

Mathematical Methods II

Weekly problem set 4

- (1) Find all the singular points of the following equations

$$(1 - x^2)y'' - 2xy' + \left[\ell(\ell + 1) - \frac{m^2}{1 - x^2} \right] y = 0 \quad (\text{associated Legendre equation})$$

$$x^2y'' + xy' + (x^2 - \nu^2)y = 0 \quad (\text{Bessel equation})$$

and classify them. (Do not forget to consider potential singularities at infinity). Here ℓ, m and ν are constants.

- (2) The aim of this question is to find two power series solutions about $x = 0$ of the differential equation

$$(1 - x^2)y'' - 3xy' + \lambda y = 0, \tag{1}$$

where λ is a constant. Begin by checking that $x = 0$ is an ordinary point.

This series terminates at a particular value of λ , for order N . Deduce the value of λ for which the corresponding power series becomes a finite N -th order polynomial $y_N(x)$ - i.e. express λ in terms of N .

Construct y_1 and y_2 from terms up to the 3rd power, and show by substitution that y_1 and y_2 satisfy the differential equation.