CENG 384 - Signals and Systems for Computer Engineers
Spring 2018-2019
Written Assignment 3

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1. (a)

N=4

$x[n] = \sum_{k=0}^{3} a_k e^{jk(2\pi/4)n}$

$a_k = \frac{1}{4}\sum_{n=0}^{3} x[n]e^{-jk(2\pi/4)n}$

$= \frac{1}{4}(x[0](e^{-jk(\pi/2).0}) + x[1](e^{-jk(\pi/2)}) + x[2](e^{-jk\pi}) + x[3](e^{-jk(3\pi/2)}))$

$= \frac{1}{4}(0 + e^{-jk\pi/2} + 2e^{-jk\pi} + e^{-jk3\pi/2})$

$= \frac{1}{4}(e^{-jk\pi/2} + 2e^{-jk\pi} + e^{-jk3\pi/2}) \qquad 0 \leq k \leq 3$ for every 4 period

$a_0 = 1$

$a_1 = -1/2$

$a_2 = 0$

$a_3 = -1/2$

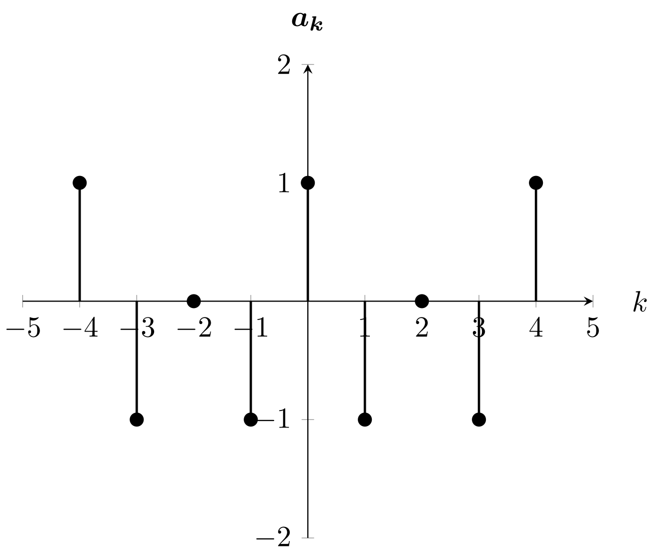

Figure 1: a_k

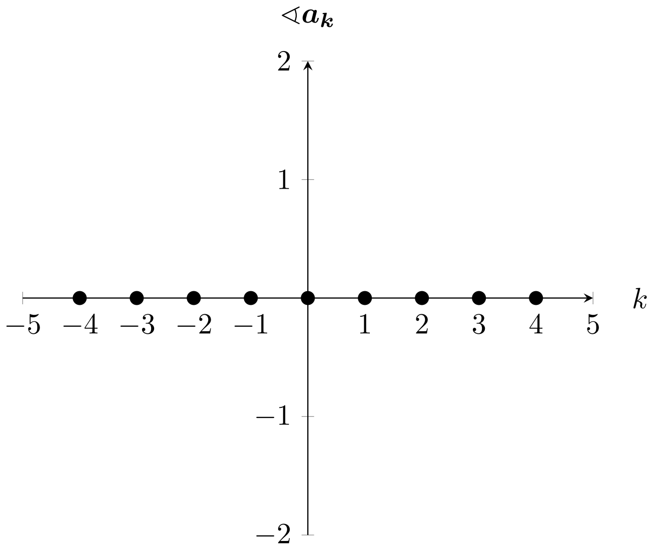

Figure 2: Phase of a_k

(b)

i. $y[n] = x[n] - \sum_{k=-\infty}^{\infty} \delta[n-4k+1]$

ii.
$$y[n] = \sum_{k=0}^{3} a_k e^{jk(2\pi/4)n}$$
$$a_k = \frac{1}{4}\sum_{n=0}^{3} y[n]e^{-jk(2\pi/4)n}$$
$$= \frac{1}{4}(y[0](e^{-jk(\pi/2).0}) + y[1](e^{-jk(\pi/2)}) + y[2](e^{-jk\pi}) + y[3](e^{-jk(3\pi/2)}))$$
$$= \frac{1}{4}(0 + e^{-jk\pi/2} + 2e^{-jk\pi} + 0)$$
$$= \frac{1}{4}(e^{-jk\pi/2} + 2e^{-jk\pi})$$
$$a_0 = 3/4$$
$$a_1 = \frac{-j-2}{4}$$
$$a_2 = 1/4$$
$$a_3 = \frac{j-2}{4}$$

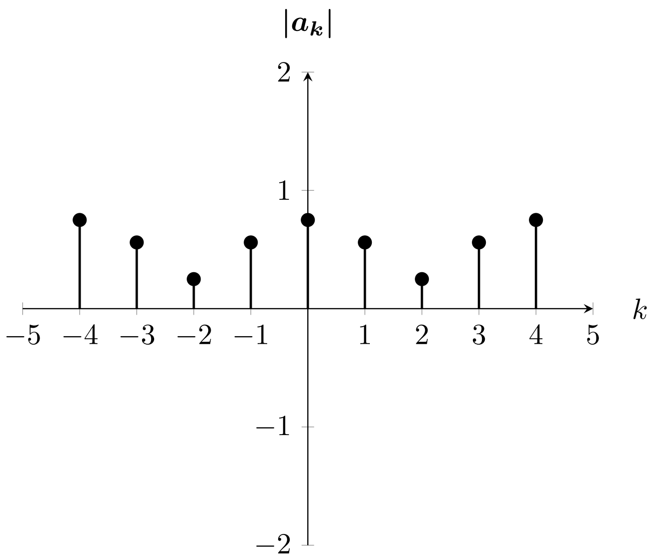

Figure 3: Magnitude of a_k

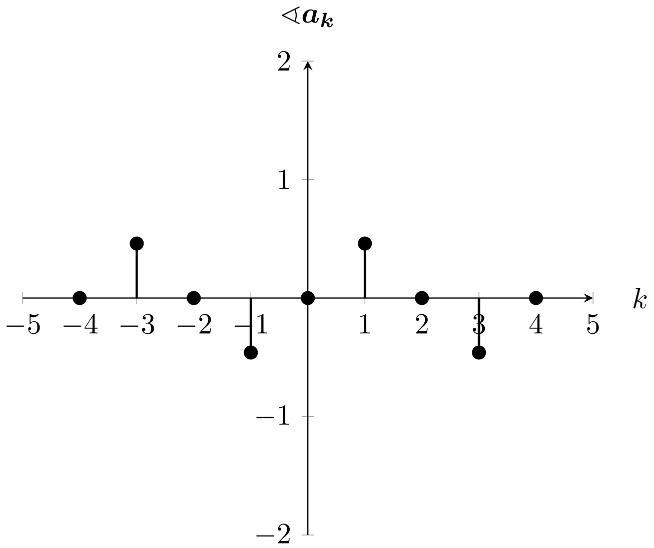

Figure 4: Phase of a_k

2.

$\sum_{k=-3}^{4} = 8$ ..from part b

Since period is 4,

$x[-3] = x[1], x[-2] = x[2], x[-1] = x[3], x[0] = x[4]$

Therefore; (1) $x[1] + x[2] + x[3] + x[4] = 4$

$\sum_{0}^{3} x[k](e^{-jk\pi/2} + e^{-jk3\pi/2}) = 4$ ..from part e

$=x[0]2 + x[1](e^{-j\pi/2} + e^{-j3\pi/2}) + x[2](e^{-j\pi} + e^{-j3\pi}) + x[3](e^{-j3\pi/2} + e^{-j9\pi/2})$

$=2x[0] - 2x[2]$

$=4$

$x[0] - x[2] = 2$

since period is 4

Therefore; (2) $x[4] - x[2] = 2$

from part c

$a_{-3} = a_{-15} = a_1$ and $a_{11} = a_3$ because of periodicty of a_k

(3) $|a_1 - a_3| = 1$

$$a_1 = \frac{1}{4}\sum_{n=1}^{4} x[n]e^{-jn\pi/2}$$
$$= \frac{1}{4}(x[1](-j) + x[2](-1) + x[3](j) + x[4](1))$$

$$a_3 = \frac{1}{4}\sum_{n=1}^{4} x[n]e^{-jn3\pi/2}$$
$$= \frac{1}{4}(x[1](j) + x[2](-1) + x[3](-j) + x[4](1))$$

Since (3) $|a_1 - a_3| = 1$

$$a_1 - a_3 = \frac{2j}{4}(x[3] - x[1])$$

(4a) $x[3] - x[1] = 2$ or (4b) $x[3] - x[1] = -2$ can be.

Therefore; (5a) $a_1 = \frac{j}{2} + \frac{1}{2}$ or (5b) $a_1 = \frac{-j}{2} + \frac{1}{2}$

(6a) $a_3 = \frac{-j}{2} + \frac{1}{2}$ or (6b) $a_1 = \frac{j}{2} + \frac{1}{2}$

$$a_4 = \frac{1}{4}\sum_{n=1}^{4} x[n]e^{-jn2\pi}$$
$$= \frac{1}{4}(x[1](1) + x[2](1) + x[3](1) + x[4](1))$$ from (1) $x[1] + x[2] + x[3] + x[4] = 4$

(7) $a_4 = 1$

Since one of the coefficients is zero

(8) $a_2 = 0$

$a_2 = \dfrac{1}{4}\sum_{n=1}^{4} x[n]e^{-jn\pi}$
$\quad = \dfrac{1}{4}(x[1](-1) + x[2](1) + x[3](-1) + x[4](1))$
$(x[1](-1) + x[2](1) + x[3](-1) + x[4](1)) = 0$ and from (1) $x[1] + x[2] + x[3] + x[4] = 4$

(9) $x[3] + x[1] = 1$

and from (4a) $x[3] - x[1] = 2$

** $x[3] = 2$
** $x[1] = 0$

$x[2] = \sum_{k=1}^{4} a_k e^{jk\pi}$
$\quad = a_1(-1) + a_2(1) + a_3(-1) + a_4(1)$
**$x[2] = 0$

from (2) $x[4] - x[2] = 2$
** $x[4] = 2$

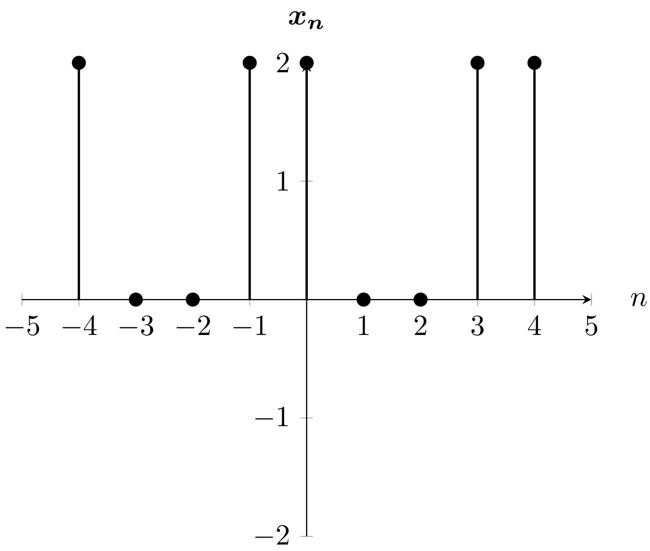

Figure 5: x_n

and from (4b) $x[3] - x[1] = -2$

** $x[3] = 0$
** $x[1] = 2$

$x[2] = \sum_{k=1}^{4} a_k e^{jk\pi}$
$\quad = a_1(-1) + a_2(1) + a_3(-1) + a_4(1)$
**$x[2] = 0$

from (2) $x[4] - x[2] = 2$
** $x[4] = 2$

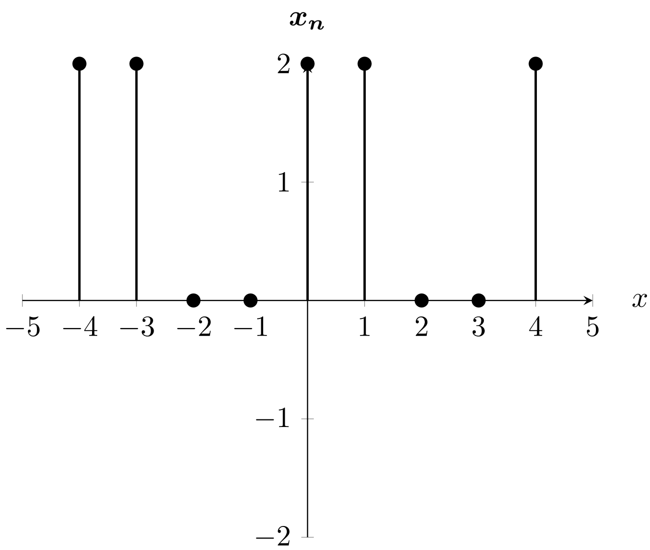

Figure 6: x_n

3.

$$x(t) = h(t) * x(t) + h(t) * r(t) \qquad \frac{-K2\pi}{2} \leq w \leq \frac{K2\pi}{2}$$
$$X(jw) = H(jw)X(jw) + H(jw)R(jw) \qquad \frac{-K2\pi}{2} \leq w \leq \frac{K2\pi}{2}$$
$$R(jw) = 0 \text{ for } \frac{-K2\pi}{2} \leq w \leq \frac{K2\pi}{2}$$
$$X(jw) = H(jw)X(jw) \qquad \frac{-K2\pi}{2} \leq w \leq \frac{K2\pi}{2}$$
$$H(jw) = 1 \qquad \frac{-K2\pi}{2} \leq w \leq \frac{K2\pi}{2}$$
$$h(t) = \delta(t) \qquad \frac{-K2\pi}{2} \leq w \leq \frac{K2\pi}{2}$$

4.

(a)

$$y(t) = \int(4x(t) - 5y(t) + (\int(x(t) - 6y(t))dt))dt$$
$$\frac{dy(t)}{dt} = 4x(t) - 5y(t) + \int(x(t) - 6y(t))dt$$
$$\frac{d^2y(t)}{dt^2} = \frac{4dx(t)}{dt} - \frac{5dy(t)}{dt} + x(t) - 6y(t)$$
$$\ddot{y}(t) + 5\dot{y}(t) - 6y(t) = x + 4\dot{x}(t)$$

$$Y(jw) = H(jw)X(jw)$$
$$H(jw) = \frac{1 + 4(jw)}{-6 + 5(jw) + (jw)^2}$$
$$H(jw) = \frac{1 + 4(jw)}{(jw - 3)(jw - 2)}$$

(b)

$$x(t) = \delta(t)$$
$$X(jw) = 1$$
$$H(jw) = \frac{1 + 4(jw)}{(jw - 3)(jw - 2)} = \frac{-13}{3 - jw} + \frac{9}{2 - jw}$$
$$h(t) = (-13e^{3t} + 9e^{2t})u(-t)$$

(c)

$$X(jw) = \frac{1}{4}\frac{1}{jw + 1/4} = \frac{1}{(4jw + 1)}$$
$$Y(jw) = H(jw)X(jw)$$
$$Y(jw) = \frac{1 + 4(jw)}{-6 + 5(jw) + (jw)^2} . \frac{1}{(4jw + 1)}$$

$$Y(jw) = \frac{1}{(jw-3)(jw-2)} = \frac{-1}{3-jw} + \frac{1}{2-jw}$$
$$y(t) = (e^{2t} - e^{3t})u(-t)$$