Brief Article

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ICF for Face recognition
Assumption on basis (Bias of Maximum Likelihood Estimation)
The MLE of the variance σ^2 is biased

$$\xi[\frac{1}{n}\sum_{n=1}^{n}(x_i-\bar{x})^2]=\frac{n-1}{n}\sigma^2\neq\sigma^2 \tag{1}$$

Considering the univariate case, let μ and σ^2 be the mean and variance of the Gaussian

$$\sigma_n^2=\Xi[\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\mu)^2] \tag{2}$$

$$=\frac{1}{n-1}\Xi[\sum_{i=1}^{n}\{(x_i-\mu)-(\hat{\mu}-\mu)\}^2] \tag{3}$$

$$\frac{1}{n-1}\Xi[\sum_{i=1}^{n}\{(x_i-\mu)-2(x_i-\mu)(\hat{\mu}-\mu)+(\bar{\mu}-\mu)^2\}] \tag{4}$$

$$\frac{1}{n-1}[\sum_{i=1}^{n}\{\Xi(x_i-\mu)-2\xi(x_i-\mu)(\hat{\mu}-\mu)+\Xi(\bar{\mu}-\mu)^2\}] \tag{5}$$

$$\frac{1}{n-1}[\sum_{i=1}^{n}\{\Xi(x_i-\mu)-2\xi(x_i-\mu)(\hat{\mu}-\mu)+\Xi(\bar{\mu}-\mu)^2\}] \tag{6}$$

$$\tag{7}$$

$$\Xi[(x_i-\mu)(\hat{\mu}-\mu)] \tag{8}$$

The MLE derivation is different. The unbiased estimator for Σ is given by equation 3-21 (page 90). Where C is the sample covariance matrix and $\hat{\Sigma}$ which equals:

$$\hat{\Sigma}=\frac{n-1}{n}C$$

Asymptotically unbias (page 90)
PCA issue from hand out document