Student Information

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Answer 1

$C = \{(x, y, z) \mid (x - c_x)^2 + (y - c_y)^2 + (z - c_z)^2 = r^2\}$

$$T = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \qquad t = \begin{bmatrix} c_x \\ c_y \\ c_z \end{bmatrix}$$

$$\begin{bmatrix} x - c_x & y - c_y & z - c_z & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -r^2 \end{bmatrix} \begin{bmatrix} x - c_x \\ y - c_y \\ z - c_z \\ 1 \end{bmatrix} = 0 \qquad \text{is a sphere.} \qquad ..1$$

$$\begin{bmatrix} x' - c'_x \\ y' - c'_y \\ z' - c'_z \\ 1 \end{bmatrix} = T \begin{bmatrix} x - c_x \\ y - c_y \\ z - c_z \\ 1 \end{bmatrix} \qquad ..2$$

$$\begin{bmatrix} x - c_x \\ y - c_y \\ z - c_z \\ 1 \end{bmatrix} = T^{-1} \begin{bmatrix} x' - c'_x \\ y' - c'_y \\ z' - c'_z \\ 1 \end{bmatrix} \qquad ..3$$

In 1. equation, putting 2. and 3. equations

$$\begin{bmatrix} x' - c'_x & y' - c'_y & z' - c'_z & 1 \end{bmatrix} (T^{-1})^T \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -r^2 \end{bmatrix} (T^{-1}) \begin{bmatrix} x' - c'_x \\ y' - c'_y \\ z' - c'_z \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} x' - c'_x & y' - c'_y & z' - c'_z & 1 \end{bmatrix} \begin{bmatrix} R & 0 \\ -t^T R & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -r^2 \end{bmatrix} \begin{bmatrix} R^T & -R^T t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x' - c'_x \\ y' - c'_y \\ z' - c'_z \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} x' - c'_x & y' - c'_y & z' - c'_z & 1 \end{bmatrix} \begin{bmatrix} R & 0 \\ -t^T R & -r^2 \end{bmatrix} \begin{bmatrix} R^T & -R^T t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x' - c'_x \\ y' - c'_y \\ z' - c'_z \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} x' - c'_x & y' - c'_y & z' - c'_z & 1 \end{bmatrix} \begin{bmatrix} RR^T & -RR^T t \\ -t^T RR^T & t^T RR^T t - r^2 \end{bmatrix} \begin{bmatrix} x' - c'_x \\ y' - c'_y \\ z' - c'_z \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} x'-c'_x & y'-c'_y & z'-c'_z & 1 \end{bmatrix} \begin{bmatrix} I & -t \\ -t^T & c_x^2+c_y^2+c_z^2-r^2 \end{bmatrix} \begin{bmatrix} x'-c'_x \\ y'-c'_y \\ z'-c'_z \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} I & -t \\ -t^T & c_x^2+c_y^2+c_z^2-r^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -c_x \\ 0 & 1 & 0 & -c_y \\ 0 & 0 & 1 & -c_z \\ -c_x & -c_y & -c_z & c_x^2+c_y^2+c_z^2-r^2 \end{bmatrix}$$

After row1*c_x+row4$->$row4, row2*c_y+row4$->$row4 and row3*c_z+row4$->$row4

$$\begin{bmatrix} 1 & 0 & 0 & -c_x \\ 0 & 1 & 0 & -c_y \\ 0 & 0 & 1 & -c_z \\ -c_x & -c_y & -c_z & c_x^2+c_y^2+c_z^2-r^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -c_x \\ 0 & 1 & 0 & -c_y \\ 0 & 0 & 1 & -c_z \\ 0 & 0 & 0 & -r^2 \end{bmatrix}$$

Answer 2

a.

$\hat{w} = \begin{bmatrix} 0 & -w_z & w_y \\ w_z & 0 & -w_x \\ -w_y & w_x & 0 \end{bmatrix}$ skew-symmetric.

$(w - I\lambda) = \begin{bmatrix} -\lambda & -w_z & w_y \\ w_z & -\lambda & -w_x \\ -w_y & w_x & -\lambda \end{bmatrix} \qquad \det(w - I\lambda) = 0$

$-\lambda(\lambda^2 + w_x^2) - (-w_z)(-\lambda w_z - w_y w_x) + w_y(w_z w_x - \lambda w_y) = -\lambda^3 - \lambda w_x^2 - \lambda w_y^2 - \lambda w_z^2$

$\lambda(\lambda^2 + (w_x^2 + w_y^2 + w_z^2)) = 0$

since w is a unit vector $(w_x^2 + w_y^2 + w_z^2) = 1$

$\lambda(\lambda^2 + 1) = 0$

roots(eigenvalues of w) are 0,i,-i.

For $\lambda = 0$

$\hat{w}v_1 = \lambda v_1$

$\begin{bmatrix} 0 & -w_z & w_y \\ w_z & 0 & -w_x \\ -w_y & w_x & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -w_y & w_x & 0 \\ 0 & -w_z & w_y \\ 0 & 0 & 0 \end{bmatrix}$

$v_1 = \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix}$

For $\lambda = i, -i$

$\begin{bmatrix} -i & -w_z & w_y \\ w_z & -i & -w_x \\ -w_y & w_x & -i \end{bmatrix} \rightarrow \begin{bmatrix} -w_y & w_x & -i \\ 0 & -w_z - iw_x/w_y & w_y^-1 \\ 0 & 0 & 0 \end{bmatrix}$

$$v_2 = \begin{bmatrix} -w_z \\ 0 \\ w_x \end{bmatrix} + i \begin{bmatrix} w_x w_y \\ w_y^2 - 1 \\ w_y w_z \end{bmatrix} \; v_3 = \begin{bmatrix} -w_z \\ 0 \\ w_x \end{bmatrix} - i \begin{bmatrix} w_x w_y \\ w_y^2 - 1 \\ w_y w_z \end{bmatrix}$$

b.

$e^A = I + A + \frac{A^2}{2!}...$
$e^{w\theta} v_i = [I + w\theta + \frac{w\theta^2}{2!}...] v_i$
$e^{w\theta} v_i = [I + (\lambda\theta) + \frac{(\lambda\theta)^2}{2!}...] v_i$
$= e^{\lambda_i \theta} v_i$
Therefore eigenvalues of R are $e^{\lambda_1}, e^{\lambda_2}, e^{\lambda_3} (1, e^{i\theta}, e^{-i\theta})$. Eigenvector of eigenvalue which is 1 is the(from a part):
$v_1 = \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix}$

c.

$det(R) = 1$(properties of rotational matrices)
$RR^T = I$
$R = \begin{bmatrix} r_1 & r_2 & r_3 \end{bmatrix}$
Since $r_1, r_2 and r_3$ are unit length and $r_1 \perp r_2 \perp r_3$:

$(r_2 \text{ x } r_3) // r_1$
$r_1^T (|r_2||r_3| sin(\pi/2)) cos(0) = 1$

Answer 3

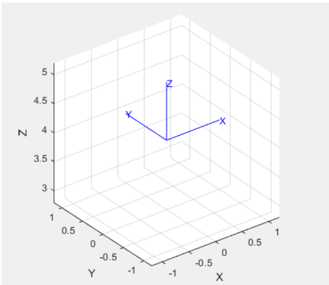
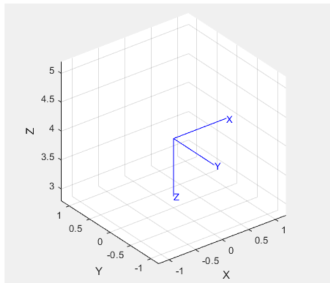
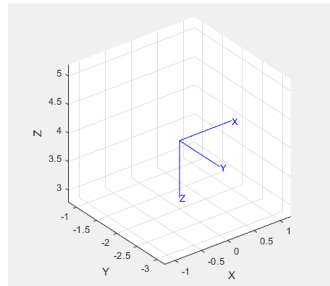
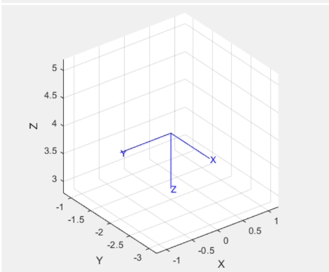
a.

$O_1 = (0, 0, 4)$ $T1$
$O_2 = (0, 0, 4)$After rotation $T1R1$
$O_3 = (0, -2, 4)$After translation $T1R1T2$
$O_4 = (0, -2, 4)$ After rotation $T1R1T2R2$

$$TF = \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & -2 \\ 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T1 = transl(0, 0, 4)$
$R1 = trotx(-pi)$
$T2 = transl(0, 2, 0)$

$R2 = trotz(pi/2)$
$TF = T1 * R1 * T2 * R2$

b.

$q_0 = 1 < 0, 0, 0 >$ $\quad t_0 = (0, 0, 0)$
$q_1 = 0 < 1, 0, 0 >$ after $-\pi$ rotation around x $\quad t_1(0, 0, 4)$
$q_f = q_1 * 0 < 0.70711, -0.70711, 0 > \pi/2$ rotation
$q_f = 0 < 0.70711, -0.70711, 0 >$ after $-\pi$ rotation around x $\quad t_3(0, -2, 4)$
$${}^{w}t_a = \begin{bmatrix} 0 \\ -2 \\ 4 \end{bmatrix}$$
$q0 = UnitQuaternion()$
$r = rotx(-pi)$
$q2 = UnitQuaternion(r)$
$r2 = rotz(pi/2)$
$q2 = UnitQuaternion(r2)$
$qf = q1 * q2$

Answer 4

$${}^{0}T_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0.7071 & 0 & -0.7071 & 0 \\ -0.7071 & 0 & -0.7071 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$t1 = transl(0, 0, 1)$
$R1 = trotx(pi)$
$R2 = trotz(-pi/2)$

$R3 = troty(-pi/4)$
$H1 = t1 * R1 * R2 * R3$

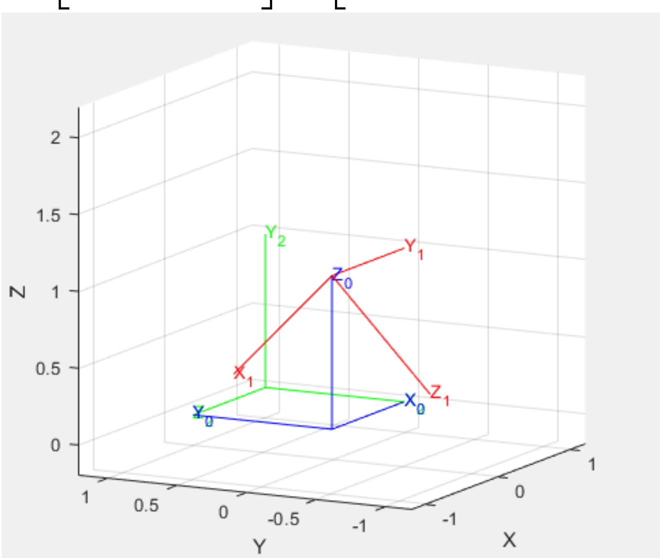
$$^0T_2 = \begin{bmatrix} 0 & 0 & -1 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$t2 = transl(1, 1, 0)$
$r1 = trotx(pi/2)$
$r2 = troty(-pi/2)$
$H2 = t2 * r1 * r2$

$$^1T_2 = \begin{bmatrix} -0.7071 & -0.7071 & 0 & 1.4142 \\ 0 & 0 & -1 & 1 \\ 0.7071 & -0.7071 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R1 = troty(-pi/4)$
$R2 = trotz(pi/2)$
$R3 = troty(-pi/2)$
$t = transl(-1, -1, -1)$
$H3 = R1 * R2 * R3 * t$

$$\begin{bmatrix} 0 & 0 & -1 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0.7071 & 0 & -0.7071 & 0 \\ -0.7071 & 0 & -0.7071 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -0.7071 & -0.7071 & 0 & 1.4142 \\ 0 & 0 & -1 & 1 \\ 0.7071 & -0.7071 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Answer 6

Matlab script : q6.m