Student Information

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Answer 1

$$
\begin{aligned}
{}^{o}p_A &= {}^{o}R_F\ {}^{F}p_A +^{o} t_F \\
&= \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} D\cos(60) \\ D\sin(60) \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix} \\
&= \text{D} \begin{bmatrix} \cos(\theta)\cos(60) - \sin(\theta) sin(60) \\ \sin(\theta) cos(60) + \cos(\theta) sin(60) \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix} \\
&= \text{D} \begin{bmatrix} \cos(\theta + 60) \\ \sin(\theta + 60) \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix} \\
{}^{o}p_B &= {}^{o}R_F\ {}^{F}p_B +^{o} t_F \\
&= \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} -D \\ 0 \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix} \\
&= \text{D} \begin{bmatrix} -\cos(\theta) \\ -\sin(\theta) \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix} \\
{}^{o}p_C &= {}^{o}R_F\ {}^{F}p_C +^{o} t_F \\
&= \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} D\cos(60) \\ -D\sin(60) \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix} \\
&= \text{D} \begin{bmatrix} \cos(\theta)\cos(60) + \sin(\theta) sin(60) \\ \sin(\theta) cos(60) - \cos(\theta) sin(60) \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix} \\
&= \text{D} \begin{bmatrix} \cos(\theta - 60) \\ \sin(\theta - 60) \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix}
\end{aligned}
$$

Answer 2

$$
\begin{aligned}
{}^{o}\dot{p}_A &= D\dot{\theta} \begin{bmatrix} -\sin(\theta + 60) \\ \cos(\theta + 60) \end{bmatrix} + \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \\
\hat{t}_A &= \begin{bmatrix} -\sin(\theta + 60) \\ \cos(\theta + 60) \end{bmatrix} \\
{}^{o}\dot{p}_A \hat{t}_A &= D\dot{\theta}\sin^2(\theta + 60) - \dot{x}\sin(\theta + 60) + D\dot{\theta}\cos^2(\theta + 60) + \dot{y}\cos(\theta + 60) \\
&= D\dot{\theta} - \dot{x}\sin(\theta + 60) + \dot{y}\cos(\theta + 60) \\
{}^{o}\dot{p}_B &= D\dot{\theta} \begin{bmatrix} \sin(\theta) \\ -\cos(\theta) \end{bmatrix} + \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \\
\hat{t}_B &= \begin{bmatrix} \sin(\theta) \\ -\cos(\theta) \end{bmatrix} \\
{}^{o}\dot{p}_B \hat{t}_B &= D\dot{\theta}\sin^2(\theta) + \dot{x}\sin(\theta) - D\dot{\theta}\cos^2(\theta) + \dot{y}\cos(\theta)
\end{aligned}
$$

$$= D\dot{\theta} + \dot{x}\sin(\theta) - \dot{y}\cos(\theta)$$

$${}^{o}\dot{p}_C = D\dot{\theta}\begin{bmatrix} -\sin(\theta-60) \\ \cos(\theta-60) \end{bmatrix} + \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$$

$$\hat{t}_C = \begin{bmatrix} -\sin(\theta-60) \\ \cos(\theta-60) \end{bmatrix}$$

$${}^{o}\dot{p}_C\hat{t}_C = D\dot{\theta}\sin^2(\theta-60) - \dot{x}\sin(\theta-60) + D\dot{\theta}\cos^2(\theta-60) + \dot{y}\cos(\theta-60)$$

$$= D\dot{\theta} - \dot{x}\sin(\theta) + \dot{y}\cos(\theta)$$

$$\mathrm{R}\begin{bmatrix} w_A \\ w_B \\ w_C \end{bmatrix} = \begin{bmatrix} -\sin(\theta+60) & \cos(\theta+60) & D \\ \sin(\theta) & -cos(\theta) & D \\ -\sin(\theta-60) & \cos(\theta-60) & D \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$

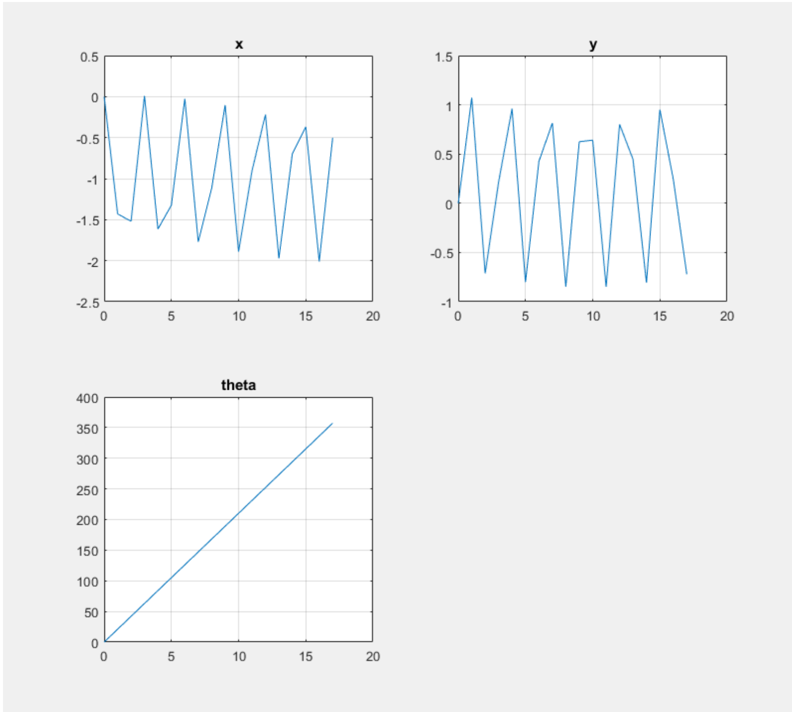
Answer 3

Matlab script : hw2_script3.m

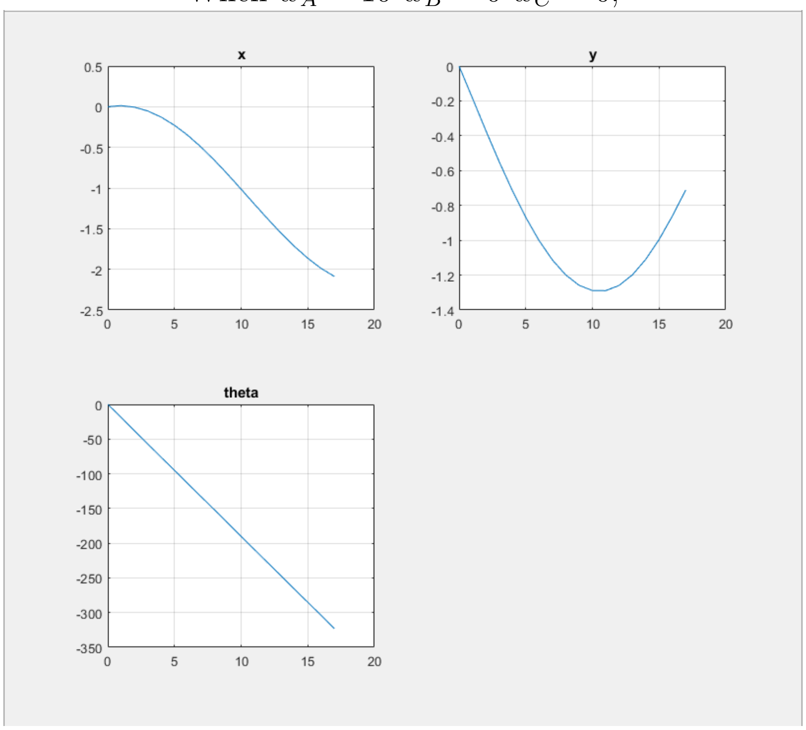
Answer 4

```
q0=[0 0 0] ;%initially x=0 y=0 theta=0
% w0=[10 0 0 ;  10 10 10  ;  10 10 30  ;  10 20 30];
[t0,x0]=ode45(@vf,[0:1:17],q0);

figure(1)
subplot(2,2,1)
plot(t0,x0(:,1))
title('x')
grid

subplot(2,2,2)
plot(t0,x0(:,2))
title('y')
grid

subplot(2,2,3)
plot(t0,x0(:,3))
title('theta')
grid

function vecDot = vf(t,q)
x=q(1);
y=q(2);
th=q(3);

wA=10;
wB=0;
wC=0;

R=0.2;
D =1;
rot_speeds=[wA*R ; wB*R ; wC*R];
A = [ -sin(th +60) cos(th +60) D; sin(th) -cos(th) D; -sin(th -60) cos(th -60) D];
inverseOfA = (inv(A));
vecDot = inverseOfA*rot_speeds;
end
```

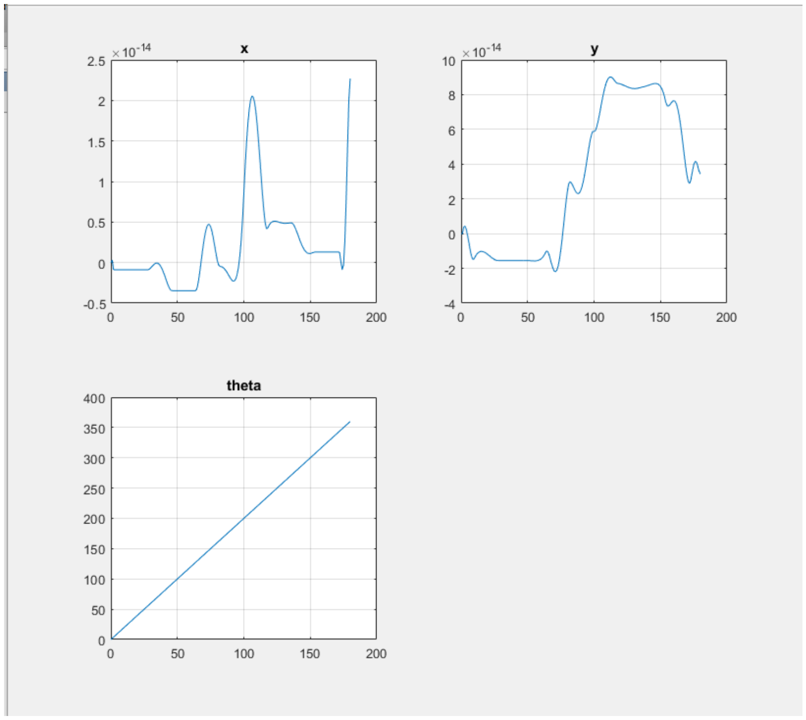
Matlab script



When $w_A = 10$ $w_B = 0$ $w_C = 0$;



When $w_A = 10$ $w_B = 10$ $w_C = 0$;



When $w_A = 10 \; w_B = 10 \; w_C = 10$;

Answer 5

$$
{}^{o}T_E = R_z(\theta_1)\; T_x(a_1)\; R_z(\theta_2)\; T_x(a_2)\; R_z(\theta_3)\; T_x(a_3)
$$

$$
= \begin{bmatrix} \cos(\theta_1) & \sin(\theta_1) & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & a_1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta_2) & \sin(\theta_2) & 0 \\ \sin(\theta_2) & \cos(\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & a_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta_3) & \sin(\theta_3) & 0 \\ \sin(\theta_3) & \cos(\theta_3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & a_3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

Firstly, we rotate world frame by a rotation of θ_1 about the z-axis followed by a translation of a_1 units along the new x-axis. Then, we repeat the same operations for θ_2, a_2, θ_3 and a_3.