

A scatter matrix S is defined by

$$\mathbf{S} = \sum_{k=1}^n (\vec{x}_k - \vec{m})(\vec{x}_k - \vec{m})^T \quad (9)$$

which happens to be the sample covariance $n - 1$ times.

We use it in

$$J_1(\vec{e}) = \sum_{k=1}^n a_k^2 - 2 \sum_{k=1}^n a_k^2 + \sum_{k=1}^n \|\vec{x}_k - \vec{m}\|^2 \quad (10)$$

$$= - \sum_{k=1}^n |\vec{e}^T(\vec{x}_k - \vec{m})|^2 + \sum_{k=1}^n \|\vec{x}_k - \vec{m}\|^2 \quad (11)$$

$$= - \sum_{k=1}^n \vec{e}^T(\vec{x}_k - \vec{m})(\vec{x}_k - \vec{m})^T \vec{e} + \sum_{k=1}^n \|\vec{x}_k - \vec{m}\|^2 \quad (12)$$

$$= - \vec{e}^T \mathbf{S} \vec{e} + \sum_{k=1}^n \|\vec{x}_k - \vec{m}\|^2 \quad (13)$$