CENG 384 - Signals and Systems for Computer Engineers
Spring 2018-2019
Written Assignment 4

Burhan, Beste
e2171395@ceng.metu.edu.tr

June 29, 2019

---

1. (a)

$$2x[n] = y[n] + \frac{y[n-2]}{8} - \frac{3y[n-1]}{4}$$

(b)

$$H(e^{jw}) = \frac{2}{1 - \frac{3e^{-jw}}{4} + \frac{e^{-2jw}}{8}} = \frac{2}{(1 - \frac{e^{-jw}}{2})(1 - \frac{e^{-jw}}{4})} = \frac{4}{1 - \frac{e^{-jw}}{2}} - \frac{2}{1 - \frac{e^{-jw}}{4}}$$

(c)

$$h[n] = 4\Big(\frac{1}{2}\Big)^n u[n] - 2\Big(\frac{1}{4}\Big)^n u[n]$$

(d)

$$X(e^{jw}) = \frac{1}{1 - \frac{e^{-jw}}{4}}$$

$$Y(e^{jw}) = H(e^{jw})X(e^{jw})$$

$$= \frac{2}{(1 - \frac{e^{-jw}}{2})(1 - \frac{e^{-jw}}{4})} . \frac{1}{(1 - \frac{e^{-jw}}{4})}$$

$$Y(e^{jw}) = \frac{8}{1 - \frac{e^{-jw}}{2}} + \frac{-4}{1 - \frac{e^{-jw}}{4}} + \frac{-2}{\Big(1 - \frac{e^{-jw}}{4}\Big)^2}$$

$$y[n] = 8\Big(\frac{1}{2}\Big)^n u[n] - 4\Big(\frac{1}{4}\Big)^n u[n] - 2(n+1)\Big(\frac{1}{4}\Big)^n u[n]$$

2.

$$h[n] = h_1[n] + h_2[n]$$

$$H(e^{jw}) = H_1(e^{jw}) + H_2(e^{jw})$$

$$H_1(e^{jw}) = \frac{1}{\Big(1 - \frac{e^{-jw}}{3}\Big)}$$

$$H_2(e^{jw}) = \frac{5e^{-jw} - 12}{e^{-2jw} - 7e^{-jw} + 12} - \frac{1}{\Big(1 - \frac{e^{-jw}}{3}\Big)}$$

$$H_2(e^{jw}) = \frac{-2}{\Big(1 - \frac{e^{-jw}}{4}\Big)} + \frac{1}{\Big(1 - \frac{e^{-jw}}{3}\Big)} + \frac{-1}{\Big(1 - \frac{e^{-jw}}{3}\Big)}$$

$$H_2(e^{-jw}) = \frac{-2}{\Big(1 - \frac{e^{-jw}}{4}\Big)}$$

$$h_2[n] = -2\Big(\frac{1}{4}\Big)^n u[n]$$

3.

(a)

$$x(t) = x_1(t) + x_2(t)$$

$x_1(t) = \frac{sin(2\pi t)}{\pi t}$

$$X_1(jw) = \begin{cases} 1 & |w| < 2\pi \\ 0 & |w| > 2\pi \end{cases} \tag{1}$$

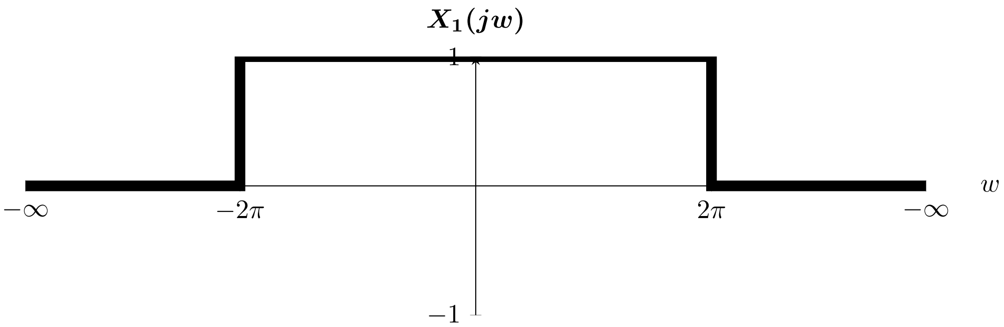

Figure 1: w vs. $X_1(jw)$.

$x_2(t) = cos(3\pi t)$
$X_2(jw) = \pi[\delta(w - 3\pi) + \delta(w + 3\pi)]$

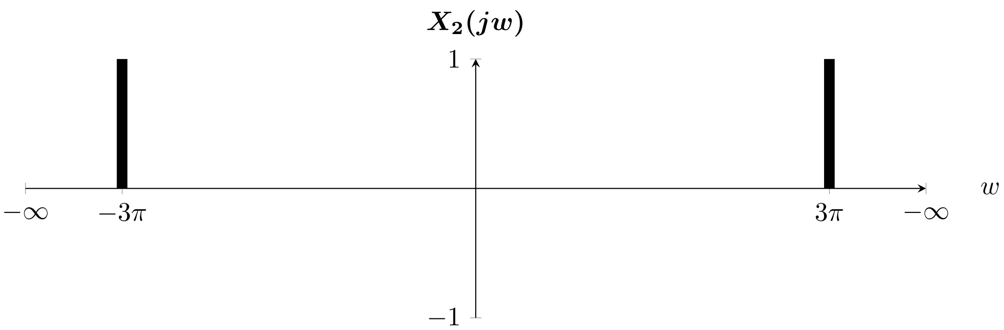

Figure 2: w vs. $X_2(jw)$.

$X(jw) = X_1(jw) + X_2(jw)$

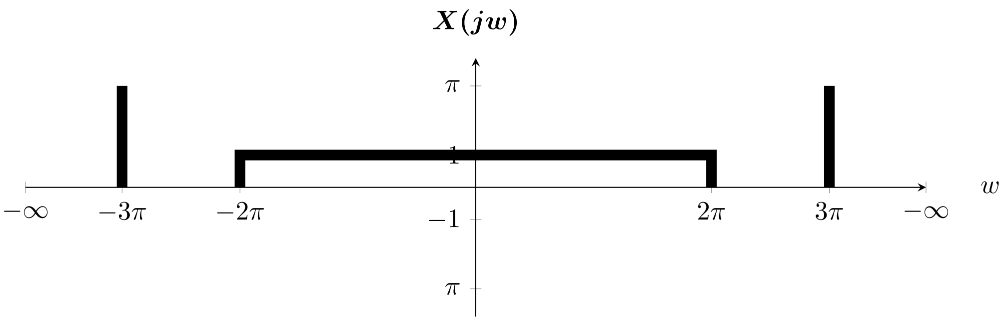

Figure 3: w vs. $X(jw)$.

(b)

$Nyquist\ rate = 2(3\pi) = 6\pi$
$Nyquist\ frequency = 3\pi$
$Nyquist\ period = \frac{2}{3}$

(c)

$x_p(t) = x(t)p(t)$
$P(jw) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(w - kw_s)$
$x_p(t) = \sum_{k=-\infty}^{\infty} x(nT)\delta(t - kT)$
$$X_p(jw) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\theta)P(j(w - \theta))d\theta$$
$$= \frac{1}{T} \sum_{k=-\infty}^{\infty} X(j(w - kw_s))$$

$= \dfrac{3}{2}\sum_{k=-\infty}^{\infty} X(j(w-k6\pi))$

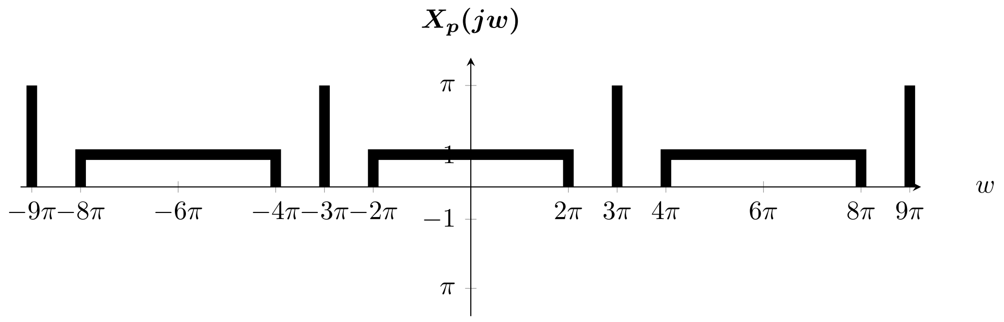

Figure 4: w vs. $X_p(jw)$.

4.

(a)

$w_s = \pi \qquad T = \dfrac{2\pi}{\pi} = 2$

$$X(jw) = \begin{cases} \dfrac{4w}{\pi} & |w| < \pi/4 \\ 0 & o.w. \end{cases} \tag{2}$$

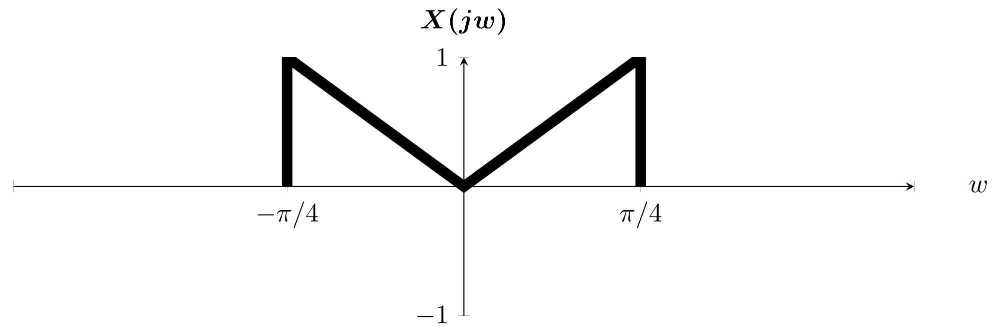

Figure 5: w vs. $X(jw)$.

$x_p(t) = \sum_{k=-\infty}^{\infty} x(nT)\delta(t-kT)$

$X_p(jw) = \dfrac{1}{2}\sum_{k=-\infty}^{\infty} X(j(w-k\pi))$

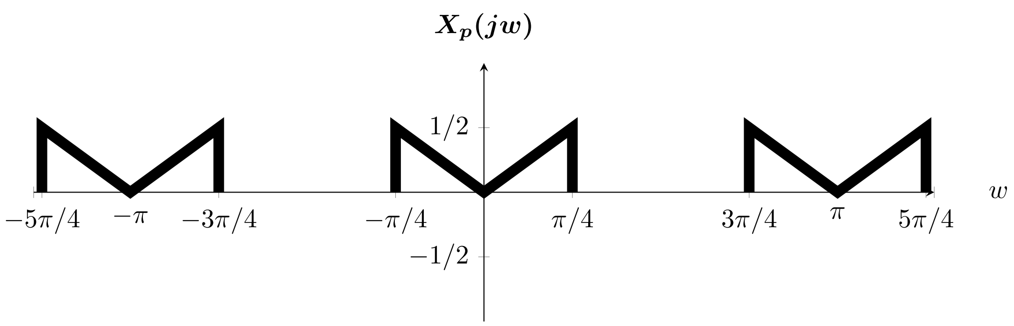

Figure 6: w vs. $X_p(jw)$.

$X_d(e^{jw}) = \dfrac{1}{2}\sum_{k=-\infty}^{\infty} X(j(w-2\pi k)/T)$

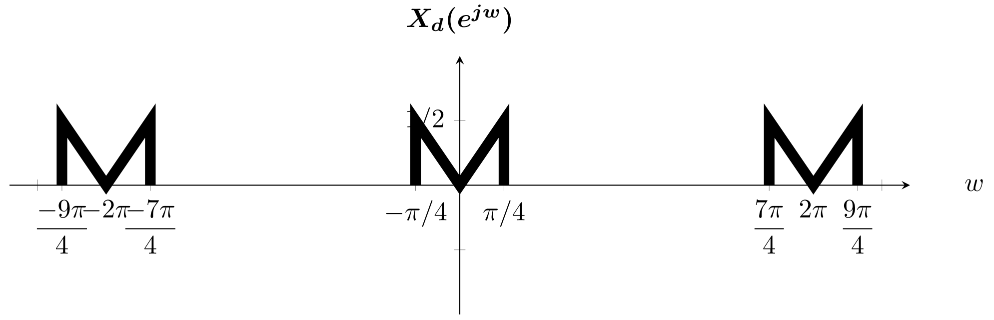

Figure 7: w vs. $X_d(e^{jw})$.

(b)

$H(e^{jw}) = \pi \sum_{k=-\infty}^{\infty} \delta(w - \pi - 2\pi k) + \delta(w + \pi - 2\pi k)$

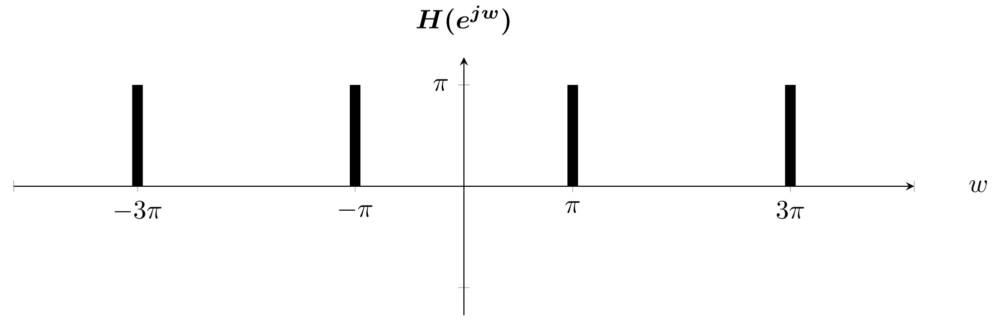

Figure 8: w vs. $H(e^{jw})$.

(c)

$$y_d[n] = x_d[n]h[n]$$

$$\begin{aligned} Y_d(e^{jw}) &= \frac{1}{2\pi} X_d(e^{jw}) * H(e^{jw}) \\ &= \frac{1}{2\pi} \int_{<2\pi>} X_d(e^{j\theta}) H(e^{j(w-\theta)}) d\theta \end{aligned}$$

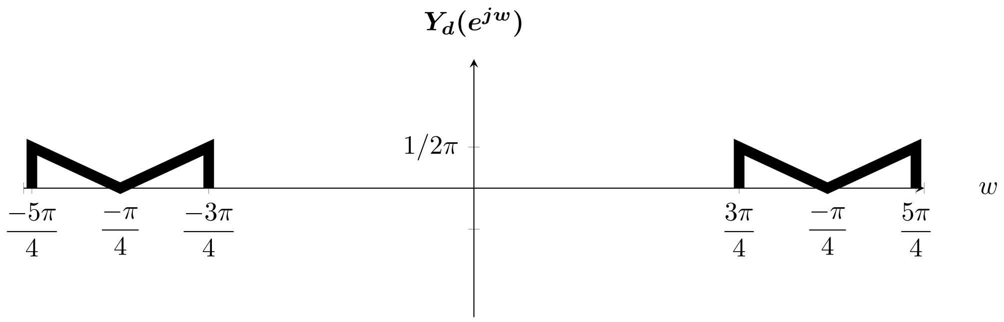

Figure 9: w vs. $Y_d(e^{jw})$.