The main things Dr. Beylkin highlighted in Wavelets and Fast Numerical Algorithms were:

- All transform methods expand vectors and operators are expanded into a basis and the computations take place in the new system of coordinates.
- Typically, the choice of the differential operator, and the basis functions are dictated by the availability of fast algorithms for expanding an arbitrary function into the basis.
- Representations in wavelet bases reduce a wide class of operators to a sparce form.

The key word here is a "sparce" system. Sparce is defined as having few elements typically where there is concentration of the bulk of the elements and the rest are insignificant.

There are ingredientss of Calderon-Zygmund theory appear in the Fast Multipole Method for computing potential interactions.

Fast Wavelet Transforms provide a system generalization of the FMM and its decendents to all Calderon-Zygmund and differential operators.

0.1 Non-standard form characteristics

- uncoupling of interaction between the scales
- explicit computation of basic operators such as derivatives, fractional deriviatives, Hilbert and Riesz transforms.

- Solutions to two-point boundary value problem for ellipic differential operators.

0.2 Multi-Resolution Analysis and Wavelet Reference Properties

Definition: A multi-resolution analysis is a decomposition of the Hilbert space $L^2(R^d)$, , $d >= 1$, into a chain of closed subspaces.

Let w_j be an orthogonal complement of V_j in V_{j-1} such that

$V_{j-1} = V_j + W_j$

and represent $L^2(R^d) = \sum_{j \in Z} W_j$ as a direct sum.

Consequences of Definition:

- The function may be described as linear combination of the basis function.
- Orthogonality is unity, in the power domain, and does not add or remove power from the original.

There are several mathematical transformation to illustrate these points:

- χ forms an orthonormal basis for V $(\chi(x-k))\forall k \in Z$
- ψ forms an orthonormal basis for W $(\psi(x-k))\forall k \in Z$

Condition for exact reconstruction for a pair of the quadrature mirror filters:

It may be a good idea to work out this proof and show results.