

$$\mathbf{R}(\alpha_i|x) \equiv \sum_{j=1}^c \lambda(\alpha_i|\omega_j)P(\omega_j|x)$$

$$R(\alpha|\vec{x}) = \int \lambda(\alpha_i|\omega_i)P(\omega_j|\vec{x})d\vec{x}$$

2-category classification

$$\mathbf{R}(\alpha_1|x) \equiv \lambda_{11}P(\omega_1|\vec{x}) + \lambda_{12}P(\omega_2|\vec{x})$$

$$\mathbf{R}(\alpha_2|x) \equiv \lambda_{21}P(\omega_1|\vec{x}) + \lambda_{22}P(\omega_2|\vec{x})$$

$$R \equiv \int_{\mathcal{R}_1} (\lambda_{11}P(\omega_1|\vec{x}) + \lambda_{12}P(\omega_2|\vec{x}))p(\vec{x})d\vec{x} + \int_{\mathcal{R}_2} (\lambda_{21}P(\omega_1|\vec{x}) + \lambda_{22}P(\omega_2|\vec{x}))p(\vec{x})d\vec{x}$$

distribution

$$R \equiv \int_{\mathcal{R}_1} (\lambda_{11}P(\omega_1|\vec{x})p(\vec{x}) + \lambda_{12}P(\omega_2|\vec{x})p(\vec{x}))d\vec{x} + \int_{\mathcal{R}_2} (\lambda_{21}P(\omega_1|\vec{x})p(\vec{x}) + \lambda_{22}P(\omega_2|\vec{x})p(\vec{x}))d\vec{x}$$

probability reversal

$$R \equiv \int_{\mathcal{R}_1} (\lambda_{11}p(\vec{x}|\omega_1)P(\vec{\omega}_1) + \lambda_{12}p(\vec{x}|\omega_2)P(\vec{\omega}_2))d\vec{x} + \int_{\mathcal{R}_2} (\lambda_{21}p(\vec{x}|\omega_1)P(\vec{\omega}_1) + \lambda_{22}p(\vec{x}|\omega_2)P(\vec{\omega}_2))d\vec{x}$$

associativity

$$R \equiv \int_{\mathcal{R}_1} \lambda_{11}p(\vec{x}|\omega_1)P(\vec{\omega}_1)d\vec{x} + \int_{\mathcal{R}_1} \lambda_{12}p(\vec{x}|\omega_2)P(\vec{\omega}_2)d\vec{x} + \int_{\mathcal{R}_2} \lambda_{21}p(\vec{x}|\omega_1)P(\vec{\omega}_1)d\vec{x} + \int_{\mathcal{R}_2} \lambda_{22}p(\vec{x}|\omega_2)P(\vec{\omega}_2)d\vec{x}$$

$$\text{Identity of probabilities } P(\omega_2) = 1 - P(\omega_1), \int_{\mathcal{R}_1} p(x|\omega_1)d\vec{x} \equiv 1 - \int_{\mathcal{R}_2} p(x|\omega_1)d\vec{x}$$

Substitutions

$$R \equiv \int_{\mathcal{R}_1} \lambda_{11}p(\vec{x}|\omega_1)P(\vec{\omega}_1)d\vec{x} - \int_{\mathcal{R}_1} \lambda_{12}p(\vec{x}|\omega_2)P(\vec{\omega}_1)d\vec{x} + \int_{\mathcal{R}_2} \lambda_{21}p(\vec{x}|\omega_1)P(\vec{\omega}_1)d\vec{x} + \int_{\mathcal{R}_1} \lambda_{12}p(\vec{x}|\omega_2)d\vec{x} + \int_{\mathcal{R}_2} \lambda_{22}p(\vec{x}|\omega_2)d\vec{x} - \int_{\mathcal{R}_2} \lambda_{22}p(\vec{x}|\omega_2)P(\omega_1)d\vec{x}$$

$$R \equiv \int_{\mathcal{R}_1} \lambda_{11}p(\vec{x}|\omega_1)P(\vec{\omega}_1)d\vec{x} - \int_{\mathcal{R}_1} \lambda_{12}p(\vec{x}|\omega_2)P(\vec{\omega}_1)d\vec{x} + \int_{\mathcal{R}_2} \lambda_{21}p(\vec{x}|\omega_1)P(\vec{\omega}_1)d\vec{x} + 1 - \int_{\mathcal{R}_2} \lambda_{12}p(\vec{x}|\omega_2)d\vec{x} + \int_{\mathcal{R}_2} \lambda_{22}p(\vec{x}|\omega_2)d\vec{x} - \int_{\mathcal{R}_2} \lambda_{22}p(\vec{x}|\omega_2)P(\omega_1)d\vec{x}$$

Combining like terms

$$R \equiv \int_{\mathcal{R}_1} \lambda_{11}p(\vec{x}|\omega_1)P(\vec{\omega}_1)d\vec{x} - \int_{\mathcal{R}_1} \lambda_{12}p(\vec{x}|\omega_2)P(\vec{\omega}_1)d\vec{x} + \int_{\mathcal{R}_2} \lambda_{21}p(\vec{x}|\omega_1)P(\vec{\omega}_1)d\vec{x} + 1 + \int_{\mathcal{R}_2} (\lambda_{22} - \lambda_{12})p(\vec{x}|\omega_2)d\vec{x} - \int_{\mathcal{R}_2} \lambda_{22}p(\vec{x}|\omega_2)P(\omega_1)d\vec{x}$$

$$R \equiv \int_{\mathcal{R}_2} \lambda_{11}p(\vec{x}|\omega_1)P(\vec{\omega}_1)d\vec{x} + \int_{\mathcal{R}_2} \lambda_{21}p(\vec{x}|\omega_1)P(\vec{\omega}_1)d\vec{x} - \int_{\mathcal{R}_1} \lambda_{12}p(\vec{x}|\omega_2)P(\vec{\omega}_1)d\vec{x} + 1 + \int_{\mathcal{R}_2} (\lambda_{22} - \lambda_{12})p(\vec{x}|\omega_2)d\vec{x} - \int_{\mathcal{R}_2} \lambda_{22}p(\vec{x}|\omega_2)P(\omega_1)d\vec{x}$$