MAHARASHTRA STATE BOARD OF TECHNICAL EDUCATION

(Autonomous) (ISO/IEC - 27001 - 2005 Certified)

WINTER – 2017 EXAMINATION

Model Answer

Subject Code:

22103

Important Instructions to Examiners:

- 1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
- 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
- 3) The language errors such as grammatical, spelling errors should not be given more importance (Not applicable for subject English and Communication Skills).
- 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answer and model answer.
- 6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
- 7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		Attempt any five of the following:	10
	a)	Evaluate $\log_3 81$	02
	Ans	$\log_3 81$	1/2
		$=\log_3 3^4$	1/2
		$=4\log_3 3$	1/2
		=4(1)	
		= 4	1/2
		OR	
		$\log_3 81$	
		$=\frac{\log 81}{\log 3}$	1/2
		$\log 3$	1/2
		$=\frac{\log 3^4}{\log 3}$	
		$-4\log 3$	1/2
		$-\log 3$	1/2
		=4	
	b)	Show that the points $(8,1)$ $(3,-4)$ and $(2,-5)$ are collinear using determinant.	02
	Ans	Consider $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$	



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		Subject Code. ZZ	
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1.	b)	$\begin{vmatrix} 8 & 1 & 1 \\ = 3 & -4 & 1 \end{vmatrix}$	1/2
		$ \begin{vmatrix} -1 & 1 & 1 \\ 3 & -4 & 1 \\ 2 & -5 & 1 \end{vmatrix} $	1/4
		=8(-4+5)-1(3-2)+1(-15+8)	1/2
		= 0 ∴ Points are collinear	1/2
			1/2
	c)	Without using calculator find the value of $\sin(105^{\circ})$	02
	Ans	$\sin(105^{\circ})$	
		$=\sin(60^{\circ}+45^{\circ})$	1/2
		$= \sin 60^{\circ} \cos 45^{\circ} + \cos 60^{\circ} \sin 45^{\circ}$	1/2
		$= \frac{\sqrt{3}}{2} \frac{1}{\sqrt{2}} + \frac{1}{2} \frac{1}{\sqrt{2}}$	1/2
		$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	72
		$= \frac{\sqrt{3} + 1}{2\sqrt{2}} \text{OR} 0.9659$	1/2
	d)	Find area of rhombus whose diagonals are of length 10 cm and 8.2 cm	02
	Ans	Area of rhombus = $\frac{1}{2} \times d_1 \times d_2$	
		$=\frac{1}{2}\times10\times8.2$	1
		=41 sq. cm	1
	e)	If the volume of a sphere is $\frac{4\pi}{3}$ cm ³ . Find its surface area	02
	Ans	Volume of sphere = $\frac{4}{3}\pi r^3$	
		$\therefore \frac{4\pi}{3} = \frac{4}{3}\pi r^3$	1/2
		$1 = r^3$	1/
		$\therefore r = 1$	1/2
		Surface area of sphere = $4\pi r^2$ = $4\pi (1)^2$	1/2
		$= 4\pi \text{ OR } 12.56 \text{ cm}^2$	1/2



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1.	f)	Find the range and coefficient of range of the data:	02
1.	1)		02
	A	50, 90, 120, 40, 180, 200, 80.	
	Ans	Range = $L - S = 200 - 40$	1
		=160	1
		Coefficient of range = $\frac{L-S}{L+S}$	
		L+S	
		$=\frac{200-40}{200+40}$	1/2
		-200 + 40	
		$=\frac{2}{3}$ OR 0.667	1/2
		3	, -
	(a)		
	g)	If the coefficient of variation of certain data is 5 and mean is 60. Find the standard	02
		deviation.	
	Ans	Coefficient of variation = $\frac{S.D.}{M} \times 100$	
		Mean	
		$\therefore 5 = \frac{S.D.}{60} \times 100$	1
			_
		$\therefore \frac{5 \times 60}{100} = S.D.$	
		$\therefore S.D. = 3$	1
2.		Attempt any three of the following:	12
	a)	If $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}$ whether AB is singular or non-singular matrix?	04
	Ans	$A_{B} = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix}$	
	Alls	$AB = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}$	
		$\therefore AB = \begin{bmatrix} 2+3 & 4-2 \\ 0+9 & 0-6 \end{bmatrix}$	1
		$\therefore AB = \begin{bmatrix} 5 & 2 \\ 9 & -6 \end{bmatrix}$	
		$AB = \begin{bmatrix} 9 & -6 \end{bmatrix}$	1
		$ \therefore AB = \begin{vmatrix} 5 & 2 \\ 9 & -6 \end{vmatrix} = -30 - 18 = -48$	
			1
		$ \therefore AB \neq 0$	1/2
		$\therefore AB$ is non-singular matrix	1/2
		x+3	04
	b)	Resolve into partial fractions: $\frac{x+3}{(x-1)(x+1)(x+5)}$	V -1
		()()	
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2.	b)	x+3	1/2
	Ans	$\frac{x+3}{(x-1)(x+1)(x+5)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+5}$	
	7 1113	$\therefore x+3=A(x+1)(x+5)+B(x-1)(x+5)+C(x-1)(x+1)$	
		Put $x = 1$	
		4 = A(2)(6)	
		4=12A	
		$\therefore A = \frac{1}{3}$	1
		Put $x = -1$	
		-1+3=B(-2)(4)	
		2 = -8B	
		$\therefore B = -\frac{1}{4}$	1
		Put $x = -5$	
		-5+3=C(-6)(-4)	
		-2 = 24C	
		$\therefore C = \frac{-1}{12}$	1
		$\frac{x+3}{(x-1)(x+1)(x+5)} = \frac{\frac{1}{3}}{x-1} + \frac{-\frac{1}{4}}{x+1} + \frac{-\frac{1}{12}}{x+5}$	1/2
	c)	Using Cramers rule solve $x - y - 2z = 1$; $2x + 3y + 4z = 4$; $3x - 2y - 6z = 5$	04
	Ans		
	7 1113	$D = \begin{vmatrix} 1 & -1 & -2 \\ 2 & 3 & 4 \\ 3 & -2 & -6 \end{vmatrix}$	
		$\begin{bmatrix} 2 & 3 & -2 & -6 \end{bmatrix}$	
		=1(-18+8)+1(-12-12)-2(-4-9)	
		=-8	1
		$\begin{vmatrix} 1 & -1 & -2 \end{vmatrix}$	
		$D_{x} = \begin{vmatrix} 1 & -1 & -2 \\ 4 & 3 & 4 \\ 5 & -2 & -6 \end{vmatrix}$	
		$\begin{vmatrix} 5 & -2 & -6 \end{vmatrix}$	
		=1(-18+8)+1(-24-20)-2(-8-15)	
		=-8	
		$\therefore x = \frac{D_x}{D} = \frac{-8}{-8} = 1$	1
		D = -8	
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	G 1		7.1:
Q. No.	Sub Q. N.	Answer	Marking Scheme
2.		$D_{y} = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 4 & 4 \\ 3 & 5 & -6 \end{vmatrix}$ $= 1(-24 - 20) - 1(-12 - 12) - 2(10 - 12)$ $= -16$ $\therefore y = \frac{D_{y}}{D} = \frac{-16}{-8} = 2$ $D_{z} = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 3 & 4 \\ 3 & -2 & 5 \end{vmatrix}$ $= 1(15 + 8) + 1(10 - 12) + 1(-4 - 9)$ $D_{z} = 8$	1
		$z = \frac{D_z}{D} = \frac{8}{-8} = -1$	1
	d)	Compute the standard deviation for 15, 22, 27, 11, 9, 21, 14, 9.	04
	Ans		2
		Daga No	



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Q.	Sub		Marking
No.	Q. N.	Answer	Scheme
2.	d)	120	1
2.	(a)	Mean $\bar{x} = \frac{128}{8} = 16$	1
		8	
		Standard deviation $\sigma = \sqrt{\frac{\sum d_i}{d_i}}$	
		Standard deviation $\sigma = \sqrt{\frac{\sum d_i^2}{n}}$ $= \sqrt{\frac{310}{8}}$ $= 6.22$	
		= 6.22	1
		<u>OR</u>	
		15 225	
		27 729	
		11 121	
		9 81	2
		21 441	2
		14 196	
		9 81	
		128 2358	
		$\operatorname{Mean} \frac{1}{x} = \frac{\sum x_i}{n}$	
			1
		$\frac{1}{x} = \frac{128}{8} = 16$	1
		Standard deviation $\sigma = \sqrt{\frac{\sum x_i^2}{N} - (\bar{x})^2}$	
		$=\sqrt{\frac{2358}{8}-(16)^2}$	
		= 6.22	1
		Page No.	0.6/0.1



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		<u>Woder Answer</u> Subject Code. 22	105
Q. No.	Sub Q. N.	Answer	Marking Scheme
3.		Attempt any three of the following:	12
	a)	If $\tan(x+y) = \frac{3}{4}$ and $\tan(x-y) = \frac{8}{15}$. Prove that $\tan 2x = \frac{77}{36}$	04
	Ans	Consider	
	7 1113	2x = x + y + x - y	
		$\tan 2x = \tan (x + y + x - y)$	1
		$= \frac{\tan(x+y) + \tan(x-y)}{1 - \tan(x+y)\tan(x-y)}$	1
		$\frac{3+8}{2+8}$	
		$=\frac{4\cdot 15}{3\cdot 8}$	1
		$=\frac{\frac{3}{4} + \frac{8}{15}}{1 - \frac{3}{4} \frac{8}{15}}$	
		$=\frac{77}{36}$	1
		$\therefore \tan 2x = \frac{77}{36}$	
		OR	
		Let $x + y = A$	
		x - y = B	
		$\therefore \tan A = \frac{3}{4} , \tan B = \frac{8}{15}$	
		$\therefore 2x = A + B = x + y + x - y$	1
		$\tan 2x = \tan (A+B)$	
		$=\frac{\tan A + \tan B}{\tan A}$	1
		$1 - \tan A \tan B$	
		$\frac{3}{4} + \frac{8}{15}$	1
		$=\frac{4 + 15}{1 - \frac{3}{4} \frac{8}{15}}$	1
		$=\frac{77}{36}$	1
		$\therefore \tan 2x = \frac{77}{36}$	
		$\frac{1}{2}$ $\frac{1}{36}$	
	b)	If $A = 30^{\circ}$, verify that	04
		$i) \sin 2A = 2 \sin A \cos A$	
		$ii)\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$	
		$1 + \tan^2 A$	
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Q.	Sub		Marking
No.	Q. N.	Answer	Scheme
3.	b)	$i)L.H.S. = \sin 2A$	
	Ans	$= \sin 2(30^{\circ})$	
	7 1113	$=\sin 60^{0}$	
		$=\frac{\sqrt{3}}{2}$	
			1
		$R.H.S. = 2\sin A\cos A$	
		$=2\sin 30^{\circ}\cos 30^{\circ}$	
		$=2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$	
		$=\frac{\sqrt{3}}{2}$	1
		$\therefore \sin 2A = 2\sin A \cos A$	
		$ii)L.H.S. = \cos 2A = \cos 2(30^{\circ})$	
		$=\cos 60^{\circ}$	
		$=\frac{1}{2}$	1
		$R.H.S. = \frac{1 - \tan^2 A}{1 + \tan^2 A}$	
		$=\frac{1-\tan^2 30^0}{1+\tan^2 30^0}$	
		$1-\left(\frac{1}{\sqrt{3}}\right)$	
		$=\frac{\sqrt{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$	
		1	1
		$=\frac{1}{2}$	1
		$\therefore \cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$	
		$1 + \tan A$	
		1	
	c)	Prove that $\cos 20 \cos 40 \cos 60 \cos 80 = \frac{1}{16}$	04
	Ans	$L.H.S. = \cos 20 \cos 40 \cos 60 \cos 80$	
		$=\cos 20\cos 40\frac{1}{2}\cos 80$	1/2
		1 (2 22 22 22 22 22 22 22 22 22 22 22 22	
		$=\frac{1}{4}(2\cos 20\cos 40)\cos 80$	1/2
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	Model Miswel Subject Code. ZZ	103
Sub Q. N.	Answer	Marking Scheme
c)	L.H.S. = $\frac{1}{4} (\cos 60 + \cos 20) \cos 80$ = $\frac{1}{4} (\frac{1}{2} + \cos 20) \cos 80$ = $\frac{1}{4} (\frac{1}{2} \cos 80 + \cos 20 \cos 80)$ = $\frac{1}{4} (\cos 80 + 2 \cos 20 \cos 80)$	1
	$= \frac{1}{8} (\cos 80 + \cos 100 + \cos 60)$ $= \frac{1}{8} (\cos 80 + \cos 100 + \frac{1}{2})$	1/2
	_/	1/2
	$=\frac{1}{16}$	1/2
	= R.H.S.	
d)	Prove that $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$	04
Ans	$\therefore \cos A = \frac{4}{5}$ $\therefore \sin A = \sqrt{1 - \cos^2 A}$ $= \sqrt{1 - \frac{16}{25}}$ $= \frac{3}{5}$ Put $\cos^{-1}\left(\frac{12}{13}\right) = B$ $\therefore \cos B = \frac{12}{13}$ $\therefore \sin B = \sqrt{1 - \cos^2 B}$ $= \sqrt{1 - \frac{144}{169}}$	1
	Q. N. c)	Sub Q. N. C) L.H.S. = $\frac{1}{4}(\cos 60 + \cos 20)\cos 80$ = $\frac{1}{4}(\frac{1}{2} + \cos 20)\cos 80$ = $\frac{1}{4}(\frac{1}{2} \cos 80 + \cos 20\cos 80)$ = $\frac{1}{8}(\cos 80 + \cos 20\cos 80)$ = $\frac{1}{8}(\cos 80 + \cos 100 + \cos 60)$ = $\frac{1}{8}(\cos 80 + \cos 100 + \frac{1}{2})$ = $\frac{1}{8}(\cos 80 + \cos 100 + \frac{1}{2})$ = $\frac{1}{8}(\cos 80 - \cos 80 + \frac{1}{2})$ = $\frac{1}{8}(\cos 80 - \cos 80 + \frac{1}{2})$ = $\frac{1}{16}$ = $RH.S.$ Prove that $\cos^{-1}(\frac{4}{5}) + \cos^{-1}(\frac{12}{13}) = \cos^{-1}(\frac{33}{65})$ Put $\cos^{-1}(\frac{4}{5}) = A$ $\therefore \cos A = \frac{4}{5}$ $\therefore \sin A = \sqrt{1 - \cos^2 A}$ = $\sqrt{1 - \frac{16}{25}}$ = $\frac{3}{5}$ Put $\cos^{-1}(\frac{12}{13}) = B$ $\therefore \cos B = \frac{12}{13}$ $\therefore \sin B = \sqrt{1 - \cos^2 B}$



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3.	d)	$\therefore \sin B = \frac{5}{13}$	1
		13 Consider,	
		$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$	
		$\cos(A+B) = \left(\frac{4}{5}\right)\left(\frac{12}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{5}{13}\right)$	1
		$\therefore \cos(A+B) = \frac{33}{65}$	1/2
		$\therefore A + B = \cos^{-1}\left(\frac{33}{65}\right)$	1/2
		$\therefore \cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$	
		<u>OR</u>	
		Let $\cos^{-1}\left(\frac{4}{5}\right) = A$ 5	
		$\therefore \cos A = \frac{4}{5}$ A B 12	
		$\therefore \tan A = \frac{3}{4}$	
		$A = \tan^{-1}\left(\frac{3}{4}\right)$	
		$\therefore \cos^{-1}\left(\frac{4}{5}\right) = \tan^{-1}\left(\frac{3}{4}\right)$	1
		Let $\cos^{-1}\left(\frac{12}{13}\right) = B$	
		$\therefore \cos B = \frac{12}{13}$	
		$\therefore \tan B = \frac{5}{12}$	
		$\therefore B = \tan^{-1}\left(\frac{5}{12}\right)$	
		$\cos^{-1}\left(\frac{12}{13}\right) = \tan^{-1}\left(\frac{5}{12}\right)$	1
		$L.H.S. = \cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right)$	
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		Niodei Aliswer Subject Code. 22	103
Q. No.	Sub Q. N.	Answer	Marking Scheme
3.	d)	$L.H.S. = \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{5}{12}\right)$	
		$= \tan^{-1} \left(\frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \frac{5}{12}} \right)$	1/2
		$= \tan^{-1} \left(\frac{56}{33} \right)$ Let $\tan^{-1} \left(\frac{56}{33} \right) = C$	1/2
		$\therefore \tan C = \frac{56}{33}$ $\therefore \cos C = \frac{33}{65}$	
		$\therefore C = \cos^{-1}\left(\frac{33}{65}\right)$ 56	
		$\therefore \tan^{-1}\left(\frac{56}{33}\right) = \cos^{-1}\left(\frac{33}{65}\right) = R.H.S.$	1
4.		Attempt any three of the following:	12
	a)	If $A = \begin{bmatrix} 2 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 1 \\ 0 & 4 \\ 5 & 7 \end{bmatrix}$. Verify that $(AB)^T = B^T A^T$	04
	Ans	$AB = \begin{bmatrix} 2 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 6 & 1 \\ 0 & 4 \\ 5 & 7 \end{bmatrix}$	
		$AB = \begin{bmatrix} 12+0+30 & 2+20+42 \\ 0+0+10 & 0+4+14 \end{bmatrix}$	1
		$AB = \begin{bmatrix} 42 & 64 \\ 10 & 18 \end{bmatrix}$	1/2
		$ \left(AB\right)^T = \begin{bmatrix} 42 & 10 \\ 64 & 18 \end{bmatrix} $	1/2
		$B^{T}A^{T} = \begin{bmatrix} 6 & 0 & 5 \\ 1 & 4 & 7 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 5 & 1 \\ 6 & 2 \end{bmatrix}$	1/2
			1 1 10 1



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Q. No.	Sub Q. N.	Answer	Marking Scheme
4.	a)	$B^{T}A^{T} = \begin{bmatrix} 12+0+30 & 0+0+10 \\ 2+20+42 & 0+4+14 \end{bmatrix}$ $B^{T}A^{T} = \begin{bmatrix} 42 & 10 \\ 64 & 18 \end{bmatrix}$ $\therefore (AB)^{T} = B^{T}A^{T}$	1 1/2
	b)	Resolve into partial fraction $\frac{x^2 - x + 3}{(x-2)(x^2+1)}$	04
	Ans	$\frac{x^2 - x + 3}{(x - 2)(x^2 + 1)} = \frac{A}{x - 2} + \frac{Bx + C}{x^2 + 1}$	1/2
		$\therefore x^{2} - x + 3 = (x^{2} + 1)A + (x - 2)(Bx + C)$ Put $x = 2$ $5 = 5A$	
		A = 1 Put $x = 0$	1
		$3 = A - 2C$ $\therefore C = -1$	1
		Put $x = 1$ 3 = 2A + (-1)(B + C)	
		$3 = 2 - B + 1$ $\therefore B = 0$	1
		$\frac{x^2 - x + 3}{(x - 2)(x^2 + 1)} = \frac{1}{x - 2} + \frac{(0)x - 1}{x^2 + 1}$ $\frac{x^2 - x + 3}{(x - 2)(x^2 + 1)} = \frac{1}{x - 2} - \frac{1}{x^2 + 1}$	1/2
		$(x-2)(x^2+1)^{-1}x-2^{-1}x^2+1$	
	c)	Prove that : $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$	04
	Ans	$\sin(A+B)\sin(A-B)$ $=(\sin A\cos B + \cos A\sin B)(\sin A\cos B - \cos A\sin B)$	1
		$=\sin^2 A\cos^2 B - \cos^2 A\sin^2 B$	1
		$= \sin^2 A \left(1 - \sin^2 B\right) - \left(1 - \sin^2 A\right) \sin^2 B$	1
		$= \sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B$	1
		$=\sin^2 A - \sin^2 B$	1
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No.	Q. N.	Answer	Scheme
4.	d)	If $\sin A = \frac{1}{2}$ find the value of $\sin 3A$.	04
	Ans	$\sin 3A = 3\sin A - 4\sin^3 A$	1
		$=3\left(\frac{1}{2}\right)-4\left(\frac{1}{2}\right)^3$	1
		= 1	2
	e)	Prove that $\frac{\sin 4A + \sin 5A + \sin 6A}{\cos 4A + \cos 5A + \cos 6A} = \tan 5A$	04
	Ans	$L.H.S. = \frac{\sin 4A + \sin 5A + \sin 6A}{\cos 4A + \cos 5A + \cos 6A}$	
		$= \frac{\sin 4A + \sin 6A + \sin 5A}{\cos 4A + \cos 6A + \cos 5A}$	
		$= \frac{2\sin\left(\frac{4A+6A}{2}\right)\cos\left(\frac{4A-6A}{2}\right)+\sin 5A}{2\cos\left(\frac{4A+6A}{2}\right)\cos\left(\frac{4A-6A}{2}\right)+\cos 5A}$	1
		$= \frac{2\sin 5A\cos(-A) + \sin 5A}{2\cos 5A\cos(-A) + \cos 5A}$	1
		$= \frac{\sin 5A \left(2\cos\left(-A\right)+1\right)}{\cos 5A \left(2\cos\left(-A\right)+1\right)}$	1
		$= \tan 5A$ $= R.H.S.$	1
5.		Attempt any two of the following:	12
5.	a) (i)	Find the equation of straight line passes through the points $(3,5)$ and $(4,6)$.	03
	Ans	Equation of line is $\frac{y - y_1}{y} = \frac{x - x_1}{y - y_1}$	
		$y_1 - y_2 \qquad x_1 - x_2$ $\frac{y - 5}{5 - 6} = \frac{x - 3}{3 - 4}$ $\frac{y - 5}{-1} = \frac{x - 3}{-1}$	2
		$\begin{vmatrix} -1 & -1 \\ x - y + 2 = 0 \end{vmatrix}$	1
	(ii) Ans	Find the distance between the parallel lines $3x - y + 7 = 0$ and $3x - y + 16 = 0$ For $3x - y + 7 = 0$	03
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Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	a) (ii)	$a = 3, b = -1, c_1 = 7$	
	u) (11)	For $3x - y + 16 = 0$	
		$a = 3, b = -1, c_2 = 16$	
		∴ distance between two parallel lines is	
		$= \left \frac{c_2 - c_1}{\sqrt{a^2 + b^2}} \right = \left \frac{16 - 7}{\sqrt{3^2 + (-1)^2}} \right $	2
		$=\left \frac{9}{\sqrt{10}}\right $	
		$=\frac{9}{\sqrt{10}}$ OR 2.846	1
	b) (i)	Find the acute angle between the lines $2x+3y+5=0$ and $x-2y-4=0$	03
	Ans	For $2x + 3y + 5 = 0$	0.5
		slope $m_1 = -\frac{a}{b} = -\frac{2}{3}$	1/2
		For $x - 2y - 4 = 0$,	
		slope $m_2 = -\frac{a}{b} = -\frac{1}{-2} = \frac{1}{2}$	1/2
		$\therefore \tan \theta = \left \frac{m_1 - m_2}{1 + m_1 m_2} \right $	
		$= \frac{-\frac{2}{3} - \frac{1}{2}}{1 + \left(-\frac{2}{3}\right) \cdot \left(\frac{1}{2}\right)}$	1
		$=\frac{7}{4}$	
		$\therefore \theta = \tan^{-1}\left(\frac{7}{4}\right) OR 60.26^{\circ}$	1
	(ii)	Find the equation of the line through the point of intersection of lines, $4x+3y=8$; and $x+y=1$ and parallel to the line $5x-7y=3$	03
	Ans	$\therefore 4x + 3y = 8$	
		$\underline{x+y=1}$	
		$\therefore 4x + 3y = 8$	
		-4x+4y=4	
		Paga No	4 4 4 6 4



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Q.	Sub		Marking
No.	Q. N.	Answer	Scheme
5.	b) (ii)	-y=4	
		y = -4	
		$\therefore x-4=1$	
		$\therefore x = 5$	
		$\therefore Point of intersection = (5, -4)$	1
		Slope of the line $5x - 7y = 3$ is,	
		$m = -\frac{a}{b} = -\frac{5}{-7} = \frac{5}{7}$	1/2
		∴ Slope of the required line is,	
		$m=\frac{5}{7}$	1/2
		∴ equation required line is,	
		$y - y_1 = m(x - x_1)$	
		$\therefore y + 4 = \frac{5}{7}(x - 5)$ $\therefore 5x - 7y - 53 = 0$	1/2
		7 (3 2)	1/2
		$\therefore 5x - 7y - 53 = 0$	72
	c) (i)	The area of a rectangular courtyard is 3000 sq.m. Its sides are in the ratio 6:5.Find	03
		the perimeter of courtyard.	
	Ans	Area of rectangular courtyard is = length \times breadth	
		Given $l:b=6:5$	
		i.e. $\frac{l}{b} = \frac{6}{5}$	
		$\therefore l = \frac{6}{5}b$	
		3	
		$\therefore A = l \times b$	
		$3000 = \frac{6}{5}b \times b$	
		$\frac{15000}{6} = b^2$	
		$2500 = b^2$	1
		$\therefore b = 50$	
		$\therefore l = \frac{6}{5}b = \frac{6}{5} \times 50$	1
		$\therefore l = 60$	
		Perimeter of rectangular courtyard is = $2(l+b)$	
		=2(60+50)	
		$= 220 \ m.$	1
		Dago No.	



WINTER – 2017 EXAMINATION **Model Answer**

		Wodel Answer Subject Code. 22	
Q. No.	Sub Q. N.	Answer	Marking Scheme
5.	c) (i)	OR	
		Sides are in the ratio 6:5	
		Let <i>x</i> be the common multiple	
		\therefore Sides are $6x$ and $5x$	
		$\therefore A = 3000$	
		$\therefore 6x \times 5x = 3000$	
		$\therefore 30x^2 = 3000$	
		$\therefore x^2 = 100$	1
		$\therefore x = 10$	
		$\therefore \text{ Sides are } 6x = 60 = l \text{ and } 5x = 50 = b$	1
		Perimeter of rectangular courtyard is = $2(l+b)$	
		=2(60+50)	
		$= 220 \ m.$	1
	c) (ii)	A circus tent is cylindrical to height 3 m and conical above it . If its diameter is	02
		105 m and slant height of cone is 5 m,calculate the area of total canvas required.	03
	Ans	Given $h = 3m$, $d = 105m$: $r = \frac{105}{2} = 52.5m$, $l = 5m$	
		2	
		curved surface area of cylinder = $2\pi rh$	1
		$= 2 \times 3.14 \times 52.5 \times 3 = 989.1 \ sq.m.$	1
		curved surface area of cone = πrl	
		$= 3.14 \times 52.5 \times 5 = 824.25 \text{ sq.m.}$	1
		∴ Area of total canvas required = 989.1+824.25	
		=1813.35 sq.m.	1
6.		Attempt any two:	12
	a)	Using matrix inversion method, solve	06
		x+y+z=3; $x+2y+3z=4$; $x+4y+9z=6$	
	Ans	$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}$	
	ZIIIS	Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$	
		$\begin{vmatrix} A = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{vmatrix}$	
		$\begin{bmatrix} A - \begin{vmatrix} 1 & 2 & 3 \\ 1 & A & 9 \end{vmatrix}$	
		A = 1(18-12)-1(9-3)+1(4-2)	
		$\begin{bmatrix} 1/1 & 1/2 & 1/$	
		Dogo No.	16/21



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Q.	Sub		Marking
No.	Q. N.	Answer	Scheme
6.	a)	$\therefore A = 2 \neq 0$	1
		$\therefore A^{-1}$ exists	
		Matrix of minors = $\begin{bmatrix} \begin{vmatrix} 2 & 3 & 1 & 3 & 1 & 2 \\ 4 & 9 & 1 & 9 & 1 & 4 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 4 & 9 & 1 & 9 & 1 & 4 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 3 & 1 & 3 & 1 & 2 \end{bmatrix}$	
		$= \begin{bmatrix} 6 & 6 & 2 \\ 5 & 8 & 3 \\ 1 & 2 & 1 \end{bmatrix}$	1
		Matrix of cofactors = $\begin{bmatrix} 6 & -6 & 2 \\ -5 & 8 & -3 \\ 1 & -2 & 1 \end{bmatrix}$	1/2
		OR $(-1)^{1+1} \begin{vmatrix} 2 & 3 \end{vmatrix}$ $(-1)^{1+2} \begin{vmatrix} 1 & 3 \end{vmatrix}$ $(-1)^{1+3} \begin{vmatrix} 1 & 2 \end{vmatrix}$	
		$\begin{vmatrix} c_{11} = (-1)^{1+1} \begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix} = 6, \ c_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 3 \\ 1 & 9 \end{vmatrix} = -6, \ c_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} = 2,$ $\begin{vmatrix} c_{21} = (-1)^{2+1} \begin{vmatrix} 1 & 1 \\ 4 & 9 \end{vmatrix} = -5, \ c_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 1 \\ 1 & 9 \end{vmatrix} = 8, \ c_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} = -3,$ $\begin{vmatrix} c_{31} = (-1)^{3+1} \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 1, \ c_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = -2, \ c_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1,$	1
		$ \text{Matrix of cofactors} = \begin{bmatrix} 6 & -6 & 2 \\ -5 & 8 & -3 \\ 1 & -2 & 1 \end{bmatrix} $	1/2
		$\therefore AdjA = \begin{bmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{bmatrix}$	1/2
		$A^{-1} = \frac{1}{ A } \text{Adj.} A = \frac{1}{2} \begin{bmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{bmatrix}$	1
		$X = A^{-1}B$ $\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 6 & -5 & 1 \\ -6 & 8 & -2 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}$	
	<u> </u>	Dogo No	17/21



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Q. No.	Sub Q. N.	Answer											Marking Scheme
6.	a)	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 18 - 20 + 6 \\ -18 + 32 - 12 \\ 6 - 12 + 6 \end{bmatrix}$ $\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$										1	
	b)	$\therefore x = 2$ Find mea	0, $y =$ an, star	ndard de	eviatio						nce of the following:		1 06
		Clas	s:	0-10	10-2	0 20-	30	30)-40	40-	50		
		Freque	ncy:	3	5	8	}		3	1			
	Ans	C.I.	X_i	f_i		$f_i x_i$	X_i	2	f_i	r ²			
			i	J 1		Jiri			J i	i			
		0-10	5	3		15	2	5	7	5			
		10-20	15	5		75	22	25	112	25			
		20-30	25	8		200	62	25	500	00			
		30-40	35	3		105	12	25	36	75			2
		40-50	45	1		45	20.	25	202	25			
				N=20		$\sum_{i} f_i x_i = 440$			$\sum f_i x_i$ 1190				
		Mean, $\bar{x} = \frac{\sum f_i x_i}{N} = \frac{440}{20} = 22$										1	
		$S.D. = \sqrt{\frac{1}{2}}$	$\sum f x^2$										
		S.D. =	11900 20	$-(22)^2$									1
		S.D. = 10											1



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22103

Q.	Sub	Answer											
No.	Q. N.				7 1115 W C1			Scheme					
6.	b)	Coefficie OR	Coefficient of variance = $\frac{S.D.}{Mean} \times 100$ = $\frac{10.54}{22} \times 100$ = 47.91%										
		Class 0-10 10-20 20-30 30-40 40-50 Mean, \bar{x}	$\begin{array}{c c} x_i \\ 5 \\ 15 \\ 25 \\ 35 \\ 45 \\ \end{array}$	8 0 3 1 1 2 N=20	$ \begin{array}{c cccc} f_i d_i & d_i^2 \\ -6 & 4 \\ -5 & 1 \\ 0 & 0 \\ 3 & 1 \\ 2 & 4 \\ -6 & \\ \end{array} $ $= 25 + \left(\frac{-6}{20}\right) \times 10 = 0$	$ \begin{array}{c c} f_i d_i^2 \\ \hline 12 \\ \hline 5 \\ 0 \\ \hline 3 \\ 4 \\ \hline 24 \end{array} $		2					
		$S.D. = \sigma$	Mean, $\overline{x} = A + \left(\frac{\sum f_i d_i}{N}\right) \times h = 25 + \left(\frac{-6}{20}\right) \times 10 = 22$ $S.D. = \sigma = \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2} \times h$ $= \sqrt{\frac{24}{20} - \left(\frac{-6}{20}\right)^2} \times 10$										
		=	$\sqrt{20}$	$\overline{0} = \left(\frac{1}{20}\right)^{-1}$				1					
		$= 10.54$ Coefficient of variance = $\frac{S.D.}{Mean} \times 100$ $= \frac{10.54}{22} \times 100$ $= 47.91\%$											
		OR		.,									
		C.I.	X_i	f_i	$f_i x_i$	$\left(x_i - \overline{x}\right)^2$	$f_i(x_i-\overline{x})^2$						
		0-10	5	3	15	289	867						
		10-20	15	5	75	49	245						
		20-30	25	8	200	9	72	2					
		30-40	35	3	105	169	507						
		40-50	45	1	45	529	529						
				$\sum f_i = 20$	$\sum f_i x_i = 440$		$\sum f_i \left(x_i - \overline{x} \right)^2 = 2220$						
							Dogg	No.19/21					



WINTER – 2017 EXAMINATION **Model Answer**

	T ~ .	Π								L			
Q. No.	Sub Q. N.					An	swer					Marking Scheme	
6.	b)	Mean,	Mean, $\bar{x} = \frac{\sum f_i x_i}{N} = \frac{440}{20} = 22$ $S.D. = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i (x_i - \bar{x})^2}}$										
			V 2	$\int f_i$	_							1	
		S.D. =	V 20									1	
		S.D. =			C D							-	
		Coeffic	cient of	varianc	$e = \frac{S.D.}{Mean}$								
					$=\frac{10.54}{22}$	×100							
					=47.91							1	
	c) (i)		Calculate the range and coefficient of range for the following data:									03	
		Ci	ass:	21-25	26-30	31-35	36-40	41-45					
		Frequ	iency:	4	16	38	12	10					
								<u> </u>					
	Ans	C.I.	20.5-2	5.5	25.5-30.5	30.5-35	.5 35.	5-40.5	40.5-45.5				
		f_i	4		16	38		12	10				
				= 25								1	
		Coeffic	cient of	range =	$=\frac{L-S}{L+S}$								
					$\frac{45.5 - 20.5}{45.5 + 20.5}$	5						1	
				= -	25 66 OR	0.379						1	
	c) (ii)	The tw	The two sets of observations are given below. Which of them is more consistent?										
		Set I			Set II							03	
		$\bar{x} = 82.$	5	$\frac{-}{x}$	= 48.75								
		$\sigma = 7.3$	3	σ =	= 8.35								
	<u> </u>	<u> </u>									o No 2		



WINTER - 2017 EXAMINATION **Model Answer**

	Sub		Morking
Q. No.	Q. N.	Answer	Marking Scheme
6.	Q. N. c) (iii)	Set-I Coefficient of variation $V_1 = \frac{S.D.}{Mean} \times 100$ $V_1 = \frac{7.3}{82.5} \times 100$ $V_1 = 8.848$ Set-II Coefficient of variation $V_2 = \frac{S.D.}{Mean} \times 100$ $V_2 = \frac{8.35}{48.75} \times 100$ $V_2 = 17.128$ $\therefore V_1 < V_2$ \therefore Set I is more consistent In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.	