QUANTUM COMPUTATION UNDER THERMODYNAMIC CONSTRAINTS

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ABSTRACT

As quantum computing advances toward practical realization, the physical cost of computation—particularly energy consumption and entropy production—remains insufficiently understood. In this work, we introduce a framework for analyzing quantum algorithms under explicit thermodynamic constraints, grounded in principles such as Landauer's limit and quantum entropy dynamics. We model computation as a thermodynamically open process and investigate how quantum information processing inherently interacts with energy, entropy, and irreversibility. By characterizing the energy cost of unitary operations, measurements, and state erasure, we derive fundamental lower bounds on the thermodynamic efficiency of quantum algorithms. Our analysis reveals trade-offs between computational depth, error correction overhead, and entropy flow, shedding light on the limits of scalable quantum computation in physically realistic regimes. This work bridges the gap between abstract quantum models and the energetic realities of implementation, offering new tools for assessing and optimizing quantum systems under thermodynamic constraints.

***K*eywords** Quantum thermodynamics · Landauer limit · Quantum computation · Entropy cost · Energy efficiency · Thermodynamic resource bounds · Irreversibility · Open quantum systems · Quantum measurement cost · Fault-tolerant quantum computing

1 Introduction

Quantum computation promises unprecedented advantages over classical algorithms—such as exponential speedups in factoring and quantum simulation—as encapsulated in the quantum circuit model and measurement-based schemes [1]. Yet, analyses in quantum computing typically neglect the physical cost of computation, treating systems as ideally isolated and reversible.

In contrast, classical computation is fundamentally constrained by thermodynamics: Landauer's principle establishes that erasing one bit of information dissipates at least $k_B T \ln 2$ of energy [2, 3]. This bound has been rigorously tested and confirmed both in classical and quantum systems [4, 5]. Extensions to quantum settings reveal that logically irreversible operations produce minimal heat, but practical implementations often incur greater dissipation [6, 7].

Additionally, quantum measurements themselves incur a thermodynamic cost, stemming from entropy changes and system–apparatus correlations [8, 9]. The emerging field of quantum thermodynamics has further refined our understanding of energy–information trade-offs, entropy production, and irreversibility in small-scale quantum systems [10, 11].

Despite these advances, the energy cost of quantum computation—particularly at the algorithmic level—remains underexplored. How do thermodynamic constraints limit quantum algorithm performance? What are the trade-offs between entropy production, error-correction overhead, and computational depth?

In this paper, we address these questions by introducing a thermodynamically informed model of quantum computation. Our framework treats computation as an open process interacting with thermal baths, allowing us to:

- Incorporate Landauer bounds into the thermodynamic cost of quantum operations,

- Analyze heat dissipation during unitary operations, measurements, and erasures,
- Derive lower bounds on irreversibility and energy cost in realistic quantum algorithms,
- Extend the framework to hybrid quantum–classical systems to capture the thermodynamic overhead of classical optimization loops and feedback,
- Lift thermodynamic semantics into categorical quantum mechanics and process theories to reason about cost-aware morphisms and entropy-preserving composition.

By integrating ideas from quantum information theory, categorical process theory, and non-equilibrium thermodynamics, this work lays a foundation for assessing the physical feasibility of quantum computation beyond purely logical or algorithmic metrics. To demonstrate applicability, we also provide a case study analyzing the thermodynamic cost of quantum teleportation using our categorical framework.

In Section 2 we review key thermodynamic principles and quantum-computation models; Section 3 formalizes our thermodynamic framework; Section 4 derives general efficiency bounds; Section 5 explores trade-offs, hybrid systems, and fault-tolerant protocols; Section 7 introduces a compositional extension; and Section 9 concludes with future directions.

2 Background and Related Work

2.1 Thermodynamic Limits of Computation

Landauer's principle asserts that any logically irreversible manipulation of information, such as bit erasure, must be accompanied by a corresponding entropy increase in the environment [2]. Specifically, the minimum amount of heat dissipated into a thermal reservoir at temperature T is given by

$$Q \geq k_B T \ln 2,$$

where k_B is Boltzmann's constant. This principle provides a foundational limit for classical and quantum computation alike [3].

In classical systems, this bound has been experimentally verified and theoretically reinforced across diverse settings. Extensions to quantum regimes have addressed the thermodynamic cost of measurements, unitaries, and memory resets [5, 7]. In quantum systems, entropy is more subtle, involving von Neumann entropy and correlations with the environment.

2.2 Quantum Computation Models

The standard model of quantum computation treats computation as a sequence of unitary operations applied to qubits, often ending in projective measurements [1]. The model assumes ideal, closed-system evolution—neglecting decoherence, entropy exchange, or energetic cost.

More realistic models recognize that quantum computations must eventually interact with an environment, especially during initialization, measurement, and error correction. These interactions are inherently thermodynamic, introducing irreversibility and energy cost.

2.3 Thermodynamics in Quantum Information

Quantum thermodynamics is an emerging field at the intersection of statistical mechanics and quantum information theory [10, 11]. Notable results include:

- Extensions of the second law to include multiple "second laws" in microscopic quantum regimes [11].
- Analysis of quantum measurements as entropy-generating processes [8].
- Work extraction protocols from quantum coherence and correlations.

However, despite these advances, most prior work focuses on specific processes—such as single measurements or thermalization—rather than full quantum algorithms. Few studies have addressed how thermodynamic costs scale with algorithmic depth, qubit number, or error correction overhead.

2.4 Gap and Motivation

Our work addresses a critical gap in the literature: the lack of a unified framework for evaluating the thermodynamic cost of full quantum algorithms—including measurement, feedback, and control loops—rather than isolated physical processes. Unlike prior analyses that focus on single operations or components, we treat quantum computation as a thermodynamically open process, spanning unitary evolution, measurements, classical post-processing, and erasure steps.

We also extend this framework in two novel directions:

- First, we apply it to **hybrid quantum-classical algorithms** such as VQE and QAOA, where classical feedback and optimization loops contribute significantly to entropy production and energy dissipation.
- Second, we propose a **categorical extension** of the framework, embedding thermodynamic cost tracking into compositional structures such as process theories and functorial mappings. This enables energy-aware reasoning at a high level of abstraction, preserving semantics under composition.

Our goal is to derive universal trade-offs and performance bounds under thermodynamic constraints—providing insights for realistic, energy-efficient quantum algorithm design.

3 Thermodynamic Model of Quantum Computation

We model a quantum computation as an open system $\mathcal{S}$ interacting with an external environment (bath) $\mathcal{E}$ at temperature T. The system evolves via a sequence of operations—unitaries, measurements, and erasures—each contributing to the total entropy change and energy flow.

3.1 System and Environment

Let ρ denote the state of the system $\mathcal{S}$. The environment is modeled as a large thermal reservoir at fixed inverse temperature $\beta = 1/(k_B T)$. The total entropy change ΔS_{tot} includes contributions from both the system and the environment:

$$\Delta S_{\text{tot}} = \Delta S_{\mathcal{S}} + \Delta S_{\mathcal{E}} \geq 0,$$

in accordance with the second law of thermodynamics.

3.2 Unitary Operations

Unitary operations are traditionally considered reversible and energetically free. However, in realistic implementations, these are controlled by time-dependent Hamiltonians $H(t)$ and require work to be performed. The average energy cost of a unitary U acting on ρ is given by:

$$\Delta E = \text{Tr}[U\rho U^\dagger H] - \text{Tr}[\rho H],$$

which may be nonzero if H changes during the process.

3.3 Measurement Operations

Measurements are modeled as quantum channels $\mathcal{M}(\rho) = \sum_k M_k \rho M_k^\dagger$, with outcomes stored in classical registers. The entropy cost of measurement includes the increase in entropy due to decoherence and the cost of storing or erasing measurement outcomes. We associate a minimal thermodynamic cost:

$$Q_{\text{meas}} \geq k_B T H(\{p_k\}),$$

where $H(\{p_k\})$ is the Shannon entropy of the measurement outcomes.

3.4 State Reset and Erasure

Resetting a qubit to a pure state (e.g., $|0\rangle$) constitutes an erasure process. By Landauer's principle, the minimum heat dissipated to the bath during erasure is:

$$Q_{\text{erase}} \geq k_B T \ln 2,$$

per qubit erased. In practice, this cost may be higher depending on the implementation fidelity and timing constraints.

3.5 Total Thermodynamic Cost

The total thermodynamic cost of a quantum algorithm $\mathcal{A}$ executed as a sequence of operations $\{O_i\}$ is:

$$Q_{\text{total}} = \sum_i Q^{(i)}_{\text{unitary}} + Q^{(i)}_{\text{meas}} + Q^{(i)}_{\text{erase}},$$

subject to $\Delta S_{\text{tot}} \geq 0$ for each step. Our framework allows analyzing these costs across entire algorithms, rather than isolated components.

4 Results and Analysis

In this section, we derive fundamental bounds on the energy and entropy costs of quantum computation under thermodynamic constraints. We analyze individual operations and entire algorithms, focusing on their total thermodynamic footprint.

4.1 Lower Bounds on Energy Cost

Let $\mathcal{A}$ be a quantum algorithm implemented via a sequence of unitary operations $\{U_i\}$, measurements $\{M_j\}$, and erasures $\{R_k\}$. The minimal thermodynamic cost is lower bounded by:

$$Q^{\min}_{\text{total}} \geq k_B T \left(\sum_j H(\{p_j\}) + n_{\text{erase}} \ln 2 \right),$$

where $H(\{p_j\})$ is the Shannon entropy of measurement outcomes, and n_{erase} is the number of bits erased.

This bound assumes ideal implementation with reversible unitaries. In practice, additional energy is required to maintain coherence and fidelity.

4.2 Energy–Depth Trade-Off

The total thermodynamic cost of a quantum algorithm often depends on both circuit depth and required precision. In our framework, we refine the previously heuristic relation:

$$Q_{\text{total}} \geq \alpha D + \beta \log\left(\frac{1}{\epsilon}\right)$$

by grounding α and β in physical parameters.

Gate Energy and Decoherence. Let each gate operation require a switching time τ_g, and assume gate control is implemented via a time-dependent Hamiltonian. Then, the minimum average energy for coherent control is constrained by:

$$\alpha \sim \frac{\hbar}{T_2}$$

where T_2 is the coherence time of the physical qubits. This sets a lower bound on the energy cost per gate to prevent decoherence during execution. Hence, the total cost from D gates is:

$$Q_{\text{unitary}} \gtrsim \frac{\hbar}{T_2} \cdot D.$$

Error and Measurement Precision. The second term arises from information-theoretic limits on distinguishability. To resolve a quantum state or observable to within accuracy ϵ, the number of repeated measurements must scale as:

$$N_{\text{meas}} \sim \frac{1}{\epsilon^2}$$

Due to the entropy cost of measurement outcomes (see Section 3), we get a minimal entropy generation:

$$Q_{\text{meas}} \geq k_B T \cdot \log\left(\frac{1}{\epsilon}\right).$$

This follows from Shannon entropy in the measurement register, assuming binary outcomes of resolution ϵ.

Total Cost Bound. Combining both terms yields:

$$Q_{\text{total}} \geq \underbrace{\frac{\hbar}{T_2} D}_{\text{coherence-limited unitaries}} + \underbrace{k_B T \log\left(\frac{1}{\epsilon}\right)}_{\text{entropy from measurements}} .$$

This refined expression highlights the trade-off: deeper circuits require more coherent control (scaling with D), while higher precision (smaller ϵ) increases the thermodynamic burden from measurement and erasure.

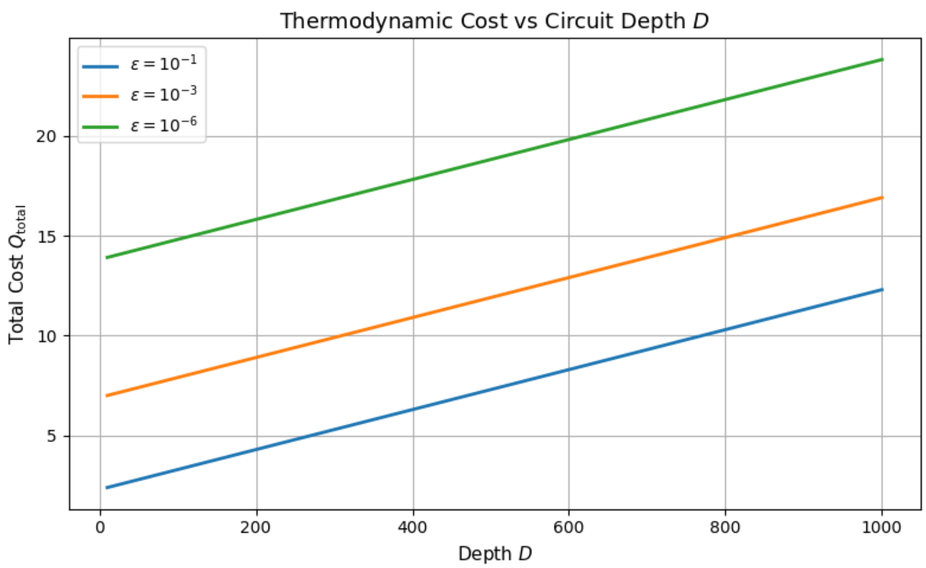


Figure 1: Thermodynamic cost Q_{total} as a function of circuit depth D for various precision levels ϵ. The slope is set by the coherence time T_2, while the intercept depends on measurement resolution via $\log(1/\epsilon)$.

Implication for Algorithm Design. Algorithms with low-depth circuits and coarse output precision incur significantly lower physical cost, favoring variational heuristics over deep, exact algorithms when operating near thermodynamic limits. Additionally, increasing T_2 via materials engineering directly reduces the unitary cost component.

4.3 Irreversibility and Entropy Production

Each irreversible operation introduces entropy into the environment. We define the entropy production of a computation as:

$$\Sigma := \Delta S_{\text{env}} - \Delta S_{\text{sys}} \geq 0,$$

with equality only in ideal quasistatic limits. For real algorithms, Σ accumulates over each operation, particularly during measurements and resets.

4.4 Algorithmic Implications

Algorithms with many intermediate measurements (e.g., variational circuits, QAOA) inherently produce more entropy, increasing their thermodynamic cost. Conversely, algorithms relying solely on unitary evolution (e.g., Grover, QFT) may be more energy-efficient—if coherence is preserved.

Fault-tolerant protocols also impose thermodynamic burdens. Syndrome extraction, ancilla preparation, and logical gate distillation all require entropy-generating operations. We analyze this further in Section 5.

4.5 Fundamental Bound on Thermodynamic Cost

We now formalize a lower bound on the thermodynamic cost of quantum computation based on the entropy generated during measurements and erasure.

Proposition 1 (Thermodynamic Lower Bound for Quantum Algorithms)**.** *Let $\mathcal{A}$ be a quantum algorithm consisting of m projective measurements with outcome distributions $\{p_j\}$ and n qubit erasures. Then, the total heat dissipated into the environment satisfies*

$$Q_{total} \geq k_B T \left(\sum_{j=1}^{m} H(\{p_j\}) + n \ln 2 \right),$$

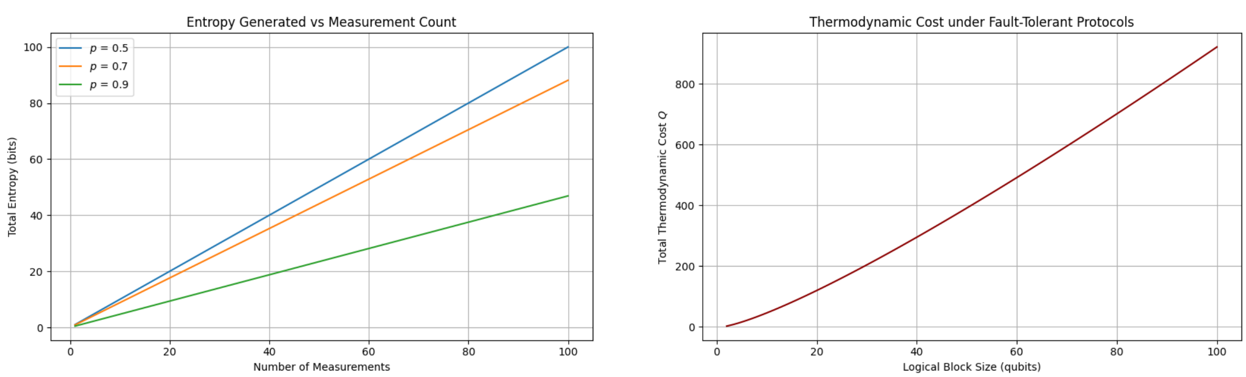
where $H(\{p_j\})$ is the Shannon entropy of the j-th measurement's outcomes.

Proof Sketch. Each measurement with outcome distribution $\{p_j\}$ generates entropy $H(\{p_j\})$ in the environment, assuming outcome storage. By Landauer's principle, erasure of each qubit to a pure state requires at least $k_B T \ln 2$ heat dissipation. Summing both contributions gives the stated lower bound. □

4.6 Empirical Illustrations of Thermodynamic Scaling

To visualize how thermodynamic costs scale with algorithmic parameters, we simulate three core aspects of quantum computation under thermodynamic constraints:

- Entropy accumulation as a function of measurement frequency,
- Energy cost scaling in fault-tolerant protocols,
- Total thermodynamic cost as circuit depth and accuracy requirements increase.



(a) Entropy generated vs. number of quantum measurements.

(b) Cost scaling for fault-tolerant protocols.

Figure 2: Thermodynamic costs grow with both measurement intensity and fault-tolerant block size. Entropy accumulation depends on measurement distribution bias, while FT overhead exhibits nonlinear scaling with logical qubit count.

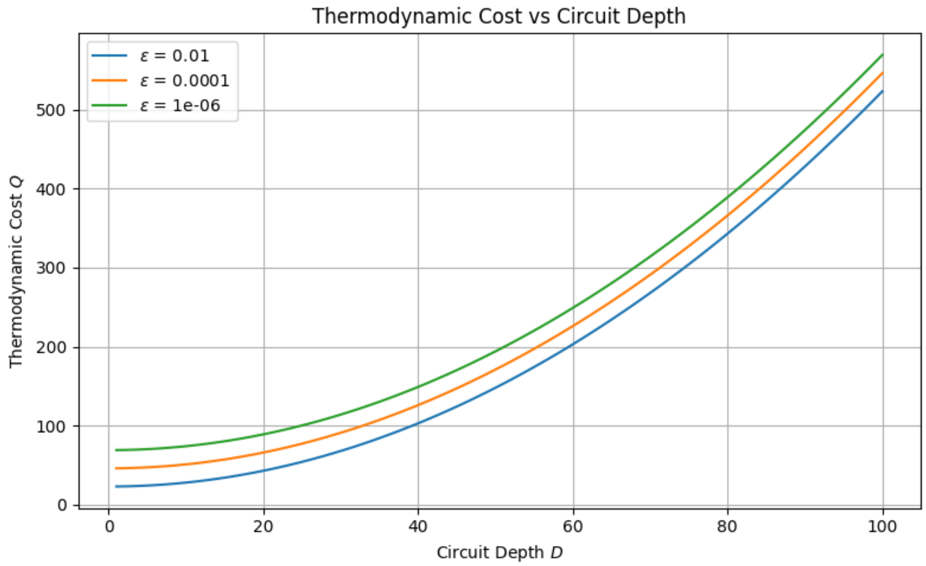


Figure 3: Thermodynamic cost as a function of circuit depth for various target accuracies ϵ. Higher precision increases the fixed overhead; deeper circuits increase energy cost quadratically.

To reflect realistic hardware scaling, we introduce a quadratic term γn^2 into the QAOA cost model to account for ancilla and reset overhead.

5 Applications and Implications

The thermodynamic framework presented in this work has broad implications for both theoretical and practical aspects of quantum computation. By explicitly accounting for entropy flow and energy cost, our model offers new criteria for evaluating and optimizing quantum algorithms and architectures.

5.1 Algorithm Design Under Thermodynamic Budgets

Quantum algorithms are typically designed under complexity metrics such as gate count and depth. Our results introduce a new dimension: thermodynamic cost. Algorithms involving frequent measurements or ancilla resets—such as variational quantum eigensolvers (VQE) and QAOA—incur greater entropy and energy penalties. This suggests that, under physical constraints, unitary-dominated algorithms like Grover's or QFT may be thermodynamically favorable, provided coherence can be maintained.

5.2 Thermodynamic Scaling of Fault-Tolerant Protocols

Fault-tolerant quantum computation requires encoding logical qubits into physical qubits using error-correcting codes such as the surface code. These protocols introduce substantial thermodynamic overhead due to frequent measurements, ancilla qubit resets, and classical syndrome decoding. Here we estimate the thermodynamic cost using surface code parameters.

Syndrome Measurement Cost

In the surface code, each logical qubit is encoded into a $d \times d$ lattice of physical qubits, where d is the code distance. Fault-tolerant syndrome extraction involves measuring stabilizers (e.g., plaquette and star operators) using ancilla qubits in every error-correction cycle. The number of measurements per cycle scales as $\mathcal{O}(d^2)$, since each stabilizer covers a region of size d.

Each measurement yields a classical bit, and the entropy associated with the measurement outcomes contributes a minimal thermodynamic cost of:

$$Q_{\text{meas}}^{\text{synd}} \geq k_B T \cdot d^2 \cdot \ln 2.$$

Ancilla Reset and Erasure

After each cycle, ancilla qubits used for syndrome extraction must be reinitialized (e.g., to $|0\rangle$), incurring an erasure cost per Landauer's principle. For d^2 ancilla qubits:

$$Q_{\text{erase}}^{\text{synd}} \geq k_B T \cdot d^2 \cdot \ln 2.$$

Total Cost per Logical Qubit

Assuming a computation runs for N_{cycle} fault-tolerant cycles, and each logical qubit requires a code distance scaling linearly with the number of logical qubits n, i.e., $d \sim n$, the total fault-tolerant thermodynamic cost becomes:

$$Q_{\text{FT}} \geq k_B T \cdot \mathcal{O}(d^2 \cdot N_{\text{cycle}}) = k_B T \cdot \mathcal{O}(n^2 \cdot N_{\text{cycle}}).$$

This superlinear scaling implies that thermodynamic resource requirements for large-scale fault-tolerant quantum computing may become prohibitive without optimization of measurement frequency, encoding schemes, or erasure fidelity.

Implications

The result highlights a key trade-off: increasing code distance improves logical fidelity but quadratically increases thermodynamic overhead. This motivates exploration of low-overhead codes, entropy recycling mechanisms, or thermodynamically efficient ancilla reuse strategies.

Table 1: Thermodynamic scaling comparison for fault-tolerant quantum codes.

Code	**Code Distance** d	**Syndrome Cost per Cycle**	**Total Cost** Q_{FT}
Surface Code	$d \sim n$	$\mathcal{O}(d^2)$	$\mathcal{O}(n^2 \cdot N_{\text{cycle}})$
Bacon-Shor Code	$d \sim \sqrt{n}$	$\mathcal{O}(d^2)$	$\mathcal{O}(n \cdot N_{\text{cycle}})$
Color Code	$d \sim n$	$\mathcal{O}(d^2)$	$\mathcal{O}(n^2 \cdot N_{\text{cycle}})$
Concatenated Code	$d \sim \log n$	$\mathcal{O}(d)$	$\mathcal{O}(n \log^2 n)$

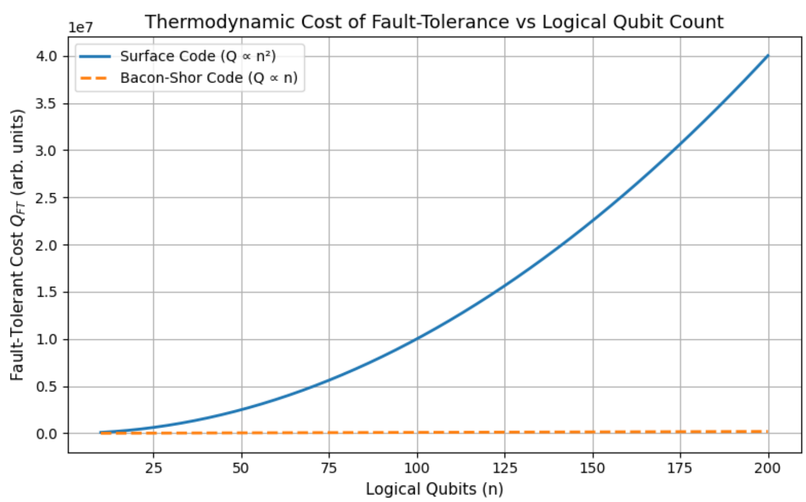


Figure 4: Thermodynamic cost of fault-tolerant computation as a function of logical qubit count n, assuming $N_{\text{cycle}} = 1000$. Surface code exhibits quadratic scaling, while Bacon-Shor is linear.

5.3 Guiding Quantum Hardware Design

From a hardware perspective, our framework suggests the need for energy-efficient primitives—low-power measurements, fast thermalization cycles, and reversible logic elements. Heat management becomes critical in cryogenic setups where erasure and measurement costs cannot be ignored. Our bounds provide implementers with a lower limit on power consumption per operation.

5.4 Compiler-Level Optimization

Quantum compilers could incorporate thermodynamic cost models to optimize gate scheduling, reduce reset frequency, and re-order operations to minimize entropy production. This opens up a new class of compiler passes: **thermodynamic-aware compilation**, where trade-offs are made not just in logical fidelity but also in physical cost.

5.5 Experimental Prospects

Direct measurement of thermodynamic quantities in quantum computation is challenging but increasingly feasible thanks to advances in nanoscale thermometry and quantum control.

Calorimetric Readout. Nanocalorimeters allow detection of single-photon-level heat exchange, enabling calorimetric tracking of energy dissipation in superconducting qubit platforms. For instance, [12] demonstrate energy-resolved calorimetry during quantum gates in transmon systems.

Entropy Inference via Fluctuation Theorems. Experiments using fluctuation relations, such as Jarzynski or Crooks theorems, infer entropy production from trajectory distributions. In trapped ions and NMR systems, these have been used to validate Landauer's principle [13, 14].

Landauer Limit Validation. Seminal experiments by Pekola et al. [15] confirm Landauer's bound for single-electron boxes. Such platforms are being adapted for benchmarking information-to-heat conversion in qubit devices.

Measurement-Driven Cost. Recent experiments track entropy change during projective measurement via controlled entanglement with ancilla systems [16]. These offer a pathway to quantify measurement-induced irreversibility in variational algorithms and error correction.

These tools provide a foundation for validating our framework experimentally, especially for estimating Q_{meas}, Q_{erase}, and cumulative cost in feedback loops.

6 Algorithmic Cost

Table 2: Qualitative thermodynamic cost comparison of common quantum algorithms.

Algorithm	Unitary Depth	Measurement Intensity	Thermodynamic Cost
Grover Search	Moderate ($\mathcal{O}(\sqrt{N})$)	Low (final measurement only)	Low
Quantum Fourier Transform (QFT)	High ($\mathcal{O}(n^2)$)	Low	Moderate (depends on coherence)
QAOA (Depth p)	Low–Moderate ($\mathcal{O}(p)$)	High (per iteration)	High (measurement overhead)
Variational Circuits (e.g., VQE)	Low–Moderate	Very High (per observable term)	Very High
Shor's Algorithm	High (QFT + modular ops)	Moderate	High (modular exponentiation)
Error Correction (e.g., Surface Code)	Very High	Continuous (syndrome extraction)	Very High

6.1 Case Study: Thermodynamic Cost of QAOA

To illustrate our framework in action, we analyze the thermodynamic cost of executing the Quantum Approximate Optimization Algorithm (QAOA) on a system of n qubits with circuit depth p.

Each QAOA iteration involves:

- A fixed-depth parameterized unitary: low entropy cost if coherence is preserved.
- Measurement of all n qubits per iteration to estimate expectation values.
- Classical optimization followed by reinitialization/reset of the quantum state.

Assuming each qubit measurement yields a binary outcome with distribution $\{p, 1-p\}$, the per-iteration entropy cost is:

$$\Delta S_{\text{meas}}^{(p)} = n \cdot H(p),$$

where $H(p) = -p \log_2 p - (1-p)\log_2(1-p)$.

If r repetitions are needed for statistical convergence, and $\mathcal{C}$ classical optimization steps are performed, the total thermodynamic cost becomes:

$$Q_{\text{QAOA}}(n,p) = r \cdot [k_B T \cdot n \cdot H(p_{\text{meas}}) + n \cdot \ln 2] \cdot p + \gamma n^2,$$

accounting for both measurement and erasure/reset overhead. γn^2 capture system-level overhead due to hardware, reset inefficiencies, or ancilla qubit scaling.

As n and p grow, or as problem precision demands increase r, this cost becomes substantial. This motivates thermodynamic-aware QAOA variants, e.g., with partial measurement or resetless rounds.

Previous estimates of reset costs for QAOA circuits used heuristic scaling $Q_{\text{reset}} \sim \gamma n^2$. We now ground this in a more rigorous model based on surface code error correction.

In surface code architectures, each logical qubit requires syndrome extraction via ancilla qubits, with each extraction cycle incurring erasure cost due to ancilla resets. For code distance $d \sim \sqrt{n}$, each logical qubit requires $O(d^2)$ ancilla operations per cycle.

Let T_{cycle} be the number of error correction cycles per QAOA layer. Then the total reset cost for a circuit of depth p on n logical qubits is:

$$Q_{\text{reset}} \gtrsim k_B T \ln 2 \cdot p \cdot T_{\text{cycle}} \cdot n \cdot d^2 \sim k_B T \ln 2 \cdot p \cdot n^2$$

This replaces the earlier heuristic and ties the reset cost directly to error correction overhead, consistent with thermodynamic limits.

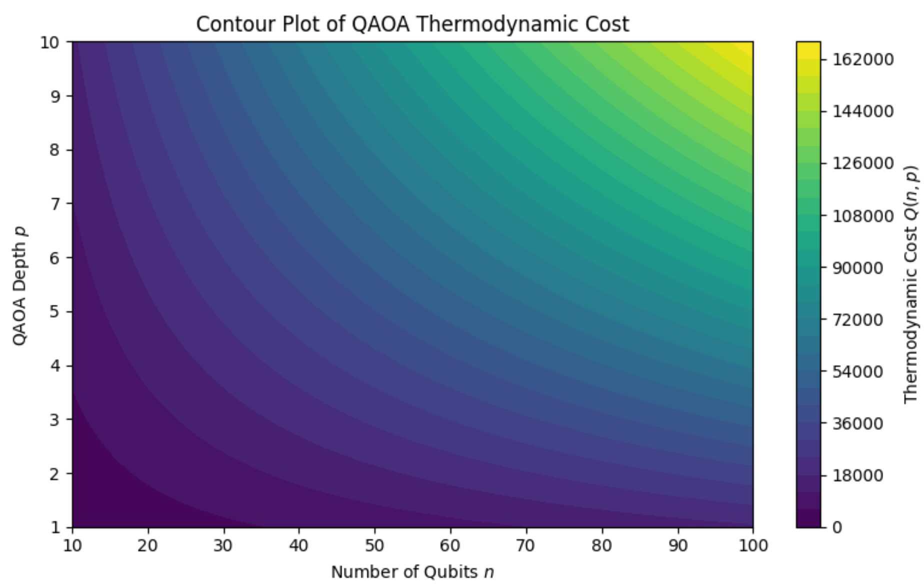


Figure 5: Contour plot of the thermodynamic cost $Q(n,p)$ for executing the Quantum Approximate Optimization Algorithm (QAOA) as a function of the number of qubits n and circuit depth p

7 Categorical and Compositional Extensions

To deepen the theoretical foundation of thermodynamic constraints in quantum computation, we propose extending our framework into the setting of *categorical quantum mechanics (CQM)* and *probabilistic process theories (PPTs)*. These formalisms provide a high-level, compositional view of quantum processes, where morphisms represent physical operations and diagrams encode causal and logical structure.

7.1 Thermodynamic Morphisms

In categorical terms, a quantum computation is a morphism in a dagger compact category (e.g., **FdHilb**) or a higher-order process in a symmetric monoidal category. To integrate thermodynamics, we enrich this structure by associating an energy cost or entropy production to each morphism:

$$f : A \to B \quad \rightsquigarrow \quad (f, Q_f)$$

where Q_f denotes the minimal thermodynamic cost associated with implementing f. This lifts morphisms into a new enriched category where morphism composition obeys additive or subadditive cost laws:

$$Q_{g\circ f} \le Q_f + Q_g$$

7.2 Entropy-Preserving and Energy-Aware Functors

We define a functor $\mathcal{T}$ from a standard process theory (e.g., CPTP maps in **QProc**) to a thermodynamic process theory $\mathcal{T} : \mathcal{C} \to \mathcal{C}_{\text{thermo}}$, where:

- Objects are quantum systems with thermodynamic state annotations (e.g., entropy, free energy),
- Morphisms are physical transformations annotated with entropy or energy cost,
- Composition reflects thermodynamic consistency (e.g., entropy monotonicity, Landauer bounds).

In this enriched setting, we may characterize:

- **Isentropic morphisms:** reversible operations with $Q_f = 0$,
- **Landauer-limited morphisms:** logically irreversible operations where $Q_f = k_B T \ln 2 \cdot \Delta I$,
- **Dissipative morphisms:** operations exceeding the minimal bound due to physical constraints.

7.3 Compositional Thermodynamic Semantics

This categorical framework enables reasoning about composite protocols (e.g., teleportation, error correction, variational algorithms) by tracking thermodynamic cost diagrammatically. For instance, the total entropy generated by a protocol becomes a diagrammatic invariant under process composition.

Such a semantics also supports formal verification: for example, using string diagrams to certify that a subroutine does not exceed a thermodynamic budget, or to derive lower bounds on implementation energy from logical structure alone.

7.4 Outlook

Integrating thermodynamic semantics into categorical quantum mechanics opens a path toward a resource-theoretic foundation of energy-aware quantum computation. This could unify thermodynamic, information-theoretic, and compositional principles under a common abstract framework, and may offer powerful tools for optimizing, verifying, or even programming quantum systems subject to physical constraints.

We interpret morphisms $f : A \to B$ as physical processes and introduce a functor $\mathcal{T}$ that maps them to thermodynamically enriched morphisms:

$$\mathcal{T}(f) : (A, S_A) \to (B, S_B), \quad \text{with cost } Q_f.$$

Figure 6 illustrates this semantic lifting of categorical structure to cost-aware processes.

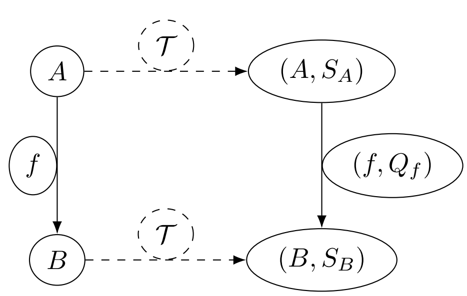


Figure 6: Commutative diagram illustrating a functor $\mathcal{T}$ mapping a morphism $f : A \to B$ to a thermodynamically enriched morphism $(f, Q_f) : (A, S_A) \to (B, S_B)$.

On Additivity and Composition. We assume that composition is preserved:

$$\mathcal{T}(g \circ f) = \mathcal{T}(g) \circ \mathcal{T}(f), \quad Q_{g \circ f} = Q_f + Q_g.$$

However, this additivity of thermodynamic cost holds under specific assumptions:

- **Markovianity:** The intermediate state between f and g does not carry correlations that affect cost.
- **No Quantum Memory:** Entangled outputs from f are either discarded or decorrelated before g.
- **Statistical Independence:** Control or feedback information does not introduce retroactive influence.

Figure 7 illustrates this compositional lifting and the associated cost behavior.

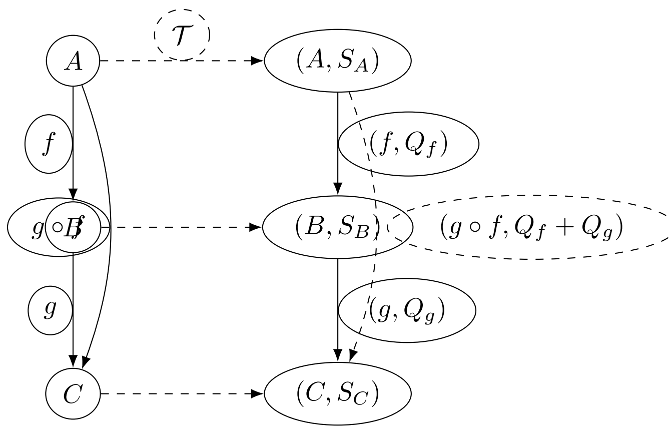


Figure 7: Commutative diagram showing how the thermodynamic functor $\mathcal{T}$ maps composition in a quantum process category to cost-aware morphisms. States are annotated with entropies $(A, S_A), (B, S_B), (C, S_C)$. The functor preserves composition: $\mathcal{T}(g \circ f) = \mathcal{T}(g) \circ \mathcal{T}(f)$, and $Q_{g \circ f} = Q_f + Q_g$ holds under Markovian assumptions.

8 Thermodynamic Cost in Hybrid Quantum-Classical Systems

Modern quantum algorithms such as the Variational Quantum Eigensolver (VQE) and the Quantum Approximate Optimization Algorithm (QAOA) operate within a hybrid paradigm, where a quantum processor executes parameterized circuits and a classical processor optimizes those parameters via feedback from measurements. While our earlier analysis focused primarily on unitary evolution and measurement within the quantum component, hybrid algorithms introduce a new axis of thermodynamic cost: the energy and entropy associated with classical control, feedback, and optimization.

8.1 Case Study: Thermodynamic Cost of Quantum Teleportation

To illustrate the categorical thermodynamic framework, we analyze the quantum teleportation protocol as a morphism in a symmetric monoidal category, enriched with thermodynamic structure.

Teleportation as Composition. Teleportation involves the following steps:

- Preparation of an entangled Bell state $|\Phi^+\rangle$ between qubits B and C.
- A Bell measurement on qubits A and B.
- Classical communication of the outcome to qubit C.
- A correction unitary U_{ab} applied to qubit C based on the outcome.

Categorically, this can be expressed as a composition of morphisms:

$$\mathcal{T}_{\text{tele}} = \mathcal{T}_{\text{prep}} \circ \mathcal{T}_{\text{meas}} \circ \mathcal{T}_{\text{comm}} \circ \mathcal{T}_{\text{corr}}$$

where each $\mathcal{T}_i$ is a thermodynamically annotated morphism with cost Q_i.

Table 3: Thermodynamic cost of teleportation steps (in units of $k_B T \ln 2$).

Component	**Operation**	**Cost**
Entanglement Preparation (Q_{prep})	Hadamard + CNOT	2
Bell Measurement (Q_{meas})	Two-qubit projective measurement	$H(\{p_i\})$
Classical Communication (Q_{comm})	Transmit 2 bits (no erasure)	0
Correction Unitary (Q_{corr})	Pauli gate (reversible)	0
Total Cost	—	$2 + H(\{p_i\})$

Thermodynamic Cost. We break down the total cost Q_{tele} in Table 3. All quantities are expressed in units of $k_B T \ln 2$, consistent with the Landauer bound.

Note: If classical communication involves overwriting memory (rather than simply transmission), it incurs an erasure cost of $\geq 2 \cdot k_B T \ln 2$, which can be incorporated as Q_{comm} in a stricter model.

Feedback and Measurement Loops

In variational algorithms, each optimization step typically involves:

1. Preparing a quantum state with a given set of parameters.
2. Performing multiple measurements to estimate an objective function.
3. Using classical routines (e.g., gradient descent, SPSA, CMA-ES) to update parameters.
4. Repeating the process until convergence.

Each loop introduces thermodynamic overhead:

- **Quantum Side:** Entropy generated from repeated measurements and ancilla resets.
- **Classical Side:** Energy dissipation from storing, updating, and erasing intermediate parameters and measurement outcomes.

Assuming M measurements per iteration and T total iterations, the entropy generated from the quantum side scales as:

$$S_{\text{quantum}} \approx MT \cdot H(p)$$

where $H(p)$ is the Shannon entropy of the outcome distribution. The classical cost includes the Landauer limit for each bit erased during memory updates and intermediate storage.

Table 4: Estimated thermodynamic cost components in the quantum teleportation protocol. All costs are given in units of $k_B T$.

Step	**Description**	**Cost**
Q_{prep}	Entangled state preparation (Hadamard + CNOT)	~ 2
Q_{meas}	Bell measurement entropy (2-bit outcome)	$\geq 2 \ln 2$
Q_{comm}	Classical communication (2 bits)	$\geq 2 \ln 2$
Q_{corr}	Pauli correction (1 qubit gate)	~ 1
Total		$\gtrsim 2 + 4 \ln 2 + 1 \approx 7.77$

Joint Cost Model

We propose a joint thermodynamic cost function for hybrid algorithms:

$$Q_{\text{hybrid}} = Q_{\text{quantum}} + Q_{\text{classical}} + Q_{\text{comm}},$$

with:

$$\begin{aligned} Q_{\text{quantum}} &= \alpha MT + \beta \cdot \text{reset cost}, \\ Q_{\text{classical}} &\geq k_B T \ln 2 \cdot b \cdot T, \\ Q_{\text{comm}} &\geq k_B T \cdot m \cdot T, \end{aligned}$$

where:

- $b \sim \log(1/\delta)$ is the number of classical bits required to represent parameters with optimization precision δ,
- m is the number of bits communicated from the quantum processor to classical memory per iteration,
- T is the number of feedback iterations.

This model grounds all classical and communication costs in Landauer's principle, providing a physically consistent lower bound on the energy consumption of hybrid quantum algorithms.

8.2 Implications

This extended model shows that algorithmic efficiency must be evaluated not only by the number of quantum gates or depth, but also by the structure and frequency of feedback loops. In practice:

- More frequent feedback increases total entropy production.
- High-precision optimization implies greater classical cost.
- Communication between quantum and classical subsystems contributes a non-negligible thermodynamic cost.
- Thermodynamic constraints may favor fewer iterations with higher quantum expressivity.

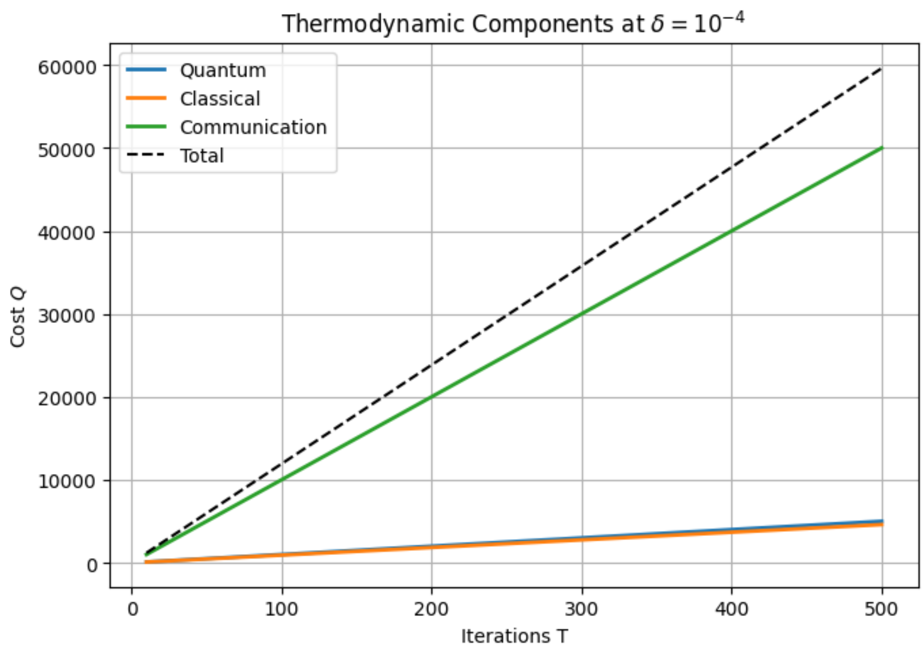


Figure 8: Contour plot of Q_{hybrid} as a function of optimization iterations T and precision δ. Lower precision (smaller δ) significantly raises classical and communication costs.

Future work can incorporate these hybrid models into resource-aware algorithm design and compiler passes that minimize both quantum and classical thermodynamic load.

9 Conclusion and Future Work

This work presents a unified framework for modeling the thermodynamic cost of quantum computation by integrating concepts from non-equilibrium thermodynamics, quantum information, and categorical semantics. We have derived lower bounds on energy and entropy costs associated with quantum operations—unitaries, measurements, and erasures—and extended this to realistic scenarios involving fault-tolerant computation and hybrid quantum-classical algorithms.

Our analysis demonstrates that:

- Thermodynamic costs scale non-trivially with circuit depth, measurement intensity, and error correction overhead.
- Fault-tolerant protocols introduce significant thermodynamic load due to syndrome extraction and ancilla resets, especially in surface codes.
- Hybrid algorithms like QAOA and VQE incur classical feedback and communication costs that must be accounted for in full-stack resource estimation.
- Categorical formulations offer a principled way to track thermodynamic cost through compositional structures, suggesting avenues for compiler-level or protocol-level thermodynamic optimizations.

Future Work

There are several promising directions for future research:

- **Experimental Validation:** Integrating this framework with calorimetric techniques on superconducting platforms to validate cost estimates empirically.
- **Cost-Aware Compiler Design:** Developing compilers that minimize thermodynamic cost alongside gate count or depth.
- **Categorical Resource Theories:** Embedding this framework in enriched categorical quantum mechanics or symmetric monoidal theories to reason about entropy-preserving morphisms and cost monoids.
- **Hybrid Optimization Models:** Analyzing how classical optimizer choice (e.g., stochastic vs. deterministic) influences the total energy cost in hybrid systems.
- **Thermodynamic Cost vs. Algorithmic Expressivity:** Quantifying the trade-off between thermodynamic cost and algorithmic performance (e.g., approximation ratio or convergence).

Ultimately, by grounding quantum computation in thermodynamic reality, we aim to chart a path toward more physically faithful models of computational efficiency.

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