

3 MDA

For c - classes problem, we consider the projection for a d -dimensional space to $(c - 1)$ dimensional space assuming $d \geq c$

$$\therefore \mathbf{S}_w = \sum_{i=1}^c \mathbf{S}_i \tag{43}$$

$$\mathbf{S}_i = \sum_{\vec{x} \in D_i} (\vec{x} - \vec{m}_i)(\vec{x} - \vec{m}_i)^T \tag{44}$$

$$\vec{m}_i = \frac{1}{n_i} \sum_{x \in D_i} \vec{x} \tag{45}$$

The generalization \mathbf{S}_B is not as direct. Define a total mean vector \vec{m} and a total scatter matrix S_T by

$$\vec{m} = \frac{1}{n} \sum_{\vec{x}} \vec{x} \tag{46}$$

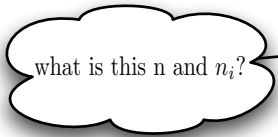
$$= \frac{1}{n} \sum_{i=1}^c n_i \vec{m}_i \tag{47}$$

$$\mathbf{S}_T = \sum_x (x - m)(x - m)^T \tag{48}$$

$$\therefore \mathbf{S}_B = \sum_{i=1}^c n_i (\vec{m}_i - \vec{m})(\vec{m}_i - \vec{m})^T \tag{49}$$

$$\therefore \mathbf{S}_T = \mathbf{S}_w + \sum_{i=1}^c n_i (\vec{m}_i - \vec{m})(\vec{m}_i - \vec{m})^T \tag{50}$$

$$= \mathbf{S}_w + \mathbf{S}_B \tag{51}$$



If one had to guess, n_i is the number of sample in each D_i class.

The $(c - 1)$ discriminant function are given by

$$y_i = \vec{w}_i^T \vec{x}, i = 1, ..., c - 1 \tag{52}$$

$$\Rightarrow y = \vec{w}^T \vec{x}, \tag{53}$$

where y is vector with y_i components and w is a matrix $[dx(c - 1)]$ with w_i are the column.