CENG 384 - Signals and Systems for Computer Engineers
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Written Assignment 1

Öztürk, Kürşat
e2171874@ceng.metu.edu.tr

Burhan, Beste
e2171395@ceng.metu.edu.tr

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1. (a)

$y(t) = \int (x(t) - 4y(t))dt$
$\frac{dy(t)}{dt} = x(t) - 4y(t)$
$\dot{y}(t) + 4y(t) = x(t)$

(b)

$y(t) = y_h(t) + y_p(t)$
Since initially at rest ;
$y(0) = \dot{y}(0) = \ddot{y}(0) = .. = 0$
Homogeneous solution;
$y_h(t) = Ke^{\alpha t}$
$(K\alpha + 4K)e^{\alpha t}$
$\alpha = -4$
$y_h(t) = Ke^{-4t}$
Particular solution;
$x(t) = (e^{-t} + e^{-2t})u(t)$
$y_p(t) = ae^{-t} + be^{-2t}$, for $t > 0$
$-ae^{-t} - 2be^{-2t} + 4ae^{-t} + 4be^{-2t} = e^{-t} + e^{-2t}$
$3ae^{-t} + 2be^{-2t} = e^{-t} + e^{-2t}$
$a = 1/3 \quad b = 1/2$
$y_p(t) = \frac{1}{3}e^{-t} + \frac{1}{2}e^{-2t},$ for $t > 0$

$y(t) = Ke^{-4t} + \frac{1}{3}e^{-t} + \frac{1}{2}e^{-2t},$ for $t > 0$
$y(0) = 0$
$0 = K + \frac{1}{3} + \frac{1}{2}$
$K = -5/6$
$y(t) = \left[\frac{-5}{6}e^{-4t} + \frac{1}{3}e^{-t} + \frac{1}{2}e^{-2t}\right]u(t)$

2.

(a)

$x[n] = \delta[n-1] - 3\delta[n-2] + \delta[n-3]$
$h[n] = \delta[n+1] + 2\delta[n] - 3\delta[n-1]$

$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$
$y[n] = h[-1]x[n+1] + h[0]x[n] + h[1]x[n-1]$
$= x[n+1] + 2x[n] + 3x[n-1]$
$y[n] = (\delta[n] - 3\delta[n-1] + \delta[n-2]) + (2\delta[n-1] - 6\delta[n-2] + 2\delta[n-3]) + (-3\delta[n-2] + 9\delta[n-3] - 3\delta[n-4])$
$= \delta[n] - \delta[n-1] - 8\delta[n-2] + 11\delta[n-3] - 3[n-4]$

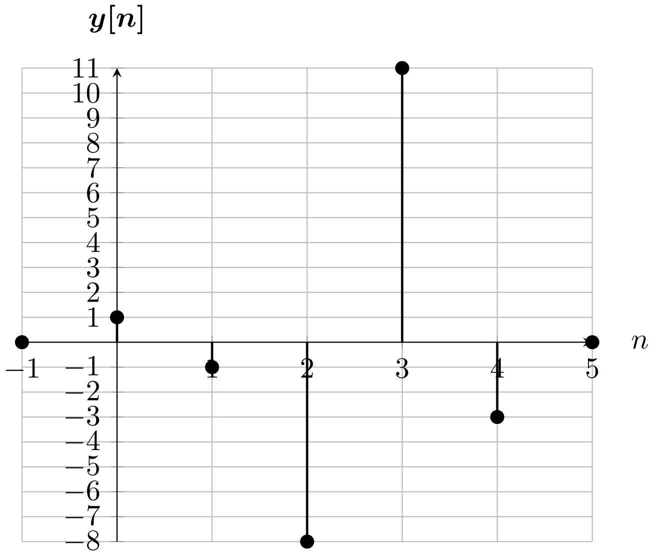

Figure 1: $y[n] = \delta[n] - \delta[n-1] - 8\delta[n-2] + 11\delta[n-3] - 3[n-4]$.

(b)

$x(t) = u(t) + u(t-1)$
$h(t) = (e^{-2t}cost)u(t)$
$\frac{dx(t)}{dt} = \delta(t) + \delta(t-1)$
$y(t) = \frac{dx(t)}{dt} * h(t) = \int_0^\infty e^{-2\tau}cos\tau\delta(t-\tau)d\tau + \int_0^\infty e^{-2\tau}cos\tau\delta(t-1-\tau)d\tau$
$= e^{-2t}costu(t) + e^{-2(t-1)}cos(t-1)u(t-1)$

3.

(a)

$x(t) = e^{-t}u(t)$
$h(t) = e^{-3t}u(t)$

$y(t) = x(t) * h(t) = \int_0^t e^{-\tau}e^{-3(t-\tau)}d\tau$
$= e^{-3t}\int_0^t e^{2\tau}d\tau$
$= e^{-3t}\left(\frac{e^{2t}}{2} - \frac{1}{2}\right)$
$y(t) = \left[\frac{e^{-t}}{2} - \frac{e^{-3t}}{2}\right]u(t)$

(b)

$x(t) = u(t-1) - u(t-2)$
$h(t) = e^t u(t)$

$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$
$= \int_0^\infty e^\tau(u(t-\tau-1) - u(t-\tau-2))d\tau$
$t \leq 1$,this convolution evalutates zero.
For $1 < t \leq 2$
$y(t) = \int_0^{t-1} e^\tau d\tau = e^{t-1} - 1$
For $t > 2$
$y(t) = \int_{t-2}^{t-1} e^\tau d\tau = e^{t-1} - e^{t-2}$

$$y(t) = \begin{cases} 0, & -\infty < t \leq 1 \\ e^{t-1} - 1, & 1 < t \leq 2 \\ e^{t-1} - e^{t-2} & 2 < t < \infty \end{cases}$$

4.

(a)

$y[n] - 15y[n-1] + 26y[n-2] = 0 \qquad y[0] = 10, y[1] = 42$

Characteristic equation;

$r^2 - 15r + 26 = 0$

$r_1 = 13,\ r_2 = 2$

$y[n] = A(2)^n + B(13)^n$

$y[0] = A + B = 10$

$y[1] = A2 + B13 = 42$

$B = 2, A = 8$

$y[n] = 8.2^n + 2.13^n$

(b)

$y[n] - 3y[n-1] + y[n-2] = 0 \qquad y[0] = 1,\ y[1] = 2$

Characteristic equation;

$r^2 - 3r + 1 = 0$

$r_1 = \dfrac{3-\sqrt{5}}{2},\ r_2 = \dfrac{3+\sqrt{5}}{2}$

$y[n] = A\Big(\dfrac{3-\sqrt{5}}{2}\Big)^n + B\Big(\dfrac{3+\sqrt{5}}{2}\Big)^n$

$y[0] = A + B = 1$

$y[1] = A\Big(\dfrac{3-\sqrt{5}}{2}\Big) + B\Big(\dfrac{3+\sqrt{5}}{2}\Big) = 2$

$B = \dfrac{5+\sqrt{5}}{10},\ A == \dfrac{5-\sqrt{5}}{10}$

$y[n] = \dfrac{5-\sqrt{5}}{10}\Big(\dfrac{3-\sqrt{5}}{2}\Big)^n + \dfrac{5+\sqrt{5}}{10}\Big(\dfrac{3+\sqrt{5}}{2}\Big)^n$

5.

(a)

$\ddot{y}(t) + 6\dot{y}(t) + 8y(t) = 2x(t)$

$x(t) = \delta(t)$

$\ddot{h}(t) + 6\dot{h}(t) + 8h(t) = 2\delta(t)$

$h(t) = 0$ for $t < 0 \qquad h(0^-) = 0,\ \dot{h}(0^-) = 0$

For $t = 0$

$\int_{0^-}^{0^+} \ddot{h}(t)dt + 6\int_{0^-}^{0^+} \dot{h}(t)dt + 8\int_{0^-}^{0^+} h(t)dt = \qquad 2\int_{0^-}^{0^+} \delta(t)dt$

$\dot{h}(0^+) - \dot{h}(0^-) + 6h(0^+) - 6h(0^-) + 0 = 2$

$\dot{h}(0^+) - 6h(0^+) = 2$

$\dot{h}(0^+) = 2$

$h(0^+) = 0$ since the position $y(t)$ is not changed at $t = 0$ so $y(0^+) = y(0^-)$

For $t > 0$ it must be solve;

$\ddot{h} + 6\dot{h} + 8h = 0,\ \dot{0^+} = 2,\ h(0^+) = 0$

Characteristic equation;

$r^2 + 6r + 8 = 0 \qquad r_1 = -4,\ r_2 = -2$

$h(t) = K_1 e^{-4t} + K_2 e^{-2t} \qquad$ for $t > 0$

$0 = h(0^+) = K_1 + K_2$

$2 = \dot{h}(0^+) = -4K_1 - 2K_2$

$K_1 = -1,\ K_2 = 1$

$h(t) = \Big[-e^{-4t} + e^{-2t}\Big]u(t)$

(b)

$h(t) = \Big[-e^{-4t} + e^{-2t}\Big]u(t)$

i.

Causal because $h(t) = 0$ for $t < 0$

ii.

It has memory because when $\exists t \neq 0\ h(t) \neq 0$. For example $h(1) = -e^{-4} + e^{-2} \neq 0$

iii.

Stable because $\int_{-\infty}^{\infty} |h(t)|dt = 3/4 < \infty$

iv.

It is invertible