Student Information

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Answer 1

Table 1: Truth Table for $(\neg q \land (p \rightarrow q)) \rightarrow \neg p$

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p	q	$\neg q$	$p \rightarrow q$	$\neg q \land (p \to q)$	$\neg p$	$(\neg q \land (p \to q)) \to \neg p$
Τ	Т	F	Т	F	F	Т
T	\mathbf{F}	Γ	F	F	F	T
F	${ m T}$	F	Т	F	Т	T
F	\mathbf{F}	Т	T	${ m T}$	Τ	T

Table 2: Truth Table for $((p \lor q) \land (\neg p \lor r)) \rightarrow (q \lor r)$

p	q	r	$p \lor q$	$\neg p$	$\neg p \lor r$	$(p \lor q) \land (\neg p \lor r)$	$q \lor r$	$((p \lor q) \land (\neg p \lor r)) \to (q \lor r)$
T	Τ	Τ	Т	F	Т	T	Т	Т
T	${\rm T}$	F	Т	F	F	F	Т	${ m T}$
T	\mathbf{F}	${ m T}$	${ m T}$	F	Τ	T	Т	${ m T}$
T	\mathbf{F}	F	Т	F	F	F	F	${ m T}$
F	${\rm T}$	\mathbf{T}	Т	Т	Т	T	Т	${ m T}$
F	${\rm T}$	F	Т	Т	Т	T	Т	${ m T}$
F	\mathbf{F}	\mathbf{T}	F	T	Т	F	Т	T
F	\mathbf{F}	\mathbf{F}	F	T	Т	F	F	T

Answer 2

 $(p \to q) \lor (p \to r) \equiv (\neg p \lor q) \lor (p \to r)$ from Table 7 K. Rosen page 28.

 $(\neg p \lor q) \lor (p \to r) \equiv (\neg p \lor q) \lor (\neg p \lor r)$ from Table 7 K.Rosen page 28.

 $(\neg p \vee q) \vee (\neg p \vee r) \equiv \neg p \vee (q \vee r)$ from Table 6 K. Rosen page 27, Distributive Laws.

 $\neg p \lor (q \lor r) \equiv p \to (q \lor r)$ from Table 7 K.Rosen page 28.

 $p \to (q \lor r) \equiv \neg (q \lor r) \to \neg p$ from Table 7 K.Rosen page 28.

 $\neg (q \lor r) \to \neg p \equiv (\neg q \land \neg r) \to \neg p$ from Table 6 K.Rosen page 27, De Morgan's Laws.

Answer 3

- **1-a)** For all animal x, if x is a cat, then there exists an animal y such that y is a dog and x and y are friend.
- **1-b)** There exists an animal x such that x is a cat and x is friend with all dog.

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2-a) \forall x \exists y (Eats(x,y) \rightarrow Customer(x))

2-b) \exists x \exists y (Chef(x) \land \neg Cooks(x.y))

2-c) \exists x \forall y (Customer(x) \land Eats(x,y) \land \exists z_1 \forall z_2 (Cooks(z_1,y) \land (Cooks(z_2,y) \rightarrow (z_2=z_1))))

2-d) \forall x \exists c (Chef(x) \rightarrow (Chef(c) \land Knows(x,c) \land \exists z (Cooks(c,z) \land \neg Cooks(x,z))))
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Answer 4

When p is false and q is true, premises which are $(p \to q)$ and $(\neg p)$ become true. Since q is true, $\neg q$ is false. Therefore, by counterexample, the invalidity of the conclusion is proven.

Answer 5

1	$p \to q$	premise
2	$q \to r$	premise
3	$r \to p$	premise
4	q	assumption
5	r	$\rightarrow e 2, 4$
6	p	$\rightarrow e \ 3, 5$
7	$q \to p$	\rightarrow i 4 – 6
8	$p \to r$	\rightarrow i 6, 5
9	$p \leftrightarrow q$	\leftrightarrow i 1, 7
10	$p \leftrightarrow r$	\leftrightarrow i 3,8
11	$(p \leftrightarrow q) \land (p \leftrightarrow r)$	$\wedge i 9, 10$

Answer 6

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1
                \forall x(Q(x) \to R(x))
                                             premise
2
                \exists x (P(x) \to Q(x))
                                             premise
3
                \forall x P(x)
                                             premise
4
         x_0 \quad P(x_0) \to Q(x_0)
                                             assumption \\
5
                P(x_0)
                                             \forall xe \ 3
6
                Q(x_0)
                                             \rightarrow e 4, 5
7
                Q(x_0) \to R(x_0)
                                             \forall xe 1
8
                R(x_0)
                                             \rightarrow e 6, 7
9
                P(x_0) \to R(x_0)
                                             \rightarrowi 5, 8
10
                \exists x (P(x) \to R(x))
                                             \exists x i 9
                \exists x (P(x) \to R(x))
11
                                             \exists xe \ 2, 4 - 10
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