Lecture 8:
Stellar structure -

Getting the energy out: energy transport

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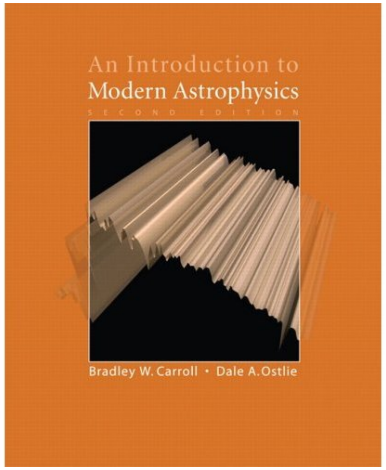
Chapters 9 and 10 of Carroll and Ostlie



The structure of stars

(1) Getting the energy out - radiation and opacity (lecture 8)

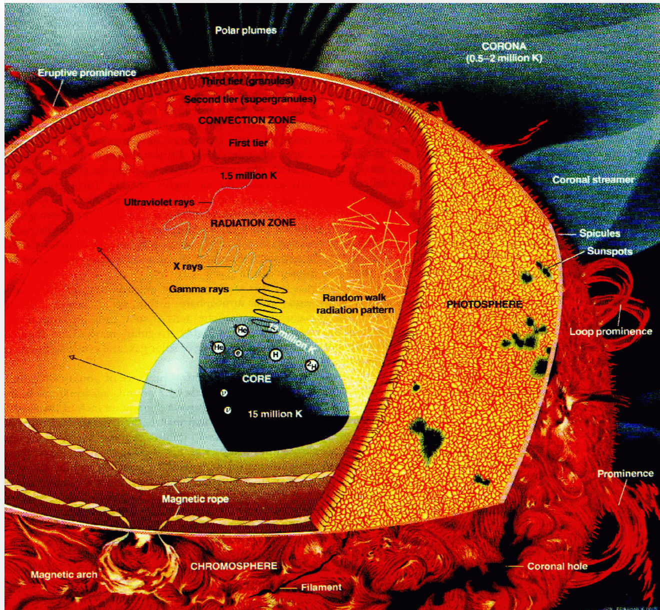
(2) Mean free path - random walk of photons (lecture 8)

(3) Opacity and optical depth
- sources of opacity (lecture 9)

(4) Getting the energy out
- convection in stars (lecture 10)

(5) Review of stars (lecture 11)
- stellar models
- minimum and maximum masses of stars

(6) Pulsating stars (lecture 12)
- stellar types
- physics of pulsating stars



Aims of lecture

Key concept: random walk of photons

Aims:

- Understand the basis behind energy transport in stars and be able to show:

$$\frac{dL}{dr} = 4\pi r^2 \rho \varepsilon$$

Energy conservation

- Understand the random walk of photons and be able to show:

$$\ell = \frac{1}{n\sigma}$$

Mean free path

$$N = \left(\frac{d}{\ell}\right)^2$$

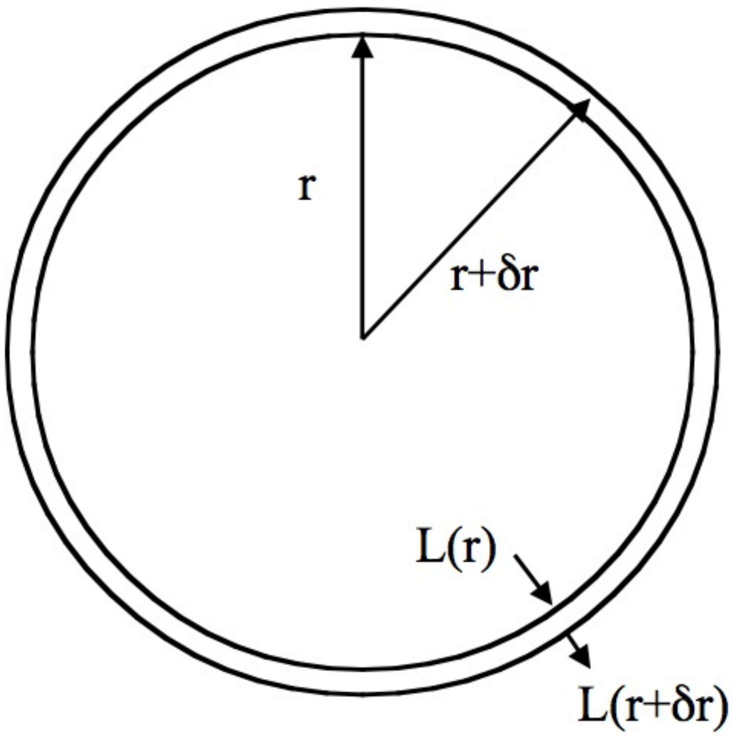
Number of scattering events

The production and conservation of energy

We can define energy release as

$$dL = \varepsilon dm$$

where ε is the energy released (W kg $^{-1}$) by nuclear reactions (e.g., $\varepsilon_{ix} = \varepsilon_0' X_i X_x \rho^{\alpha} T^{\beta}$). In some stars there is significant additional energy release (e.g., gravitational at some evolutionary stages).



For a spherically symmetric star the mass of a thin shell of depth *dr* is

$dm = \rho dV$ which is

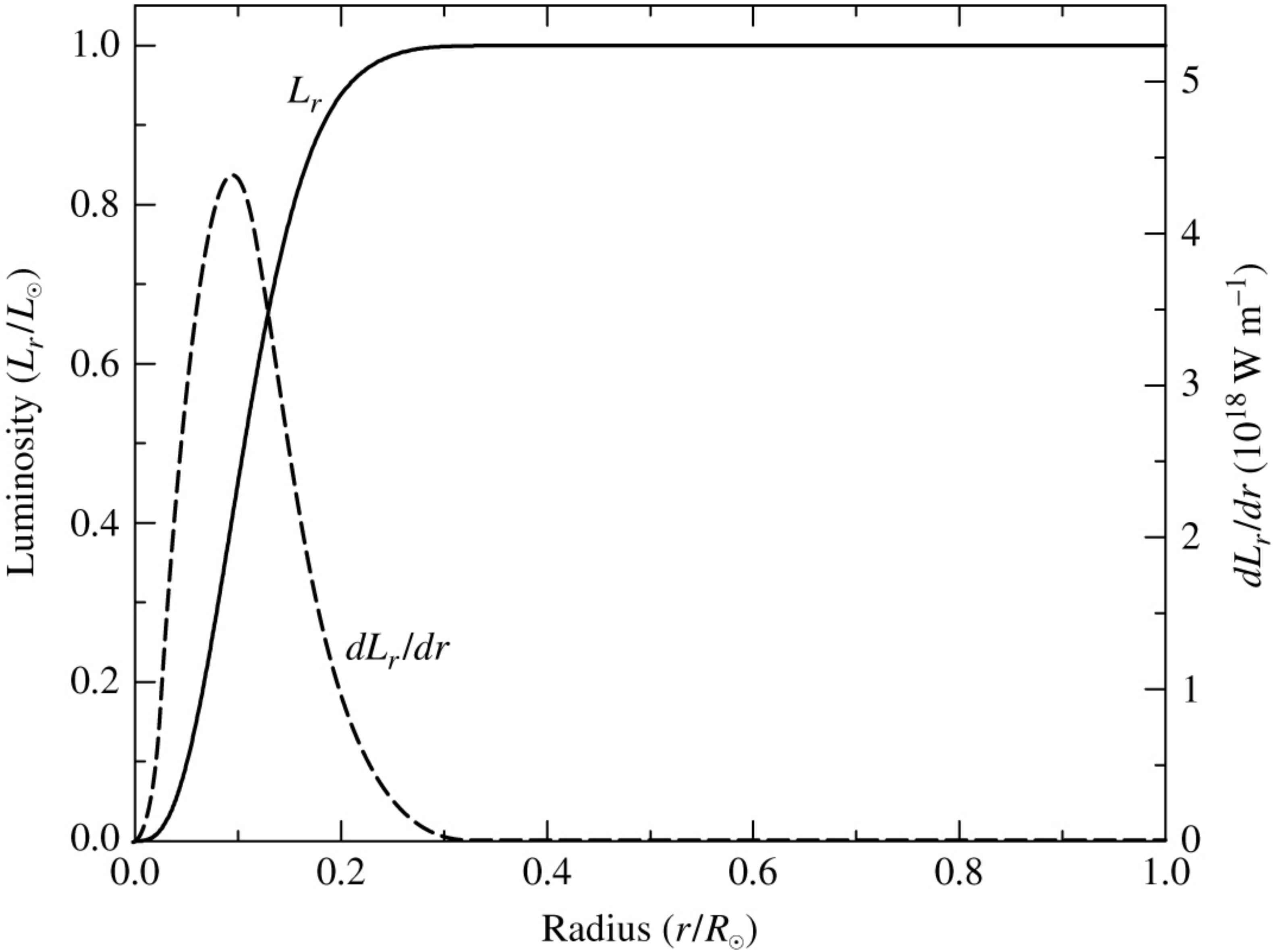
$dm = 4\pi r^2 \rho dr$ and therefore

$$\frac{dL}{dr} = 4\pi r^2 \rho \varepsilon \qquad \textbf{Equation 15}$$

This is the equation of energy conservation

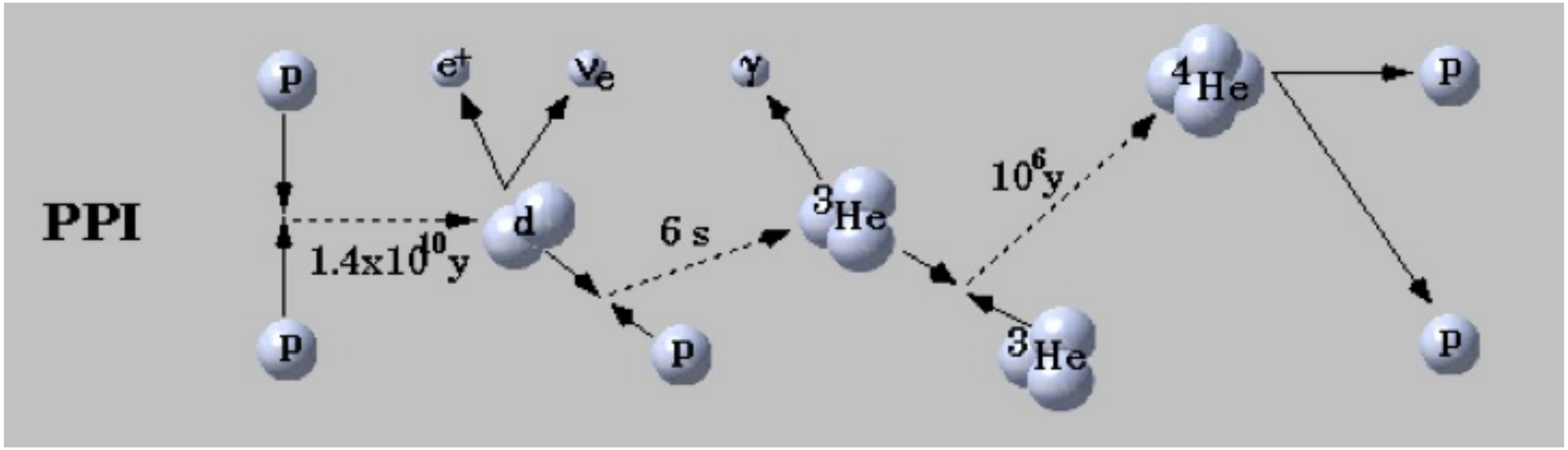
In a typical star dL/dr is positive through the regions of energy generation (stellar core) but will drop to zero outside of the core – energy is conserved but no additional energy is generated

Energy production in the Sun



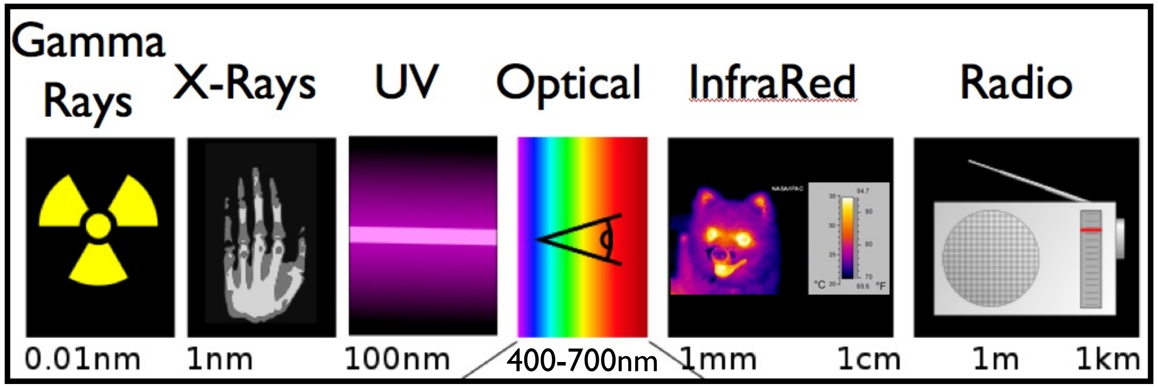
Energy produced at the Suns core

The power source behind stars is (as we know) nuclear fusion, which releases energy as (1) photons, (2) particles, (3) neutrinos, and (4) kinetic energy

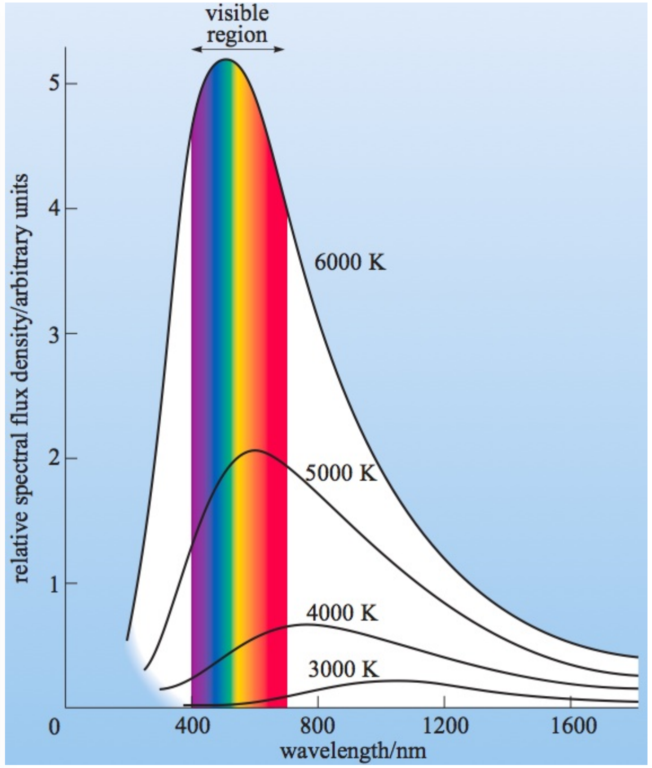


As an example, in the PP1 chain the photon-producing reaction yields <u>5.49 MeV</u> (which includes both the photon and kinetic energy). Assuming all of the energy goes into the photon, what waveband will it be produced at?

$E=hc/\lambda$ or
$\lambda=hc/E$



Energy we see from the Suns surface



What can we infer has happened to the emission since it was produced at the stellar core?

Opacity drives the transfer of photon energy

Electrons, protons, ions, atoms

In most stellar interiors the opacity is determined by all of the processes that scatter and absorb photons, which (1) remove the photon from its original direction and/or (2) sometimes degrade the photon energy (the emission being split into >1 photons of lower energy or the photon energy going into the gas kinetic energy through particle interactions). Remember, the total energy is always conserved in any given process.

We explore different sources of opacity in the next lecture:
bound bound (excitation), bound free (ionisation), free free, and electron scatering

Transporting energy in stars

Opacity effects the transfer of energy. Therefore we also need to investigate the mechanisms of energy transport.

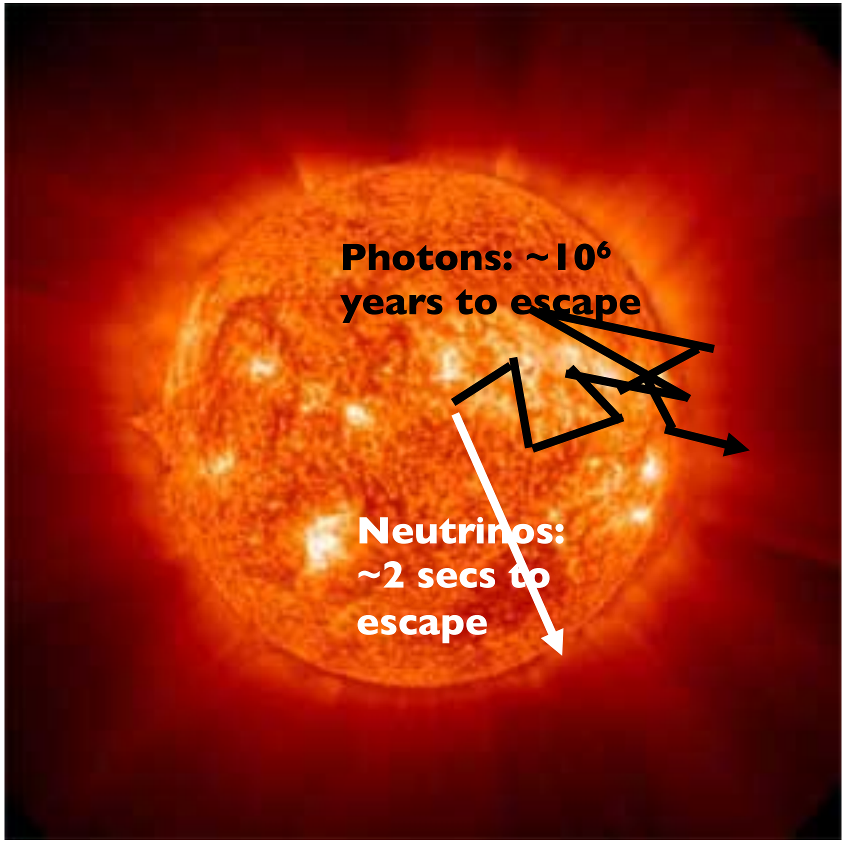
The main methods of energy transport in stars are:

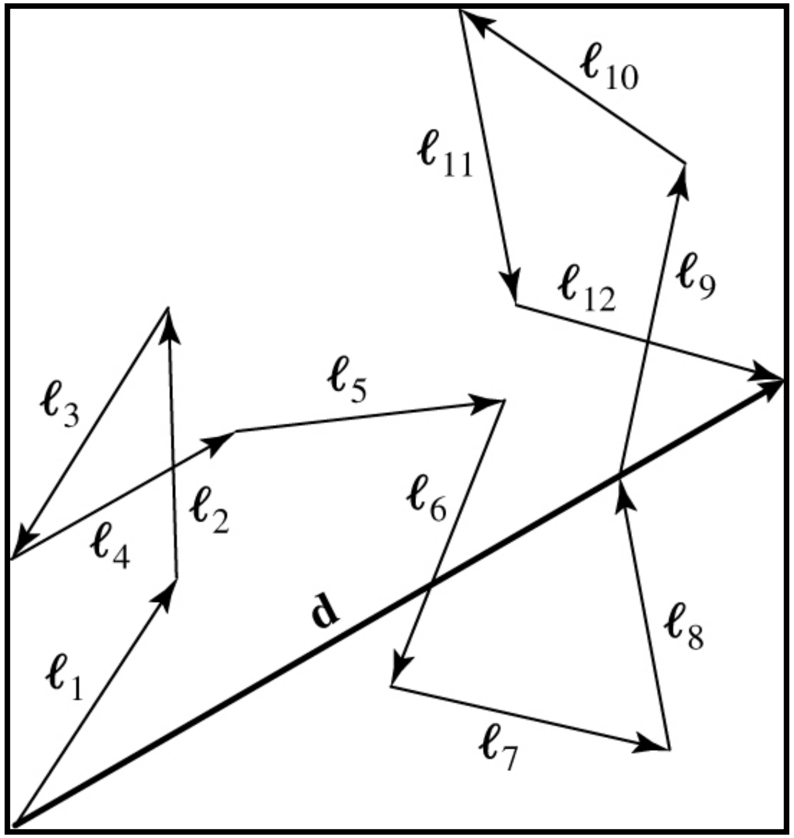
(1) Radiation (lectures 8 and 9)
- energy transport by the emission and absorption of photons

(2) Convection (lecture 10)
- efficient energy transport by the mass motions of gas

Another method of energy transport is conduction (energy exchange through collisions between particles). However, conduction is only important in dense degenerate stars (such as white dwarfs) and we will ignore it here.

Energy transport: random walk of photons





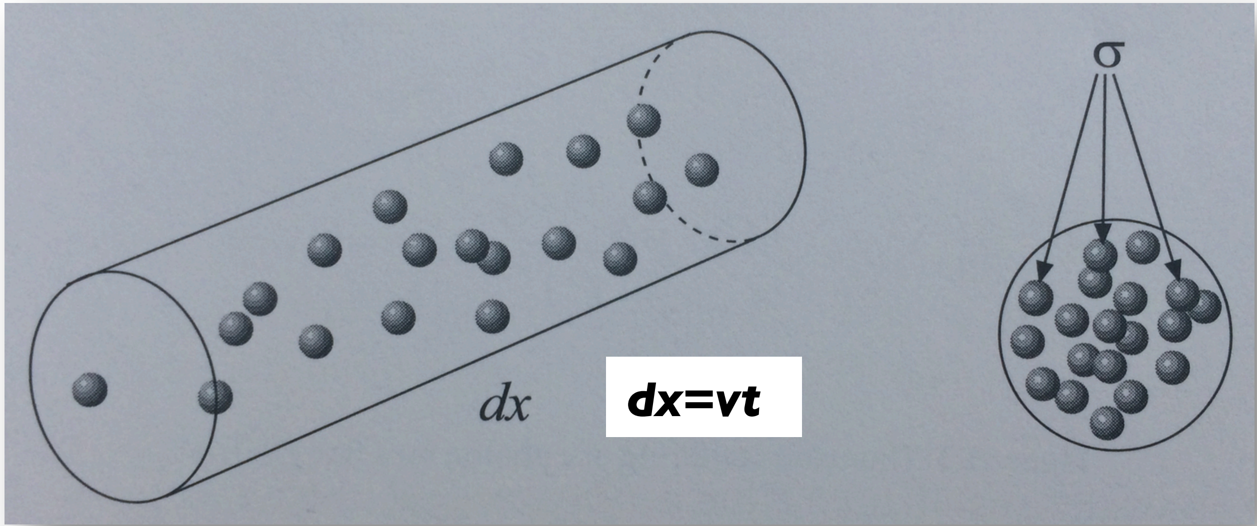
The photons make a random walk (at the speed of light!) through the Sun, being scattered and absorbed by the material (the opacity) that they encounter

We will estimate how long, on average, it takes for photons to travel from the stellar core to the surface by calculating (1) the mean-free path and (2) the number of photon scattering events

Energy transport: mean free path

Cross section: probability of a collision

Electron scattering:
$\sigma_T = 6.65 \times 10^{-29}$ m^2



The mean-free path between collisions

The mean-free path is then calculated as:

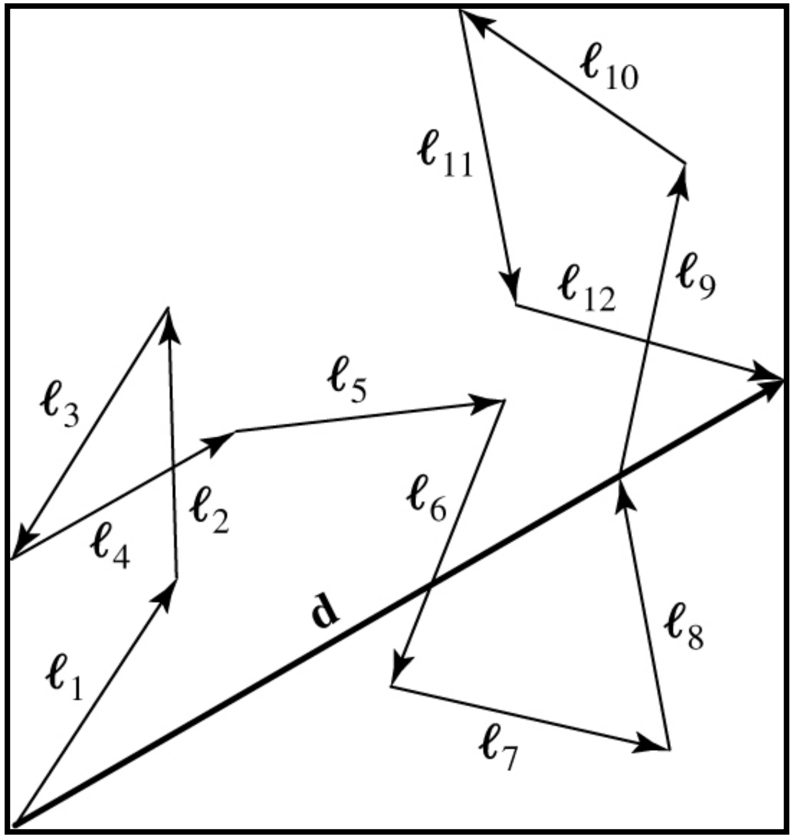
$$\ell = \frac{vt}{n\sigma vt} \quad \text{therefore} \quad \ell = \frac{1}{n\sigma}$$

Equation 16

Mean free path: average distance travelled before 1 collision/interaction

Energy transport: number of scattering events

The figure indicates a photon that undergoes a net vector displacement d as a result of making a large number N of randomly directed steps, each of length ℓ, the mean free path:



$$d = \ell_1 + \ell_2 + \ell_3 + \cdots + \ell_N$$

$$d^2 = \ell_1^2 + \ell_2^2 + \cdots + \ell_N^2 + 2(\ell_1 \cdot \ell_2 + \ell_1 \cdot \ell_3 + \cdots)$$

The expectation value of the term in parentheses is zero since it is a sum over many vector dot products, each with a random angle, and hence with both positive and negative directions contributing equally. Therefore:

$d^2 = N\ell^2$ and hence $N = \left(\dfrac{d}{\ell}\right)^2$ **Equation 17**

Energy transport: photon-escape time

A photon escaping from the sun

For the random walk of photons with a mean free path of ℓ, the number of scattering events for a photon to cover the distance d is given as:

$$N = \left(\frac{d}{\ell}\right)^2$$

The time it takes for a photon to escape from the Sun therefore needs to consider the total distance that the photon travelled.

However, we also need to consider the time it takes for a photon to be re-emitted after each absorption (i.e., scattering event), which is $\sim 10^{-8}$ s.

Now let's try out an example

Energy transport: radiation

We have from lecture 4 the equation for radiation pressure:

$$P_{rad} = \frac{1}{3} a T^4 \qquad \text{and therefore} \qquad \frac{dP_{rad}}{dr} = \frac{4aT^3}{3} \frac{dT}{dr}$$

This pressure differential can also be expressed as:

$$\frac{dP_{rad}}{dr} = -\frac{\kappa \rho F_{rad}}{c}$$

which is related to the pressure that photons exert on the gas (opacity/mean free path) – this is referred to as radiation pressure

κ **expresses the opacity in units of m^2 kg^{-1}; we will explore opacity in more detail in the next lecture**

The connection between opacity, cross section, and mean free path length:

$$\ell = \frac{1}{n\sigma} \qquad \text{and} \qquad \ell = \frac{1}{\kappa\rho} \qquad \text{then } \kappa = \frac{n\sigma}{\rho}$$

Energy transport: radiation

By combining these two equations we can therefore determine a temperature differential:

$$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\kappa\rho F_{rad}}{T^3}$$

or expressed in luminosity units

$$\frac{dT}{dr} = -\frac{3}{16\pi ac}\frac{\kappa\rho}{T^3}\frac{L_r}{r^2}$$

Equation 18

What are the dominant factors that dictate the temperature gradient?

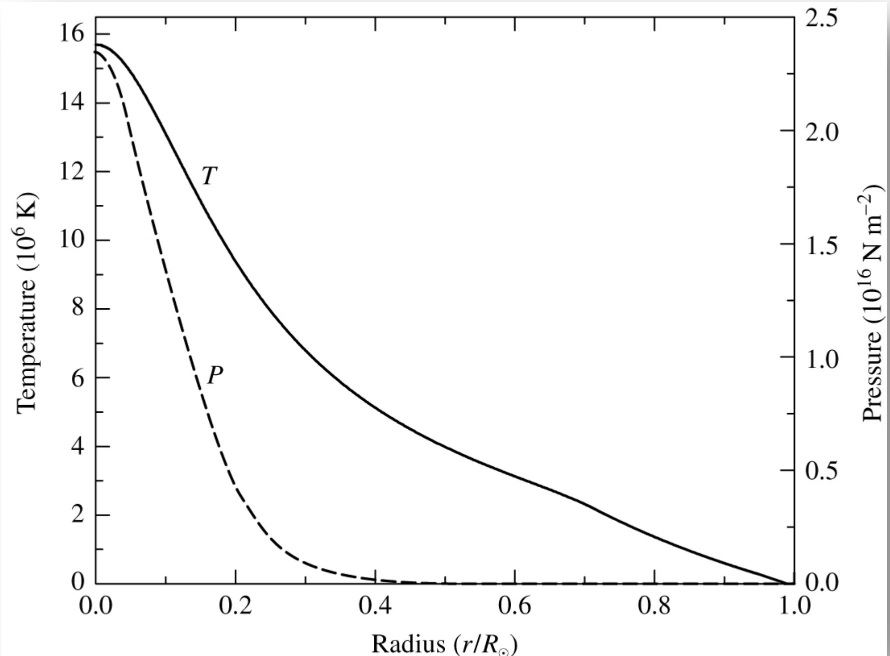
Radiation pressure:

The absorption and emission of photons by an atom causes a small recoil – transfer of momentum

Energy transport through radiation

Given the huge number of times photons are scattered and absorbed as they escape from the stellar interior it may seem incredible that they get out at all.

The key is the temperature, pressure, and density gradient (dT/dr; dP/dr; dρ/dr) which decrease with radius from the centre (as we'd expect): this gradient ultimately dictates the migration of photons to the surface, effectively like a slow "photon wind".



The random walk of photons:

The random walk of photons causes radiation pressure: the absorption and emission of photons by an atom causes a small recoil (transfer of momentum).

The emission events balance out (random directions) while the absorption events are from a preferential direction.

In the next lecture we will explore the four different sources of opacity in stars, two of which we have already encountered in lecture 2