

Parsers: A set notes from Dan Cooke's Manual and Alfred Aho, Ravi Sethi, and Jeffrey Ulman's Compilers Principles, Techniques and Tools

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The Role of the Parser “The parser obtains a string of tokens from the lexical analyzer, and verifies that the string can be generated by the grammar for the source language. ” It is expected that the parser should recover from commonly occurring errors so that it can continue processing the remainder of its input.

The parser is typically applied to context free grammars.

“Why use regular expressions to define the lexical syntax of a language?”

1. The lexical rules of a language are frequently quite simple, and to describe them we do not need a notation as powerful as grammars
2. Regular expression generally provide a more concise and easier to understand notation for tokens than grammars.
3. More efficient lexical analyzers can be constructed automatically from regular expressions than from arbitrary grammars.
4. Separating the syntactic structure of a language into lexical and non-lexical parts provides a convenient way of modularizing the front end of a compiler into two manageable-sized components.
5. Regular expressions are most useful for describing the structure of lexical constructs such as identifiers, constants, keywords, and so forth.
6. Grammars are useful in describing nested structures such as balanced parentheses, matching begin and end statements, corresponding if-then-else statements, and the like.

Errors are expected to be reported if they occur lexically, syntactically, semantically, or logically. Three types of parsers are common for compilers. The first compiler type is for universal parsing as done by the Cocke-Younger-Kasami algorithm. Universal implies that it will compile any grammar; however, it is inefficient for commercial compilers. Top-down, or bottom up compilers handle LL and LR grammars which is good enough in most cases. However, not all grammars are LL or LR. An error handler has a simple to state set of goals:

- Should report the presence of errors clearly and accurately.
- It should recover from errors quickly enough to detect subsequent errors.
- Should not slow down the processing of correct programs.

LL and LR grammar(s) may allow the compiler to detect immediately an error produced by a incorrect program.

One common mistake in compiler design is the avalanche of spurious errors. This is an inadequate job of recovery.

Error Recovery strategies include:

1. Panic mode
2. Phrase mode
3. Error Productions
4. Global Corrections

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Elimination of Left Recursion Eliminating Left Recursion is one of the steps for preparing a CFG to be parsed. The general rule is for mapping $A \rightarrow A\alpha|\beta$ to a non-left recursive production

- $A \rightarrow \beta A'$
- $A' \rightarrow \alpha A'|\epsilon$

Algorithm for eliminating left recursion:

Left Factoring Why is left factoring useful as a grammar transformation? When it is not clear which of two alternative productions to use to expand a nonterminal A , we may be able to re-wrtie the A -productions to defer the decision until we have seen enough of the input to make the right choice. Left-factoring is also known as removing common prefixes.

Given a production of the form $A \rightarrow \alpha\beta_1|\alpha\beta_2|\gamma$

Transformed to:

- $A \rightarrow \alpha A'|\gamma$
- $A' \rightarrow \beta_1|\beta_2$

Algorithm for Left Factoring a grammar:

Input: Grammar G

Output: An equivalent left-factored grammar.

Method: For each non-terminal A find the longest prefix α common to two or more of its alternatives. If $\alpha \neq \epsilon$ i.e., there is a nontrivial common prefix, replace all the A productions $A \rightarrow \alpha\beta_1|\alpha\beta_2|\gamma$ where γ represents all alternatives that do not begin with α by

- $A \rightarrow \alpha A'|\gamma$
- $A' \rightarrow \beta_1|\beta_2$

Here A' is a new non-terminal. Repeatedly apply this transformation until no two alternatives for a non-terminal have a common prefix.

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Top Down Parsing Notes Purpose: To introduce the basic ideas behind top down parsing and show how to construct an efficient non-backtracking form of top-down parser called a predictive parser.

Goal: Construct a parse tree for an input string from the root and create the nodes of the parse tree in pre-order.

Buzz words

- Predictive parsing
- Recursive descent

Backtracking

- Seldom necessary for context free grammar.
- inefficient on CSG

A left-recursive grammar can cause a recursive-descent parser, even one with back-tracking, to go into an infinite loop.

Predictive Parser Qualities:

- Eliminates left recursion
- left factors
- Proper alternatives are detected by examining the first symbols derives.

Transition Diagrams for Parsers

Differences between lexical analyzer and predictive parser transition

- One diagram for each non-terminal
- Labels are both tokens and non-terminals
- Transition on tokens next symbols
- Transition on non-terminal equates to a procedure calls

To build a syntax analyzer

1. Apply left recursion removal
2. Left factor the grammar
3. Create initial and final (return) state
4. For each production, $A \rightarrow X_1, X_2, \dots, X_n$ create a path from the initial to the final state with edges labeled X_1, X_2, \dots, X_n .

What happens for a predictive parser which works off the transition diagram?

1. It begins in the start state for the start symbol.
2. After some actions, the parser arrives in state S .
 - Possibility: There is an edge to t labeled a . If the next input symbol is a , then the transition to t occurs.

- Possibility: There is an edge to t labeled by non-terminal A
 - The parser goes to the start state of A without moving the input cursor.
 - If a final state in A is reached, the transition is made
- Possibility: There is an edge to t labeled by ϵ . Transition is made without advancing the input cursor.

reference: page 184 **Big idea** “A predictive parsing program based on a transition diagram attempts to match terminal symbols against the input, and makes a potentially recursive procedure call, whenever it has to follow an edge labeled by a nonterminal.”

- For non-determinism this does not work
- For deterministic automata this can work.

Non-recursive Predictive Parsing A stack may be used explicitly to build a non-recursive predictive parser.

- Problems determining the production to be applied
- Solution: parse table look up- table driven predictive parser.

First, Follow, and Selection Sets

- First and follow sets form functions associated with the grammar and aids in the construction of a predictive parser by filling in the predictive parsing table.
- The $FIRST(\alpha)$ is the set of terminals that begin the set of strings derived from α
- The $FOLLOW(A)$ for a non-terminal A , is the set of terminals that can appear immediately to the right of A in some sentential form. The set of terminals of a such that there exists a derivation of the form $S \rightarrow^* \alpha A a \beta$ for some α and β

Process to Compute $FIRST(X)$

1. If X is terminal, then $FIRST(X) = \{X\}$
2. If $X \rightarrow \epsilon$ is a production, then add ϵ to $FIRST(X)$.
3. If X is a non-terminal and $X \rightarrow Y_1 Y_2 \dots Y_k$ is a production then $\forall Y_i \in X \rightarrow Y_i FIRST(X) + = FIRST(Y_i)$.

Process to compute $FOLLOW(A)$

1. Place end of string $\$$ in $FOLLOW(S)$ where S is the start symbol.

2. If there is a production $A \rightarrow \alpha B \beta$ where $\epsilon \notin FIRST(\beta)$ then make the following statement so: $FOLLOW(B) = FOLLOW(B) \cup FIRST(\beta)$
3. If there is a production $A \rightarrow \alpha B \beta$ where $\epsilon \in FIRST(\beta)$ then make the following statement so: $FOLLOW(B) = FOLLOW(A) \cup FOLLOW(B)$

Selection Sets (reference Cooke page IV-24)

1. The selection set of a non- ϵ right hand side option is the option's first set.
2. The selection set of a ϵ right hand side option is the option's follow set.

“To complete the syntax analysis routines, one follows clearly defined rules for implementing the grammar specification”

Given Reserved Words and Symbols: $\alpha_0 = \text{program}$ $\alpha_1 = \text{array}$ $\alpha_2 = \text{integer}$ $\alpha_3 = \text{read}$ $\alpha_4 = \text{write}$ $\alpha_5 = \text{rdln}$ $\alpha_6 = \text{wrln}$ $\alpha_7 = \text{when}$ $\alpha_8 = \text{until}$ $\alpha_9 = \text{from}$ $\alpha_{10} = ..$ $\alpha_{11} = \text{eof}$ $\alpha_{12} = \{ \alpha_{13} = \}$

1.1 Given grammar:

$$P \rightarrow \alpha_0 D_1 \alpha_{12} S_L \alpha_{13} \quad D_1 \rightarrow I_L \alpha_{31} D \quad D_1 \rightarrow \epsilon \quad D \rightarrow \alpha_1 \alpha_{14} \alpha_{-2} D_2 \alpha_{15} \alpha_{16} D_1 \quad D \rightarrow \alpha_2 \alpha_{16} D_1$$

$$D_2 \rightarrow \alpha_{17} \alpha_{-2} D_2 \quad D_2 \rightarrow \epsilon \quad I_L \rightarrow \alpha_{-1} I_L \quad I_{L_1} \rightarrow \alpha_{17} I_L \quad I_{L_1} \rightarrow \epsilon$$

Note that I_D and S_u requires left factoring $S_u \rightarrow \alpha_{-1} \quad S_u \rightarrow \alpha_{-2} \quad S_u \rightarrow \alpha_{-1} \alpha_{17} S_L$

$$T \rightarrow I_D \quad T \rightarrow \alpha_{11} \quad T \rightarrow \alpha_{-2}$$

$$\text{Left Factor Candidate } S \rightarrow \alpha_{12} S_L \alpha_{13} S' \quad S \rightarrow \alpha_5 \quad S \rightarrow \alpha_6 \quad S \rightarrow \alpha_{18} I_L \alpha_{19} S''$$

$$S' \rightarrow \alpha_7 C \quad S' \rightarrow \alpha_8 C \quad S' \rightarrow \alpha_{-1} \alpha_9 \alpha_{14} E \alpha_{10} E \alpha_{15}$$

$$S'' \rightarrow \alpha_{18} E_L \alpha_{19} \alpha_{20} \quad S'' \rightarrow \alpha_3 \quad S'' \rightarrow \alpha_4$$

$$S_L \rightarrow S \quad S_L \rightarrow S \alpha_{16}$$

$$\text{Left Factor Candidate: } C \rightarrow E_L \alpha_{16} E_L C' \quad C' \rightarrow \alpha_{21} C' \quad C' \rightarrow \alpha_{22} C' \quad C' \rightarrow \alpha_{23} C' \quad C' \rightarrow \alpha_{24} C' \\ C' \rightarrow \alpha_{25} C' \quad C' \rightarrow \alpha_{26} C' \quad C'' \rightarrow \alpha_{16} C C''' \quad C''' \rightarrow \alpha_{16} C \alpha_{31} C'' \quad C''' \rightarrow \alpha_{16} C \alpha_{32} C'' \quad C''' \rightarrow \alpha_{16} C \alpha_{33} C'' \\ C''' \rightarrow \alpha_{16} C \alpha_{34} C'' \quad C''' \rightarrow \epsilon$$

$$E \rightarrow \alpha_{18} E_L \alpha_{19} \alpha_{18} E_L \alpha_{19} E' \quad E \rightarrow T \quad E' \rightarrow \alpha_{27} \quad E' \rightarrow \alpha_{28} \quad E' \rightarrow \alpha_{29} \quad E' \rightarrow \alpha_{30} \quad E' \rightarrow \alpha_{35}$$

$$E_L \rightarrow E E'_L \quad E_L \rightarrow E E_L \quad E_L \rightarrow \epsilon$$

$$C \rightarrow (E_L)(E_L)C'' \ C' \rightarrow; CC''' \ C'' \rightarrow < C' | \dots | not C' | \epsilon \ C'' \rightarrow \alpha_{21}C' \ C'' \rightarrow \alpha_{22}C' \ C'' \rightarrow \alpha_{23}C' \\ C'' \rightarrow \alpha_{24}C' \ C'' \rightarrow \alpha_{25}C' \ C'' \rightarrow \alpha_{26}C' \ C'' \rightarrow \alpha_{32}C' \ C''' \alpha_{33}C' \ C''' \alpha_{34}C'$$

$$I_D \rightarrow \alpha_{-1} \ I_D \rightarrow \alpha_{14}S_u\alpha_{15}$$

$$S_u \rightarrow \alpha_{-1}S'_u \ S_u \rightarrow \alpha_{-2}S''_u \ S'_u \rightarrow \alpha_{17}S_u \ S''_u \rightarrow \alpha_{17}S_u$$

1.2 FIRST Sets:

$$P \rightarrow \alpha_0 D_1 \alpha_{12} S_L \alpha_{13} \text{ has the following: } \alpha_0$$

$$D_1 \rightarrow I_L \alpha_{31} D \ D_1 \rightarrow \epsilon \text{ has } \epsilon, \alpha_{-1}, \alpha_{31}$$

$$D \rightarrow \alpha_1 \alpha_{14} \alpha_{-2} D_2 \alpha_{15} \alpha_{16} D_1 \ D \rightarrow \alpha_2 \alpha_{16} D_1 \text{ has } \alpha_1, \alpha_2$$

$$D_2 \rightarrow \alpha_{17} \alpha_{-2} D_2 \ D_2 \rightarrow \epsilon \text{ has } \epsilon, \alpha_{17}$$

$$I_L \rightarrow \alpha_{-1} I_L \text{ has } \alpha_{-1}$$

$$I_{L_1} \rightarrow \alpha_{17} I_L \ I_{L_1} \rightarrow \epsilon \text{ has } \alpha_{17} \ \epsilon$$

$$I_D \rightarrow \alpha_{-1} I'_D \ \alpha_{-1} \ I'_D \rightarrow [S_u] \ I'_D \rightarrow \epsilon \text{ has } \alpha_{-1}, \alpha_{-2}, \epsilon$$

$$T \rightarrow I_D \ T \rightarrow \alpha_{11} \ T \rightarrow \alpha_{-2} \text{ has: } \alpha_{11}, \alpha_{-2}, \alpha_{-1}$$

$$S_u \rightarrow \alpha_{-1} S'_u \ S_u \rightarrow \alpha_{-2} S''_u \ S'_u \rightarrow \alpha_{-1} \alpha_{17} S_u \ S''_u \rightarrow \alpha_{-2} \alpha_{17} S_u \text{ has } \alpha_{-1}, \alpha_{-2}$$

$$S \rightarrow \alpha_{12} S_L \alpha_{13} S' \ S \rightarrow \alpha_5 \ S \rightarrow \alpha_6 \ S \rightarrow \alpha_{18} I_L \alpha_{19} S'' \text{ has } \alpha_5, \alpha_6, \alpha_{12}, \alpha_{18}$$

$$S' \rightarrow \alpha_7 C \ S' \rightarrow \alpha_8 C \ S' \rightarrow \alpha_{-1} \alpha_9 \alpha_{14} E \alpha_{10} E \alpha_{15}$$

$$\text{has } \alpha_7, \alpha_8, \alpha_{-1}$$

$$S'' \rightarrow \alpha_{18} E_L \alpha_{19} \alpha_{20} \ S'' \rightarrow \alpha_3 \ S'' \rightarrow \alpha_4 \text{ has } \alpha_{18}, \alpha_3, \alpha_4$$

$$E \rightarrow \alpha_{18} E_L \alpha_{19} \alpha_{18} E_L \alpha_{19} E' \ E \rightarrow T \text{ has: } \alpha_{18}, \alpha_{11}, \alpha_{-2}, \alpha_{-1}$$

$$E' \rightarrow \alpha_{27} \ E' \rightarrow \alpha_{28} \ E' \rightarrow \alpha_{29} \ E' \rightarrow \alpha_{30} \ E' \rightarrow \alpha_{35} \text{ has } \alpha_{27}, \alpha_{28}, \alpha_{29}, \alpha_{30}, \alpha_{35}$$

$$E_L \rightarrow E E'_L \ E_L \rightarrow \epsilon \text{ has } \alpha_{18}, \alpha_{11}, \alpha_{-2}, \alpha_{-1} \ \epsilon \ E'_L \rightarrow \alpha_{17} E_L \text{ has } \alpha_{17}$$

$$C \rightarrow \alpha_{18} E_L \alpha_{19} \alpha_{18} E_L \alpha_{19} C'' \text{ has } \alpha_{18}$$

$$C' \rightarrow \alpha_{16} C C''' \text{ has } \alpha_{16}$$

$C'' \rightarrow \alpha_{21} C' C'' \rightarrow \alpha_{22} C' C'' \rightarrow \alpha_{23} C' C'' \rightarrow \alpha_{24} C' C'' \rightarrow \alpha_{25} C' C'' \rightarrow \alpha_{26} C'$ has $\alpha_{21}, \alpha_{22}, \alpha_{23}, \alpha_{24}, \alpha_{25}, \alpha_{26}$

$C''' \rightarrow \alpha_{32} C' C''' \rightarrow \alpha_{33} C' C''' \rightarrow \alpha_{34} C'$ has $\alpha_{32}, \alpha_{33}, \alpha_{34}$

$S_L \rightarrow SS'_L S'_L \rightarrow \alpha_{16} S'_L \rightarrow \epsilon$

$\alpha_5, \alpha_6, \alpha_{12}, \alpha_{18}, \alpha_{16}, \epsilon$

Follow Sets: $P = \$$, $D_1 = \alpha_{12} \cup FOLLOW(D_1)$ $D = \alpha_{12} \cup FOLLOW(D)$ $D_2 = \alpha_{15}$ $I_L = \alpha_{31}, \alpha_{19}, \alpha_{12}$ $I_{L_1} = FOLLOW(I_{L_1})$ $S = \alpha_{13} \cup FOLLOW(S)$ $S' = \alpha_{13}$ $S'' = \alpha_{13}$ $S_u = \alpha_{15} \cup FOLLOW(S'_u) \cup FOLLOW(S''_u)$ $S'_u = FOLLOW(S_u)$ $S''_u = FOLLOW(S_u)$ $T = FOLLOW(E)$ $I'_D = FOLLOW(I_D)$ $I_D = FOLLOW(T)$

$E = \alpha_{10}, \alpha_{15}, \alpha_{16}, \alpha_{17}, \alpha_{19}$ $E = \cup FOLLOW()$ $E' = \cup FOLLOW(E)$ $E' = \alpha_{15}, \alpha_{16}, \alpha_{17}, \alpha_{19}$ $E_L = \alpha_{19}$ $E_L = \cup FOLLOW(E'_L)$ $E'_L = \alpha_{32}, \alpha_{33}, \alpha_{34}, \alpha_{13}$ $E'_L = \cup FOLLOW(E_L)$ $C = \alpha_{32}, \alpha_{33}, \alpha_{34}, \alpha_{13}$ $C' = \alpha_{32}, \alpha_{33}, \alpha_{34}, \alpha_{13}$ $C' = \cup FOLLOW(C'')$ $C'' = \cup FOLLOW(C)$ $C''' = \alpha_{32}, \alpha_{33}, \alpha_{34}, \alpha_{13}$ $C''' = \cup FOLLOW(C')$ $S_L = \alpha_{13}, \alpha_{19}, \alpha_{16}$ $S'_L = \alpha_{13}$ $S'_L = \cup FOLLOW(S_L)$

1.3 Selection Sets

1.4 Syntax Analysis Completion

\forall LHS (or element of C) there is a boolean function: \forall RHS option of a given LHS there will be an if-statement block and if a given RHS option has multiple symbols, there will be, in that block, a nested if for each symbol. For a given RHS symbol there are three possible actions:

1. In the case of matching an element of the Vocabulary, V such that if $lex \in V$: , consume the element (invoke the LA) and if the last item in the RHS option set the LHS symbol to true. If you were supposed to match an element of V and did not, set LHS to false and output an error message.
2. In the case where the RHS is an element of the syntactic category, C : invoke C's function to see if it is true. If true and if at the end of the RHS option, set the LHS symbol to true otherwise set the LHS to false.
3. In the case of an epsilon production, check to see if lex is an element of the selection set. If it is, set the LHS to true. Otherwise set LHS to false and put out an error message.

Table 1: Selection Sets for Given Grammar

Production	First	Follow	Selection
$P \rightarrow \alpha_0 D_1 \alpha_{12} S_L \alpha_{13}$	α_0	$\$$	α_0
$D_1 \rightarrow I_L \alpha_{31} D$	$\epsilon, \alpha_{-1}, \alpha_{31}$	α_{12}	$\epsilon, \alpha_{-1}, \alpha_{31}$
$D_1 \rightarrow \epsilon$			
$D \rightarrow \alpha_1 \alpha_{14} \alpha_{-2} D_2 \alpha_{15} \alpha_{16} D_1$	α_1, α_2	α_{12}	α_1
$D \rightarrow \alpha_2 \alpha_{16} D_1$	α_2		α_2
$D_2 \rightarrow \alpha_{17} \alpha_{-2} D_2$	ϵ, α_{17}	α_{15}	α_{17}
$D_2 \rightarrow \epsilon$		α_{15}	
$I_L \rightarrow \alpha_{-1} I_{L_1}$	α_{-1}	α_{19}, α_{31}	α_{-1}
$I_{L_1} \rightarrow \alpha_{17} I_L$	$\alpha_{17} \epsilon$	α_{19}, α_{31}	α_{17}
$I_{L_1} \rightarrow \epsilon$			α_{19}, α_{31}
$S_u \rightarrow \alpha_{-1} S'_u$	α_{-1}, α_{-2}	α_{15}	α_{-1}
$S_u \rightarrow \alpha_{-2} S''_u$			α_{-2}
$S'_u \rightarrow \alpha_{17} S_u$	α_{17}		α_{17}
$S''_u \rightarrow \alpha_{17} S_u$	α_{17}		α_{17}
$S \rightarrow \alpha_{12} S_L \alpha_{13} S'$	$\alpha_5, \alpha_6, \alpha_{12}, \alpha_{18}$	α_{13}	α_{12}
$S \rightarrow \alpha_5$	α_5	α_{13}	α_5
$S \rightarrow \alpha_6$	α_6	α_{13}	α_6
$S \rightarrow \alpha_{18} I_L \alpha_{19} S''$	α_{18}		α_{18}
$S' \rightarrow \alpha_7 C$	$\alpha_7, \alpha_8, \alpha_{-1}$	α_{13}	α_7
$S' \rightarrow \alpha_8 C$	α_8	α_{13}	α_8
$S' \rightarrow \alpha_{-1} \alpha_9 \alpha_{14} E \alpha_{10} E \alpha_{15}$	α_{-1}	α_{13}	α_{-1}
$S'' \rightarrow \alpha_{18} E_L \alpha_{19} \alpha_{20}$	$\alpha_{18}, \alpha_3, \alpha_4$	α_{13}	α_{18}
$S'' \rightarrow \alpha_3$	α_3	α_{13}	α_3
$S'' \rightarrow \alpha_4$	α_4	α_{13}	α_4
$T \rightarrow I_D$	α_{-1}, α_{-2}	$\alpha_{10}, \alpha_{15}, \alpha_{17}, \alpha_{19}$	α_{-1}
$T \rightarrow \alpha_{11}$	α_{11}	$\alpha_{10}, \alpha_{15}, \alpha_{17}, \alpha_{19}$	α_{11}
$T \rightarrow \alpha_{-2}$	α_{-2}	$\alpha_{10}, \alpha_{15}, \alpha_{17}, \alpha_{19}$	α_{-2}
$I_D \rightarrow \alpha_{-1} I'_D$	α_{-1}	$\alpha_{10}, \alpha_{15}, \alpha_{17}, \alpha_{19}$	α_{-1}
$I'_D \rightarrow \alpha_{14} S_u \alpha_{15}$	$\alpha_{-1}, \alpha_{-2}, \epsilon$	$\alpha_{10}, \alpha_{15}, \alpha_{17}, \alpha_{19}$	$\alpha_{-1}, \alpha_{-2}, \epsilon$
$I'_D \rightarrow \epsilon$		$\alpha_{10}, \alpha_{15}, \alpha_{17}, \alpha_{19}$	$\alpha_{10}, \alpha_{15}, \alpha_{17}, \alpha_{19}$
$E \rightarrow \alpha_{18} E_L \alpha_{19} \alpha_{18} E_L \alpha_{19} E'$	α_{18}	$\alpha_{15}, \alpha_{16}, \alpha_{17}, \alpha_{19}$	α_{18}
$E \rightarrow T$	$\alpha_{11}, \alpha_{-2}, \alpha_{-1}$	$\alpha_{15}, \alpha_{16}, \alpha_{17}, \alpha_{19}$	$\alpha_{11}, \alpha_{-2}, \alpha_{-1}$
$E' \rightarrow \alpha_{27}$	α_{27}	$\alpha_{15}, \alpha_{16}, \alpha_{17}, \alpha_{19}$	α_{27}
$E' \rightarrow \alpha_{28}$	α_{28}		α_{28}
$E' \rightarrow \alpha_{29}$	α_{29}		α_{29}
$E' \rightarrow \alpha_{30}$	α_{30}		α_{30}
$E' \rightarrow \alpha_{35}$	$\alpha_{27}, \alpha_{28}, \alpha_{29}, \alpha_{30}, \alpha_{35}$	$\alpha_{15}, \alpha_{16}, \alpha_{17}, \alpha_{19}$	α_{27}
$E_L \rightarrow E E'_L$	$\alpha_{18} \alpha_{11}, \alpha_{-2}, \alpha_{-1}$		$\alpha_{18}, \alpha_{11}, \alpha_{-2}, \alpha_{-1}$
$E_L \rightarrow \epsilon$	$\alpha_{11}, \alpha_{-2}, \alpha_{-1} \epsilon$	α_{19}	α_{19}
$E'_L \rightarrow \alpha_{17} E_L$	α_{17}	$\alpha_{11}, \alpha_{-2}, \alpha_{-1}, \alpha_{19}$	α_{17}
$E'_L \rightarrow \epsilon$			$\alpha_{11}, \alpha_{-2}, \alpha_{-1}, \alpha_{19}$
$C \rightarrow \alpha_{18} E_L \alpha_{19} \alpha_{18} E_L \alpha_{19} C''$	α_{18}	$\alpha_{32}, \alpha_{33}, \alpha_{34}, \alpha_{13}$	α_{18}

Table 2: Selection Sets for Given Grammar

Production	First	Follow	Selection
$C' \rightarrow \alpha_{16}CC'''$	α_{16}	$\alpha_{32}, \alpha_{33}, \alpha_{34}, \alpha_{13}$	α_{16}
$C'' \rightarrow \alpha_{21}C'$	α_{21}		α_{21}
$C''' \rightarrow \alpha_{22}C'$			α_{22}
$C''' \rightarrow \alpha_{23}C'$			α_{23}
$C''' \rightarrow \alpha_{24}C'$			α_{24}
$C''' \rightarrow \alpha_{25}C'$			α_{25}
$C''' \rightarrow \alpha_{26}C'$	$\alpha_{21}, \alpha_{22}, \alpha_{23}, \alpha_{24}, \alpha_{25}, \alpha_{26}$	$\alpha_{32}, \alpha_{33}, \alpha_{34}, \alpha_{13}$	α_{26}
$C'''' \rightarrow \alpha_{32}C'$	α_{32}		α_{32}
$C'''' \rightarrow \alpha_{33}C'$	α_{33}		α_{33}
$C'''' \rightarrow \alpha_{34}C'$	$\alpha_{32}, \alpha_{33}, \alpha_{34}$	$\alpha_{32}, \alpha_{33}, \alpha_{34}, \alpha_{13}$	α_{34}
$S_L \rightarrow SS'_L$	$\alpha_5, \alpha_6, \alpha_{12}, \alpha_{18}$	α_{13}	$\alpha_5, \alpha_6, \alpha_{12}, \alpha_{18}$
$S'_L \rightarrow \alpha_{16}$	α_{16}	α_{13}	α_{16}
$S'_L \rightarrow \epsilon$	ϵ	α_{13}	α_{13}