilon_i$. SSE(F) = 4,248.84, $df_F = 42$, SSE(R) = 4,427.7, $df_R = 44$, $F^* = [(4427.7 4248.84)/2] \div (4,248.84/42) = .8840$, F(.975; 2,42) = 4.0327. If $F^* \leq 4.0327$ conclude H_0 , otherwise H_a . Conclude H_0 .
- 7.13. $R_{Y1}^2 = .0431, \ R_{Y2}^2 = .0036, \ R_{12}^2 = .0072, \ R_{Y1|2}^2 = 0.0415, \ R_{Y2|1}^2 = 0.0019, \ R_{Y2|13}^2 = .0067 \ R^2 = .6883$
- 7.14. a. $R_{Y1}^2 = .6190, R_{Y1|2}^2 = .4579, R_{Y1|23}^2 = .4021$
 - b. $R_{Y2}^2 = .3635, R_{Y2|1}^2 = .0944, R_{Y2|13}^2 = .0189$
- 7.17. a. $\hat{Y}^* = .17472X_1^* .04639X_2^* + .80786X_3^*$
 - b. $R_{12}^2 = .0072, R_{13}^2 = .0021, R_{23}^2 = .0129$

c.
$$s_Y = 249.003, s_1 = 55274.6, s_2 = .87738, s_3 = .32260 \ b_1 = \frac{249.003}{55274.6}(.17472) = .00079, \ b_2 = \frac{249.003}{.87738}(-.04639) = -13.16562, \ b_3 = \frac{249.003}{5.32260}(.80786) = 623.5572, \ b_0 = 4363.04 - .00079(302,693) + 13.16562(7.37058) - 623.5572(0.115385) = 4149.002.$$

7.18. a.
$$\hat{Y}^* = -.59067X_1^* - .11062X_2^* - .23393X_3^*$$

b.
$$R_{12}^2 = .32262, R_{13}^2 = .32456, R_{23}^2 = .44957$$

c.
$$s_Y = 17.2365, s_1 = 8.91809, s_2 = 4.31356, s_3 = .29934, b_1 = \frac{17.2365}{8.91809}(-.59067) = -1.14162, b_2 = \frac{17.2365}{4.31356}(-.11062) = -.44203, b_3 = \frac{17.2365}{.29934}(-.23393) = -13.47008, b_0 = 61.5652 + 1.14162(38.3913) + .44203(50.4348) + 13.47008(2.28696) = 158.4927$$

7.25. a.
$$\hat{Y} = 4079.87 + 0.000935X_2$$

c. No,
$$SSR(X_1) = 136,366, SSR(X_1|X_2) = 130,697$$

d.
$$r_{12} = .0849$$

7.26. a.
$$\hat{Y} = 156.672 - 1.26765X_1 - 0.920788X_2$$

c. No,
$$SSR(X_1) = 8,275.3$$
, $SSR(X_1|X_3) = 3,483.89$
No, $SSR(X_2) = 4,860.26$, $SSR(X_2|X_3) = 708$

d.
$$r_{12} = .5680, r_{13} = .5697, r_{23} = .6705$$

MODELS FOR QUANTITATIVE AND QUALITATIVE PREDICTORS

- 8.4. a. $\hat{Y} = 82.9357 1.18396x + .0148405x^2$, $R^2 = .76317$
 - b. H_0 : $\beta_1 = \beta_{11} = 0$, H_a : not both β_1 and $\beta_{11} = 0$. MSR = 5915.31, MSE = 64.409, $F^* = 5915.31/64.409 = 91.8398$, F(.95; 2, 57) = 3.15884. If $F^* \leq 3.15884$ conclude H_0 , otherwise H_a . Conclude H_a .
 - c. $\hat{Y}_h = 99.2546$, $s\{\hat{Y}_h\} = 1.4833$, t(.975; 57) = 2.00247, $99.2546 \pm 2.00247(1.4833)$, $96.2843 \le E\{Y_h\} \le 102.2249$
 - d. $s\{\text{pred}\} = 8.16144, 99.2546 \pm 2.00247(8.16144), 82.91156 \le Y_{h(\text{new})} \le 115.5976$
 - e. H_0 : $\beta_{11} = 0$, H_a : $\beta_{11} \neq 0$. $s\{b_{11}\} = .00836$, $t^* = .0148405/.00836 = 1.7759$, t(.975;57) = 2.00247. If $|t^*| \leq 2.00247$ conclude H_0 , otherwise H_a . Conclude H_0 . Alternatively, $SSR(x^2|x) = 203.1$, $SSE(x,x^2) = 3671.31$, $F^* = (203.1/1) \div (3671.31/57) = 3.15329$, F(.95;1,57) = 4.00987. If $F^* \leq 4.00987$ conclude H_0 , otherwise H_a . Conclude H_0 .
 - f. $\hat{Y} = 207.350 2.96432X + .0148405X^2$
 - g. $r_{X,X^2} = .9961, r_{x,x^2} = -.0384$
- 8.5. a. $\frac{i:}{e_i:}$ 1 2 3 ... 58 59 60 $\frac{1}{e_i:}$ -1.3238 -4.7592 -3.8091 ... -11.7798 -.8515 6.22023
 - b. H_0 : $E\{Y\} = \beta_0 + \beta_1 x + \beta_{11} x^2$, H_a : $E\{Y\} \neq \beta_0 + \beta_1 x + \beta_{11} x^2$. MSLF = 62.8154, MSPE = 66.0595, $F^* = 62.8154/66.0595 = 0.95$, F(.95; 29, 28) = 1.87519. If $F^* \leq 1.87519$ conclude H_0 , otherwise H_a . Conclude H_0 .
 - c. $\hat{Y} = 82.92730 1.26789x + .01504x^2 + .000337x^3$
 - H_0 : $\beta_{111} = 0$, H_a : $\beta_{111} \neq 0$. $s\{b_{111}\} = .000933$, $t^* = .000337/.000933 = .3612$, t(.975;56) = 2.00324. If $|t^*| \leq 2.00324$ conclude H_0 , otherwise H_a . Conclude H_0 . Yes. Alternatively, $SSR(x^3|x,x^2) = 8.6$, $SSE(x,x^2,x^3) = 3662.78$, $F^* = (8.6/1) \div (3662.78/56) = .13148$, F(.95;1,56) = 4.01297. If $F^* \leq 4.01297$ conclude H_0 , otherwise H_a . Conclude H_0 . Yes.
- 8.19. a. $\hat{Y} = 2.81311 + 14.3394X_1 8.14120X_2 + 1.77739X_1X_2$

b. $H_0: \beta_3 = 0, H_a: \beta_3 \neq 0.$ $s\{b_3\} = .97459, t^* = 1.77739/.97459 = 1.8237, t(.95; 41) = 1.68288.$ If $|t^*| \leq 1.68288$ conclude H_0 , otherwise H_a . Conclude H_a . Alternatively, $SSR(X_1X_2|X_1,X_2) = 255.9, SSE(X_1,X_2,X_1X_2) = 3154.44, F^* = (255.9/1) \div (3154.44/41) = 3.32607, F(.90; 1, 41) = 2.83208.$ If $F^* \leq 2.83208$ conclude H_0 , otherwise H_a . Conclude H_a .

BUILDING THE REGRESSION MODEL I: MODEL SELECTION AND VALIDATION

	Variables in Model	R_p^2	AIC_p	C_p	$PRESS_p$
	None	0	262.916	88.16	13,970.10
	X_1	.6190	220.529	8.35	$5,\!569.56$
	X_2	.3635	244.131	42.11	9,254.49
9.9.	X_3	.4155	240.214	35.25	8,451.43
	X_1, X_2	.6550	217.968	5.60	5,235.19
	X_1, X_3	.6761	215.061	2.81	4,902.75
	X_2, X_3	.4685	237.845	30.25	8,115.91
	X_1, X_2, X_3	.6822	216.185	4.00	5,057.886

9.10. b.

c.
$$\hat{Y} = -124.3820 + .2957X_1 + .0483X_2 + 1.3060X_3 + .5198X_4$$

9.11. a.

Subset	$R_{a,p}^2$
X_1, X_3, X_4	.9560
X_1, X_2, X_3, X_4	.9555
X_1, X_3	.9269
X_1, X_2, X_3	.9247

9.17. a.
$$X_1, X_3$$

c.
$$X_1, X_3$$

d.
$$X_1, X_3$$

9.18. a.
$$X_1, X_3, X_4$$

9.21.
$$PRESS = 760.974, SSE = 660.657$$

9.22. a.

b.

	Model-building	Validation
	data set	data set
$\overline{b_0}$:	-127.596	-130.652
$s\{b_0\}$:	12.685	12.189
b_1 :	.348	.347
$s\{b_1\}$:	.054	.048
b_3 :	1.823	1.848
$s\{b_3\}$:	.123	.122
MSE:	27.575	21.446
R^2 :	.933	.937

c.
$$MSPR = 486.519/25 = 19.461$$

d.
$$\hat{Y} = -129.664 + .349X_1 + 1.840X_3, s\{b_0\} = 8.445, s\{b_1\} = .035, s\{b_3\} = .084$$

BUILDING THE REGRESSION MODEL II: DIAGNOSTICS

10.10.a&f.

t(.9995192;47)=3.523. If $|t_i|\leq 3.523$ conclude no outliers, otherwise outliers. Conclude no outliers.

b.
$$2p/n = 2(4)/52 = .15385$$
. Cases 3, 5, 16, 21, 22, 43, 44, and 48.

c.
$$\mathbf{X}'_{\text{new}} = [1 \quad 300,000 \quad 7.2 \quad 0]$$

$$(\mathbf{X}'\mathbf{X})^{-1} = \begin{bmatrix} 1.8628 & -.0000 & -.1806 & .0473 \\ & .0000 & -.0000 & -.0000 \\ & & .0260 & -.0078 \\ & & & .1911 \end{bmatrix}$$

 $h_{\text{new, new}} = .01829$, no extrapolation

d.

		DFBETAS						
	DFFITS	b_0	b_1	b_2	b_3	D		
Case 16:	554	2477	0598	.3248	4521	.0769		
Case 22:	.055	.0304	0253	0107	.0446	.0008		
Case 43:	.562	3578	.1338	.3262	.3566	.0792		
Case 48:	147	.0450	0938	.0090	1022	.0055		
Case 10:	.459	.3641	1044	3142	0633	.0494		
Case 32:	651	.4095	.0913	5708	.1652	.0998		
Case 38:	.386	0996	0827	.2084	1270	.0346		
Case 40 :	.397	.0738	2121	.0933	1110	.0365		

e. Case 16: .161%, case 22: .015%, case 43: .164%, case 48: .042%, case 10: .167%, case 32: .227%, case 38: .152%, case 40: .157%.

t(.998913;41) = 3.27. If $|t_i| \le 3.27$ conclude no outliers, otherwise outliers. Conclude no outliers.

b.
$$2p/n = 2(4)/46 = .1739$$
. Cases 9, 28, and 39.

c.
$$\mathbf{X}'_{\text{new}} = [1 \ 30 \ 58 \ 2.0]$$

$$(\mathbf{X}'\mathbf{X})^{-1} = \begin{bmatrix} 3.24771 & .00922 & -.06793 & -.06730 \\ & .00046 & -.00032 & -.00466 \\ & & .00239 & -.01771 \\ & & & .49826 \end{bmatrix}$$

 $h_{\text{new, new}} = .3267$, extrapolation

d.

		DFBETAS						
	DFFITS	b_0	b_1	b_2	b_3	D		
Case 11:	.5688	.0991	3631	1900	.3900	.0766		
Case 17:	.6657	4491	4711	.4432	.0893	.1051		
Case 27:	6087	0172	.4172	2499	.1614	.0867		

e. Case 11: 1.10%, case 17: 1.32%, case 27: 1.12%.

10.16.b.
$$(VIF)_1 = 1.0086, (VIF)_2 = 1.0196, (VIF)_3 = 1.0144.$$

10.17.b.
$$(VIF)_1 = 1.6323, (VIF)_2 = 2.0032, (VIF)_3 = 2.0091$$

10.21a.
$$(VIF)_1 = 1.305, (VIF)_2 = 1.300, (VIF)_3 = 1.024$$

b&c.

i:	1	2	3	 32	33
e_i :	13.181	-4.042	3.060	 14.335	1.396
$e(Y \mid X_2, X_3)$:	26.368	-2.038	-31.111	 6.310	5.845
$e(X_1 \mid X_2, X_3)$:	330	050	.856	 .201	.111
$e(Y \mid X_1, X_3)$:	18.734	-17.470	8.212	 12.566	-8.099
$e(X_2 \mid X_1, X_3)$:	-7.537	18.226	-6.993	 2.401	12.888
$e(Y \mid X_1, X_2)$:	11.542	-7.756	15.022	 6.732	-15.100
$e(X_3 \mid X_1, X_2)$:	-2.111	-4.784	15.406	 -9.793	-21.247
Exp. value:	11.926	-4.812	1.886	 17.591	940

10.22a.
$$\hat{Y}' = -2.0427 - .7120X_1' + .7474X_2' + .7574X_3', \text{ where } Y' = \log_e Y, X_1' = \log_e X_1, \\ X_2' = \log_e (140 - X_2), X_3' = \log_e X_3$$

b.

$$i$$
: 1 2 3 \cdots 31 32 33 e_i : $-.0036$ 0.005 0.00

c.
$$(VIF)_1 = 1.339, (VIF)_2 = 1.330, (VIF)_3 = 1.016$$

d&e.

t(.9985;28)=3.25. If $|t_i|\leq 3.25$ conclude no outliers, otherwise outliers. Conclude no outliers.

f.

		DFBETAS					
Case	DFFITS	b_0	b_1	b_2	b_3	D	
28	.739	.530	151	577	187	.120	
29	719	197	310	133	.420	.109	

BUILDING THE REGRESSION MODEL III: REMEDIAL MEASURES

b. $SSR^* = 123,753.125, SSE = 2,316.500,$ $X_{BP}^2 = (123,753.125/2)/(2,316.500/12)^2 = 1.66, \ \chi^2(.90;1) = 2.71.$ If $X_{BP}^2 \leq 2.71$ conclude error variance constant, otherwise error variance not constant. Conclude error variance constant.

e.
$$\hat{Y} = -6.2332 + .1891X$$

f.

$$\hat{Y} = -6.2335 + .1891X$$

11.10a.
$$\hat{Y} = 3.32429 + 3.76811X_1 + 5.07959X_2$$

d.
$$c = .07$$

e.
$$\hat{Y} = 6.06599 + 3.84335X_1 + 4.68044X_2$$

11.11a.
$$\hat{Y} = 1.88602 + 15.1094X \text{ (47 cases)}$$

$$\hat{Y} = -.58016 + 15.0352X$$
 (45 cases)

b. $\frac{i:}{u_i:}$ 1 2 ... 46 47 $\frac{1}{u_i:}$ -1.4123 -.2711 ... 4.6045 10.3331

smallest weights: .13016 (case 47), .29217 (case 46)

- c. $\hat{Y} = -.9235 + 15.13552X$
- d. 2nd iteration: $\hat{Y} = -1.535 + 15.425X$

3rd iteration: $\hat{Y} = -1.678 + 15.444X$

smallest weights: .12629 (case 47), .27858 (case 46)

AUTOCORRELATION IN TIME SERIES DATA

12.6. $H_0: \rho = 0, H_a: \rho > 0.$ $D = 2.4015, d_L = 1.29, d_U = 1.38.$ If D > 1.38 conclude H_0 , if D < 1.29 conclude H_a , otherwise the test is inconclusive. Conclude H_0 .

12.9. a.
$$\hat{Y} = -7.7385 + 53.9533X$$
, $s\{b_0\} = 7.1746$, $s\{b_1\} = 3.5197$

c. $H_0: \rho = 0, H_a: \rho > 0.$ $D = .857, d_L = 1.10, d_U = 1.37.$ If D > 1.37 conclude H_0 , if D < 1.10 conclude H_a , otherwise the test is inconclusive. Conclude H_a .

12.10. a.
$$r = .5784$$
, $2(1 - .5784) = .8432$, $D = .857$

b.
$$b'_0 = -.69434, b'_1 = 50.93322$$

$$\hat{Y}' = -.69434 + 50.93322X'$$

$$s\{b_0'\}=3.75590,\,s\{b_1'\}=4.34890$$

c. $H_0: \rho = 0, H_a: \rho > 0.$ $D = 1.476, d_L = 1.08, d_U = 1.36.$ If D > 1.36 conclude H_0 , if D < 1.08 conclude H_a , otherwise the test is inconclusive. Conclude H_0 .

d.
$$\hat{Y} = -1.64692 + 50.93322X$$

$$s\{b_0\} = 8.90868, s\{b_1\} = 4.34890$$

- f. $F_{17} = -1.64692 + 50.93322(2.210) + .5784(-.6595) = 110.534$, $s\{\text{pred}\} = .9508$, t(.975; 13) = 2.160, $110.534 \pm 2.160(.9508)$, $108.48 \le Y_{17(\text{new})} \le 112.59$
- g. $t(.975; 13) = 2.160, 50.93322 \pm 2.160(4.349), 41.539 \le \beta_1 \le 60.327.$

12.11. a.
$$\frac{\rho\colon \ .1}{SSE\colon \ 11.5073} \ \ \frac{.2}{10.4819} \ \ \frac{.3}{9.6665} \ \ \frac{.4}{9.0616} \ \ \frac{.5}{8.6710}$$

$$\frac{\rho\colon \ .6}{SSE\colon \ 8.5032} \ \ \frac{.7}{8.5718} \ \ \frac{.9}{8.8932} \ \ \frac{.10}{9.4811} \ \ \frac{.03408}{10.3408}$$

$$\rho=.6$$

- b. $\hat{Y}' = -.5574 + 50.8065X', s\{b'_0\} = 3.5967, s\{b'_1\} = 4.3871$
- c. $H_0: \rho = 0, H_a: \rho > 0.$ $D = 1.499, d_L = 1.08, d_U = 1.36.$ If D > 1.36 conclude H_0 , if D < 1.08 conclude H_a , otherwise test is inconclusive. Conclude H_0 .
- d. $\hat{Y} = -1.3935 + 50.8065X$, $s\{b_0\} = 8.9918$, $s\{b_1\} = 4.3871$
- f. $F_{17} = -1.3935 + 50.8065(2.210) + .6(-.6405) = 110.505$, $s\{\text{pred}\} = .9467$, t(.975; 13) = 2.160, $110.505 \pm 2.160(.9467)$, $108.46 \le Y_{17(\text{new})} \le 112.55$
- 12.12. a. $b_1 = 49.80564$, $s\{b_1\} = 4.77891$
 - b. $H_0: \rho=0,\ H_a: \rho\neq 0.\ D=1.75$ (based on regression with intercept term), $d_L=1.08,\ d_U=1.36.$ If D>1.36 and 4-D>1.36 conclude H_0 , if D<1.08 or 4-D<1.08 conclude H_a , otherwise the test is inconclusive. Conclude H_0 .
 - c. $\hat{Y} = .71172 + 49.80564X$, $s\{b_1\} = 4.77891$
 - e. $F_{17} = .71172 + 49.80564(2.210) .5938 = 110.188$, $s\{pred\} = .9078$, t(.975; 14) = 2.145, $110.188 \pm 2.145(.9078)$, $108.24 \le Y_{17(new)} \le 112.14$
 - f. $t(.975; 14) = 2.145, 49.80564 \pm 2.145(4.77891), 39.555 \le \beta_1 \le 60.056$

INTRODUCTION TO NONLINEAR REGRESSION AND NEURAL NETWORKS

13.1. a. Intrinsically linear

$$\log_e f(\mathbf{X}, \, \boldsymbol{\gamma}) = \gamma_0 + \gamma_1 X$$

- b. Nonlinear
- c. Nonlinear
- 13.3. b. 300, 3.7323
- 13.5. a. $b_0 = -.5072512$, $b_1 = -0.0006934571$, $g_0^{(0)} = 0$, $g_1^{(0)} = .0006934571$, $g_2^{(0)} = .6021485$ b. $g_0 = .04823$, $g_1 = .00112$, $g_2 = .71341$
- 13.6. a. $\hat{Y} = .04823 + .71341 \exp(-.00112X)$

	City A							
i:	1	2	3	4	5			
\hat{Y}_i :	.61877	.50451	.34006	.23488	.16760			
e_i :	.03123	04451	00006	.02512	.00240			
Exp. value:	.04125	04125	00180	.02304	.00180			
i:	6	7	8					
\hat{Y}_i :	.12458	.07320	.05640	•				
e_i :	.02542	01320	01640					
Exp. value:	.02989	01777	02304					
		City I	3					
i:	9	10	11	12	13			
\hat{Y}_i :	.61877	.50451	.34006	.23488	.16760			
e_i :	.01123	00451	04006	.00512	.02240			
Exp. value:	.01327	00545	02989	.00545	.01777			

13.7. $H_0: E\{Y\} = \gamma_0 + \gamma_2 \exp(-\gamma_1 X), H_a: E\{Y\} \neq \gamma_0 + \gamma_2 \exp(-\gamma_1 X).$

SSPE = .00290, SSE = .00707, MSPE = .00290/8 = .0003625,

 $MSLF = (.00707 - .00290)/5 = .000834, F^* = .000834/.0003625 = 2.30069, F(.99; 5, 8) = 6.6318.$ If $F^* \le 6.6318$ conclude H_0 , otherwise H_a . Conclude H_0 .

13.8. $s\{g_0\} = .01456, s\{g_1\} = .000092, s\{g_2\} = .02277, z(.9833) = 2.128$

$$.04823 \pm 2.128(.01456)$$

$$.01725 \le \gamma_0 \le .07921$$

$$.00112 \pm 2.128(.000092)$$

$$.00092 \le \gamma_1 \le .00132$$

$$.71341 \pm 2.128(.02277)$$

$$.66496 \le \gamma_2 \le .76186$$

13.9. a. $g_0 = .04948$, $g_1 = .00112$, $g_2 = .71341$, $g_3 = -.00250$

b. z(.975) = 1.96, $s\{g_3\} = .01211$, $-.00250 \pm 1.96(.01211)$, $-.02624 \le \gamma_3 \le .02124$, yes, no.

13.13. $g_0 = 100.3401, g_1 = 6.4802, g_2 = 4.8155$

13.14. a. $\hat{Y} = 100.3401 - 100.3401/[1 + (X/4.8155)^{6.4802}]$

b.

i:	1	2	3	4	5	6	7	
\hat{Y}_i :	.0038	.3366	4.4654 11	1.2653 1	1.2653	23.1829	23.1829	
e_i :	.4962	1.9634 -	1.0654	.2347 -	3653	.8171	2.1171	
Expected Val.:	.3928	1.6354 -	1.0519 -	1947 -	5981	.8155	2.0516	
i:	8	9	10	11	12	13	3 14	
\hat{Y}_i :	39.3272	39.3272	56.2506	56.2506	70.530	08 70.5	308 80.88	76
e_i :	.2728	-1.4272	-1.5506	.5494	.269	-2.1	308 1.21	24
Expected Val.:	.1947	-1.3183	-1.6354	.5981	.000	00 -2.0	516 1.05	19
i:	15	16	17	18	19			
\hat{Y}_i :	80.8876	87.7742	92.1765	96.7340	98.626	3		
e_i :	2876	1.4258	2.6235	5340	-2.226	3		
Expected Val.:	3928	1.3183	2.7520	8155	-2.752	0		

13.15.
$$H_0: E\{Y\} = \gamma_0 - \gamma_0/[1 + (X/\gamma_2)^{\gamma_1}], H_a: E\{Y\} \neq \gamma_0 - \gamma_0/[1 + (X/\gamma_2)^{\gamma_1}].$$

SSPE = 8.67999, SSE = 35.71488, MSPE = 8.67999/6 = 1.4467, MSLF = (35.71488 - 8.67999)/10 = 2.7035, $F^* = 2.7035/1.4467 = 1.869$, F(.99; 10, 6) = 7.87. If $F^* \le 7.87$ conclude H_0 , otherwise H_a . Conclude H_0 .

13.16.
$$s\{g_0\} = 1.1741$$
, $s\{g_1\} = .1943$, $s\{g_2\} = .02802$, $z(.985) = 2.17$

$$100.3401 \pm 2.17(1.1741)$$
 $97.7923 \le \gamma_0 \le 102.8879$

$6.4802 \pm 2.17 (.1943)$	$6.0586 \le \gamma_1 \le$	6.9018
$4.8155 \pm 2.17(.02802)$	$4.7547 \le \gamma_2 \le$	4.8763

14.5. a.

LOGISTIC REGRESSION, POISSON REGRESSION,AND GENERALIZED LINEAR MODELS

b.
$$100$$
c. $X = 125$: $\pi = .006692851$, $\pi/(1-\pi) = .006737947$
 $X = 126$: $\pi = .005486299$, $\pi/(1-\pi) = .005516565$
 $005516565/.006737947 = .81873 = \exp(-.2)$
14.7. a. $b_0 = -4.80751$, $b_1 = .12508$, $\hat{\pi} = [1 + \exp(4.80751 - .12508X)]^{-1}$
c. 1.133
d. $.5487$
e. 47.22
14.11a.
$$\frac{j: \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6}{p_j: \quad .144 \quad .206 \quad .340 \quad .592 \quad .812 \quad .898}$$
b. $b_0 = -2.07656$, $b_1 = .13585$
 $\hat{\pi} = [1 + \exp(2.07656 - .13585X)]^{-1}$
d. 1.1455
e. $.4903$
f. 23.3726
14.14a. $b_0 = -1.17717$, $b_1 = .07279$, $b_2 = -.09899$, $b_3 = .43397$
 $\hat{\pi} = [1 + \exp(1.17717 - .07279X_1 + .09899X_2 - .43397X_3)]^{-1}$
b. $\exp(b_1) = 1.0755$, $\exp(b_2) = .9058$, $\exp(b_3) = 1.5434$
c. $.0642$
14.15a. $z(.95) = 1.645$, $s\{b_1\} = .06676$, $\exp[.12508 \pm 1.645(.06676)]$,

 $E{Y} = [1 + \exp(-20 + .2X)]^{-1}$

 $1.015 \le \exp(\beta_1) \le 1.265$

- b. $H_0: \beta_1=0, \ H_a: \beta_1\neq 0. \ b_1=.12508, \ s\{b_1\}=.06676, \ z^*=.12508/.06676=1.8736. \ z(.95)=1.645, \ |z^*|\leq 1.645, \ \text{conclude } H_0, \ \text{otherwise conclude } H_a. \ \text{Conclude } H_a. \ P-value=.0609.$
- c. $H_0: \beta_1 = 0, H_a: \beta_1 \neq 0.$ $G^2 = 3.99, \chi^2(.90; 1) = 2.7055.$ If $G^2 \leq 2.7055,$ conclude H_0 , otherwise conclude H_a . Conclude H_a . P-value=.046
- 14.17a. $z(.975) = 1.960, s\{b_1\} = .004772, .13585 \pm 1.960(.004772),$ $.1265 \le \beta_1 \le .1452, \qquad 1.1348 \le \exp(\beta_1) \le 1.1563.$
 - b. $H_0: \beta_1=0, H_a: \beta_1\neq 0.$ $b_1=.13585, s\{b_1\}=.004772, z^*=.13585/.004772=28.468.$ $z(.975)=1.960, |z^*|\leq 1.960,$ conclude H_0 , otherwise conclude H_a . Conclude H_a . P-value= 0+.
 - c. $H_0: \beta_1 = 0, H_a: \beta_1 \neq 0.$ $G^2 = 1095.99, \chi^2(.95; 1) = 3.8415.$ If $G^2 \leq 3.8415,$ conclude H_0 , otherwise conclude H_a . Conclude H_a . P-value= 0+.
- 14.20 a. $z(1-.1/[2(2)]) = z(.975) = 1.960, s\{b_1\} = .03036, s\{b_2\} = .03343, \exp\{30[.07279 \pm 1.960(.03036)]\}, 1.49 \le \exp(30\beta_1) \le 52.92, \exp\{25[-.09899 \pm 1.960(.03343)]\}, .016 \le \exp(2\beta_2) \le .433.$
 - b. $H_0: \beta_3 = 0, H_a: \beta_3 \neq 0.$ $b_3 = .43397, s\{b_3\} = .52132, z^* = .43397/.52132 = .8324.$ $z(.975) = 1.96, |z^*| \leq 1.96$, conclude H_0 , otherwise conclude H_a . Conclude H_0 . P-value= .405.
 - c. $H_0: \beta_3 = 0, H_a: \beta_3 \neq 0.$ $G^2 = .702, \chi^2(.95; 1) = 3.8415.$ If $G^2 \leq 3.8415$, conclude H_0 , otherwise conclude H_a . Conclude H_0 .
 - d. $H_0: \beta_3 = \beta_4 = \beta_5 = 0$, $H_a:$ not all $\beta_k = 0$, for k = 3, 4, 5. $G^2 = 1.534$, $\chi^2(.95;3) = 7.81$. If $G^2 \leq 7.81$, conclude H_0 , otherwise conclude H_a . Conclude H_0 .
- 14.22a. X_1 enters in step 1; X_2 enters in step 2; no variables satisfy criterion for entry in step 3.
 - b. X_{11} is deleted in step 1; X_{12} is deleted in step 2; X_3 is deleted in step 3; X_{22} is deleted in step 4; X_1 and X_2 are retained in the model.
 - c. The best model according to the AIC_p criterion is based on X_1 and X_2 . $AIC_3 = 111.795$.
 - d. The best model according to the SBC_p criterion is based on X_1 and X_2 . $SBC_3 = 121.002$.

14.23.

j:	1	2	3	4	5	6
O_{j1} :	72	103	170	296	406	449
E_{j1} :	71.0	99.5	164.1	327.2	394.2	440.0
O_{j0} :	428	397	330	204	94	51
E_{i0} :	429.0	400.5	335.9	172.9	105.8	60.0

$$H_0: E\{Y\} = [1 + \exp(-\beta_0 - \beta_1 X)]^{-1},$$

$$H_a: E\{Y\} \neq [1 + \exp(-\beta_0 - \beta_1 X)]^{-1}.$$

 $X^2 = 12.284$, $\chi^2(.99;4) = 13.28$. If $X^2 \le 13.28$ conclude H_0 , otherwise H_a . Conclude H_0 .

14.25.a.

Class j	$\hat{\pi}^{'}$ Interval	Midpoint	n_{j}	p_{j}
1	-1.1 - under 4	75	10	.3
2	4 - under $.6$.10	10	.6
3	.6 - under 1.5	1.05	10	.7

b.

14.28.a.

$$j$$
:
 1
 2
 3
 4
 5
 6
 7
 8

 O_{j1} :
 0
 1
 0
 2
 1
 8
 2
 10

 E_{j1} :
 .2
 .5
 1.0
 1.5
 2.4
 3.4
 4.7
 10.3

 O_{j0} :
 19
 19
 20
 18
 19
 12
 18
 10

 E_{j0} :
 18.8
 19.5
 19.0
 18.5
 17.6
 16.6
 15.3
 9.7

b.
$$H_0: E\{Y\} = [1 + \exp(-\beta_0 - \beta_1 X_1 - \beta_2 X_2 - \beta_3 X_3)]^{-1},$$

$$H_a: E\{Y\} \neq [1 + \exp(-\beta_0 - \beta_1 X_1 - \beta_2 X_2 - \beta_3 X_3)]^{-1}.$$

 $X^2 = 12.116$, $\chi^2(.95; 6) = 12.59$. If $X^2 \le 12.59$, conclude H_0 , otherwise conclude H_a . Conclude H_0 . P-value = .0594.

c.

 $14.29 \, a.$

$$i:$$
 1 2 3 \cdots 28 29 30 $h_{ii}:$.1040 .1040 .1040 \cdots .0946 .1017 .1017

b.

i:	1	2	3	 28	29	30
ΔX_i^2 :	.3885	3.2058	.3885	 4.1399	.2621	.2621
Δdev_i :	.6379	3.0411	.6379	 3.5071	.4495	.4495
D_i :	.0225	.1860	.0225	 .2162	.0148	.0148

14.32 a.

b.

i:
 1
 2
 3
 ...
 157
 158
 159

$$\Delta X_i^2$$
:
 .1340
 .1775
 1.4352
 ...
 .0795
 .6324
 2.7200

 Δdev_i :
 .2495
 .3245
 1.8020
 ...
 .1478
 .9578
 2.6614

 D_i :
 .0007
 .0008
 .0395
 ...
 .0016
 .0250
 .0191

14.33a. $z(.95) = 1.645, \ \hat{\pi}'_h = .19561, \ s^2\{b_0\} = 7.05306, \ s^2\{b_1\} = .004457, \ s\{b_0, b_1\} = -.175353, \ s\{\hat{\pi}'_h\} = .39428, \ .389 \le \pi_h \le .699$

b.

Cutoff	Renewers	Nonrenewers	Total
.40	18.8	50.0	33.3
.45	25.0	50.0	36.7
.50	25.0	35.7	30.0
.55	43.8	28.6	36.7
.60	43.8	21.4	33.3

c. Cutoff = .50. Area = .70089.

14.36a. $\hat{\pi}_h' = -1.3953, \ s^2\{\hat{\pi}_h'\} = .1613, \ s\{\hat{\pi}_h'\} = .4016, \ z(.95) = 1.645. \ L = -1.3953 - 1.645(.4016) = -2.05597, \ U = -1.3953 + 1.645(.4016) = -.73463. \\ L^* = [1 + \exp(2.05597)]^{-1} = .11345, \ U^* = [1 + \exp(.73463)]^{-1} = .32418.$

b.

Cutoff	Received	Not receive	Total
.05	4.35	62.20	66.55
.10	13.04	39.37	52.41
.15	17.39	26.77	44.16
.20	39.13	15.75	54.88

c. Cutoff = .15. Area = .82222.

14.38a. $b_0 = 2.3529, b_1 = .2638, s\{b_0\} = .1317, s\{b_1\} = .0792, \hat{\mu} = \exp(2.3529 + .2638X).$

b.

$$i: 1 2 3 \cdots 8 9 10$$

 $dev_i: .6074 -.4796 -.1971 \cdots .3482 .2752 .1480$

c.

$$X_h$$
:
 0
 1
 2
 3

 Poisson:
 10.5
 13.7
 17.8
 23.2

 Linear:
 10.2
 14.2
 18.2
 22.2

e.
$$\hat{\mu}_h = \exp(2.3529) = 10.516$$

$$P(Y \le 10 \mid X_h = 0) = \sum_{Y=0}^{10} \frac{(10.516)^Y \exp(-10.516)}{Y!}$$
$$= 2.7 \times 10^{-5} + \dots + .1235 = .5187$$

f. $z(.975) = 1.96, .2638 \pm 1.96(.0792), .1086 \le \beta_1 \le .4190$