Student Information

Full Name : Beste Burhan
Id Number : 2171395

Answer 1

a.

Table 1: Membership Table for $A \cap B \subseteq (A \cup \overline{B}) \cap (\overline{A} \cup B)$

A	B	$\overline{A}$	$\overline{B}$	$A \cap B$	$A \cup \overline{B}$	$\overline{A} \cup B$	$(A \cup \overline{B}) \cap (\overline{A} \cup B)$
1	1	0	0	1	1	1	1
1	0	0	1	0	1	0	0
0	1	1	0	0	0	1	0
0	0	1	1	0	1	1	1

b.

Table 2: Membership Table for $\overline{A} \cap \overline{B} \subseteq (A \cup \overline{B}) \cap (\overline{A} \cup B)$

A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \cap \overline{B}$	$A \cup \overline{B}$	$\overline{A} \cup B$	$(A \cup \overline{B}) \cap (\overline{A} \cup B)$
1	1	0	0	0	1	1	1
1	0	0	1	0	1	0	0
0	1	1	0	0	0	1	0
0	0	1	1	1	1	1	1

Answer 2

Suppose that $A \cap B = \emptyset$,$f^{-1}((A \cap B) \times C) = \emptyset$. Since f is a bijection, there are no two elements with same image (so for f^{-1}). Therefore,
$f^{-1}(A \times C) \cap f^{-1}(B \times C)) = \emptyset = f^{-1}((A \cap B) \times C)$

Suppose that $t \in f^{-1}(A \times C) \cap f^{-1}(B \times C)$,then $y \in f^{-1}(A \times C)$ and $y \in f^{-1}(B \times C)$ Hence, there exist x_1,x_2 such that $f^{-1}(\{x_1, x_2\}) = y$ and there exist x_3,x_4 such that $f^{-1}(\{x_3, x_4\}) = y$. Since f is a bijection, $x_1 = x_3$, $x_2 = x_4$ and $x_1, x_3 \in A \cap B$. $f^{-1}(\{x_1, x_2\}) = y \in f^{-1}((A \cap B) \times C)$. Therefore, $f^{-1}((A \cap B) \times C) \subseteq f^{-1}(A \times C) \cap f^{-1}(B \times C))$ and so
$f^{-1}((A \cap B) \times C) = f^{-1}(A \times C) \cap f^{-1}(B \times C))$

Answer 3

a.

Since $f(-2) = f(2) = ln9$, f is not one-to-one. $(-1) \in R$ but $ln(x^2 + 5)$ can not be equal to -1 for any value of x, so f is not onto.

b.

To show that f is one to one , $f(x) = f(y) \rightarrow x = y$ should be shown

$$\begin{aligned} e^{e^{x^7}} &= e^{e^{y^7}} \\ e^{x^7} &= e^{y^7} \\ x^7 &= y^7 \\ x &= y \end{aligned}$$

so f is one to one.

$(-1) \in R$ but $e^{e^{x^7}}$ can not be equal to -1. Therefore, f is not onto.

Answer 4

a.

Since A and B are countable, $A \rightarrow N$ and $B \rightarrow N$ are injections. Therefore, there exist an injection $f : A \times B \rightarrow N^2$

if I take $g : N^2 \rightarrow N$ and $g(x, y) = 3^x.5^y$, assume that $a, b, c, d \in N$

$$\begin{aligned} f(a, b) &= f(c, d) \\ 3^a.5^b &= 3^c.5^d \end{aligned}$$

if and only if when $a = c, b = d$, so g is an injection.
Therefore, $f \circ g : A \times B \rightarrow N$ is an injection.

b.

Assume that B is countable.Because $A \subseteq B$ and b is countable , I can list elements of A. It means that A is countable, but it is not. There is a contradiction. Hence, B is uncountable.

c.

There is an injection $f : B \rightarrow N$. Assume that $g : A \rightarrow B$, then g is an injection. Since f and g are injections, then $f \circ g : A \rightarrow N$ is an injection.

Answer 5

$f_1(x) \leq C f_2(x)$

Assume that $f_1(x), \quad f_2(x) = x$ as a increasing functions when $x > 1$

$0 < f_1(x) \leq cx$

a.

$0 < ln(f_1(x)) \leq ln(cx) = lnc + lnx$ since lnc is a constant $ln(f_1(x))$ is $\mathcal{O}(lnx)$

b.

$0 < 3^{f_1(x)} \leq 3^{cx} \leq 3^c.3^x$ $(C = 3^c)$ so $3^{f_1(x)}$ is $\mathcal{O}(3^x)$

Answer 6

a.

$$(3^x - 1)mod(3^y - 1) = 3^{(xmody)} - 1$$
$$(3^x - 1 - 3^{xmody} + 1)mod(3^y - 1) = 0$$
$$(3^x - 3^{xmody})mod(3^y - 1) = 0$$
$$3^y - 1 \mid (3^x - 3^{xmody})$$
$$x = ty + d \quad (for \quad x \quad mod \quad y)$$
$$3^y - 1 \mid 3^{ty+d} - 3^d$$
$$3^y - 1 \mid 3^d(3^{ty} - 1)$$
$$(3^y - 1) \mid 3^d((3^y - 1).(3^{t-1} + 3^{t-2} + ..1))$$

b.

$$277 = 2.123 + 31$$
$$123 = 3.31 + 30$$
$$31 = 1.30 + 1$$
$$30 = 30.1$$

since 1 divides 30, gcd(277,123) = gcd(123,31) = gcd(31,30) = gcd(30,1) = 1.