Now

$$\tilde{m} = \frac{1}{n} \sum_{i=1}^{c} n_i \tilde{m_i}$$

$$egin{aligned} ilde{\mathbf{S_w}} &= \sum_{i=1}^c \sum_{y \in Y_i} (y - ilde{m}_i) (y - ilde{m}_i)^T \ ilde{\mathbf{S_p}} &= \sum_{i=1}^c n_i ( ilde{m}_i - ilde{m}_i) ( ilde{m}_i - ilde{m}_i)^T \end{aligned}$$

$$\tilde{\mathbf{S}_{\mathbf{B}}} = \sum_{i=1}^{c} n_i (\tilde{m}_i - \tilde{m}) (\tilde{m}_i - \tilde{m})^T$$

$$\mathbf{S}_{\mathbf{W}} = \sum_{i=1}^{n_i} n_i (m_i - m_i) (m_i - m_i)$$

$$\therefore \tilde{\mathbf{S}}_{\mathbf{W}} = \mathbf{W}^T \mathbf{S}_{\mathbf{W}}$$

$$egin{aligned} & \hat{\mathbf{S}_{\mathbf{w}}} = \mathbf{W}^T \mathbf{S}_{\mathbf{W}} \mathbf{V} \\ & \tilde{\mathbf{S}_{\mathbf{B}}} = \mathbf{W}^T \mathbf{S}_{\mathbf{B}} \mathbf{V} \end{aligned}$$

$$egin{aligned} & ilde{\mathbf{S}}_{\mathbf{w}} = \mathbf{W}^T \mathbf{S}_{\mathbf{W}} \mathbf{W} \ & ilde{\mathbf{S}}_{\mathbf{B}} = \mathbf{W}^T \mathbf{S}_{\mathbf{B}} \mathbf{W} \end{aligned}$$

$$\mathbf{S}_{\mathbf{W}} = \mathbf{W}^T \mathbf{S}_{\mathbf{W}} \mathbf{W}$$
 $\mathbf{S}_{\mathbf{B}} = \mathbf{W}^T \mathbf{S}_{\mathbf{B}} \mathbf{W}$ 
 $\mathbf{S}_{\mathbf{B}} = \mathbf{W}^T \mathbf{S}_{\mathbf{B}} \mathbf{W}$ 

 $\tilde{m}_i = \frac{1}{n_i} \sum_{y \in Y_i} y$ 

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(57)

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(60)

$$J(\mathbf{W}) = rac{| ilde{S_B}|}{| ilde{S_{\cdots}}|} = rac{|w^T S_B w|}{|w^T S_B w|}$$