Is vagueness rational?

Presentation of Bachelor-Thesis

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11.06.2018

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Misinterpretation

(https://xkcd.com/1984/)

Problem Statement

- Why is language vague?
- Which processes enable us to understand vague adjectives like *tall*?
- What is the exact semantics of such vague terms?
- Is vagueness rational?

Contents

Pragmatics and Game Theory

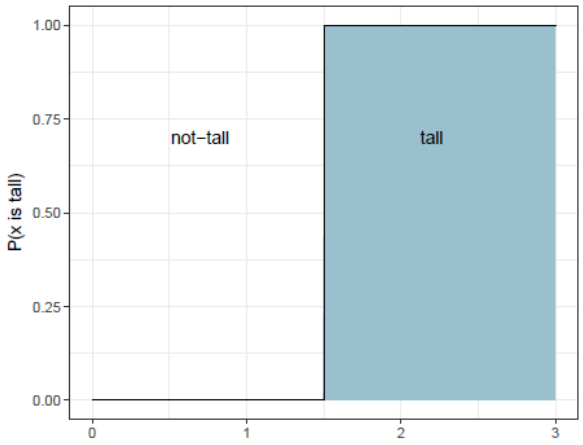
- **Pragmatics** is a subfield of linguistics.
- **(Evolutionary) Game theory** analyzes strategic interaction between individuals/agents.
- An **evolutionary stable strategy** cannot be further improved by other strategies in a population.

Adjectival vagueness in language use

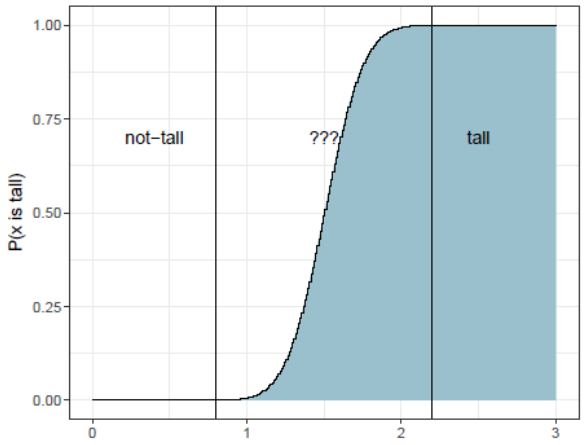
Characteristics of vague adjectives:

- Existence of **borderline cases**.
- **Threshold** semantics.

Schematic presentation of crisp and vague denotations

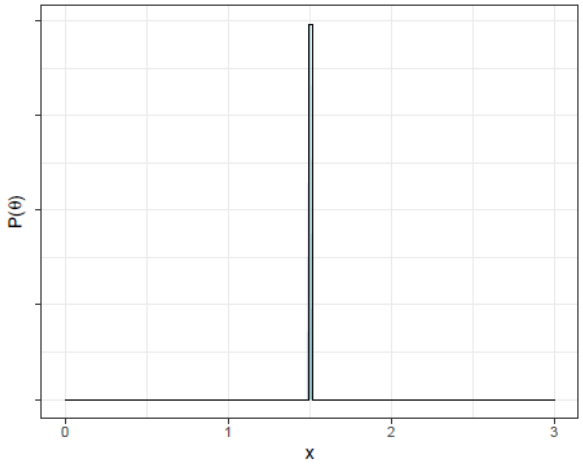


(a) Crisp meaning function, according to $P(\theta)$ in figure (c).

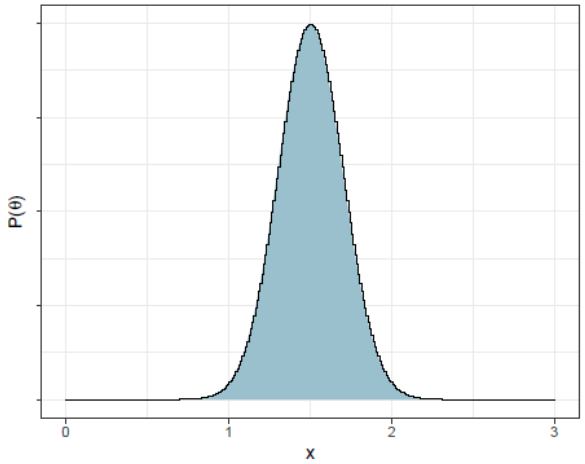


(b) Vague meaning function with borderline cases, according to $P(\theta)$ in figure (d).

$$\rightarrow P(x \text{ is tall}) = \Phi(P(\theta))$$
$$= [\![\text{"tall"}]\!]^x$$



(c) Exact belief about the value of θ.



(d) Uncertainty about the exact value of θ.

$$\rightarrow P(\theta)$$

RSA - Model

- The **rational speech acts model** (RSA model) is a cognitive model of language-understanding and -production.
- **Bayes' theorem:** $P(A \mid B) \propto P(B \mid A) \cdot P(A)$.
- An **informative speaker** chooses utterances, depending on their **informativity** for a hypothetical **literal listener**.
- A **pragmatic listener** infers world states (given a message) by reasoning about the speaker model and taking into account alternative messages.

Extension to RSA by Bergen & Goodman (2012)

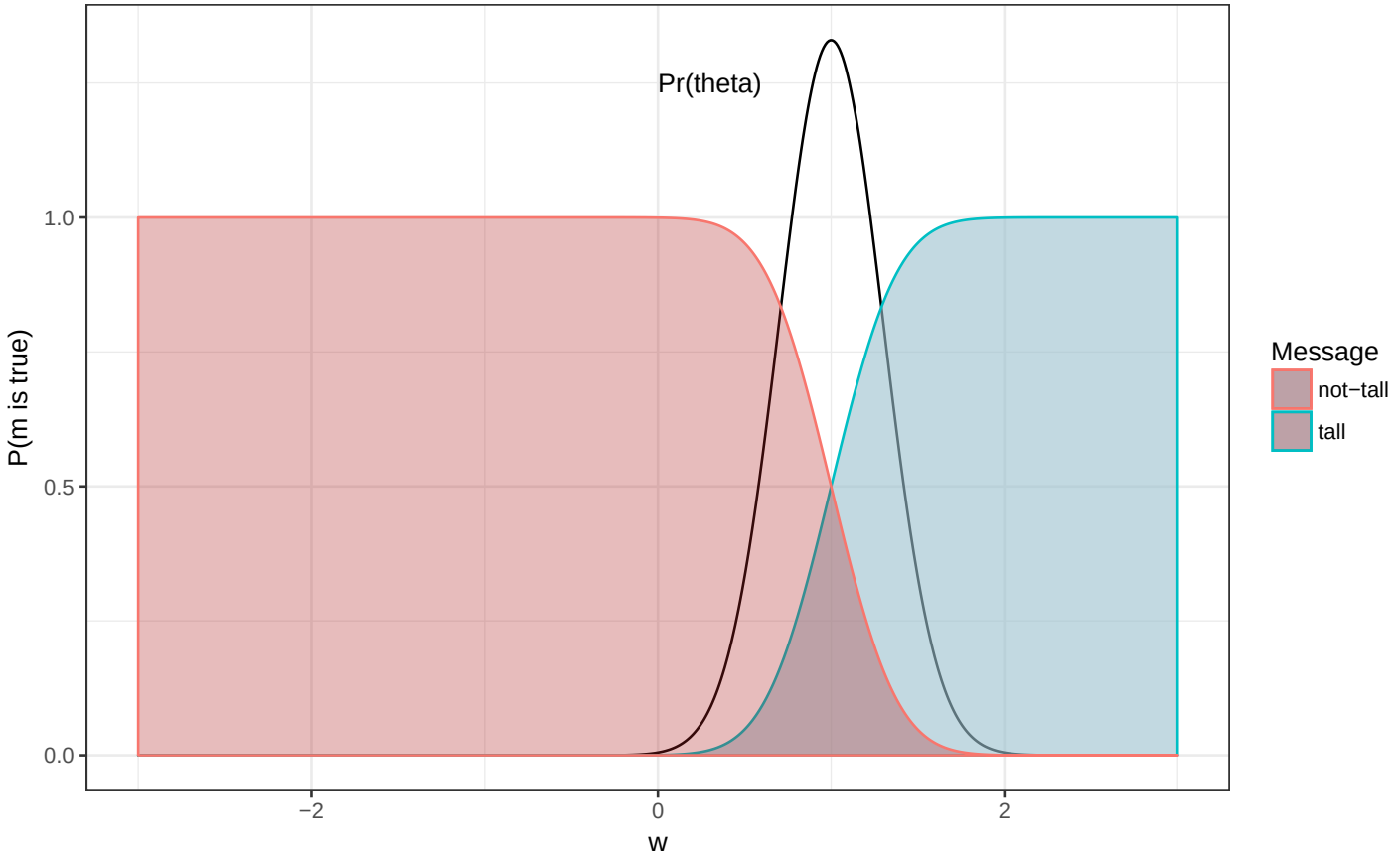
Agents are defined by **types**, that represent the semantic understanding:

- **Literal listener**:
  $P_{L0}(w \mid m, [\mu, \sigma, \alpha]) = [\![m]\!]^{w,\mu,\sigma} \cdot Pr(w)$
- **Informative speaker**:
  $P_{S1}(m \mid w, [\mu, \sigma, \alpha]) \propto exp(\alpha \cdot log(P_{L0}(w \mid m, [\mu, \sigma, \alpha])))$
- **Pragmatic listener**:
  $P_{L_1}(w \mid m, [\mu, \sigma, \alpha]) \propto P_{S_1}(m \mid w, [\mu, \sigma, \alpha]) \cdot Pr(w)$:

With: $w =$ world state (e.g. height),
$m =$ message,
$Pr =$ Prior,
$\mu, \sigma =$ Threshold parameters,
$\alpha =$ "Rationality" parameter

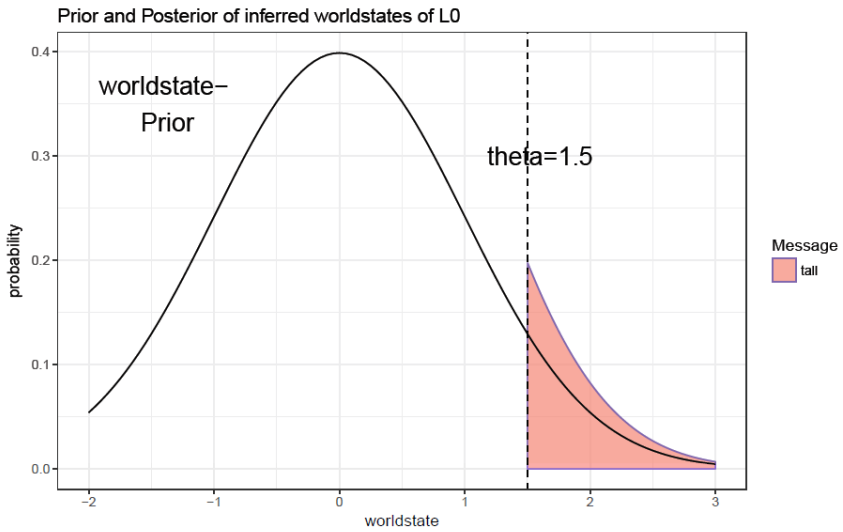
Implementation of vagueness

Literal vague meaning of *tall* and *not-tall*:

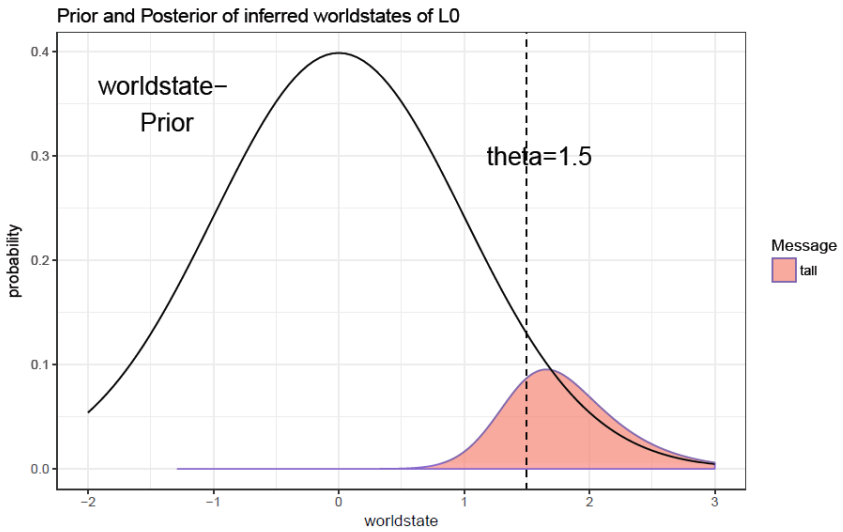


Literal listener L_0 - Posterior

$$P_{L0}(w \mid m, [\mu, \sigma, \alpha]) = [\![m]\!]^{w,\mu,\sigma} \cdot Pr(w):$$



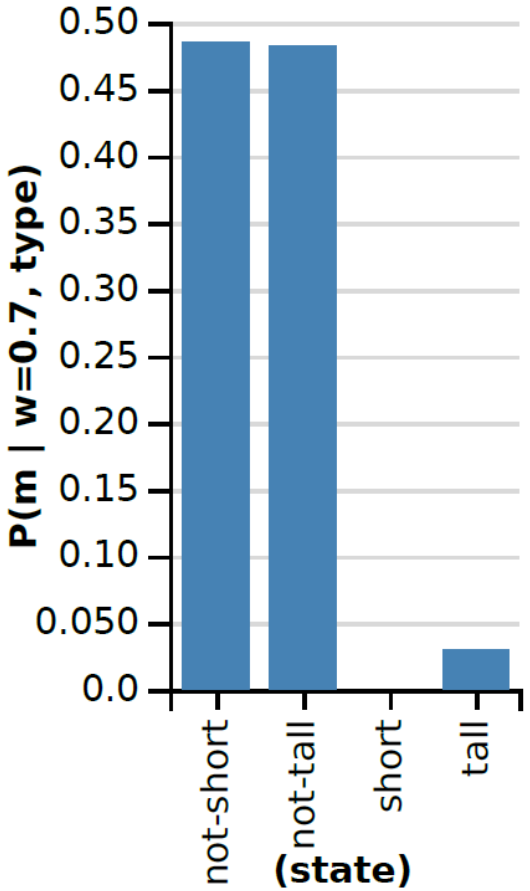
(a) L0-type: $\mu = 1.5, \sigma = 0.0001, \alpha = 10$



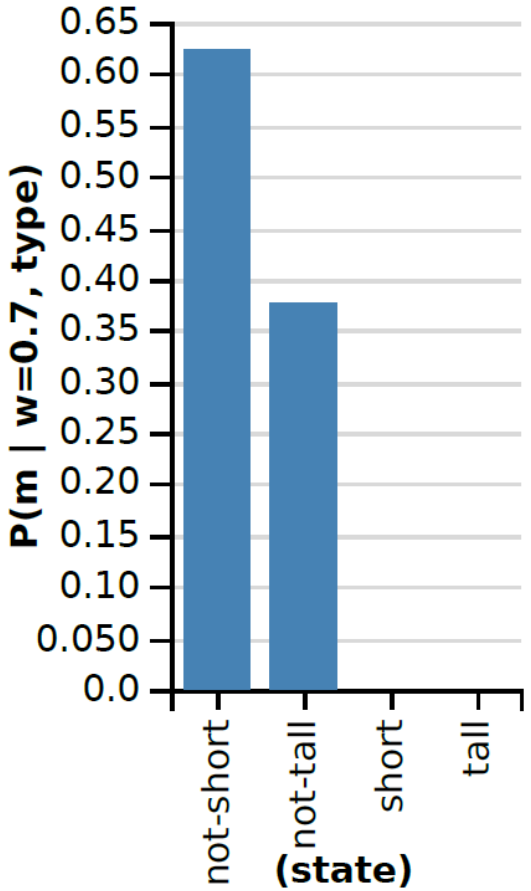
(b) L0-type: $\mu = 1.5, \sigma = 0.3, \alpha = 10$

Informative speaker S_1 - Posterior

$$P_{S1}(m \mid w, [\mu, \sigma, \alpha]) \propto exp(\alpha \cdot log(P_{L0}(w \mid m, [\mu, \sigma, \alpha])))$$:



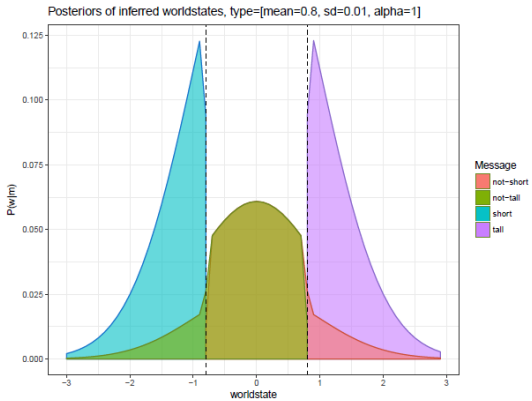
(a) S1-type: $\mu = 1.5, \sigma = 0.3, \alpha = 1$



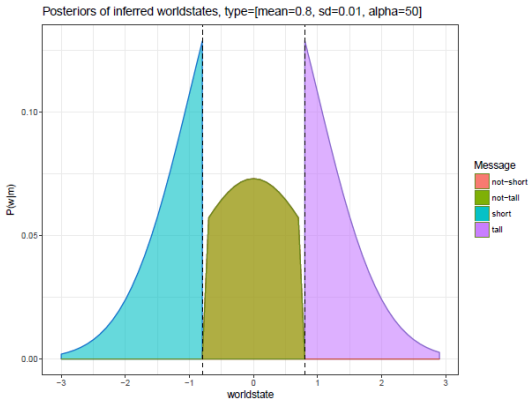
(b) S1-type: $\mu = 1.5, \sigma = 0.3, \alpha = 100$

Pragmatic listener L_1 - Posterior

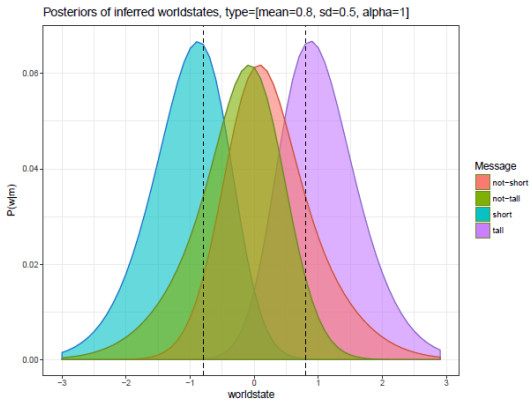
$$P_{L_1}(w \mid m, [\mu, \sigma, \alpha]) \propto P_{S_1}(m \mid w, [\mu, \sigma, \alpha]) \cdot Pr(w):$$



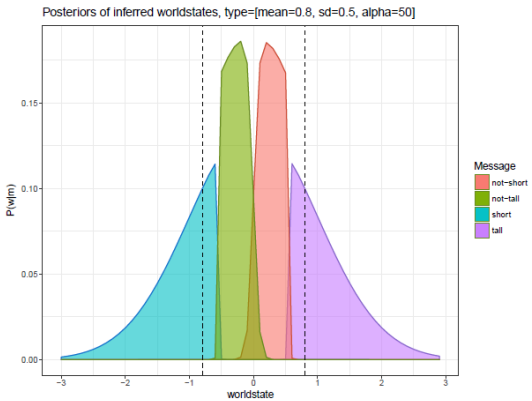
(a) L1-type: $\mu = 0.8, \sigma = 0.01, \alpha = 1$



(b) L1-type:$\mu = 0.8, \sigma = 0.01, \alpha = 50$



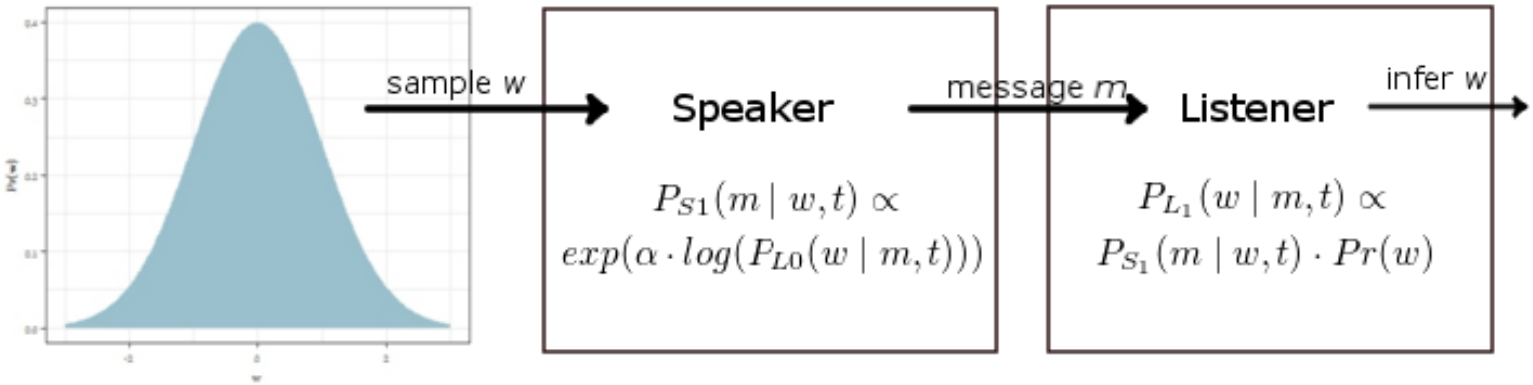
(c) L1-type:$\mu = 0.8, \sigma = 0.5, \alpha = 1$



(d) L1-type:$\mu = 0.8, \sigma = 0.5, \alpha = 50$

Goal of simulation

- Agents behave according to RSA.
- Examine effect of different semantic beliefs.
- Find out best strategy.



Measure of communicative success: Expected Utility

The **Expected Utility (EU)** is calculated as followed:

$$EU(t_1, t_2) = \sum_w \sum_m 0.5 \cdot \big[P_{S_1}(m \mid w,\ t_1) \cdot P_{L_1}(w \mid m,\ t_2) \cdot Pr(w) + P_{S_1}(m \mid w,\ t_2) \cdot P_{L_1}(w \mid m,\ t_1) \cdot Pr(w)\big]$$

Simulation set-up

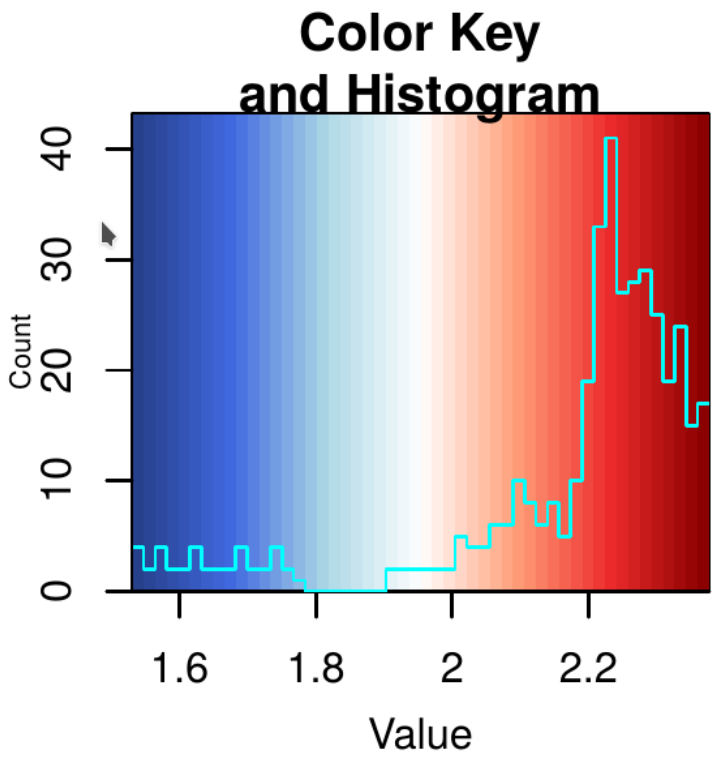
In the simulation, the types are combined from the following parameter spaces:

$$\mu \sim \{0, 0.1, 0.2, ..., 1.9\}$$
$$\sigma \sim \{0.001, 0.1, 0.2, ..., 1.9\}$$
$$\alpha \sim \{1, 5, 10, 50, 100\}$$

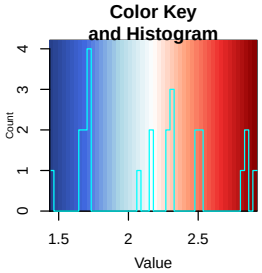
$$M = \{short, not - short, tall, not - tall\}$$

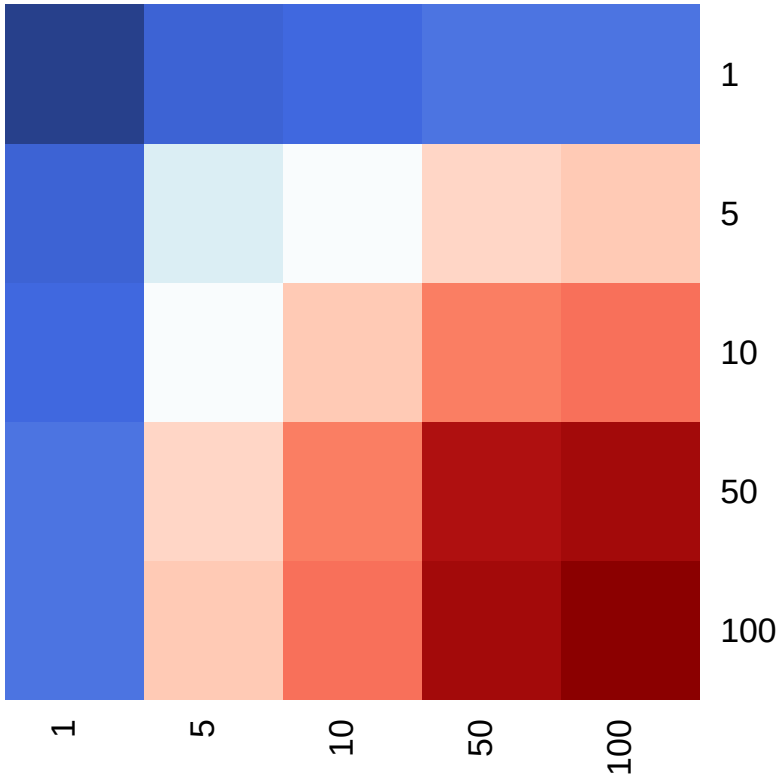
Simulation Results / EU data

The **Expected Utility** values are displayed in a **heatmap** visualization.
Color-key-mapping:

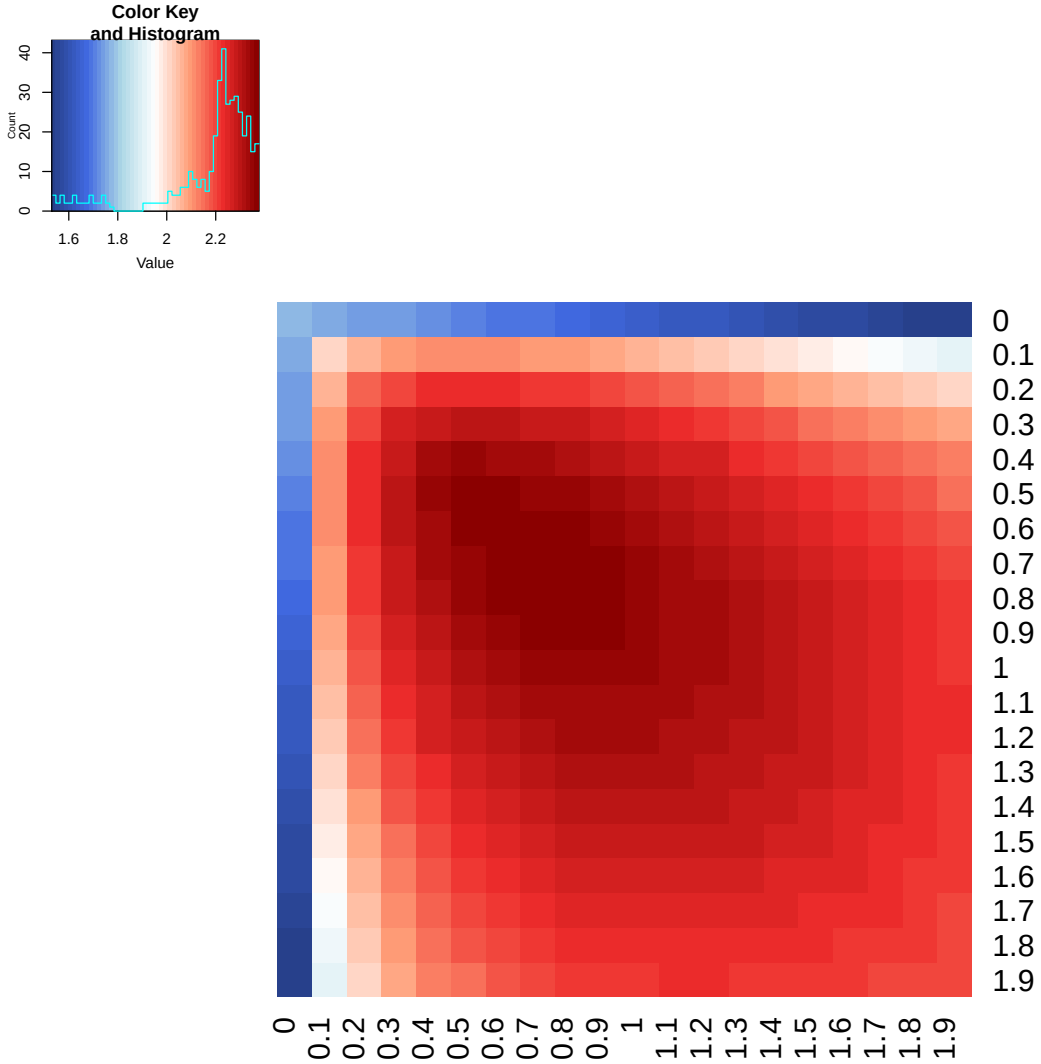


Effect of parameter α

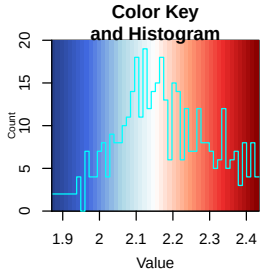


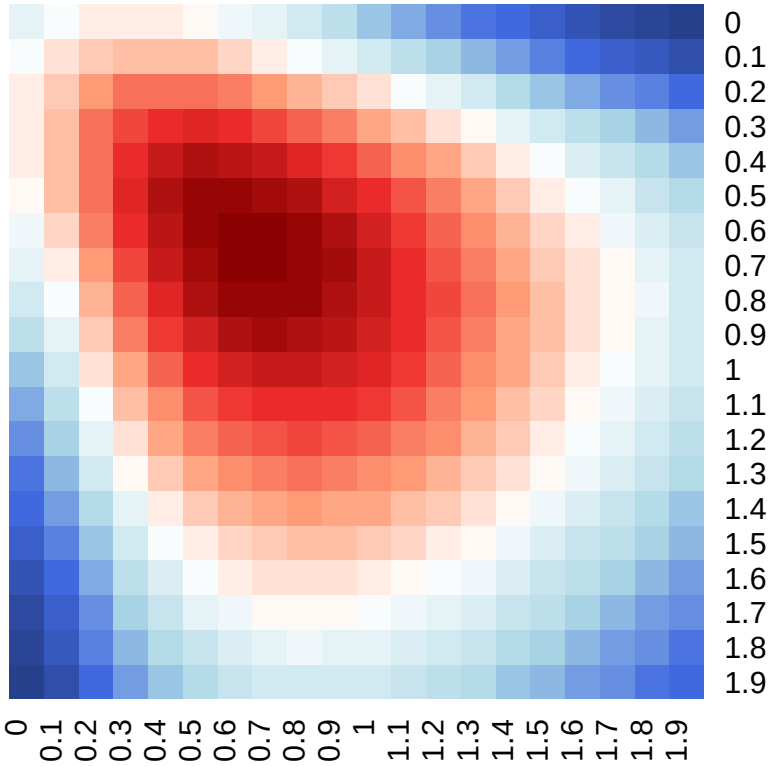


Effect of parameter μ

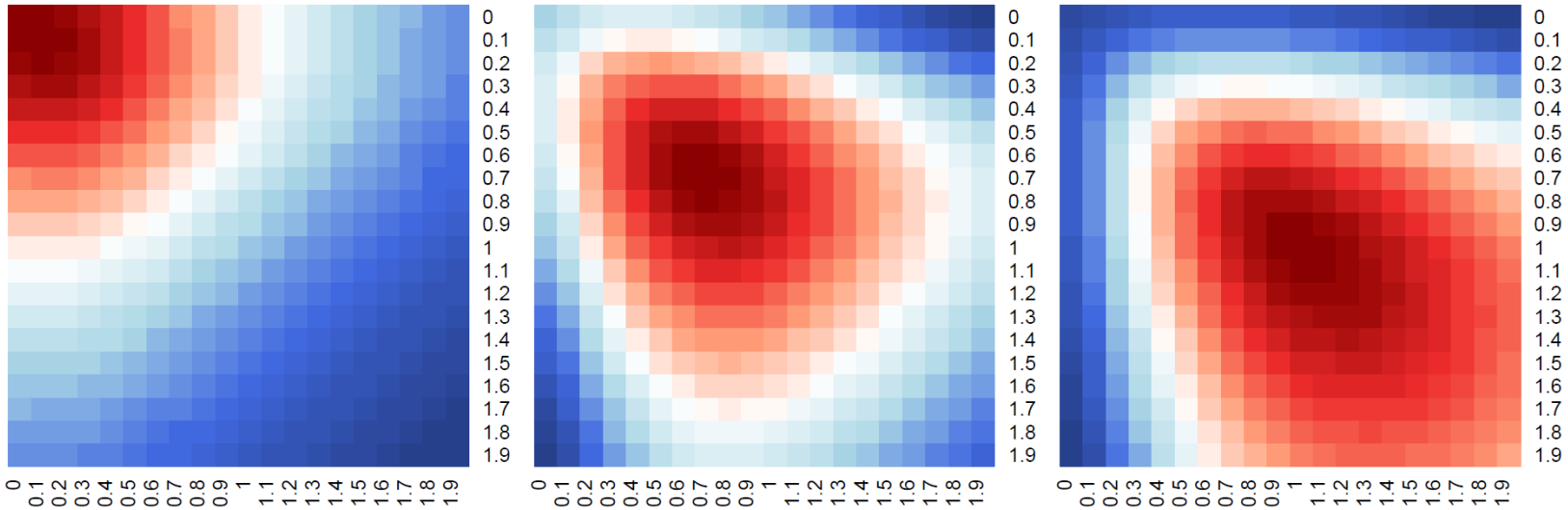


Effect of parameter σ





Interaction effect of parameters α and σ



(a) Effect of σ on EU-scores with $\alpha = 1$.

(b) Effect of σ on EU-scores with $\alpha = 10$.

(c) Effect of σ on EU-scores with $\alpha = 50$.

Evolutionary stable strategies

With S = set of possible strategies.
Strategy s_i is an ESS, if for all $s_j \neq s_i \in S$:

$$1. EU(s_i, s_i) \geq EU(s_j, s_i) \quad \textbf{and}$$
$$2. EU(s_i, s_j) > EU(s_j, s_j)$$

The only ESS is:

$$type_{opt} = \left[\mu = 0.8, \sigma = 0.5, \alpha = 100\right]$$

Discussion and Conclusion

- Pragmatic recursive reasoning allows for interpretation of vague adjectives.
- Rational agents can make use of vagueness.
- Vagueness indeed seems to be rational.

Thank you!