University of Durham

EXAMINATION PAPER

Examination session:	Year:	Examination code:
May/June	2019	PHYS2591-WE01

Title:
Foundations of Physics 2B

Time allowed:	3 hours		
Additional material provided:	None		
Materials permitted:	None		
Calculators permitted:	Yes	Models permitted:	Casio fx-83 GTPLUS or Casio fx-85 GTPLUS
Visiting students may use dictionaries:	No		

Instructions to candidates:

- Attempt **all** questions. The short-answer questions at the start of each section carry 50% of the total marks for the paper. The remaining 50% of the marks are carried by the longer questions, which are equally weighted.
- The marks shown in brackets for the main parts of each question are given as a guide to the weighting the markers expect to apply.
- **ANSWER EACH SECTION IN A SEPARATE ANSWER BOOK.**
- Do **not** attach your answer booklets together with a treasury tag, unless you have used more than one booklet for a single section.
- Slip your booklets for Sections B and C, in order, inside your booklet for Section A, before they are collected by the invigilator.

Information

Section A: Thermodynamics
Section B: Optics
Section C: Condensed Matter Physics

A list of physical constants is provided on the next page.

Revision:	

Information

Elementary charge:	$e = 1.60 \times 10^{-19}$ C
Speed of light:	$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Boltzmann constant:	$k_\mathrm{B} = 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}}$
Bohr magneton:	$\mu_\mathrm{B} = 9.27 \times 10^{-24}\ \mathrm{J\,T^{-1}}$
Electron mass:	$m_\mathrm{e} = 9.11 \times 10^{-31}$ kg
Gravitational constant:	$G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$
Proton mass:	$m_\mathrm{p} = 1.67 \times 10^{-27}$ kg
Planck constant:	$h = 6.63 \times 10^{-34}\ \mathrm{J\,s}$
Permittivity of free space:	$\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F\,m^{-1}}$
Magnetic constant:	$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Molar gas constant:	$R = 8.31\ \mathrm{J\,K^{-1}\,mol^{-1}}$
Avogadro's constant:	$N_\mathrm{A} = 6.02 \times 10^{23}\ \mathrm{mol^{-1}}$
Gravitational acceleration at Earth's surface:	$g = 9.81\ \mathrm{m\,s^{-2}}$
Stefan-Boltzmann constant:	$\sigma = 5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$
Astronomical Unit:	$\mathrm{AU} = 1.50 \times 10^{11}$ m
Parsec:	$\mathrm{pc} = 3.09 \times 10^{16}$ m
Solar Mass:	$M_\odot = 1.99 \times 10^{30}$ kg
Solar Luminosity:	$L_\odot = 3.84 \times 10^{26}$ W

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SECTION A: THERMODYNAMICS

1. (a) When a change dB is made to the applied magnetic field for a sample with a magnetic moment m, an amount of work $W = -mdB$ is done. The *Gibbs function* of this magnet is given by $G = U + mB - TS$, where all other symbols have their usual meanings. Show that in a first order phase transition, when the alignment of the moments breaks down with increasing temperature, the entropy change is discontinuous, and sketch how both G and S change with temperature across the boundary. [4 marks]

   (b) When counting distinguishable particles at thermal equilibrium, the following three conditions are found:

   (i) $\sum_j dn_j = 0$;
   (ii) $\sum_j \varepsilon_j dn_j = 0$;
   (iii) $\sum_j \ln(n_j)\, dn_j = 0$;

   where n and ε have their usual meanings. Explain where the three terms originate from. Then, use Lagrange multipliers to show that the Boltzmann distribution must result. [4 marks]

   (c) By considering two engines that operate between hot and cold reservoirs at T_H and T_L respectively, taking in heat Q_H, show that $\oint \delta Q/T \leq 0$ must hold for both the case of a Carnot cycle and some real, less efficient engine. [4 marks]

   (d) Two equal mass blocks, m have specific heat capacities given by c_1 and c_2. If the hotter block, with specific heat capacity c_1, is at 500 K and the cooler block is at 300 K the process irreversibility is found to be $I = (24.2 \text{ K})\, mc_2$. Determine the temperature of the environment, T_0 if $c_1 = 5c_2$. [4 marks]

   (e) Briefly explain the two processes (adiabatic and Joule-Kelvin) that can be used to cool a gas from room temperature until the gas is liquefied, with reference to appropriate pV and pT diagrams, respectively. [4 marks]

2. (a) One mole of *van der Waals* gas has equation of state given by

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT,$$

where all the symbols have their usual meanings. Show that the isothermal compressibility, $\kappa_T = -(1/V)\,(\partial V/\partial p)_T$ is given by

$$\kappa_T = \frac{V^2(V-b)^2}{RTV^3 - 2a(V-b)^2}.$$

The critical isotherm is at temperature, $T_c = 8a/27Rb$. Determine the volume at the critical point (point of inflection), V_c. Hence evaluate the compressibility at a volume $V_n = \frac{2}{3}V_c$, and comment on your result. How does the compressibility behave as $V \to V_c$? [8 marks]

(b) The *Helmholtz* and *Gibbs functions* are defined as $F = U - TS$ and $G = U - TS + pV$ respectively, where all the symbols have their usual meanings. Confirm that the following is a possible Helmholtz function of the above gas

$$F = -RT\ln(V - b) - \frac{a}{V} + f(T),$$

where $f(T)$ is some function of T. Further, by showing that

$$G = F - V\left(\frac{\partial F}{\partial V}\right)_T,$$

determine the corresponding Gibbs function of this gas. [5 marks]

(c) A further refinement of the equation of state for a gas is via a *Virial expansion*,

$$pV = RT + \frac{bRT}{V} + \frac{cRT}{V^2},$$

for some constants b and c, where the other symbols have their usual meanings. Using the first TdS equation,

$$TdS = C_V dT + T\left(\frac{\partial p}{\partial T}\right)_V dV,$$

where C_V is the heat capacity at constant volume, determine the equation of an adiabatic for such a gas, and compare it to the corresponding form to that which an ideal gas has when the volume becomes large. [7 marks]

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SECTION B: OPTICS

3. (a) The electric field amplitude of two light fields can be written as $\mathcal{E}_1 = \mathcal{E}_0 e^{i(k_x x + k_z z)}$ and $\mathcal{E}_2 = \mathcal{E}_0 e^{i(-k_x x + k_z z)}$, respectively. Derive an expression for the total intensity given by $\mathcal{I} = \frac{1}{2} c\epsilon_0 |\mathcal{E}_1 + \mathcal{E}_2|^2$. Write your answer in terms of the intensity of each field individually, $\mathcal{I}_0 = \frac{1}{2} c\epsilon_0 \mathcal{E}_0^2$. [4 marks]

   (b) An opaque screen containing four small holes in a line along the horizontal axis is illuminated normally by monochromatic light. The far-field intensity along the horizontal axis is found to contain regions with high and low intensity. Draw phasors diagrams corresponding to two distinct positions where the intensity is observed to be zero. [4 marks]

   (c) The electric field along the x axis in the $z = 0$ plane for a paraxial spherical wave propagating along the z axis with source point at $(0, 0, -f)$ is $\mathcal{E} = [\mathcal{E}_0/(ikf)]e^{ikx^2/2f}$. A thin lens with focal length f is placed in the $z = 0$ plane. Derive an expression for the field in a plane at a distance z downstream of the lens. State any assumptions you make. [4 marks]

   (d) Calculate the Rayleigh range of a laser guide star with wavelength $\lambda = 589$ nm, and initial beam size, on Earth, of $w_0 = 1.00$ m. Estimate the size of the laser guide star in the atmospheric sodium layer 90 km above the Earth's surface. [4 marks]

   (e) Sketch the Fraunhofer diffraction pattern for an aperture with the shape of the letter X. [4 marks]

4. (a) (i) Draw a right-handed coordinate system with the z axis going from upper left to lower right. [2 marks]

   (ii) On your sketch add the electric vectors for right-circularly polarized light at $t = 0$ and positions $z = 0$, $\lambda/4$, $\lambda/2$, $3\lambda/4$, and λ. [4 marks]

   (b) A linearly polarized laser beam propagating along z enters a sugar solution at $z = 0$.

   (i) If no light scattering is observed along the x axis at $z = 0$ which is the initial polarization state of the laser? [2 marks]

   (ii) Draw a sketch to show the left and right-handed components of the polarization vector at $z = 0$ for $t = 0$. [2 marks]

   (iii) Repeat the above sketch at $z = 0$ for $t = \lambda/(4c)$. [3 marks]

   (c) At a distance $z = \Lambda/2$, no scattering is observed along the y axis.

   (i) Sketch the left- and right-hand components for $t = 0$. Assume that $n_{\mathrm{L}}k\Lambda/2 = 2m\pi$, where n_{L} is the refractive index for left-circularly polarized light and m is an integer. [3 marks]

   (ii) Repeat the above sketch for $t = \lambda/(4c)$. [2 marks]

   (d) Explain, briefly, how you could distinguish whether the sugar solution contains left- or right-handed molecules. [2 marks]

SECTION C: CONDENSED MATTER PHYSICS

5. (a) For a primitive cubic lattice, sketch the set of planes with the Miller indices (101) and (221). Include the x, y and z axes in your diagram. If the lattice constant, a, is 0.50 nm, determine the spacing between the planes for each of these two sets of Miller indices. [4 marks]

   (b) State Bragg's Law and explain, with the aid of a simple sketch, how this describes the diffraction of x-rays from a crystal structure. [4 marks]

   (c) Sketch the general form of the phonon dispersion relation for the first Brillouin zone in a one-dimensional crystal with a two-atom basis. Give the names of the different phonon dispersion curves and describe the relative motion of the atoms in each case. Describe the Debye approximation and state the physical principle used in the approximation. [4 marks]

   (d) Briefly describe the Wiedemann-Franz Law. Explain why the Drude model is able to predict the observed temperature variation of the ratio of the thermal to electrical conductivities in different metals. [4 marks]

   (e) Sketch an energy dispersion curve, $E(k)$, for the lowest energy band in the first Brillouin zone as predicted by the nearly-free electron model. Explain how the shape of this curve can be used to determine the electron effective mass, m_e^*. Indicate on your sketch where holes are likely to occur. [4 marks]

6. (a) Explain, with the aid of a sketch, the role of the energy density of states function and the Fermi-Dirac distribution function in determining the electron energy distribution in a metal as described by the free-electron model. What is the significance of the Fermi energy? [7 marks]

   (b) The experimentally determined Hall coefficient, R_{H}, of silver measured at room temperature is -9.00×10^{-11} m^3 C^{-1}. Use this to determine the density of free electron carriers in silver. [3 marks]

   (c) Use your result from part (b) to calculate both the Fermi energy of silver and the Fermi velocity at room temperature. [6 marks]

   (d) Compare the values in part (c) with the thermal energy and velocity of free electrons at 300 K and explain the reasons for the differences. [4 marks]