3 What is independence?

3.1 Definition and fundamental properties

To define the concept of independence, consider two scalar-valued random variables y_1 and y_2. Basically, the variables y_1 and y_2 are said to be independent if information on the value of y_1 does not give any information on the value of y_2, and vice versa. Above, we noted that this is the case with the variables s_1, s_2 but not with the mixture variables x_1, x_2.

Technically, independence can be defined by the probability densities. Let us denote by $p(y_1, y_2)$ the joint probability density function (pdf) of y_1 and y_2. Let us further denote by $p_1(y_1)$ the marginal pdf of y_1, i.e. the pdf of y_1 when it is considered alone:

$$p_1(y_1) = \int p(y_1, y_2) dy_2, \tag{9}$$

and similarly for y_2. Then we define that y_1 and y_2 are independent if and only if the joint pdf is *factorizable* in the following way:

$$p(y_1, y_2) = p_1(y_1) p_2(y_2). \tag{10}$$