CENG 460 - Introduction to Robotics
Spring 2018

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Homework 2 - Mobile Robot Kinematics

Assigned - Nov 12, 2018, **Due - Nov 21, 2018**

1. Consider the mobile robot with three omni-direction wheels illustrated in Figure 1. The black point marks the "front" of the robot. The three wheels are located symmetrically, 120^o from each other and all at a distance D from the center of the robot, and the wheels A and C are symmetric with respect to the front of the robot. The position and orientation of the robot body are denoted with $[x, y, \theta]$ in the world frame O.

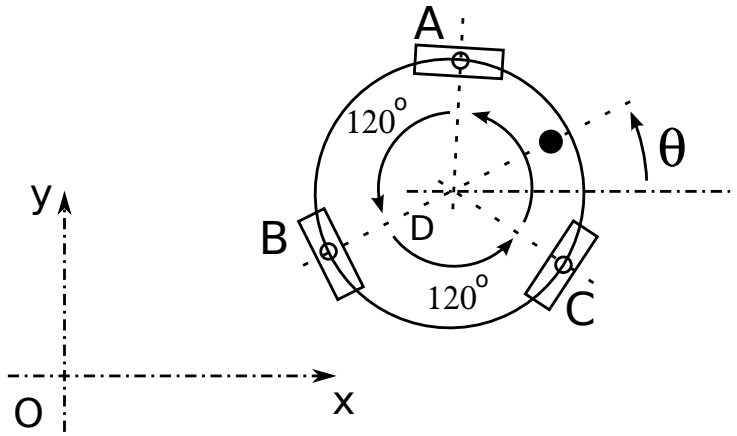

Figure 1: Planar omnidirectional mobile robot with three wheels.

Now, suppose that a body frame F is defined on the robot, located at the center point such that the x_F points towards the front of the robot and y_F towards the left side. Given a particular body pose with $[x, y, \theta]$, derive formulas for the positions of the centers of all wheels p_A, p_B and p_C in the world frame O.

2. Suppose that the robot (and hence the frame F) is moving with a translational velocity $\dot{x}$ and $\dot{y}$ in O as well as a rotational velocity $\dot{\theta}$. Using kinematic equations from earlier chapters, derive expressions for the translational velocities of the centers of all wheels, $\dot{p}_A$, $\dot{p}_B$ and $\dot{p}_C$ in the world frame O. Assuming these wheels are simple omnidirecional wheels with sideways rollers and radius R, find expressions for the rotational speeds w_A, w_B and w_C for all three wheels. Write the (linear) function from the variables $\dot{x}$, $\dot{y}$ and $\dot{\theta}$ to w_A, w_B and w_C in matrix form.

3. Write a Matlab script, `hw3_script3.m`, that implements a function taking three wheel rotational speeds w_A, w_B and w_C as a column vector, returning the associated robot velocities $\dot{x}$, $\dot{y}$ and $\dot{\theta}$ as a column vector using your derivations above.

4. Use Matlab's `ode45` function to simulate a robot with $D = 1$, $R = 0.2$ for different combinations of rotational velocities for the wheels. Include figures in your report to illustrate and discuss the resulting robot motion in each case. For example, what happens if all wheels rotate at the same speed? What happens if only one of the three wheels is rotating?

5. Consider the three-DOF planar robot arm given in Figure 2, which consists of three links with lengths a_1, a_2 and a_3 connected with three revolute joints θ_1, θ_2 and θ_3. Derive the planar (2D) forward kinematics for this arm to compute the position and orientation of the end effector frame E relative to the fixed inertial frame O as a function of the joint angles θ_1, θ_2 and θ_3 as defined in the figure. In other words, find a function $f : S^1 \times S^1 \times S^1 \to SE(2)$ such that ${}^{O}T_{E} = f(\theta_1, \theta_2, \theta_3)$, where the homogeneous transform ${}^{O}T_{E}$ maps points in frame E to points in O. Explain your work.

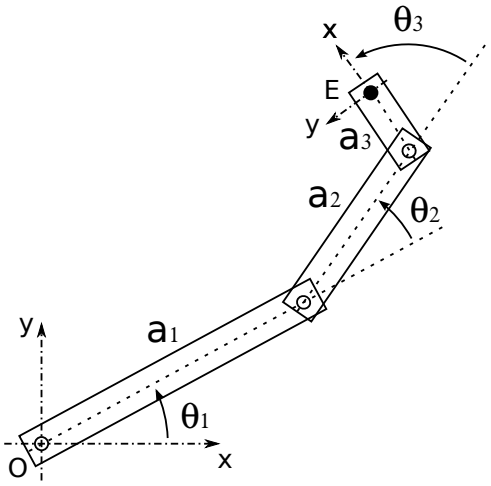

Figure 2: Three joint planar robot arm.

Submission

Submitted solutions must be typeset in a word processing environment such as LaTeX. Submissions are expected to be in the form of a ZIP file named `hw2_e1234567#.zip`, including a PDF report with answers to theoretical questions with your name and student ID indicated clearly, as well as Matlab or other source files that are requested in the homework text. Late submissions will be penalized with a deduction of $10n^2$ points where n is the number of late days.

Note: You can discuss your discoveries and knowledge with your classmates but you must write your own answers and code for all questions above. If any significant similarities are found between your answers and other homeworks, you will be audited on your understanding of your own solutions.