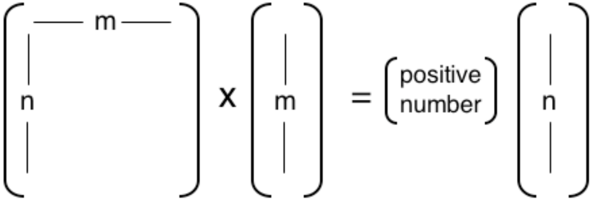
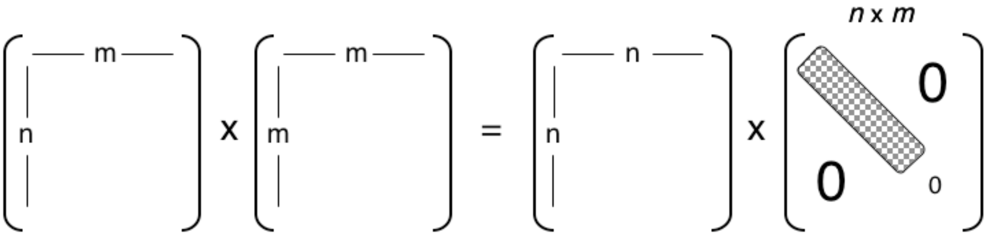
The "value" form of SVD is expressed in equation 7.

$$\mathbf{X}\hat{\mathbf{v}}_i = \sigma_i \hat{\mathbf{u}}_i$$

The mathematical intuition behind the construction of the matrix form is that we want to express all n "value" equations in just one equation. It is easiest to understand this process graphically. Drawing the matrices of equation 7 looks likes the following.



We can construct three new matrices $\mathbf{V}$, $\mathbf{U}$ and $\mathbf{\Sigma}$. All singular values are first rank-ordered $\sigma_{\tilde{1}} \geq \sigma_{\tilde{2}} \geq \ldots \geq \sigma_{\tilde{r}}$, and the corresponding vectors are indexed in the same rank order. Each pair of associated vectors $\hat{\mathbf{v}}_\mathbf{i}$ and $\hat{\mathbf{u}}_\mathbf{i}$ is stacked in the i^{th} column along their respective matrices. The corresponding singular value σ_i is placed along the diagonal (the ii^{th} position) of $\mathbf{\Sigma}$. This generates the equation $\mathbf{XV} = \mathbf{U\Sigma}$, which looks like the following.



The matrices $\mathbf{V}$ and $\mathbf{U}$ are $m \times m$ and $n \times n$ matrices respectively and $\mathbf{\Sigma}$ is a diagonal matrix with a few non-zero values (represented by the checkerboard) along its diagonal. Solving this single matrix equation solves all n "value" form equations.

FIG. 4 How to construct the matrix form of SVD from the "value" form.