3 MDA

For c- classes problem, we consider the projection for a d-dimensional space to $(c-1)$ dimensional space assuming $d \geq c$

$$\therefore \mathbf{S_w} = \sum_{i=1}^{c} \mathbf{S_i} \tag{43}$$

$$\mathbf{S_i} = \sum_{\vec{x} \in D_i} (\vec{x} - \vec{m}_i)(\vec{x} - \vec{m}_i)^T \tag{44}$$

$$\vec{m}_i = \frac{1}{n_i} \sum_{x \in D_i} \vec{x} \tag{45}$$

The generalization $\mathbf{S_B}$ is not as direct. Define a total mean vector $\vec{m}$ and a total scatter matrix S_T by

$$\vec{m} = \frac{1}{n} \sum_{\vec{x}} \vec{x} \tag{46}$$

$$= \frac{1}{n} \sum_{i=1}^{c} n_i \vec{m}_i \tag{47}$$

what is this n and n_i?

If one had to guess, n_i is the number of sample in each D_i class.

$$\mathbf{S_T} = \sum_{x} (x - m)(x - m)^T \tag{48}$$

$$\because \mathbf{S_B} = \sum_{i=1}^{c} n_i (\vec{m}_i - \vec{m})(\vec{m}_i - \vec{m})^T \tag{49}$$

$$\therefore \mathbf{S_T} = \mathbf{S_w} + \sum_{i=1}^{c} n_i (\vec{m}_i - \vec{m})(\vec{m}_i - \vec{m})^T \tag{50}$$

$$= \mathbf{S_w} + \mathbf{S_B} \tag{51}$$

The $(c-1)$ discriminant function are given by

$$y_i = \vec{w_i}^T \vec{x}, i = 1, ..., c-1 \tag{52}$$

$$\Rightarrow y = \vec{w}^T \vec{x}, \tag{53}$$

where y is vector with y_i components and w is a matrix $[dx(c-1)]$ with w_i are the column.