

“A function $\psi \in L_2(R)$ is called an orthonormal wavelet if the family $\{\psi_{j,k}\}$ defined

$$\psi_{j,k}(x) = 2^{j/2} \psi(2^j x - k) \forall j, k \in Z$$

is an orthonormal basis of $L_2(R)$ where $\langle \psi_{j,k}, \psi_{l,m} \rangle = \delta_{j,l} \delta_{k,m}, \forall j, k, l, m \in Z$ and every $f \in L_2(R)$ can be written as

$$f(x) = \sum_{j,k=-\infty}^{\infty} c_{j,k} \psi_{j,k}(x)$$

where the series converges and is

$$f(x) = \sum_{j,k=-\infty}^{\infty} c_{j,k} \psi_{j,k}(x)$$

in $L_2(R)$ such that

$$\lim_{M_1, M_2, N_1, N_2} \left\| f - \sum_{j=-M_2}^{N_2} \sum_{k=-M_1}^{N_1} c_{j,k} \psi_{j,k} \right\| = 0$$

The simplest example of orthonormal wavelets is the Haar Transform.”