Starting Equation

$$\langle f', g' \rangle = \langle f, g \rangle \tag{1}$$

Expanded

$$\langle f', g' \rangle = \sum_{k=0}^{p/2-1} (\frac{f_{2k} + f_{2k+1}}{\sqrt{2}} \cdot \frac{g_{2k} + g_{2k+1}}{\sqrt{2}}) + \sum_{k=0}^{p/2-1} (\frac{f_{2k} - f_{2k+1}}{\sqrt{2}} \cdot \frac{g_{2k} - g_{2k+1}}{\sqrt{2}}) \tag{2}$$

Simplified

$$\begin{aligned}\langle f', g' \rangle = \frac{1}{2} \sum_{k=0}^{p/2-1} (f_{2k}g_{2k} + f_{2k+1}g_{2k} + f_{2k}g_{2k+1} + f_{2k+1}g_{2k+1} \\ + f_{2k}g_{2k} - f_{2k+1}g_{2k} - f_{2k}g_{2k+1} + f_{2k+1}g_{2k+1}\end{aligned} \tag{3}$$

Equivalence

$$\langle f', g' \rangle = \frac{1}{2} \sum_{k=0}^{p/2-1} (f_{2k}g_{2k} + f_{2k+1}g_{2k+1}) \tag{4}$$

$$\langle f', g' \rangle = \frac{1}{2} \sum_{k=0}^{p-1} (f_k g_k) = \langle f, g \rangle \tag{5}$$