University of Durham

EXAMINATION PAPER

May/June 2016 Examination code: PHYS2591-WE01

FOUNDATIONS OF PHYSICS 2B

SECTION A. Thermodynamics
SECTION B. Condensed Matter Physics
SECTION C. Modern Optics

Time allowed: 3 hours

Additional material provided: None

Materials permitted: None

Calculators permitted: Yes **Models permitted:** Casio fx-83 GTPLUS or Casio fx-85 GTPLUS

Visiting students may use dictionaries: No

Instructions to candidates:

- Answer the compulsory question that heads each of sections A, B and C. These **three** questions have a total of 15 parts and carry 50% of the total marks for the paper. Answer **one** other question from **each** section. If you attempt more than the required number of questions only those with the lowest question number compatible with the rubric will be marked: **clearly delete** those that are not to be marked. The marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply.
- **ANSWER EACH SECTION IN A SEPARATE ANSWER BOOK**
- Do **not** attach your answer booklets together with a treasury tag, unless you have used more than one booklet for a single section.

Information

A list of physical constants is provided on the next page.

Information

Elementary charge:	$e = 1.60 \times 10^{-19}$ C
Speed of light:	$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Boltzmann constant:	$k_{\mathrm{B}} = 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}}$
Electron mass:	$m_{\mathrm{e}} = 9.11 \times 10^{-31}$ kg
Gravitational constant:	$G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$
Proton mass:	$m_{\mathrm{p}} = 1.67 \times 10^{-27}$ kg
Planck constant:	$h = 6.63 \times 10^{-34}\ \mathrm{J\,s}$
Permittivity of free space:	$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F\,m^{-1}}$
Magnetic constant:	$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Molar gas constant:	$R = 8.31\ \mathrm{J\,K^{-1}\,mol^{-1}}$
Avogadro's constant:	$N_{\mathrm{A}} = 6.02 \times 10^{23}\ \mathrm{mol^{-1}}$
Gravitational acceleration at Earth's surface:	$g = 9.81\ \mathrm{m\,s^{-2}}$
Stefan-Boltzmann constant:	$\sigma = 5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$
Astronomical Unit:	$\mathrm{AU} = 1.50 \times 10^{11}$ m
Parsec:	$\mathrm{pc} = 3.09 \times 10^{16}$ m
Solar Mass:	$M_{\odot} = 1.99 \times 10^{30}$ kg
Solar Luminosity:	$L_{\odot} = 3.84 \times 10^{26}$ W

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SECTION A. THERMODYNAMICS

Answer question 1 and **either** question 2 **or** question 3.

1. (a) You are told that for two samples, one an unknown gas and the other an unknown magnet, a small change in the pressure and small change in the magnetisation could be described by the following equations,

$$dp = \left(\frac{R}{V-b}\right) dT - \left(\frac{RT}{(V-b)^2} + \frac{2a}{V^3}\right) dV,$$

$$dM = \frac{\gamma}{T} dH - \frac{\gamma H}{T^3} dT,$$

where H is the magnetic field strength, a, b and γ are constants, and the other symbols have their usual meanings. Only one of these expressions corresponds to a real, physical thermodynamic property. Explain which one it is. [4 marks]

(b) Describe the thermodynamic processes which make up a Carnot Cycle and sketch a pV diagram for such a cycle. By stating the correct Carnot Theorem, explain why a heat engine operating between heat reservoirs at 300°C and 20.0°C cannot have an efficiency greater than 48.9 %. [4 marks]

(c) By sketching a temperature-entropy (TS) diagram, explain how adiabatic demagnetisation can be used to cool a sample to very low temperatures. [4 marks]

(d) What does the heat capacity tell us for a substance which undergoes a small temperature change dT, whilst some property is held constant? By considering a differential change to the heat energy in terms of the entropy, show that for a general system property, α

$$C_\alpha = T\left(\frac{\partial S}{\partial T}\right)_\alpha .$$

Why can the heat capacity for a gas at constant volume be written in terms of a small change to the internal energy, dU and a differential change to the heat energy, δQ? [4 marks]

2. (a) A 10.0 kg block of iron at a temperature of 700 K is brought into contact with a block of aluminium at 400 K, but of lower mass 7.50 kg. The two blocks are allowed to reach thermal equilibrium. Determine the irreversibility of the process, assuming that the blocks are thermally isolated as they come to equilibrium, but that the surroundings are at a temperature of 300 K. Iron has $c_p = 450$ J kg^{-1} K^{-1} and aluminium has $c_p = 910$ J kg^{-1} K^{-1}. [5 marks]

(b) A solid is made up of 1 mole of distinguishable particles. Determine the entropy of this solid if its constituent particles are arranged in the energy states $\varepsilon_1, \varepsilon_2, \varepsilon_3$ in the ratio $10 : 5 : 1$. [6 marks]

[Hint: The *Stirling Approximation* is $\ln(X!) = X \ln X - X$.]

(c) Derive the following general relation,

$$\left(\frac{\partial T}{\partial V}\right)_S = -\frac{T}{C_V}\left(\frac{\partial p}{\partial T}\right)_V .$$

Using the above, derive the general equation of state of an adiabatic line for one mole of ideal gas. [6 marks]

$$\left[\text{Hint: You may wish to use the Maxwell Relation } \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V .\right]$$

Using your result, or otherwise, prove that, for an ideal gas, an adiabatic is steeper than an isotherm on a pV diagram. [3 marks]

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3. (a) The Gibbs Function for an elastic rod of length x, when placed under tension f, is given by $G = U - TS - fx$, where the other symbols have their usual meanings. The associated work done when the rod is extended by an amount dx is $\delta W = +f dx$. Use this to derive the Maxwell relation

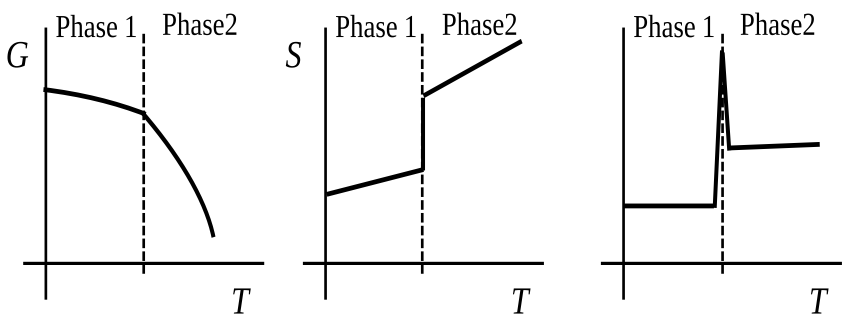
$$\left(\frac{\partial S}{\partial f}\right)_T = \left(\frac{\partial x}{\partial T}\right)_f .$$

[4 marks]
By considering the entropy of the rod as a function of its tension and temperature, derive the following equation,

$$T dS = C_f dT + T\left(\frac{\partial x}{\partial T}\right)_f df,$$

where C_f is the heat capacity at constant tension. Using the above, find an expression which describes the shape of an adiabat in the fT plane. For a rubber band, the coefficient of linear expansion, $\alpha_f = (1/x)\left(\partial x/\partial T\right)_f$, is negative. What effect does increasing the tension on the band have on its temperature, when the length is held constant? [6 marks]

(b) For a gas, the Gibbs function is $G = U - TS + pV$. The figures below represent the behaviour of a number of thermodynamic properties in a first-order phase transition. Which thermodynamic property is represented by the vertical axis of the third plot? Sketch the behaviour of the same three quantities for a second-order transition. [5 marks]



The Gibbs function of a real gas is given by

$$G = RT \ln\left(\frac{p}{p_0}\right) + Ap.$$

Find the equation of state of this gas and comment on the likely physical origin of the constant, A. Further, find an expression for the entropy change of this gas as its pressure is quadrupled, isothermally, and comment on your result. [5 marks]

SECTION B. CONDENSED MATTER PHYSICS

Answer question 4 and **either** question 5 **or** question 6.

4. (a) For a simple cubic lattice, sketch the set of planes with the Miller indices $(10\bar{1})$ and (221). Include the x, y and z axes in your diagram. If the lattice constant, a, is 0.40 nm, determine the spacing between the planes for each of these two sets of Miller indices. [4 marks]

   (b) For the case of van der Waals bonding draw a simple sketch indicating the general form of the interatomic potential curve. State the physical origin of the attractive and repulsive components and give the empirical relation used to describe this potential energy curve as a function of atomic separation, r. [4 marks]

   (c) Describe the motion of an electron in an electric field as modelled by the classical Drude model. Discuss how scattering time is used to quantify the electron motion in this model. How is this parameter related to the electrical conductivity, σ? [4 marks]

   (d) Bloch's Theorem can be written as $\psi(\underline{x} + \underline{R}) = u_k(\underline{R})\psi(\underline{x})$ where ψ is the electron wavefunction, $\underline{R}$ is any lattice vector and $\underline{x}$ is position. Describe in a few sentences what this expression means. Your answer should also include a short description of the general form of the wavefunction $\psi(\underline{x})$. [4 marks]

   (e) Sketch an energy dispersion curve, $E(k)$, for the lowest energy band in the first Brillouin zone as predicted by the nearly-free electron model. Explain how the shape of this curve can be used to determine electron effective mass, m_e^*. Indicate on your sketch where holes are likely to occur. [4 marks]

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5. (a) Starting with structure factor expression

$$S_G(hkl) = \sum_j f_j \exp\left[-i2\pi\left(hx_j + ky_j + lz_j\right)\right]$$

and using the coordinates for positions of unique atoms in the body-centred-cubic (bcc) lattice, determine the structure factor rules for a bcc lattice. [6 marks]

(b) An X-ray powder diffraction pattern is produced from a metal powder at a temperature of 290 K. The metal has a bcc lattice with a lattice constant of 0.400 nm. The X-ray wavelength is 0.150 nm. Determine the 2θ values observed for the first five peaks that are allowed by the structure factor determined in part (a). [6 marks]

(c) At a temperature of 300 K the metal undergoes a phase transition to a face-centred-cubic (fcc) lattice having the same lattice constant. Determine the 2θ values for the first five X-ray powder diffraction peaks observed at a temperature of 310 K. Clearly state any assumptions you have made. [6 marks]

(d) The metal powder is then heated to a temperature of 500 K, where it remains stable and has not experienced any further phase transitions. Describe qualitatively what changes would be observed in the X-ray diffraction pattern. [2 marks]

6. (a) For the case of a crystal which has a two atom basis, draw an appropriately labelled sketch of the phonon dispersion curve for the first Brillouin zone. Explain how the different dispersion curves relate to the motion of the atoms in the basis. State the approximations that are often used to describe the phonon dispersion curves, explaining briefly the physical meaning of each approximation. [7 marks]

(b) Beginning with the phonon dispersion relation

$$\omega(K) = \left(\frac{4C}{M}\right)^{\frac{1}{2}} \left|\sin \frac{1}{2}Ka\right|,$$

derive an expression for the velocity of sound. Using this relation, determine the speed of sound for Al, which has an atomic mass of 27 u, a lattice constant of $a = 0.40$ nm and a spring constant of $C = 2.0$ Nm^{-1}. Comment on the value you obtain. [5 marks]

$$[1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}]$$

(c) Draw a sketch indicating how the velocity of sound varies with phonon wavevector, K. What are the values of the velocity of sound at the mid-point of the first Brillouin zone and at the boundary of the first Brillouin zone? [6 marks]

(d) Describe the relative motion of the two atoms in the basis when the phonon wavevector is equal to that at the first Brillouin zone boundary. [2 marks]

SECTION C. MODERN OPTICS

Answer question 7 and **either** question 8 **or** question 9.

7. (a) Define spatial frequency and angular spatial frequency. Write an expression for a phasor in terms of both. [4 marks]

   (b) Write an equation for a spherical wave. What form does this equation take in the paraxial approximation? [4 marks]

   (c) $G(u)$ and $H(u)$ are the Fourier transforms of $g(x)$ and $h(x)$, respectively. Write expressions for the Fourier transforms of:

      (i) $g(x) - h(x)$,

      (ii) $g(x - d)$, where d is a constant,

      (iii) $g(x)h(x)$,

      (iv) $g(x)h(y)$. [4 marks]

   (d) A green laser pointer with wavelength, $\lambda = 0.50\ \mu$m, has an initial beam waist of 1.0 mm. What is the beam radius 10 m downstream? Does this correspond to the far-field? [4 marks]

   (e) A Young's double-slit experiment is performed with monochromatic light with wavelength, $\lambda = 0.50\ \mu$m, a slit spacing, $d = 1.0$ mm, and a lens with focal length $f = 10$ cm. Calculate the fringe spacing in the focal plane of the lens. You may assume that the double slit is placed in the plane of the lens. Comment on whether the fringes are visible with a camera with pixel size 5.0 μm. [4 marks]

   (f) A Fraunhofer diffraction experiment is performed with an aperture with the shape of the letter E. The aperture is illuminated normally by uniform monochromatic light. Sketch the far-field intensity distribution along the vertical axis, labelling the key features. It is observed that the 4th principal maximum in the vertical direction is missing. What does that tells us about the geometry of the aperture? [4 marks]

8. Explain, briefly, the key differences between the complex representation of a plane wave, a spherical wave, and a phasor. [4 marks]

   The spatial distribution of light in the $z = 0$ plane can be approximated by the function

$$f(x') = \mathrm{comb}_3\left(\frac{x'}{2d}\right) - \mathrm{comb}_2\left(\frac{x'}{2d}\right) \ .$$

   Write an expression for the spatial distribution of the field in an observation plane a distance z downstream of the input where $z \gg d$. Write your answer in terms of the transverse displacement in the observation plane, x, and the angular spatial frequency of the input light, k. [4 marks]

   Sketch a phasor diagram corresponding to the points $x = 0$ and $x = \lambda z/2d$. [4 marks]

   Re-write $f(x')$ as a product of $g(x') = \mathrm{comb}_5(x'/d)$ and a phasor. Comment on the Fourier transform of the product. [4 marks]

   Sketch the intensity patterns along the x axis corresponding to Fraunhofer diffraction from masks described by $f(x')$ and $g(x')$, respectively. [4 marks]

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9. Sketch the set-up of a 4f spatial filter. [4 marks]

   The field in the input plane is

$$\mathcal{E}(x) = \begin{cases} \epsilon_0 & |x| < a \ , \\ 0 & |x| > a \ . \end{cases} \tag{1}$$

   Write an expression for the field in the Fourier plane as a function of position along the x axis. You may neglect the finite size of the lens and the dependence on y. [4 marks]

   Sketch the intensity distribution along the x axis in the Fourier plane and the output plane. Label the x axis to indicate the scale. [4 marks]

   A phase plate is placed in the input plane that shifts the phase in the half plane with negative x by π. The field in the input plane is

$$\mathcal{E}(x) = \begin{cases} -\epsilon_0 & -a < x < 0 \ , \\ \epsilon_0 & 0 < x < a \ , \\ 0 & |x| > a \ . \end{cases} \tag{2}$$

   Rewrite the input field as a convolution. [2 marks]

   Using this result or other methods, find the modified field in the Fourier plane. [4 marks]

   A second identical phase plate is added to filter the field in the Fourier plane. Sketch the field distribution in a plane located immediately after the filter. Based on your sketch, what can be said about the average value of the field in the output plane? [2 marks]