Now

$$\tilde{m}_i = \frac{1}{n_i} \sum_{y \in Y_i} y \tag{54}$$

$$\tilde{m} = \frac{1}{n} \sum_{i=1}^{c} n_i \tilde{m}_i \tag{55}$$

$$\tilde{\mathbf{S_w}} = \sum_{i=1}^{c} \sum_{y \in Y_i} (y - \tilde{m}_i)(y - \tilde{m}_i)^T \tag{56}$$

$$\tilde{\mathbf{S_B}} = \sum_{i=1}^{c} n_i (\tilde{m}_i - \tilde{m})(\tilde{m}_i - \tilde{m})^T \tag{57}$$

$$\therefore \tilde{\mathbf{S_w}} = \mathbf{W}^T \mathbf{S_W} \mathbf{W} \tag{58}$$

$$\tilde{\mathbf{S_B}} = \mathbf{W}^T \mathbf{S_B} \mathbf{W} \tag{59}$$

$$J(\mathbf{W}) = \frac{|\tilde{S_B}|}{|\tilde{S_w}|} = \frac{|w^T S_B w|}{|w^T S_B w|} \tag{60}$$