Critic on Tools to detect non-Gaussianity in non-standard cosmological models

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Reason for wavelet concentration:

1. Space-frequency localization
2. Demonstration in CMB and COBE-DMR data analysis
3. Note spherical wavelets (Tenorio)

Experiments with spherical wavelets on COBE-DMR data in HEALPix pixelization. They were looking for Gaussianity of data by using

- Spherical Haar Wavelet (SHW)
- Spherical Mexican Hat Wavelet (SMHW)

They measured performance of these wavelets for discriminating between standard inflationary models and non-Gaussian models.

For astronomical data, these non-Gaussian effects are said to be attributes of the following:

- Cosmic defects
- Linear defects
- hot spots
- cold spots

Other examples referenced

- Salopek and Spergel
- Gangui et al

$\phi(\vec{n}) = \phi(\vec{n}) + f_{i,l}(\phi_L^2(n) - < \phi^2, n >)$

Statement: The model has been the gravitational potential $\phi(\vec{n})$ includes now a quadratic term with amplitude regulated by the so called non-linear coupling parameters such that:

- ϕ_L is the linear part of the gravitational potential, $f_{n,l}$
- $f_{n,l}$ is the nonlinear coupling parameter controlling the amount of non-Gaussianity introduced.
- $<>$ is a volume average.

Cayon et. al. applied a wavelet space statistic model to simulations of slow-roll inflation model to quantify the probability of non-Gaussiantity detections used by the above model:

Simple Questions:

1. What are the SMHW and SHW?

2. What are the properties of SMHW and SHW?

3. What is the performance of these methods in Gaussian conditions?

4. What is the performance of these methods in non-Gaussian conditions?

From Section 2: The Spherical Wavelets:

Claim: The Mexican Hat Wavelet family has been successfully used to extract point sources from CMB maps.

1 The Mexican Hat on S^2

Group Theory approach by Antoine and Vanderghegnst have extended to the wavelet on S^2 based on these following properties:

- The basic function is a compensated filter
- Translations
- Dialations
- Euclidean Limits for small angles

Conclusions from expansion: The stereo-graphic projection on the sphere is the appropriate one to translate the mentioned properties from plane to the sphere.

Projection definition ($\vec{x} \to (\theta, \phi)$)

$x_1 = 2 \tan \frac{\theta}{2} \cos \phi$

$x_2 = 2 \tan \frac{\theta}{2} \sin \phi$

where (θ, ϕ) represent polar coordinates in S^2, $(y = \equiv 2 \tan \frac{\phi}{2}, \phi)$ are polar coordinates in the tangent plane to the North Pole.

What does this do for an isotropic wavelet $\psi(x; R)$?

$\psi_s(\phi, R) \alpha (\cos \frac{\theta}{2})^{-4} \psi(x \equiv 2 \tan \frac{\theta}{2}; R)$

Analysis: A function on a sphere $f(\theta, \phi)$

Definition: Linear operation continuous wavelet transform with respect to $\psi_s(\theta, R)$ is defined

$\tilde{w}(\vec{X}, R) = \int d\theta\prime \sin \theta\prime \hat{f}(\vec{x} + \vec{u}) \psi_s(\theta, R)$

$\vec{x} \equiv 2 \tan \frac{\theta}{2} (\cos \theta, \sin \theta)$

$\vec{u} \equiv 2 \tan \frac{\theta\prime}{2} (\cos \theta\prime, \sin \theta\prime)$

$\hat{f}(\vec{x}) = f(\theta, \phi)$

$w = (\theta, \psi; r) \equiv \tilde{w}(\vec{x}, R)$ are wavelet coefficients dependent on 3 parameters.

Synthesis The equation $\psi = (\theta\prime; R)$ leads to the following reconstruction formula.

$f(\theta, \phi) = \bar{f} = \frac{1}{C_\psi} \int d\theta\prime d\phi\prime \sin \theta\prime \frac{dR}{R} (\vec{x} + \vec{\mu}, \vec{R})$

Example of Mexican Hat

$\psi(\theta, R) = \frac{1}{\sqrt{2\pi}RN}[1 + (\frac{1}{2})^2]^2[2 - (\frac{y}{2})^2]e^{-\frac{y^2}{2R^2]}}$

$N(R) \equiv +\sqrt{1 + \frac{R^2}{2} + \frac{R^4}{4}}$

$y \equiv 2\tan\frac{\theta}{2}$

2 Spherical Haar Wavelet

Claims

1. Introduced by Sweldens with general planar Haar Wavelet to a pixelized sphere.
2. Orthogonal
3. Adapted to pixelization of the sky
4. ? Must be hierarchical ?
5. ?Dialation and translation capability lost ?
6. SHW decomposition is based on one scaling $\phi_{j,k}$ and three wavelet functions $\psi_{m,j,k}$ at each resolution j and position on the grid k.
7. $N_{side} = 2^{s-1}$
8. The total number of pixels for a level j, w/ in an area μ_j is given by $n_j = 12 \times 4^{J-1}$
9. Each pixel resolution j ($S_{j,k}$) is divided into four pixels ($S_{j+1,k_0}, S_{j+1,k_1}, S_{j+1,k_2}, S_{j+1,k_3}$)
10. Their tool: HEALPix

Key equations:

- $\phi_{j,k}(x) = \begin{pmatrix} 1 & if\ x \in S_{j,k} \\ 0 & otherwise \end{pmatrix}$
- $\psi_{1,j,k} = \frac{\phi_{j+1,k_0}+\phi_{j+1,k_2}-\phi_{j+1,k_1}-\phi_{j+1,k_3}}{4\mu_{j+1}}$
- $\psi_{2,j,k} = \frac{\phi_{j+1,k_0}+\phi_{j+1,k_1}-\phi_{j+1,k_2}-\phi_{j+1,k_3}}{4\mu_{j+1}}$
- $\psi_{1,j,k} = \frac{\phi_{j+1,k_0}+\phi_{j+1,k_3}-\phi_{j+1,k_2}-\phi_{j+1,k_1}}{4\mu_{j+1}}$

such that k_0, k_1, k_2, k_3 are the four pixels at resolution level j+1 into which k at level j is divided.

Note similarity to 2-D average, vertical, horizontal and diagonal.

Wavelet coefficients at level j can be obtained from the four corresponding approximation at level j+1, λ_{j+1,k_i} .

- $\lambda_{j,k} = \frac{1}{4}\sum_{i=0}^{3} \lambda_{j+1,k_i}$
- $\gamma_{1,j,k} = \mu_{j+1}(\lambda_{j+1,k_0} + \lambda_{j+1,k_2} - \lambda_{j+1,k_1} - \lambda j+1, k_3)$
- $\gamma_{2,j,k} = \mu_{j+1}(\lambda_{j+1,k_0} + \lambda_{j+1,k_1} - \lambda_{j+1,k_2} - \lambda j+1, k_3)$
- $\gamma_{3,j,k} = \mu_{j+1}(\lambda_{j+1,k_0} + \lambda_{j+1,k_3} - \lambda_{j+1,k_1} - \lambda j+1, k_2)$

Question: Where do λ_{j+1,k_0} , λ_{j+1,k_1} , λ_{j+1,k_2} , and λ_{j+1,k_2} come from.

According to the paper, the generation of coefficients starts with the original map j=J, for which the coefficients are defined by original values a pixel k.

Questions:

- What is this division which is refered to?
- Is this similar to 1-D and 2-D in the fact that the elements ahead or behind the original element are used to define the next resolution components? It would make sense for a 3-D world based on spherical coordinates.