HIGH PERFORMANCE COMPUTING CENTER

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Amdahl's Law

- **Gene Myron Amdahl** (born November 16, 1922) is an American computer architect and hi-tech entrepreneur of Norwegian descent, chiefly known for his work on mainframe computers at International Business Machines (IBM) and later his own companies.
- The following presentation was borrowed from Jay Boisseau and Kent Milfeld at TACC (UT Austin)

Speedup

$$\text{Speedup} = \frac{\text{Sequential execution time}}{\text{Parallel execution time}}$$

The operations in a parallel code are one of:

1. Sequential computations
2. Parallel computations
3. Parallel overhead:
   - Communications (message passing, bus/crossbar traffic)
   - Redundant operations

Inherently Serial Code

```
for(i=0; i<n-1; i++)
    a[i+1] = f( a[i] );
```

Notice the difference between the above and

```
for(i=0; i<n-1; i++)
    a[i] = f( a[i+1] );
```

Speedup

Let σ and φ be execution times on one processor :

- $\psi(n,p)$ be speedup of solving problem of size n on p processors
- $\sigma(n)$ be the inherently serial portion of the code
- $\varphi(n)$ be portion of code that can be executed in parallel
- $\kappa(n,p)$ be parallel overhead time

Then

- $\sigma(n) + \varphi(n)$ is time required for all inherently sequential code
- $\varphi(n)/p$ is best possible execution time of a parallel code
- the general speedup is now:

$$\psi(n,p) \quad <= \quad \frac{\sigma(n) + \varphi(n)}{\sigma(n) + \varphi(n)/p + \kappa(n,p)}$$

Efficiency

$$\text{Efficiency} = \varepsilon = \frac{\text{Sequential execution time}}{\text{Parallel execution time x Processors used}}$$

Then the efficiency on a problem of size *n* on *p* processors is

$$\varepsilon(n,p) \le \frac{\sigma(n) + \varphi(n)}{(\sigma(n) + \varphi(n)/p + \kappa(n,p))\, p}$$

$$\varepsilon(n,p) \le \frac{\sigma(n) + \varphi(n)}{p\sigma(n) + \varphi(n) + p\kappa(n,p)}$$

Amdahl's Law

Remember that the speedup is:

$$\psi(n,p) \sim \frac{\sigma(n) + \varphi(n)}{\sigma(n) + \varphi(n)/p + \kappa(n,p)}$$

Since $\kappa(n,p) \geq 0$:

$$\psi(n,p) \leq \frac{\sigma(n) + \varphi(n)}{\sigma(n) + \varphi(n)/p + \kappa(n,p)} \leq \frac{\sigma(n) + \varphi(n)}{\sigma(n) + \varphi(n)/p}$$

Amdahl's Law

Now let *f* denote the inherently serial portion of the computation:

$$f = \sigma(n) / (\sigma(n) + \varphi(n))$$

Then

$$\psi(n,p) \le \frac{\sigma(n) + \varphi(n)}{\sigma(n) + \varphi(n)/p}$$

$$\psi(n,p) \le \frac{\sigma(n) / f}{\sigma(n) + \sigma(n)(1/f - 1)/p}$$

$$\psi(n,p) \le \frac{1 / f}{1 + (1/f - 1)/p}$$

$$\psi \le \frac{1}{f + (1-f)/p}$$

Gustafson's Law

Let s denote fraction of time spent in the *parallel* computation performing inherently sequential operations (different from f):

$$s = \frac{\sigma(n)}{\sigma(n) + \varphi(n)/p}$$

The fraction of time spent executing parallel computation is (1 - s)

$$(1 - s) = \frac{\varphi(n)/p}{\sigma(n) + \varphi(n)/p}$$

So

$$\sigma(n) = (\sigma(n) + \varphi(n)/p)s$$

$$\varphi(n) = (\sigma(n) + \varphi(n)/p)(1 - s)p$$

Gustafson's Law

We now have:

$$\psi(n,p) \leq \frac{\sigma(n) + \varphi(n)}{\sigma(n) + \varphi(n)/p}$$

$$\psi(n,p) \leq \frac{(\sigma(n) + \varphi(n)/p)(s + (1-s)p)}{\sigma(n) + \varphi(n)/p}$$

$$\psi(n,p) \leq s + (1-s)\,p$$

$$\psi \leq p + (1-p)\,s$$

GUSTAFSON'S LAW

In response to Amdahl's Law on limitation of effectiveness of parallel computing due to a serial (non-parallelizable) component of code, Gustafson's Law was developed to show that you can achieve over a 1000-fold speed-up using 1024 processors. It states that if the size of most problems is increased sufficiently, you can achieve any given efficiency by increasing the amount of processors.

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