CENG 460 - Introduction to Robotics
Fall 2018

Middle East Technical University
Uluç Saranlı

Homework 1 - Pose Representations

Assigned - Oct 25, 2018, **Due - Nov 2, 2018**

1. Prove that a sphere of radius r centered at (c_x, c_y, c_z) remains a sphere with the same radius once transformed through homogeneous transformation T corresponding to a rotation R and a translation t. Note that points (x, y, z) on the original sphere satisfy $(x - c_x)^2 + (y - c_y)^2 + (z - c_z)^2 - r^2 = 0$. Where is the center of the new sphere located? Hint: Try to express the equation of the sphere above in matrix form using the vector $[x, y, z, 1]^T$ as we did in class.

2. Let $R \in SO(3)$ be a rotation matrix generated by rotating about a unit vector $\hat{w}$ by θ radians. That is, R satisfies $R = e^{[\hat{w}]_\times \theta}$.

   (a) Show that the eigenvalues of $[\hat{w}]_\times$ are 0, i, and $-i$, where $i = \sqrt{-1}$. What are the corresponding eigenvectors?

   (b) Show that the eigenvalues of R are 1, $e^{i\theta}$, and $e^{-i\theta}$. What is the eigenvector whose eigenvalue is 1?

   (c) Let $R = [r_1\ r_2\ r_3]$ be a rotation matrix. Show that $\det(R) = r_1^T(r_2 \times r_3)$

3. (a) Compute the homogeneous transformation representing a translation of an initial coordinate frame by 4 units along the z-axis (${}^0T_1$) followed by a rotation of $-\pi$ about the x-axis (${}^1T_2$) followed by a translation of 2 units along the new y-axis (${}^2T_3$) and finally a rotation around the z axis by $\pi/2$ (${}^3T_4$). Plot each intermediate frame using the Matlab toolbox and include them as figures. What are the coordinates of the frame origins O_i with respect to the original frame after each step? How did you compute these positions?

   (b) Give unit quaternion and translation representations of all the transformation steps in the first part. Also give the final transformation as a unit quaternion and a translation. How did you compute these new representations? Describe alternative methods you could have used to obtain the same quaternion and translation vectors.

4. Consider the coordinate frames in Figure 1. Find the homogeneous transformations ${}^0T_1$, ${}^0T_2$, ${}^1T_2$, representing transformations among the three frames shown. Verify your results by visualizing the frames using the Matlab toolbox and include a single figure in your report with all three frames shown. Show that ${}^0T_2 = {}^0T_1{}^1T_2$.

5. Suppose that we want to model the motion of a mobile robot moving on the horizontal plane, on which a 6DOF robot arm is mounted. Assume that the robot has its own "body" frame B located at (x, y) and an orientation θ with respect to a fixed world coordinate frame W on the ground. Suppose, also, that there is another coordinate frame R at base of the robot arm is fixed on the robot at a position ${}^Br$, having the same orientation as the body frame B. Finally, assume that the end

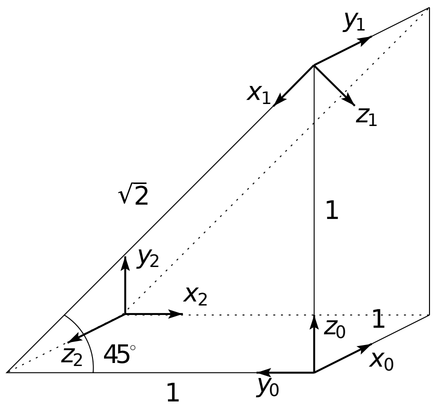

Figure 1: Illustration of coordinate frames (all right handed) for question 4.

effector of the robot arm has its own frame E which can move arbitrarily with respect to the robot arm base R.

(a) Write down the 3D homogeneous transformation ${}^{W}T_{R}$ as a function of x, y, θ and ${}^{B}r$.

(b) Given a specific robot trajectory $[x(t), y(t), \theta(t)]$, formulate an expression for what the motion of the robot arm should be such that the end effector frame stays fixed at a desired position and orientation specific by the homogeneous transformation ${}^{W}T_{desired}$ relative to the world frame.

6. Write a Matlab script to animate the motion of a cube with an edge size of $0.1m$, whose center is located at the end of a $1m$ rod, rotating clockwise with a velocity of $1rad/s$ on the floor of an elevator that is traveling upwards with $1m/s$. You can assume that at $t = 0$, the center of the rod is located at $[0, 0, 0]$ and the rod is pointing in the same direction as the x axis. You can also assume that the cube edges are aligned with the rod orientation. Explain your reasoning and derivations in the report, and submit your Matlab script $q6.m$ in your submission. Please include your name and student ID as a comment in the Matlab source file.

Submission

Submitted solutions must be typeset in a word processing environment such as LaTeX. Submissions are expected to be in the form of a ZIP file named `460_name_surname_hw#.zip`, including a PDF report with answers to theoretical questions with your name and student ID indicated clearly, as well as Matlab or other source files that are requested in the homework text. Late submissions will be penalized with a deduction of $10n^2$ points where n is the number of late days.

Note: You can discuss your discoveries and knowledge with your classmates but you must write your own answers and code for all questions above. If any significant similarities are found between your answers and other homeworks, you will be audited on your understanding of your own solutions.