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QF600 (Asset Pricing) – Homework 6

Behavioral Finance

Assume Barberis, Huang, and Santos economy where investor receives utility from consumption as well as recent financial gain or loss. Use these parameters:

$$\delta = 0.99, \quad \gamma = 1, \quad \lambda = 2$$

Consumption growth has lognormal distribution:

$$\ln \tilde{g} = 0.02 + 0.02\tilde{\varepsilon}$$

where ε is standard normal random variable. Simulate probability distribution for consumption growth with (at least) 10^4 random draws from standard normal distribution.

With these parameters, risk-free rate is around 3% per year:

$$R_f = \frac{e^{0.0198}}{0.99} = 1.0303$$

Define x as one plus dividend yield for market portfolio:

$$x = \left(1 + \frac{P}{D}\right)\frac{D}{P} = 1 + \frac{D}{P}$$

and define error term:

$$e(x) = 0.99 b_0 E[v(x\tilde{g})] + 0.99x - 1$$

where utility from recent financial gain or loss is given by:

$$v(R) = R - 1.0303 \quad \text{for} \quad R \geq 1.0303$$
$$v(R) = 2\,(R - 1.0303) \quad \text{for} \quad R < 1.0303$$

Solve for e(x) = 0 to find equilibrium value of x, using bisection search:

1. Set $x_- = 1$ and $x_+ = 1.1$, and use simulated distribution of consumption growth to confirm that $e(x_-) < 0$ and $e(x_+) > 0 \Rightarrow$ solution must lie between x_- and x_+
2. Set $x_0 = 0.5*(x_- + x_+)$ and use simulated distribution of consumption growth to calculate $e(x_0)$
3. If $|e(x_0)| < 10^{-5}$, then you have converged to solution

4. Otherwise if $e(x_0) < 0$, then solution lies between x_0 and x_+ ⇒ repeat from step 2 with $x_- = x_0$
5. Otherwise if $e(x_0) > 0$, then solution lies between x_- and x_0 ⇒ repeat from step 2 with $x_+ = x_0$

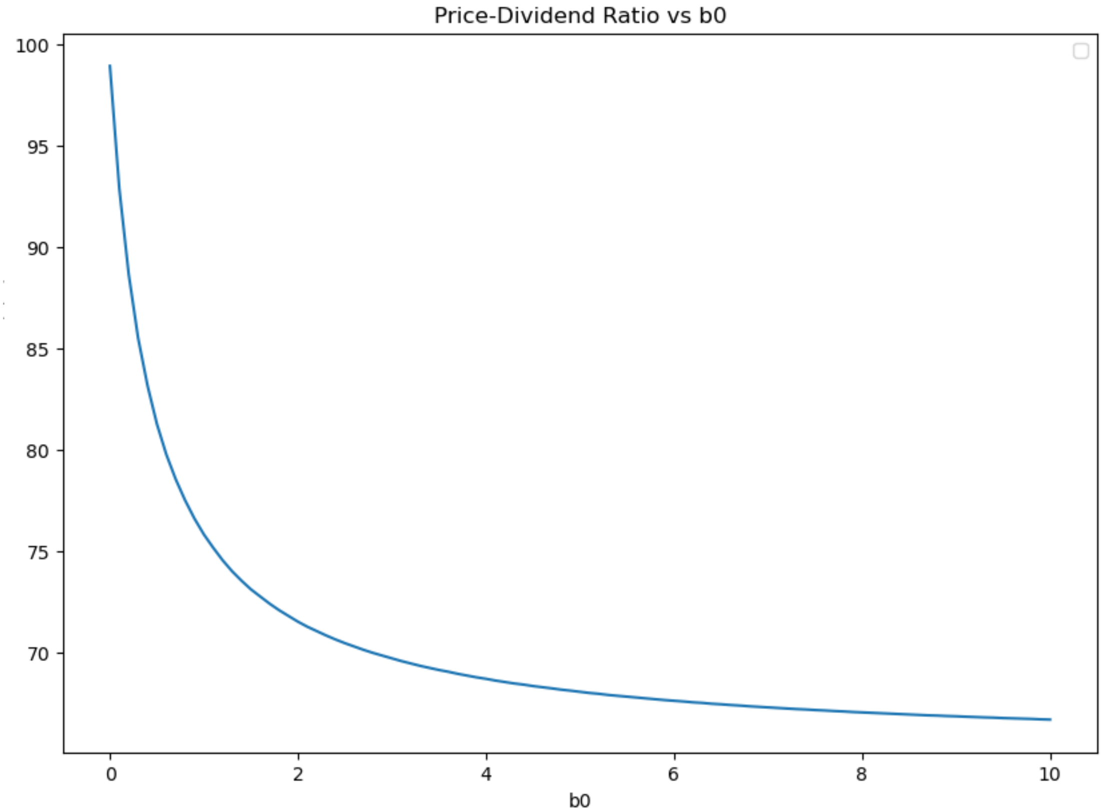
Repeat for b_0 in range from 0 to 10, in increments of 0.1 (or less).

1. Calculate price-dividend ratio for market portfolio:

$$\frac{P}{D} = \frac{1}{x-1}$$

Plot price-dividend ratio (on vertical axis) vs b_0.

Answer:

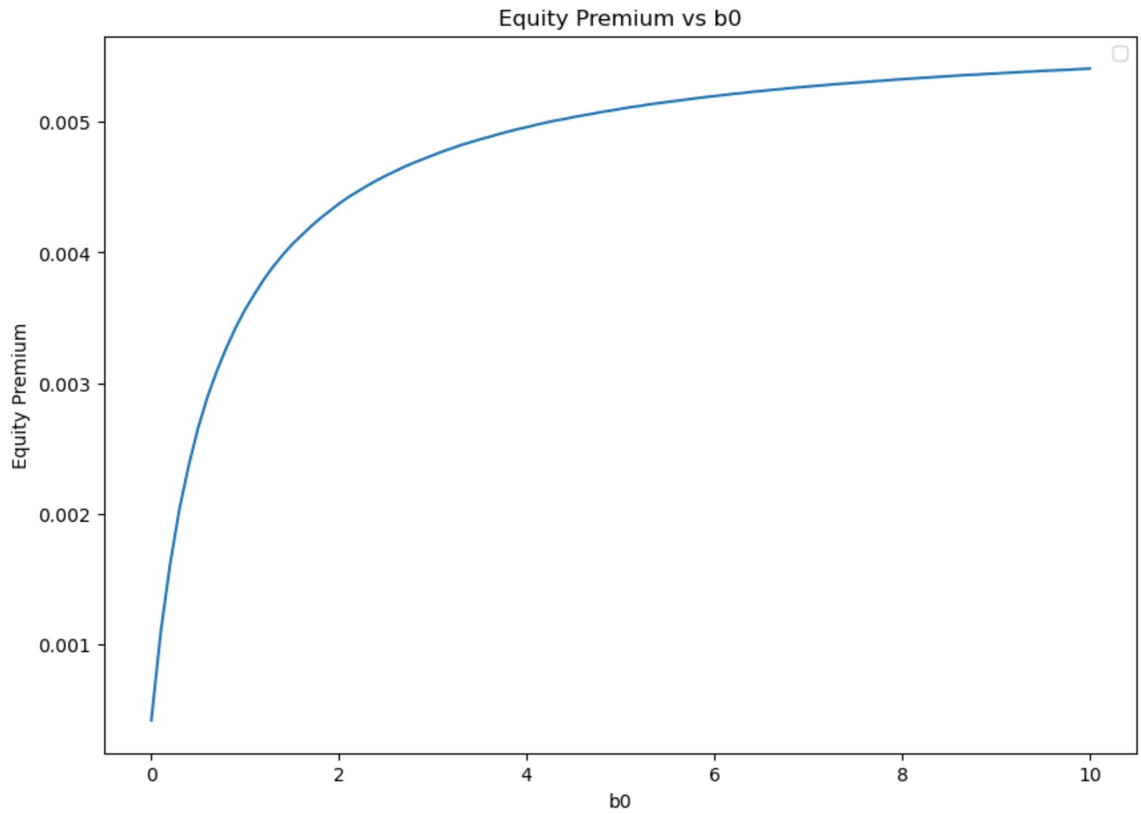


2. Calculate expected market return:

$$E\left(\tilde{R}_m\right) = E(x\tilde{g}) = xe^{0.0202}$$

Plot equity premium (on vertical axis) vs b_0.

Answer:



3. Briefly describe (in words, without using mathematical equations or formulas) main characteristics of v(·) as well as economic significance and implications of b_0 and λ.

Answer:

- *Main characteristics of utility function (v(.)):*

Utility function is piecewise-linear, with different slopes before and after X_{t+1}. The steepness / gradient of utility function is always greater than 1 when $X_{t+1} < 0$ and exactly 45 degree from the origin when $X_{t+1} >= 0$. This indicates loss aversion whereby investors are more sensitive to shortfall in financial gain (or outright financial loss).

- *Economic significance of b_0:*

$b_0 >= 0$ determines the extent to which utility from recent financial gain or loss contributes to investor's lifetime utility. b_0 can be seen as a slider to allocate weight between the contribution of consumption (1st term) and recent financial gain or loss (2nd term).

  - If $b_0 = 0$, it means no utility is derived from recent financial gain or loss
  - If b_0 = infinity, it means no utility is derived consumption

In summary, a higher b_0 means greater sensitivity to financial gain or loss. Whereas, a lower b_0 represents lower sensitivity to financial gain or loss.

- *Economic significance of λ:*

  λ represents sensitivity of an investor towards losses as opposed to gains. Empirically, in real life, the λ is approximately 2, which means that investors are twice as sensitive for financial losses compared to financial gains. The higher the λ, the more sensitive investor to losses compared to gains. Conversely, a lower λ indicates that the investor is more neutral on losses and gains, for example when λ is nearer to 1, investor is becomes more equally as sensitive to losses as they are to financial gains. In summary, the more loss averse the investor, the higher the λ.