CENG 384 - Signals and Systems for Computer Engineers
Spring 2018-2019
Written Assignment 1

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1. (a)

   $i)$
   $z = x + jy$
   $\bar{z} = x - jy$
   $3z + 4 = 2j - \bar{z}$
   $3z + \bar{z} = 2j - 4$
   $4x + 2yj = 2j - 4$
   $y = 1$
   $x = -1$
   $z = -1 + j$
   $|z|^2 = 2$

   $ii)$

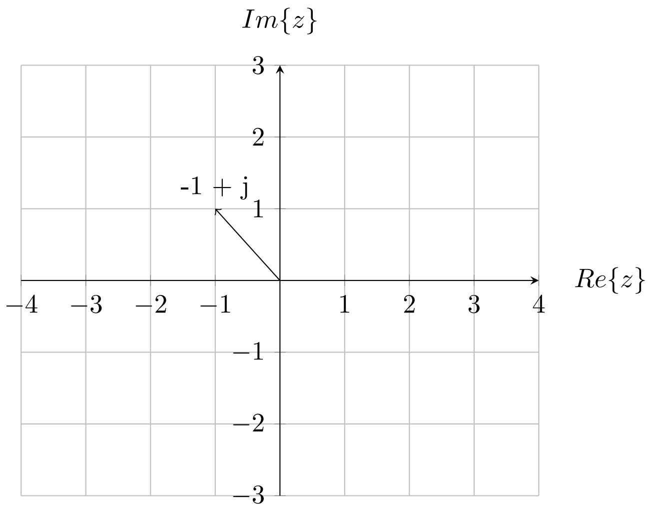


Figure 1: $Re\{z\}$ vs. $Im\{z\}$.

(b)

$z = re^{j\theta}$
$z^3 = 64j$
$z^3 = r^3 e^{3j\theta}$
$64j = r^3 e^{3j\theta}$
$64j = r^3(cos(3\theta) + jsin(3\theta))$
$64j = 64(cos(\pi/2 + 2\pi k) + jsin(\pi/2 + 2\pi k))$
$z = \sqrt[3]{64}[cos(\frac{\pi/2+2\pi k}{3}) + jsin(\frac{\pi/2+2\pi k}{3})]$
$z_1 = 4[cos(\pi/6) + jsin(\pi/6)] = 2\sqrt{3} + 2j$
$z_1 = 4[cos(5\pi/6) + jsin(5\pi/6)] = -2\sqrt{3} + 2j$
$z_1 = 4[cos(3\pi/2) + jsin(3\pi/2)] = -4j$

(c)

$z = \frac{(1-j)(1+\sqrt{3}j)}{1+j}$
$z = \frac{\sqrt{2}e^{j7\pi/4} * 2e^{j\pi/3}}{\sqrt{2} * e^{j\pi/4}}$
$z = 2e^{(7\pi/4+\pi/3-\pi/4)j}$
$z = 2e^{j11\pi/6}$
magnitude $= 2$
angle $= 11\pi/6$

(d)

$z = -je^{j\pi/2}$
$z = e^{j3\pi/2}e^{j\pi/2}$
$z = e^{j2\pi}$
$z = cos(2\pi) + jsin(2\pi)$

2.

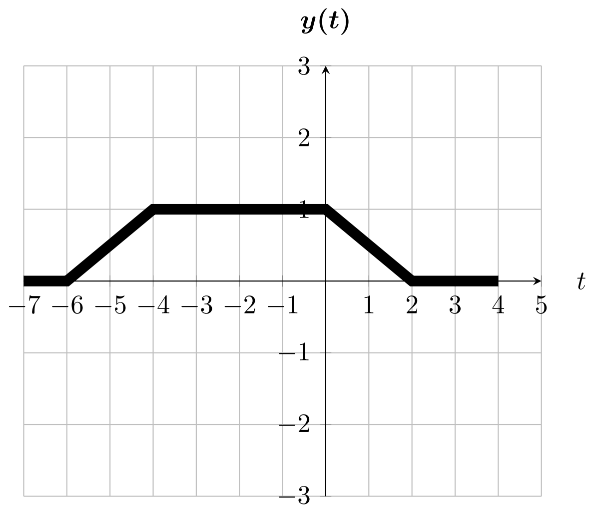


Figure 2: t vs. $y(t) = x(t/2 + 1))$.

3. (a)

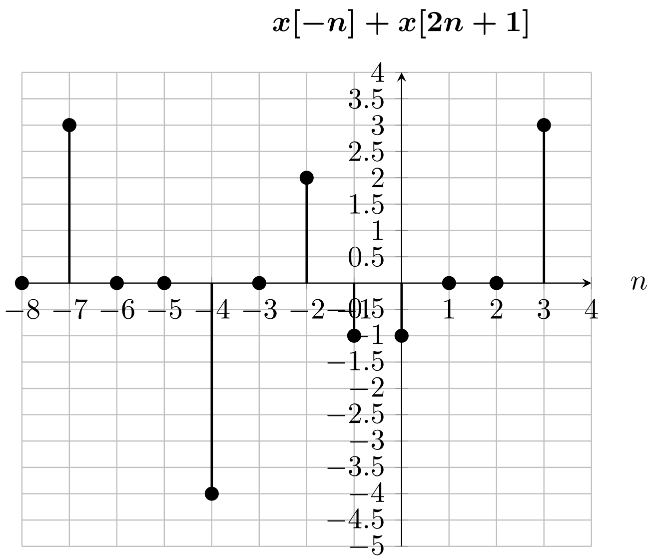


Figure 3: $x[-n] + x[2n+1]$.

(b)

$x[n] + x[2n+1] = 3\delta(n+7) - 4\delta(n+4) + 2\delta(n+2) - \delta(n+1) - \delta(n) + 3\delta(n-3)$

4.

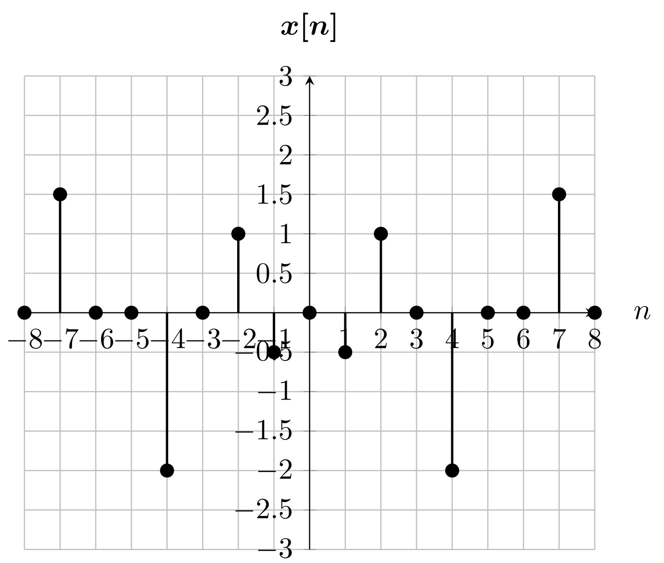
(a)
$x[n] = 3x_1[n] + 5x_2[n]$
where
$x_1[n] = cos[13\pi/10n]$ and $x_2[n] = cos[7\pi/3 - 2\pi/3]$
$w_1 = 13\pi/10$
$T_1 = 2\pi/(13\pi/10)$
$T_1 = 20/13$
$w_2 = 7\pi/3$
$T_2 = 2\pi/(7\pi/3)$
$T_2 = 6/7$
$T_1/T_2 = \frac{20/13}{6/7} = 70/39$
$T_0 = 39 * 20/13 = 60$
So it is periodic and its period is 60.

(b) $w_1 = 3$
$T_1 = 2\pi/3$
Since there is not an integer m such that $2\pi/3 * m$ is integer, y[n] is not periodic.

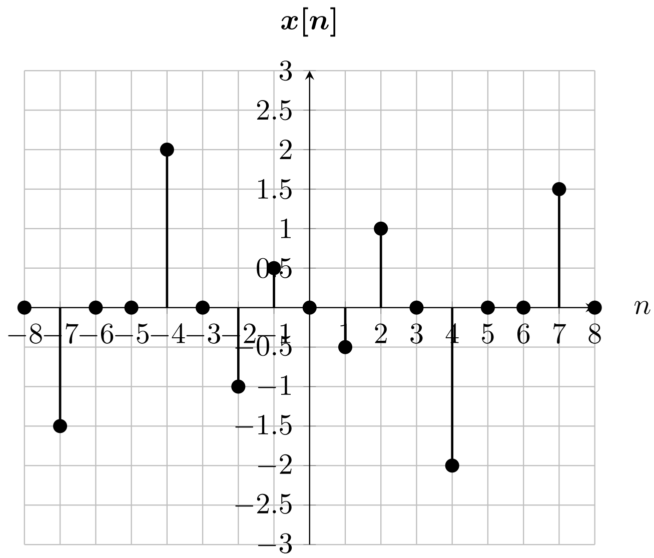
(c) $w_1 = 3\pi$
$T_1 = 2\pi/3\pi$
$T_1 = 2/3$
Since there is an integer m such that 2/3 * m is integer, y[n] is periodic and its period is 2.

(d) $x(t) = e^{(3\pi/2)j}e^{5tj}$
$x(t) = e^{(5t+3\pi/2)j}$
$w_1 = 5$
$T_1 = 2\pi/5$
$x(t)$ is periodic and its period is $2\pi/5$

5.
$Ev\{x[n]\} = 1/2\{x[n] + x[-n]\}$



$Od\{x[n]\} = 1/2\{x[n] - x[-n]\}$



6. (a)

Memoriless:

y(t) has memory because y(t) depends on previous and next values of x(t). For example: y(1) = x(-1), y(4) = x(5)

Stability:

if $|x(t)|<M$, $|x(2t-3)|<M$ and $|y(t)|<M$, Then $y(t) = x(2t-3)$ is stable.

Causality:

The value of y(.) at time=4 depends on x(.) at a future time(t=5). Hence it is not causal.

Linearity:

$y(t) = x(2t-3) = T[x(t)]$
$T[ax_1(t) + bx_2(t)] = ax_1(2t-3) + bx_2(2t-3) = aT[x_1(t)] + bT[x_2(t)]$
So, y(t) is linear

Invertibility:

y(t) is invertible. x(t) = y($\frac{t+3}{2}$)

Time-invariance:

$y(t) = x(2t-3) = T[x(t)]$
$T[x(t-T_0)] = x(2t-2T_0-3) \neq x(2t-T_0-3) = y(t-T_0)$
So y(t) is not time-invariant.

(b)

Memoryless:

y(t) is memoryless because y(t) depends only on x(t), not on previous or next values.

Stability:

It is unstable. When t goes to ∞, y(t) also goes to ∞.

Causality:

y(t) only depends on present values of x(t). So it is causal.

Linearity:

$y(t) = tx(t) = T[x(t)]$
$T[ax_1(t) + bx_2(t)] = t[ax_1(t) + bx_2(t)] = aT[x_1(t)] + bT[x_2(t)]$
So, y(t) is linear

Invertibility:

y(t) is invertible. x(t) = $\frac{1}{t}$y(t)

Time-invariance:

$y(t) = tx(t) = T[x(t)]$
$T[x(t - T_0)] = tx(t - T_0) \neq (t - T_0)x(t - T_0) = y(t - T_0)$
So y(t) is not time-invariant.

(c)

Memoriless:

y[n] has memory because y[n] depends on previous and next values of x[n]. For example: y[4] = x[5]

Stability:

if $|x[n]|<M$, $|x[2n-3]|<M$ and $|y[n]|<M$, Therefore $y[n] = x[2n-3]$ is stable.

Causality:

The value of y[.] at time=4 depends on x[.] at a future time(t=5). Hence it is not causal.

Linearity:

$y[n] = x[2n-3] = T[x[n]]$
$T[ax_1[n] + bx_2[n] = ax_1[2n-3] + bx_2[2n-3] = aT[x_1[n]] + bT[x_2[n]]$
So, y[n] is linear

Invertibility:

y[n] is not invertible. $x[n] = \delta[n] + \delta[n-1]$ and $x_2 = \delta[n]$ will give $y[n] = \delta[n]$

Time-invariance:

$y[n] = x[2n+3] = T[x[n]]$
$T[x[n - N_0]] = x[2n - 2N_0 - 3] \neq x[2n - N_0 - 3] = y[n - N_0]$
So y(t) is not time-invariant.

(d)

Memoriless:

y[n] has memory because y[n] depends on previous values of x[n] and not present values.

Stability:

y[n] is unstable. , if $x[n] \leq M$ and $y[n] \leq M \sum_{k=1}^{\infty} 1$. It is unbounded. So y[n] is not stable.

Causality:

y[n] is always depends on previous values of x[n]. It is causal.

Linearity:

$|y[n] = \sum_{k=1}^{\infty} x[n-k] = T[x[n]]|$
$T[ax_1[n] + bx_2[n]] = a\sum_{k=1}^{\infty} x_1[n-k] + b\sum_{k=1}^{\infty} x_2[n-k] = aT[x_1[n]] + bT[x_2[n]]$
So, y[n] is linear.

Invertibility:

y[n] is invertible. x[n] = y[n+2] - y[n+1]

Time-invariance:

$y[n] = \sum_{k=1}^{\infty} x[n-k] = T[x[n]]$
$T[x[n-N_0]] = \sum_{k=1}^{\infty} x[n-N_0-k] = \sum_{k=1}^{\infty} x[n-N_0-k] = y[n-N_0]$
So y[n] is time-invariant.