$$\left(\begin{array}{cc} a & b \\ c & d \end{array}\right)^2 = \begin{array}{cc} a^2+bc & ab+bd \\ ca+dc & bc+d^2 \end{array}$$

$$\psi_r\left(\begin{array}{cc} a & b \\ c & d \end{array}\right) = \begin{array}{cc} \frac{1}{\sqrt{2}}(a+b) & \frac{1}{\sqrt{2}}(a-b) \\ \frac{1}{\sqrt{2}}(c-d) & \frac{1}{\sqrt{2}}(c+d) \end{array} = \begin{array}{cc} \frac{1}{2}\sqrt{2}\,(a+b) & \frac{1}{2}\sqrt{2}\,(a-b) \\ \frac{1}{2}\sqrt{2}\,(c-d) & \frac{1}{2}\sqrt{2}\,(c+d) \end{array}$$

$$\psi_c\left(\begin{array}{cc} \frac{1}{2}\sqrt{2}\,(a+b) & \frac{1}{2}\sqrt{2}\,(a-b) \\ \frac{1}{2}\sqrt{2}\,(c-d) & \frac{1}{2}\sqrt{2}\,(c+d) \end{array}\right) = \begin{array}{cc} (\frac{1}{2}\sqrt{2})(\frac{1}{2}\sqrt{2}\,(a+b)-\frac{1}{2}\sqrt{2}\,(c-d)) & (\frac{1}{2}\sqrt{2})(\frac{1}{2}\sqrt{2}\,(a-b)-(\frac{1}{2}\sqrt{2}\,(c+d) \\ (\frac{1}{2}\sqrt{2})(\frac{1}{2}\sqrt{2}\,(a+b)+\frac{1}{2}\sqrt{2}\,(c-d)) & (\frac{1}{2}\sqrt{2})(\frac{1}{2}\sqrt{2}\,(a-b)+(\frac{1}{2}\sqrt{2}\,(c+d) \end{array}$$

$$\left(\begin{array}{cc} (\frac{1}{2}\sqrt{2})(\frac{1}{2}\sqrt{2}\,(a+b)-\frac{1}{2}\sqrt{2}\,(c-d)) & (\frac{1}{2}\sqrt{2})(\frac{1}{2}\sqrt{2}\,(a-b)-(\frac{1}{2}\sqrt{2}\,(c+d))) \\ (\frac{1}{2}\sqrt{2})(\frac{1}{2}\sqrt{2}\,(a+b)+\frac{1}{2}\sqrt{2}\,(c-d)) & (\frac{1}{2}\sqrt{2})(\frac{1}{2}\sqrt{2}\,(a-b)+(\frac{1}{2}\sqrt{2}\,(c+d))) \end{array}\right)^2 = \begin{array}{cc} \frac{1}{2}\left(\frac{1}{2}\sqrt{2}\,(a+b)-\frac{1}{2}\sqrt{2}\,(c-d)\right)^2+\frac{1}{2}\left(\frac{1}{2}\sqrt{2}\,(a-b)-\frac{1}{2}\sqrt{2}\,(c+d)\right)\left(\frac{1}{2}\sqrt{2}\,(a+b)+\frac{1}{2}\sqrt{2}\,(c- \\ \frac{1}{2}\left(\frac{1}{2}\sqrt{2}\,(a+b)+\frac{1}{2}\sqrt{2}\,(c-d)\right)\left(\frac{1}{2}\sqrt{2}\,(a+b)-\frac{1}{2}\sqrt{2}\,(c-d)\right)+\frac{1}{2}\left(\frac{1}{2}\sqrt{2}\,(a-b)+\frac{1}{2}\sqrt{2}\,(c+d)\right)\left(\frac{1}{2}\sqrt{2}\,(a+b\right. \end{array}$$

$$= \begin{array}{cc} \frac{1}{2}a^2+\frac{1}{2}ab-\frac{1}{2}ca-bc+\frac{1}{2}bd-\frac{1}{2}dc+\frac{1}{2}d^2 & \frac{1}{2}a^2-\frac{1}{2}ca-\frac{1}{2}bd-\frac{1}{2}d^2-\frac{1}{2}ab-\frac{1}{2}dc \\ \frac{1}{2}a^2+\frac{1}{2}ab+\frac{1}{2}dc-\frac{1}{2}d^2+\frac{1}{2}ca+\frac{1}{2}bd & \frac{1}{2}a^2-bc+\frac{1}{2}d^2-\frac{1}{2}ab+\frac{1}{2}ca-\frac{1}{2}bd+\frac{1}{2}dc \end{array}$$

$$\psi_c^{-1}\left(\begin{array}{cc} \frac{1}{2}a^2+\frac{1}{2}ab-\frac{1}{2}ca-bc+\frac{1}{2}bd-\frac{1}{2}dc+\frac{1}{2}d^2 & \frac{1}{2}a^2-\frac{1}{2}ca-\frac{1}{2}bd-\frac{1}{2}d^2-\frac{1}{2}ab-\frac{1}{2}dc \\ \frac{1}{2}a^2+\frac{1}{2}ab+\frac{1}{2}dc-\frac{1}{2}d^2+\frac{1}{2}ca+\frac{1}{2}bd & \frac{1}{2}a^2-bc+\frac{1}{2}d^2-\frac{1}{2}ab+\frac{1}{2}ca-\frac{1}{2}bd+\frac{1}{2}dc \end{array}\right) =$$

$$\begin{array}{cc} (\frac{1}{\sqrt{2}})((\frac{1}{2}a^2+\frac{1}{2}ab-\frac{1}{2}ca-bc+\frac{1}{2}bd-\frac{1}{2}dc+\frac{1}{2}d^2)-(\frac{1}{2}a^2+\frac{1}{2}ab+\frac{1}{2}dc-\frac{1}{2}d^2+\frac{1}{2}ca+\frac{1}{2}bd)) & (\sqrt{\frac{1}{2}})((\frac{1}{2}a^2- \\ (\frac{1}{\sqrt{2}})((\frac{1}{2}a^2+\frac{1}{2}ab-\frac{1}{2}ca-bc+\frac{1}{2}bd-\frac{1}{2}dc+\frac{1}{2}d^2)+(\frac{1}{2}a^2+\frac{1}{2}ab+\frac{1}{2}dc-\frac{1}{2}d^2+\frac{1}{2}ca+\frac{1}{2}bd)) & (\sqrt{\frac{1}{2}})((\frac{1}{2}a^2- \end{array}$$

$$= \begin{array}{cc} -\frac{1}{2}\sqrt{2}\left(ca+bc+dc-d^2\right) & -\frac{1}{2}\sqrt{2}\left(ca+d^2+dc-bc\right) \\ \frac{1}{2}\sqrt{2}\left(a^2+ab-bc+bd\right) & \frac{1}{2}\sqrt{2}\left(a^2-bd-ab-bc\right) \end{array}$$

$$\psi_r^{-1}\left(\begin{array}{cc} -\frac{1}{2}\sqrt{2}\left(ca+bc+dc-d^2\right) & -\frac{1}{2}\sqrt{2}\left(ca+d^2+dc-bc\right) \\ \frac{1}{2}\sqrt{2}\left(a^2+ab-bc+bd\right) & \frac{1}{2}\sqrt{2}\left(a^2-bd-ab-bc\right) \end{array}\right) =$$

$$\begin{array}{cc} (\sqrt{\frac{1}{2}})(-\frac{1}{2}\sqrt{2}\left(ca+bc+dc-d^2\right)-(-\frac{1}{2}\sqrt{2}\left(ca+d^2+dc-bc\right))) & (\sqrt{\frac{1}{2}})(-\frac{1}{2}\sqrt{2}\left(ca+bc+dc-d^2\right)+(-\frac{1}{2}\sqrt{} \\ (\sqrt{\frac{1}{2}})(\frac{1}{2}\sqrt{2}\left(a^2+ab-bc+bd\right)-(\frac{1}{2}\sqrt{2}\left(a^2-bd-ab-bc\right))) & (\sqrt{\frac{1}{2}})(\frac{1}{2}\sqrt{2}\left(a^2+ab-bc+bd\right)-(\frac{1}{2}\sqrt{} \end{array}$$

$$= \begin{array}{cc} -bc+d^2 & -c\,(a+d) \\ b\,(a+d) & b\,(a+d) \end{array}$$

$$\begin{array}{cc} -bc+d^2 & -c\,(a+d) \\ b\,(a+d) & b\,(a+d) \end{array} \neq \begin{array}{cc} a^2+bc & ab+bd \\ ca+dc & bc+d^2 \end{array}$$