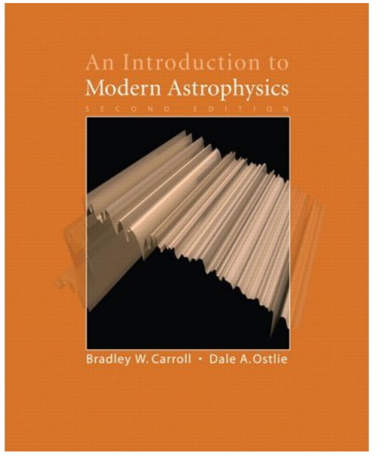
Lecture 16:
Stellar evolution –

Degenerate stars: white dwarfs and neutron stars

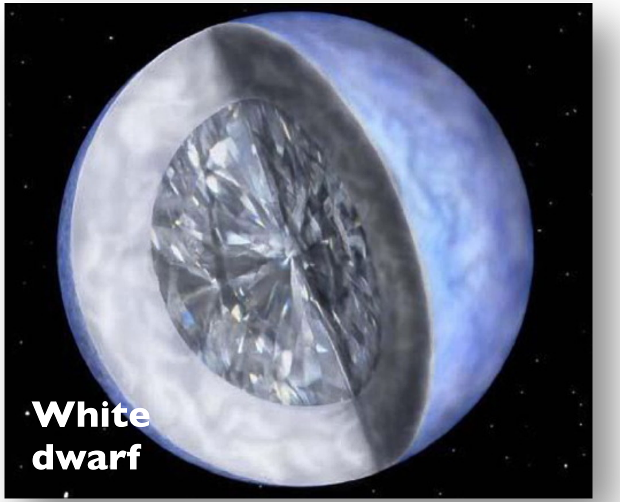
Professor David Alexander
Ogden Centre West 119

Chapter 16 of Carroll and Ostlie



Degenerate stars

What is a degenerate star? The core of a star where nuclear fusion has ceased. We consider 3 types of degenerate star; another (quark stars) is largely a theoretical idea.



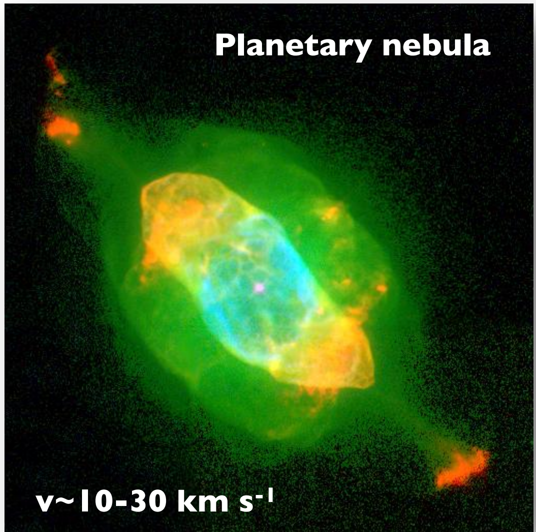
Degeneracy: at the lowest energy state

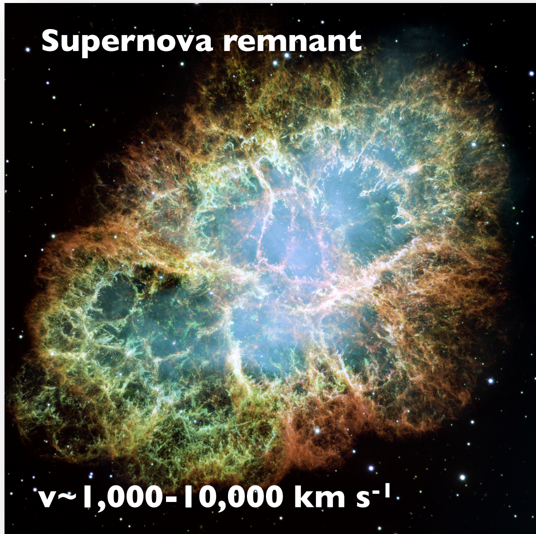
Ranges of masses for degenerate stars

Approximate range of properties expected for different degenerate stars

Degenerate state	Mass range (solar mass)	Progenitor star (solar mass)	Prior phase
White dwarf	~0.2-1.4	<8	Planetary nebula
Neutron star	~1.0-2.5	~8-25	Supernova
Black hole	>2.5	>25	Supernova

What can you tell from the big difference between progenitor and degenerate mass?





Aims of lecture

Key concept: degeneracy pressure

Aims:

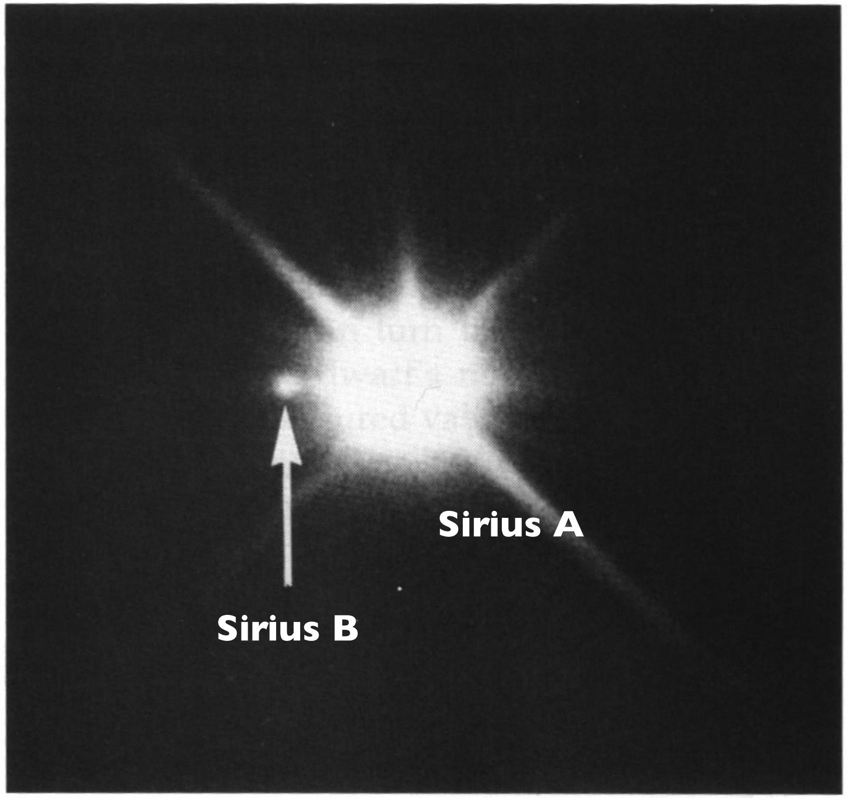
- Understand the basic properties of white dwarfs and neutron stars
- Have a basic understanding of cooling in white dwarfs
- Understand the principles behind degeneracy pressure and be able to show:

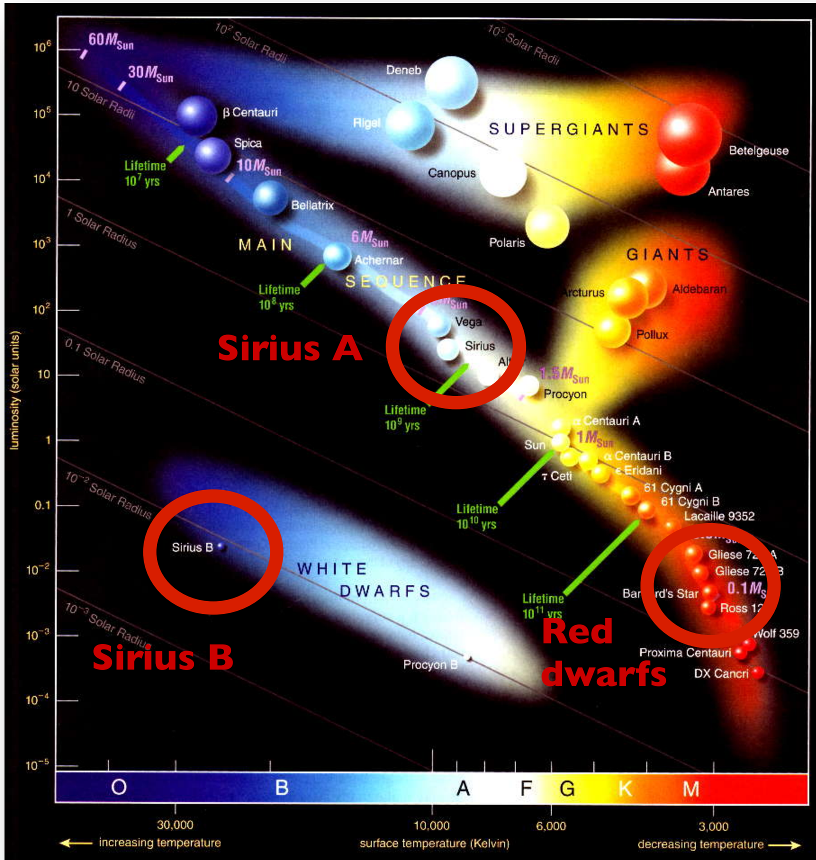
$$P = \frac{\hbar^2}{m_e}\left[\left(\frac{Z}{A}\right)\frac{\rho}{m_H}\right]^{5/3}$$

Electron-degeneracy pressure

White dwarfs: discovery

In 1844 it was noticed that the orbit of Sirius (brightest star in the sky: Sirius A) deviated from a straight line, suggesting it is a binary system (astrometric binary; lecture 3). The companion (now called Sirius B) was detected in 1862 and is ~1,000x fainter than Sirius A.

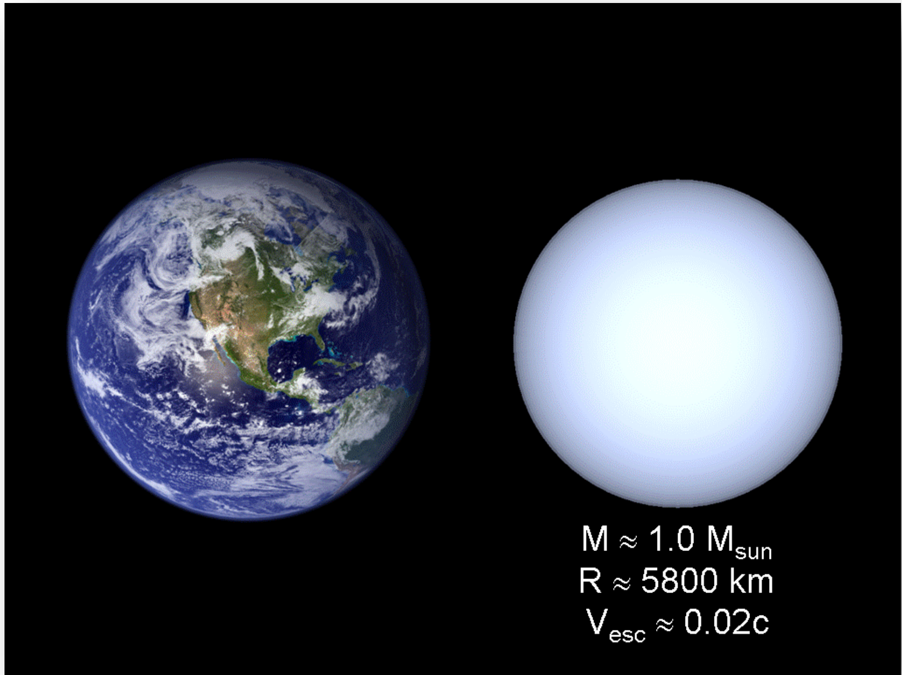




It was initially thought that Sirius B would be a cool red dwarf and therefore it was a great surprise when in 1915 it was found that Sirius B is a hot blue-white star with T~27,000K, as compared to T~9,900 K for Sirius A (and T~3,000 K for a red dwarf). **It doesn't lie on the main sequence.**

White dwarfs: basic properties

The implications of this were astounding: calculate the radius of Sirius B given that its luminosity is 0.033 times that of the sun and its temperature is 27,000 K (Sun is 5,800 K).



What would be the mass of a sugar-cube sized piece of white-dwarf material?

The mass of Sirius A is ~1 solar mass (reliably determined as it is a binary system). The implied average density of Sirius B was therefore incredible. It is not a normal star: we now know that a white dwarf is the collapsed core of the star after fusion has ceased.

What would be the chemical composition of a typical white dwarf?

White dwarfs: basic properties

The pressure can be calculated using an equation derived from lecture 4:

$$P_c = \frac{3}{8\pi}\frac{GM^2}{R^4}$$ which gives $P_c \sim 2.8 \times 10^{22}$ N m^{-2}!

These are immense pressures, $\sim 10^6$ times those found at the centre of the Sun!

We can also crudely estimate the central temperature using the temperature gradient defined in lecture 8:

$$\frac{dT}{dr} = -\frac{3}{16\pi ac}\frac{\kappa\rho}{T^3}\frac{L_r}{r^2}$$

κ=0.02 m^2 kg^{-1} (electron scattering for a metal-rich composition); a=7.57x10^{-16} J m^{-3} K^{-4}

Assuming that the surface temperature is much lower than the central temperature and that electron scattering is the dominant form of opacity then:

$$T_c \approx \left[\frac{3\kappa\rho}{16\pi ac}\frac{L_{wd}}{R_{wd}}\right]^{1/4}$$ which gives $T_c \sim 7.3 \times 10^7$ K

The temperature and pressure properties are sufficient for Hydrogen fusion; however, no fusion is occurring in a white dwarf: why not?

Electron degeneracy pressure

The density of material in a white dwarf is so high that the electrons are forced to almost be on top of each other. This leads to electron degeneracy pressure, which is a consequence of two things:

1) Pauli exclusion principle, which allows at most one electron in each quantum state

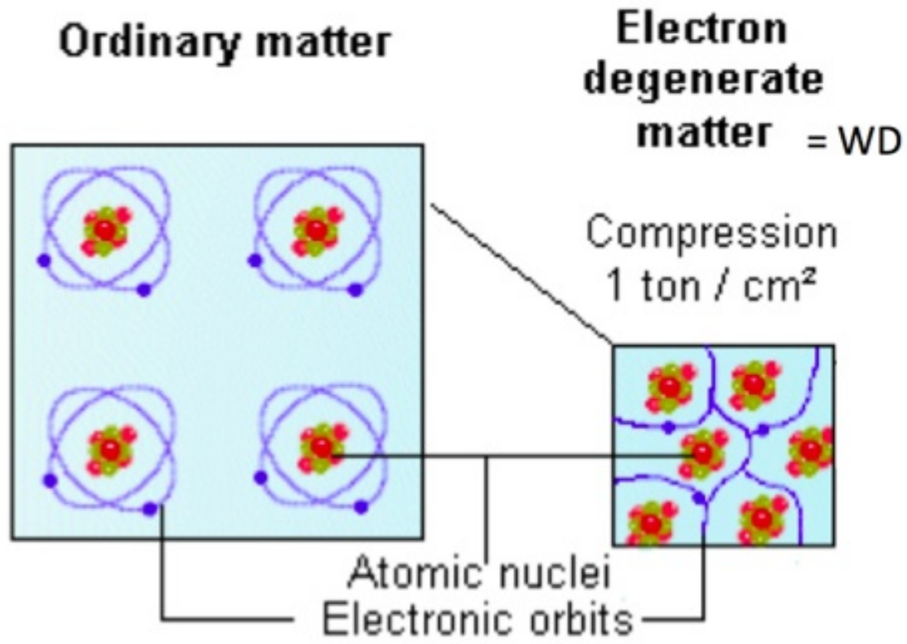
2) Heisenberg uncertainty principle, which requires that an electron confined to a small volume will have a correspondingly high uncertainty in its momentum:

$$\Delta x \Delta p_x \sim \hbar$$

and

$$p_{\min} \sim \Delta p_x$$

To satisfy Pauli exclusion: uncertainty in the position cannot be larger than the physical separation



Electron degeneracy pressure in a white dwarf

The pressure at the centre of a white dwarf can be calculated from:

$P \sim \frac{1}{3} n_e p v$ where n_e is electron number density, p is momentum, and v is velocity

(see section 10.2 of CO book; eqn 10.8 – assumes all particles have same p)

The electron number density is:

$n_e = \left(\frac{Z}{A}\right)\frac{\rho}{m_H}$ which is $\left(\frac{num\ electrons}{nucleon}\right)\frac{num\ nucleons}{volume}$

The uncertainty in the position is therefore

$\Delta x \sim n_e^{-1/3}$ and hence $p_x \sim \hbar n_e^{1/3}$ (i.e., $p_x \sim \frac{\hbar}{\Delta x}$)

However, in 3 dimensions each direction is equally likely and so

$p^2 = p_x^2 + p_y^2 + p_z^2 = 3p_x^2$ and therefore $p = \sqrt{3} p_x$

Electron degeneracy pressure in a white dwarf

We can re-write the pressure equation $P \sim \frac{1}{3} n_e p v$ in terms of electrons:

$$p \sim \sqrt{3}\hbar \left[\left(\frac{Z}{A} \right) \frac{\rho}{m_H} \right]^{1/3}$$

If the electrons are non relativistic then $p = v m_e$ and therefore

$$v \sim \frac{\sqrt{3}\hbar}{m_e} \left[\left(\frac{Z}{A} \right) \frac{\rho}{m_H} \right]^{1/3}$$

From this simple derivation the exerted pressure will therefore be:

$$P = \frac{\hbar^2}{m_e} \left[\left(\frac{Z}{A} \right) \frac{\rho}{m_H} \right]^{5/3}$$

Equation 28 **(note: the strong density dependence)**

For a white-dwarf density of $\rho = 3.80 \times 10^9$ kg m^{-3} and assuming Z/A=0.5 (for a typical Carbon-Oxygen white dwarf) we get:

$P \sim 1.5 \times 10^{22}$ N m^{-2}

Electron degeneracy pressure is consistent with that required to prevent collapse of the white dwarf

Relationship between mass and radius

For stars on the main sequence we find:

$R \propto M^{0.5}$ i.e., the radius increases as the star becomes more massive

The situation is very different for white dwarfs, as first explored by Subrahmanyan Chandrasekhar in 1931 (aged just 21!). The mass—relationship of white dwarfs is:

$R \propto M^{-2/3}$ i.e., the radius decreases as the white dwarf becomes more massive!

Value of exponent depends on the mass range but it is always an inverse relationship

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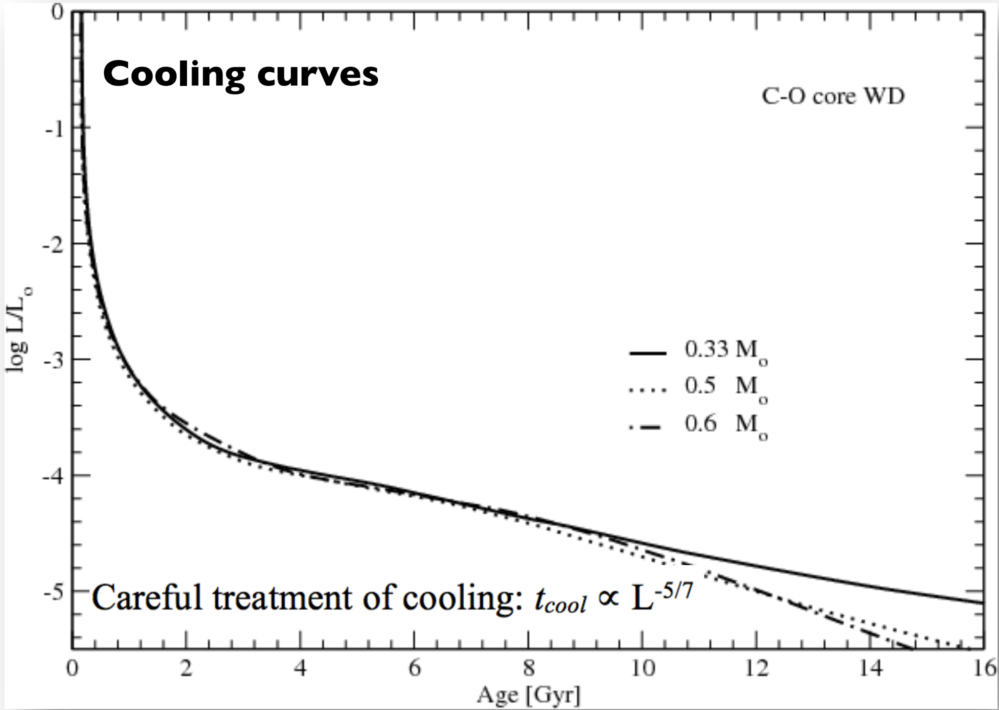
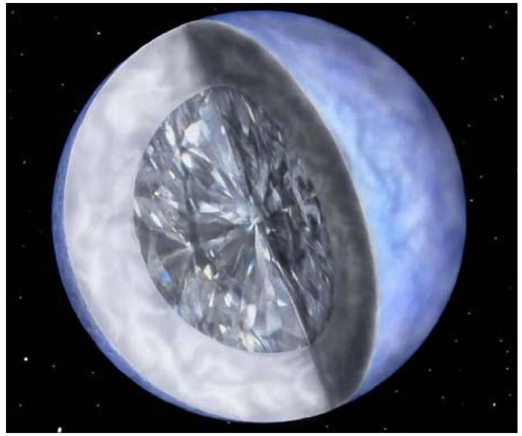
The white-dwarf radius decreases with mass: to withstand the increased gravitational force the momentum of the electrons must increase (to increase the pressure).

Clearly a limit is reached when electron degeneracy fails: this is called the Chandrasekhar limit and occurs at ~1.4 solar masses.

Evolution of white dwarfs: cooling





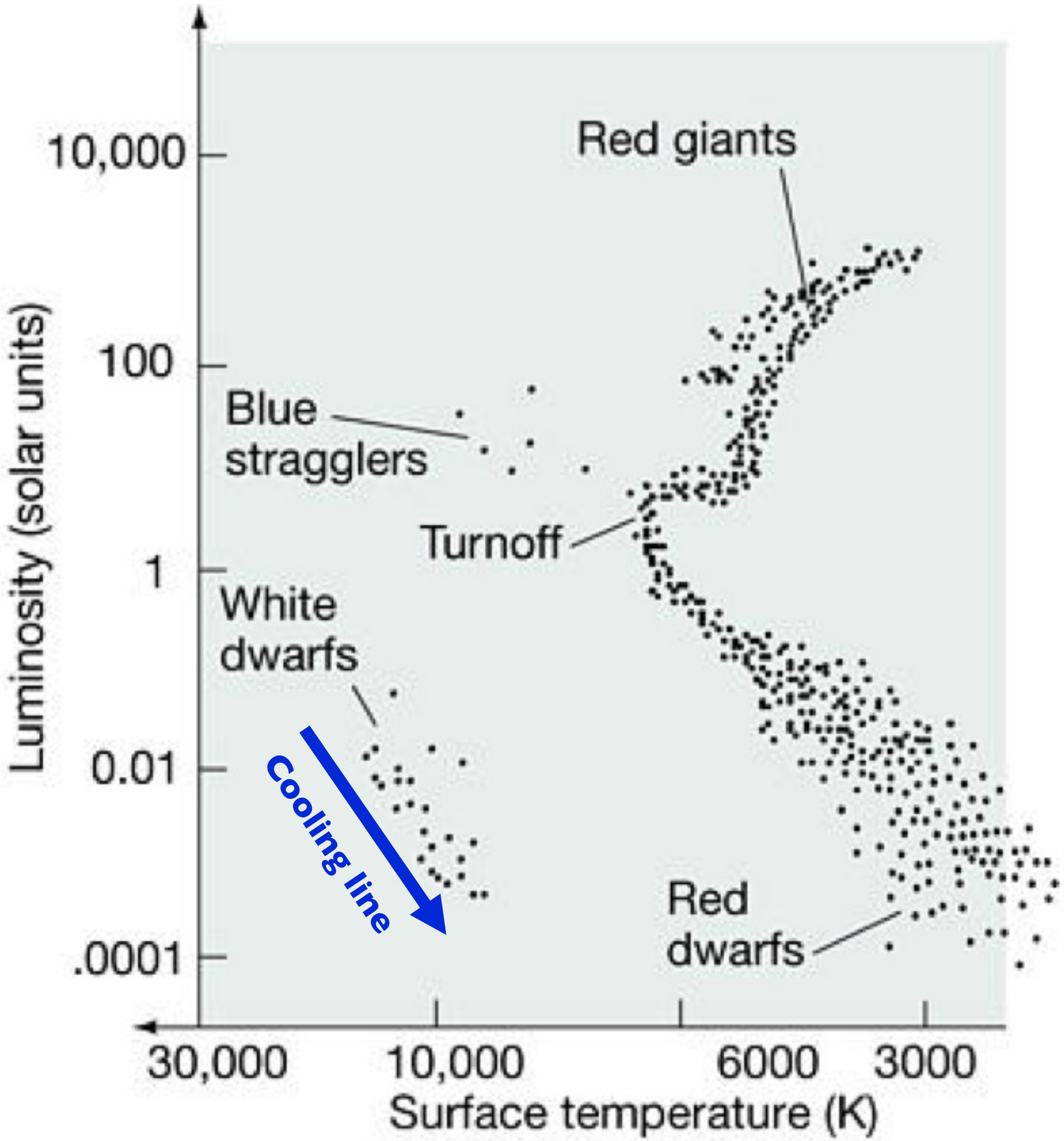
As the white dwarf cools the centre crystallises, producing a supermassive diamond!

White dwarfs cool due to the release of kinetic energy from their nuclei. A lower limit on the cooling time can therefore be estimated by:

$$t_{cool} = \frac{E_{WD}}{L_{WD}} = \left(\frac{3kT_{c,WD}}{2}\right)\left(\frac{M_{WD}}{Am_H}\right)\left(\frac{1}{L_{WD}}\right) \sim 4\times10^8\, yrs \qquad \text{for Sirius (A=12; i.e., Carbon)}$$

However, this doesn't take account the increasing cooling timescale with decreasing core temperature (T_c). Treatment of this gives cooling times of >>10^9 yrs (see above)!

Evolution of white dwarfs: cooling



As white dwarfs cool they will follow a distinct line on the HR diagram since their radius remains fixed.

This cooling line is simply calculated from:

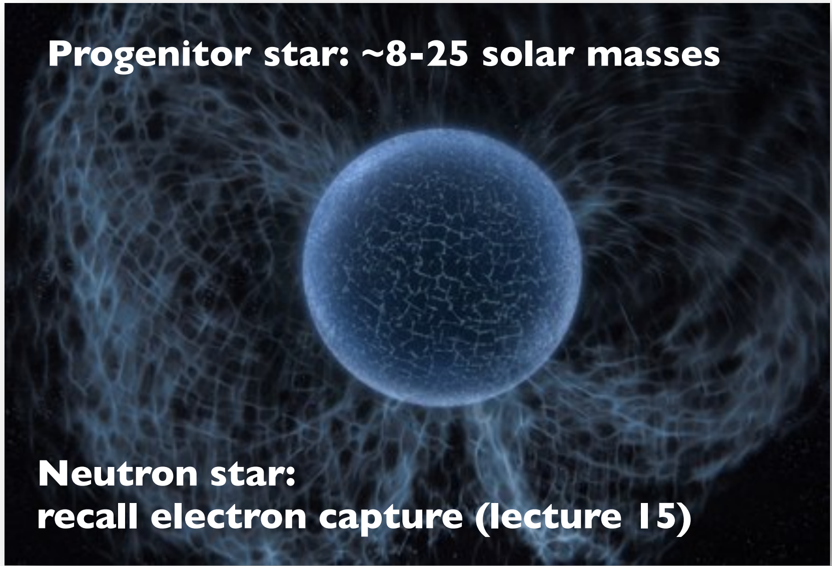
$$L = 4\pi R^2 \sigma T_e^4$$

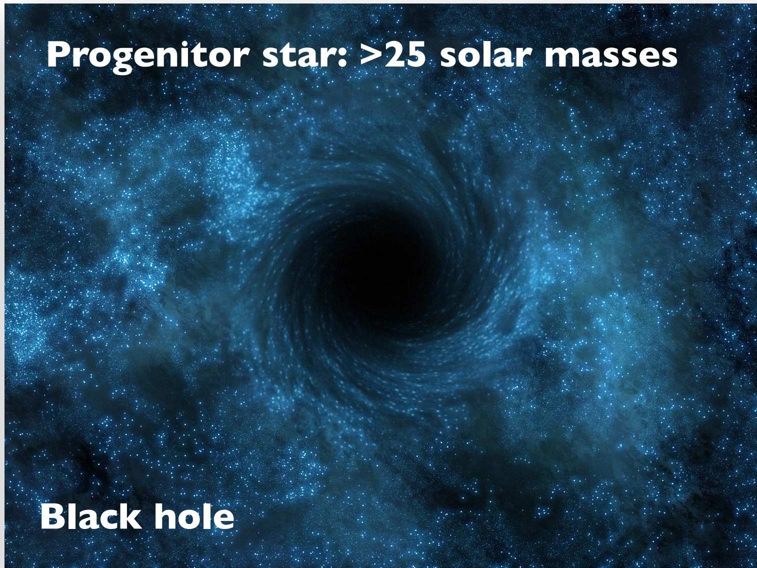
Ultimately a white dwarf will become sufficiently cool that it doesn't produce significant optical emission, at which point it is a black dwarf.

What if electron degeneracy pressure fails?
The stellar core will further collapse...

Core mass <2.5 solar masses?

Core mass >2.5 solar masses?





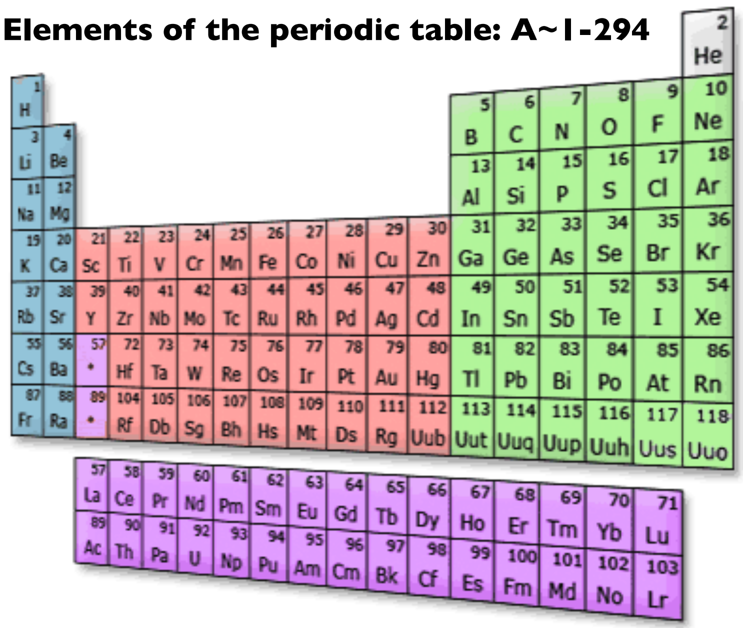
Neutron stars: basic properties

The neutron star was first proposed just 2 years after the discovery of the neutron in 1934 by Baade and Zwicky. They suggested that it would be the end product of a supernova. However, Oppenheimer and Volkoff were the first to explore the theoretical aspects of neutron stars in 1939.

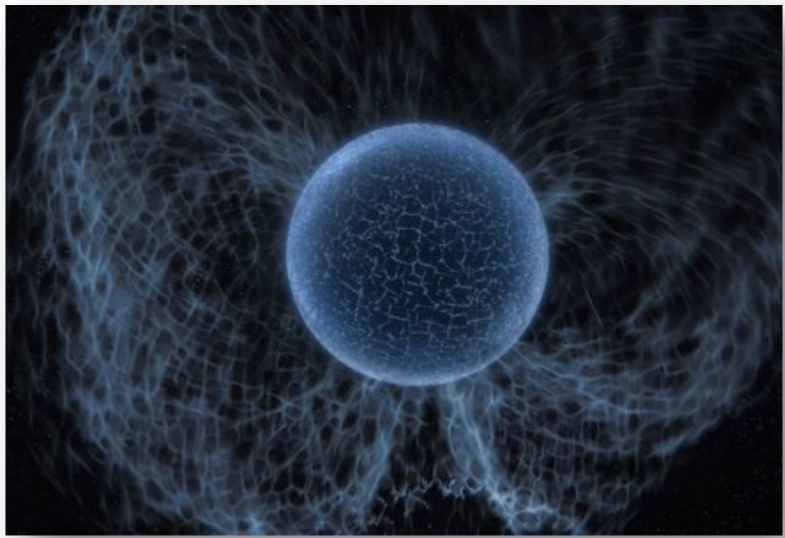
A neutron star is held together by **neutron degeneracy pressure** and repulsion from the strong force, which provide significantly more pressure than electron degeneracy and allow for a much more compressed state. **They are the most perfect spheres in the Universe!**

At some level, a neutron star can be considered a very large atomic nucleus

Elements of the periodic table: A~1-294



A neutron star: effectively A~10^{57}
(but bound by gravity not the strong force)!



Neutron stars: basic properties

A neutron star has the mass of a typical star but is compressed to the size of a city!

The gravitational force is $>10^{11}$ times larger than that on Earth. An object dropped from a height of 1 metre would hit the surface at >6 million kph!

The surface temperature of a neutron star is $\sim 10^6$ K: where would a neutron star lie on the HR diagram? At what wavelengths will the emission peak?

ρ	$10^{17} - 7\text{x}10^{18}$	kg m^{-3}
P	$10^{33} - 3\text{x}10^{35}$	Pa
M	1.0-2.5	$M_{\odot}$
R	7 - 20	km

What would be the mass of a sugar-cube sized piece of neutron-star material?

Most neutron stars are extremely difficult to detect; however, in the last lecture we will explore the subset of neutron stars that are easy to detect