6.1 FastICA for one unit

To begin with, we shall show the one-unit version of FastICA. By a "unit" we refer to a computational unit, eventually an artificial neuron, having a weight vector $\mathbf{w}$ that the neuron is able to update by a learning rule. The FastICA learning rule finds a direction, i.e. a unit vector $\mathbf{w}$ such that the projection $\mathbf{w}^T\mathbf{x}$ maximizes nongaussianity. Nongaussianity is here measured by the approximation of negentropy $J(\mathbf{w}^T\mathbf{x})$ given in (25). Recall that the variance of $\mathbf{w}^T\mathbf{x}$ must here be constrained to unity; for whitened data this is equivalent to constraining the norm of $\mathbf{w}$ to be unity.

The FastICA is based on a fixed-point iteration scheme for finding a maximum of the nongaussianity of $\mathbf{w}^T\mathbf{x}$, as measured in (25), see (Hyvärinen and Oja, 1997; Hyvärinen, 1999a). It can be also derived as an approximative Newton iteration (Hyvärinen, 1999a). Denote by g the derivative of the nonquadratic function G used in (25); for example the derivatives of the functions in (26) are:

$$\begin{aligned} g_1(u) &= \tanh(a_1 u), \\ g_2(u) &= u\exp(-u^2/2) \end{aligned} \tag{40}$$