

QF600 (Asset Pricing) – Homework 1

Industry_Portfolios.xlsx contains monthly nominal (net) returns (expressed as percentages) for ten industry portfolios, over the ten-year period from Jan 2004 through Dec 2013. Use these returns to estimate the vector of mean returns and the covariance matrix of returns for the ten industry portfolios:

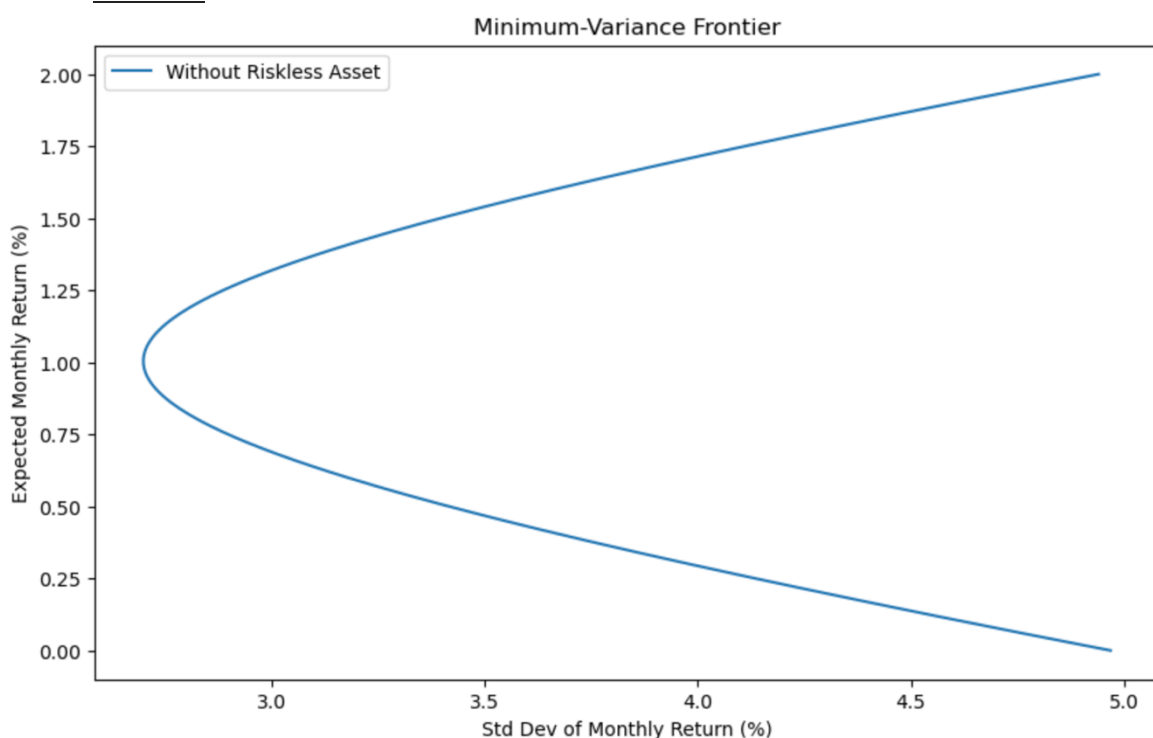
1. Create a table showing the mean return and standard deviation of return for the ten industry portfolios.

Answer:

Industry	Mean Return (%)	Std Deviation (%)
NoDur	0.902833	3.345657
Durbl	0.733333	8.361852
Manuf	1.012833	5.31027
Enrgy	1.231167	6.081524
HiTec	0.76625	5.381191
Telcm	0.881417	4.448284
Shops	0.916333	4.093786
Hlth	0.783833	3.787172
Utils	0.907167	3.701763
Other	0.489083	5.582452

2. Plot the minimum-variance frontier (without the riskless asset) generated by the ten industry portfolios:
 - This graph must have expected (monthly) return on the vertical axis vs standard deviation of (monthly) return on the horizontal axis.
 - This graph must cover the range from 0% to 2% on the vertical axis, increments of 0.1% (or less):

Answer:



3. Briefly explain (in words, without mathematical equations or formulas) the economic significance and relevance of the minimum-variance frontier to an investor.

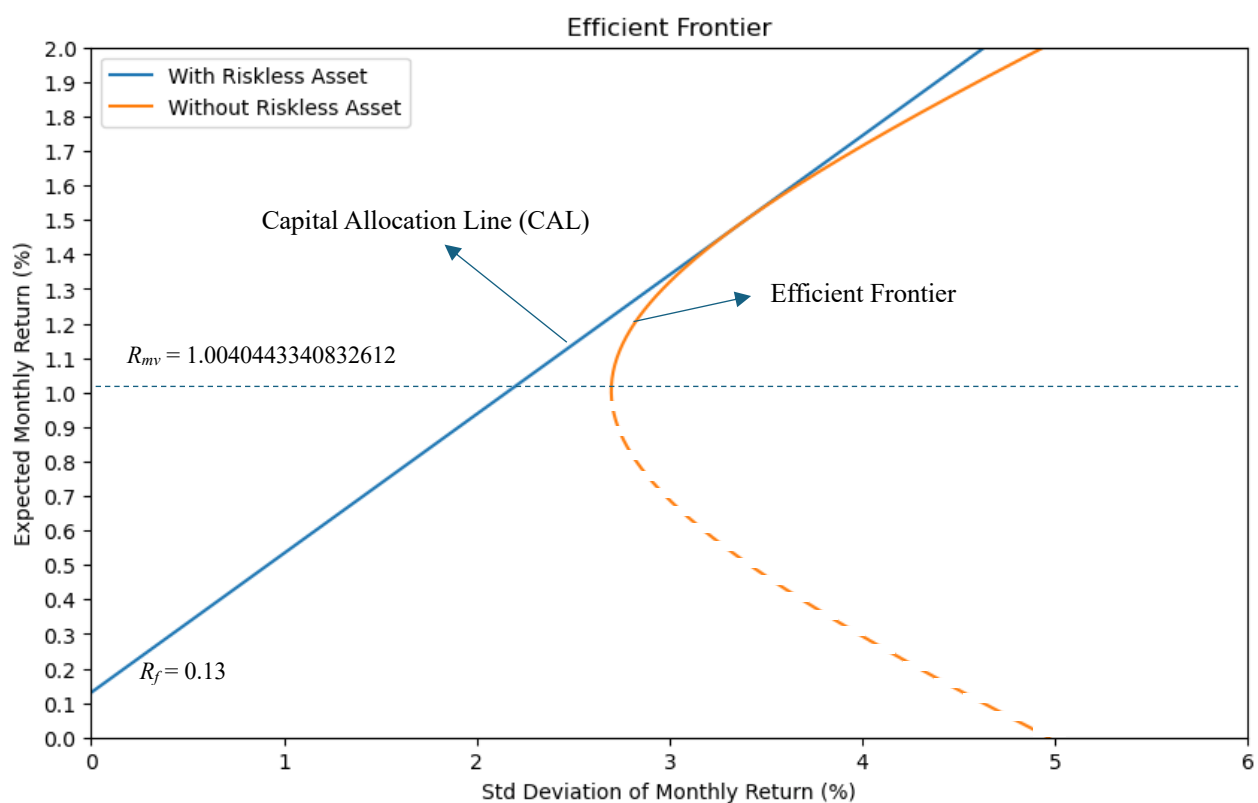
Answer:

The minimum-variance frontier is significant for investors because it consists of portfolios with the lowest amount of risk, for different values of expected return (R_p). Investors aim to optimize their portfolios by balancing risk and return, and the minimum-variance frontier helps them achieve this by identifying the most efficient portfolios. These efficient portfolios minimize risk while providing the maximum possible return for that level of risk, helping investors make informed decisions about their trade-offs between risk and return.

Now suppose that the (net) risk-free rate is 0.13% per month:

4. Plot the efficient frontier (with the riskless asset) on the same graph as the minimum-variance frontier generated by the ten industry portfolios.

Answer:



5. Briefly explain the economic significance and relevance of the efficient frontier to an investor.

Answer:

The top half of minimum-variance frontier is known as the efficient frontier. The efficient frontier is economically significant because it consists of portfolios with highest expected return for a given amount of risk. It helps investors to choose the best combinations of assets that maximizes return without taking unnecessary risk.

The two frontiers will intersect at single point: the tangency portfolio:

6. Calculate the Sharpe ratio for the tangency portfolio, and also the tangency portfolio weights for the ten industry portfolios.

Answer:

Sharpe ratio formula:

$$\frac{R_{tg} - R_f}{\sigma_{tg}} = \left[-\frac{\zeta - 2\alpha R_f + \delta R_f^2}{\delta (R_f - R_{mv})} \right] \left[-\frac{\delta (R_f - R_{mv})}{(\zeta - 2\alpha R_f + \delta R_f^2)^{\frac{1}{2}}} \right]$$

$$= (\zeta - 2\alpha R_f + \delta R_f^2)^{\frac{1}{2}}$$

Sharpe ratio = 0.40356559934950903

$$R_{tg} = R_{mv} - \frac{\zeta \delta - \alpha^2}{\delta^2 (R_f - R_{mv})} = \frac{\alpha R_f - \zeta}{\delta R_f - \alpha}$$

$R_{tg} = 1.4862735358446897$

Portfolio weight formula (w^*):

Rearrange to get linear relationship: $\mathbf{w}^* = \mathbf{a} + \mathbf{b}R_p$, where:

$$\mathbf{a} = \frac{\zeta \mathbf{V}^{-1} \mathbf{e} - \alpha \mathbf{V}^{-1} \mathbf{R}}{\zeta \delta - \alpha^2}; \quad \mathbf{b} = \frac{\delta \mathbf{V}^{-1} \mathbf{R} - \alpha \mathbf{V}^{-1} \mathbf{e}}{\zeta \delta - \alpha^2}$$

Portfolio weight (w^*):

Industry	Portfolio Weight (w^*)
NoDur	0.567972
Durbl	-0.214073

Manuf	0.714105
Enrgy	0.104087
HiTec	-0.363438
Telcm	-0.095463
Shops	0.991647
Hlth	0.07557
Utils	0.132643
Other	-0.913051

Sum of all portfolio weight = 1

Double check that sum of all portfolio weights must be equal to 1 to verify the answer.

7. Briefly explain the economic significance and relevance of the tangency portfolio to an investor.

Answer:

Tangency portfolio is a unique risky portfolio where Capital Allocation Line (CAL) tangent to efficient frontier generated by n risky assets. Tangency portfolio is economically significant because it represents the optimal portfolio on the efficient frontier when combined with a risk-free asset. It also maximizes Sharpe ratio (highest possible return per unit of risk). Thus, it helps investors achieve their preferred level of risk while obtaining the best possible return.