The definition can be used to derive a most important property of independent random variables. Given two functions, h_1 and h_2, we always have

$$E\{h_1(y_1)h_2(y_2)\} = E\{h_1(y_1)\}E\{h_2(y_2)\}. \tag{11}$$

This can be proven as follows:

$$\begin{aligned} E\{h_1(y_1)h_2(y_2)\} &= \int\int h_1(y_1)h_2(y_2)p(y_1,y_2)dy_1dy_2 \\ &= \int\int h_1(y_1)p_1(y_1)h_2(y_2)p_2(y_2)dy_1dy_2 = \int h_1(y_1)p_1(y_1)dy_1 \int h_2(y_2)p_2(y_2)dy_2 \\ &= E\{h_1(y_1)\}E\{h_2(y_2)\}. \end{aligned} \tag{12}$$