"value" version of singular value decomposition is just a restatement of the third definition.

$$\mathbf{X}\hat{\mathbf{v}}_i = \sigma_i \hat{\mathbf{u}}_i \tag{7}$$

This result says a quite a bit. $\mathbf{X}$ multiplied by an eigenvector of $\mathbf{X}^T\mathbf{X}$ is equal to a scalar times another vector. The set of eigenvectors $\{\hat{\mathbf{v}}_1, \hat{\mathbf{v}}_2, \ldots, \hat{\mathbf{v}}_r\}$ and the set of vectors $\{\hat{\mathbf{u}}_1, \hat{\mathbf{u}}_2, \ldots, \hat{\mathbf{u}}_r\}$ are both orthonormal sets or bases in r-dimensional space.