

Note that convergence means that the old and new values of \mathbf{w} point in the same direction, i.e. their dot-product is (almost) equal to 1. It is not necessary that the vector converges to a single point, since \mathbf{w} and $-\mathbf{w}$ define the same direction. This is again because the independent components can be defined only up to a multiplicative sign. Note also that it is here assumed that the data is prewhitened.

The derivation of FastICA is as follows. First note that the maxima of the approximation of the negentropy of $\mathbf{w}^T \mathbf{x}$ are obtained at certain optima of $E\{G(\mathbf{w}^T \mathbf{x})\}$. According to the Kuhn-Tucker conditions (Luenberger, 1969), the optima of $E\{G(\mathbf{w}^T \mathbf{x})\}$ under the constraint $E\{(\mathbf{w}^T \mathbf{x})^2\} = \|\mathbf{w}\|^2 = 1$ are obtained at points where

$$E\{\mathbf{x}g(\mathbf{w}^T \mathbf{x})\} - \beta \mathbf{w} = 0 \tag{41}$$