This section shows a test simple 8×8 matrices multiplied in wavelet space. One set of examples uses an upper triangular matrix and multiplies it by itself. Another set uses a matrix $\frac{1}{2}$s with the diagonal being $\frac{1}{4}$ and also multiplies itself. The third example takes the upper triangular matrix and multiplies it by the matrix of $\frac{1}{2}$s.

The values are inserted and retrieved from the program in PGM and PPM with signs retained. Also, values above 256 are retained. Each value is a quantized at values of 256 during the input and output stages. While processing, the values are computed with double floating point precision. In these case, error is showing up mostly due to quantization.

1 Matrix multiplication on 8×8 Upper Triangle

This first set of multiplications uses the matrix defined in equation **??** as the test matrix A. In equation **??**, the results of matrix multiply of A^2 is presented. A is simple, and its results in fractional and decimal form are provided. In the sub-sections that follow the one, two and three resolution wavelet transform and the multiplications for A^2 are provided. In each section, the conventional A^2 is referenced for comparison.

$$A = \frac{1}{256} \begin{array}{cccccccc} 64 & 128 & 128 & 128 & 128 & 128 & 128 & 128 \\ 0 & 64 & 128 & 128 & 128 & 128 & 128 & 128 \\ 0 & 0 & 64 & 128 & 128 & 128 & 128 & 128 \\ 0 & 0 & 0 & 64 & 128 & 128 & 128 & 128 \\ 0 & 0 & 0 & 0 & 64 & 128 & 128 & 128 \\ 0 & 0 & 0 & 0 & 0 & 64 & 128 & 128 \\ 0 & 0 & 0 & 0 & 0 & 0 & 64 & 128 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 64 \end{array}$$

Results in conventional multiplication are shown in equation **??**. Again these results are reported in intervals of 256. These results are used to compare the results of $W^{-1}((W(A))^2)$, $W^{-2}((W^2(A))^2)$, and $W^{-3}((W^3(A))^2)$.

$$A^2 = \frac{1}{256} \begin{array}{cccccccc} 1530 & 2550 & 3570 & 4590 & 5610 & 6630 & 7140 & 0 \\ 1020 & 2040 & 3060 & 4080 & 5100 & 6120 & 6630 & 0 \\ 255 & 1020 & 2040 & 3060 & 4080 & 5100 & 5610 & 0 \\ 0 & 255 & 1020 & 2040 & 3060 & 4080 & 4590 & 0 \\ 0 & 0 & 255 & 1020 & 2040 & 3060 & 3570 & 0 \\ 0 & 0 & 0 & 255 & 1020 & 2040 & 2550 & 0 \\ 0 & 0 & 0 & 0 & 255 & 1020 & 1530 & 0 \\ 0 & 0 & 0 & 0 & 0 & 255 & 765 & 255 \end{array}$$

$$A^2 = \begin{matrix} \frac{765}{128} & \frac{1275}{128} & \frac{1785}{128} & \frac{2295}{128} & \frac{2805}{128} & \frac{3315}{128} & \frac{1785}{64} & 0 \\ \frac{255}{64} & \frac{255}{32} & \frac{765}{64} & \frac{255}{16} & \frac{1275}{64} & \frac{765}{32} & \frac{3315}{128} & 0 \\ \frac{255}{256} & \frac{255}{64} & \frac{255}{32} & \frac{765}{64} & \frac{255}{16} & \frac{1275}{64} & \frac{2805}{128} & 0 \\ 0 & \frac{255}{256} & \frac{255}{64} & \frac{255}{32} & \frac{765}{64} & \frac{255}{16} & \frac{2295}{128} & 0 \\ 0 & 0 & \frac{255}{256} & \frac{255}{64} & \frac{255}{32} & \frac{765}{64} & \frac{1785}{128} & 0 \\ 0 & 0 & 0 & \frac{255}{256} & \frac{255}{64} & \frac{255}{32} & \frac{1275}{128} & 0 \\ 0 & 0 & 0 & 0 & \frac{255}{256} & \frac{255}{64} & \frac{765}{128} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{255}{256} & \frac{765}{256} & \frac{255}{256} \end{matrix}$$

$$A^2 = \begin{matrix} \frac{765}{128} & \frac{1275}{128} & \frac{1785}{128} & \frac{2295}{128} & \frac{2805}{128} & \frac{3315}{128} & \frac{1785}{64} & 0 \\ \frac{255}{64} & \frac{255}{32} & \frac{765}{64} & \frac{255}{16} & \frac{1275}{64} & \frac{765}{32} & \frac{3315}{128} & 0 \\ \frac{255}{256} & \frac{255}{64} & \frac{255}{32} & \frac{765}{64} & \frac{255}{16} & \frac{1275}{64} & \frac{2805}{128} & 0 \\ 0 & \frac{255}{256} & \frac{255}{64} & \frac{255}{32} & \frac{765}{64} & \frac{255}{16} & \frac{2295}{128} & 0 \\ 0 & 0 & \frac{255}{256} & \frac{255}{64} & \frac{255}{32} & \frac{765}{64} & \frac{1785}{128} & 0 \\ 0 & 0 & 0 & \frac{255}{256} & \frac{255}{64} & \frac{255}{32} & \frac{1275}{128} & 0 \\ 0 & 0 & 0 & 0 & \frac{255}{256} & \frac{255}{64} & \frac{765}{128} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{255}{256} & \frac{765}{256} & \frac{255}{256} \end{matrix}$$

$$A^2 = \begin{matrix} 5.9766 & 9.9609 & 13.945 & 17.93 & 21.914 & 25.898 & 27.891 & 0 \\ 3.9844 & 7.9688 & 11.953 & 15.938 & 19.922 & 23.906 & 25.898 & 0 \\ .99609 & 3.9844 & 7.9688 & 11.953 & 15.938 & 19.922 & 21.914 & 0 \\ 0 & .99609 & 3.9844 & 7.9688 & 11.953 & 15.938 & 17.93 & 0 \\ 0 & 0 & .99609 & 3.9844 & 7.9688 & 11.953 & 13.945 & 0 \\ 0 & 0 & 0 & .99609 & 3.9844 & 7.9688 & 9.9609 & 0 \\ 0 & 0 & 0 & 0 & .99609 & 3.9844 & 5.9766 & 0 \\ 0 & 0 & 0 & 0 & 0 & .99609 & 2.9883 & .99609 \end{matrix}$$

1.1 A multiplication at one resolutions

First shown is the wavelet transform of the test matrix A. $W(A)$ is shown with its equivalent fractional and decimal form, and just one resolution of transform. Elements near or at zero are candidates for sparse filtering. Contributions made by these near zero elements is small in comparison to that of the larger elements.

$$W(A)\frac{1}{256} \begin{matrix} 893 & 1021 & 1021 & 511 & 128 & 0 & 0 & -510 \\ 128 & 893 & 1021 & 511 & 128 & 128 & 0 & -510 \\ 0 & 128 & 893 & 511 & 0 & 128 & 128 & -510 \\ 0 & 0 & 128 & 511 & 0 & 0 & 128 & -255 \\ -127 & 1 & 1 & 1 & 128 & 0 & 0 & 0 \\ -127 & -127 & 1 & 1 & -127 & 128 & 0 & 0 \\ 0 & -127 & -127 & 1 & 0 & -127 & 128 & 0 \\ 0 & 0 & -127 & 1 & 0 & 0 & -127 & 256 \end{matrix}$$

$$
W(A) = \begin{matrix}
\frac{893}{256} & \frac{1021}{256} & \frac{1021}{256} & \frac{511}{256} & \frac{1}{2} & 0 & 0 & -\frac{255}{128} \\
\frac{1}{2} & \frac{893}{256} & \frac{1021}{256} & \frac{511}{256} & \frac{1}{2} & \frac{1}{2} & 0 & -\frac{255}{128} \\
0 & \frac{1}{2} & \frac{893}{256} & \frac{511}{256} & 0 & \frac{1}{2} & \frac{1}{2} & -\frac{255}{128} \\
0 & 0 & \frac{1}{2} & \frac{511}{256} & 0 & 0 & \frac{1}{2} & -\frac{255}{256} \\
-\frac{127}{256} & \frac{1}{256} & \frac{1}{256} & \frac{1}{256} & \frac{1}{2} & 0 & 0 & 0 \\
-\frac{127}{256} & -\frac{127}{256} & \frac{1}{256} & \frac{1}{256} & -\frac{127}{256} & \frac{1}{2} & 0 & 0 \\
0 & -\frac{127}{256} & -\frac{127}{256} & \frac{1}{256} & 0 & -\frac{127}{256} & \frac{1}{2} & 0 \\
0 & 0 & -\frac{127}{256} & \frac{1}{256} & 0 & 0 & -\frac{127}{256} & 1
\end{matrix}
$$

The results of $(W(A))^2$ are shown in equation **??**. One thing to be careful of in this form are the signs and how they are represented. If this transform value is stored externally, then these wide range of values have to be considered.

$$
\frac{1}{256} \begin{matrix}
3571 & 7651 & 11731 & 6886 & 1021 & 1021 & 1021 & -6885 \\
766 & 4081 & 8161 & 5101 & 511 & 1021 & 1021 & -5100 \\
0 & 766 & 4081 & 3061 & 0 & 511 & 1021 & -3060 \\
0 & 0 & 766 & 1276 & 0 & 0 & 511 & -1020 \\
-510 & -509 & -509 & -254 & 1 & 1 & 1 & 255 \\
-510 & -1020 & -1020 & -510 & -255 & 1 & 1 & 511 \\
1 & -510 & -1020 & -510 & 1 & -255 & 1 & 511 \\
0 & 1 & -510 & -255 & 0 & 1 & -255 & 511
\end{matrix}
= \begin{matrix}
\frac{3571}{256} & \frac{7651}{256} & \frac{11\,731}{256} & \frac{3443}{128} & \frac{1021}{256} \\
\frac{383}{128} & \frac{4081}{256} & \frac{8161}{256} & \frac{5101}{256} & \frac{511}{256} \\
0 & \frac{383}{128} & \frac{4081}{256} & \frac{3061}{256} & 0 \\
0 & 0 & \frac{383}{128} & \frac{319}{64} & 0 \\
-\frac{255}{128} & -\frac{509}{256} & -\frac{509}{256} & -\frac{127}{128} & \frac{1}{256} \\
-\frac{255}{128} & -\frac{255}{64} & -\frac{255}{64} & -\frac{255}{128} & -\frac{255}{256} \\
\frac{1}{256} & -\frac{255}{128} & -\frac{255}{64} & -\frac{255}{128} & \frac{1}{256} \\
0 & \frac{1}{256} & -\frac{255}{128} & -\frac{255}{256} & 0
\end{matrix}
$$

After the inverse transform is applied $(W(A))^2$, the matrix is very close to the matrix A^2.

$$
\frac{1}{256} \begin{matrix}
1531 & 2551 & 3571 & 4591 & 5611 & 6631 & 7141 & 1 \\
1021 & 2041 & 3061 & 4081 & 5101 & 6121 & 6631 & 0 \\
256 & 1021 & 2041 & 3061 & 4081 & 5101 & 5611 & 1 \\
0 & 256 & 1021 & 2041 & 3061 & 4081 & 4591 & 1 \\
0 & 0 & 256 & 1021 & 2041 & 3061 & 3571 & 0 \\
0 & 0 & 0 & 256 & 1021 & 2041 & 2551 & 0 \\
0 & 0 & 0 & 0 & 256 & 1021 & 1531 & 0 \\
0 & 0 & 0 & 0 & 0 & 256 & 766 & 256
\end{matrix}
= \begin{matrix}
\frac{1531}{256} & \frac{2551}{256} & \frac{3571}{256} & \frac{4591}{256} & \frac{5611}{256} & \frac{6631}{256} & \frac{7141}{256} \\
\frac{1021}{256} & \frac{2041}{256} & \frac{3061}{256} & \frac{4081}{256} & \frac{5101}{256} & \frac{6121}{256} & \frac{6631}{256} \\
1 & \frac{1021}{256} & \frac{2041}{256} & \frac{3061}{256} & \frac{4081}{256} & \frac{5101}{256} & \frac{5611}{256} \\
0 & 1 & \frac{1021}{256} & \frac{2041}{256} & \frac{3061}{256} & \frac{4081}{256} & \frac{4591}{256} \\
0 & 0 & 1 & \frac{1021}{256} & \frac{2041}{256} & \frac{3061}{256} & \frac{3571}{256} \\
0 & 0 & 0 & 1 & \frac{1021}{256} & \frac{2041}{256} & \frac{2551}{256} \\
0 & 0 & 0 & 0 & 1 & \frac{1021}{256} & \frac{1531}{256} \\
0 & 0 & 0 & 0 & 0 & 1 & \frac{383}{128}
\end{matrix}
$$

$$
= \begin{matrix}
5.9805 & 9.9648 & 13.949 & 17.934 & 21.918 & 25.902 & 27.895 & 3.9063 \times 10^{-3} \\
3.9883 & 7.9727 & 11.957 & 15.941 & 19.926 & 23.91 & 25.902 & 0 \\
1.0 & 3.9883 & 7.9727 & 11.957 & 15.941 & 19.926 & 21.918 & 3.9063 \times 10^{-3} \\
0 & 1.0 & 3.9883 & 7.9727 & 11.957 & 15.941 & 17.934 & 3.9063 \times 10^{-3} \\
0 & 0 & 1.0 & 3.9883 & 7.9727 & 11.957 & 13.949 & 0 \\
0 & 0 & 0 & 1.0 & 3.9883 & 7.9727 & 9.9648 & 0 \\
0 & 0 & 0 & 0 & 1.0 & 3.9883 & 5.9805 & 0 \\
0 & 0 & 0 & 0 & 0 & 1.0 & 2.9922 & 1.0
\end{matrix}
$$

The values for this operation match those of the conventional matrix multiplication a relative energy of $10^{-}15$. These slight difference are enough to generate quantization difference between the original and the transform based one. However, it should be noted that numerically, $W^{-1}(W(A)^2)$ are the same.

1.2 Upper Triangular Matrix Multiply with Two Resolution of the Wavelet Transform

After two resolutions of the wavelet transform, matrix multiplication retains a fidelity of 10^{-14} between A^2 and $W^{-2}(((W^2(A))^2)$. Unfortunately, only seven of the elements are within the epsilon threshold of $\frac{1}{512}$. and nine elements within the epsilon threshold of $\frac{3}{512}$.

$$\frac{1}{256}\begin{matrix} 1467 & 1531 & 192 & -510 & 447 & -510 & -63 & -510 \\ 64 & 1021 & 64 & -255 & 64 & 0 & 64 & -510 \\ -191 & 1 & 64 & 0 & 64 & 0 & 64 & 0 \\ -63 & -127 & -63 & 128 & -63 & 128 & -63 & 128 \\ -446 & 1 & 64 & 1 & 319 & 1 & 64 & 1 \\ -63 & -382 & -63 & 128 & -63 & 383 & -63 & 128 \\ -63 & 1 & -63 & 0 & -63 & 1 & 192 & 0 \\ 64 & 0 & 64 & 1 & 64 & 1 & 64 & 256 \end{matrix}$$

Next step is to square matrix $W^2(A)$ designated $(W^2(A))^2$. $(W^2(A))^2$ is shown in equation. $(W^2(A))^2 = \frac{1}{256}\begin{matrix} 8033 & 15938 & 1786 & -4972 & 3698 & -3952 & 256 & -7012 \\ 383 & 4591 & 256 & -1275 & 383 & -255 & 256 & -2805 \\ -1275 & -1147 & -127 & 383 & -255 & 383 & 128 & 383 \\ -255 & -1147 & -127 & 383 & -255 & 383 & -127 & 638 \\ -3187 & -2677 & -255 & 893 & -382 & 893 & 256 & 893 \\ -382 & -2550 & -255 & 766 & -382 & 766 & -255 & 1276 \\ -255 & -382 & -127 & 128 & -255 & 128 & 128 & 128 \\ 256 & 383 & 128 & -127 & 256 & -127 & 128 & 128 \end{matrix}$

After the inverse transform is applied $(W^2(A))^2$, the result is very close to the product of A^2.

$$W^-2((W^2(A))^2) = \frac{1}{256}\begin{matrix} 1531 & 2551 & 3571 & 4591 & 5611 & 6631 & 7141 & 1 \\ 1021 & 2041 & 3061 & 4081 & 5101 & 6121 & 6631 & 1 \\ 256 & 1021 & 2041 & 3061 & 4081 & 5101 & 5611 & 1 \\ 1 & 256 & 1021 & 2041 & 3061 & 4081 & 4591 & 0 \\ 1 & 1 & 256 & 1021 & 2041 & 3061 & 3571 & 0 \\ 1 & 1 & 0 & 256 & 1020 & 2041 & 2551 & 0 \\ 0 & 1 & 0 & 0 & 256 & 1021 & 1531 & 0 \\ 0 & 0 & 0 & 0 & 0 & 256 & 766 & 256 \end{matrix}$$

1.3 Multiplication on Three Resolutions of Wavelet Transform on an Upper Triangular Matrix

After three resolutions of the wavelet transform on an upper triangular matrix, six elements of the sixty-four elements are below $\frac{1}{512}$. Another ten of the sixty-four elements are below $\frac{3}{512}$. Relative fidelity is on the order of 10^-14. The steps of producing a wavelet based transform with three resolutions is shown here.

First, acquire three resolutions of the wavelet transform, shown in equation **??**.

$$W^3(A) = \frac{1}{256} \begin{matrix} 2041 & -255 & 0 & -510 & 511 & -510 & -510 & -510 \\ -191 & 64 & 64 & 64 & 64 & 64 & 64 & 64 \\ -446 & 64 & 319 & 64 & 64 & 64 & 64 & 64 \\ 0 & 0 & 1 & 256 & 1 & 1 & 1 & 1 \\ -956 & 64 & 64 & 64 & 447 & 192 & 447 & -63 \\ 0 & 1 & 1 & 1 & -127 & 128 & 128 & 128 \\ 1 & 0 & 1 & 0 & -382 & 128 & 383 & 128 \\ 64 & 64 & 64 & 64 & -63 & -63 & -63 & 192 \end{matrix}$$

Next step is to square matrix $W^3(A)$ designated $(W^3(A))^2$. $(W^3(A))^2$ is shown in equation **??**. $(W^3(A))^2 = \frac{1}{256} \begin{matrix} 14472 & -2103 & -63 & -4653 & 6057 & -4143 & -4143 & -5163 \\ -1912 & 256 & 128 & 511 & -382 & 511 & 638 & 511 \\ -4398 & 574 & 447 & 1084 & -828 & 1084 & 1212 & 1084 \\ 0 & 1 & 1 & 256 & 0 & 1 & 1 & 1 \\ -9498 & 1084 & 192 & 2104 & -1848 & 2614 & 3507 & 2104 \\ 511 & 1 & 1 & 1 & -510 & 1 & 1 & 256 \\ 1467 & -63 & -63 & -63 & -1338 & -63 & -63 & 447 \\ 638 & 1 & 128 & 1 & 128 & -255 & -382 & 1 \end{matrix}$

Last the inverse transform is applied $(W^3(A))^2$, the result is very close to the product of A^2.

$$W^-3((W^3(A))^2) = \frac{1}{256} \begin{matrix} 1531 & 2551 & 3571 & 4591 & 5611 & 6631 & 7141 & 0 \\ 1021 & 2041 & 3061 & 4081 & 5101 & 6121 & 6631 & 0 \\ 256 & 1021 & 2041 & 3061 & 4081 & 5101 & 5611 & 1 \\ 1 & 256 & 1021 & 2041 & 3061 & 4081 & 4591 & 0 \\ 1 & 1 & 256 & 1021 & 2041 & 3061 & 3571 & 0 \\ 1 & 1 & 1 & 256 & 1021 & 2041 & 2551 & 0 \\ 0 & 1 & 1 & 1 & 255 & 1021 & 1531 & 0 \\ 0 & 1 & 1 & 1 & 0 & 256 & 766 & 255 \end{matrix}$$

2 Matrix Multiply Results for 8 by 8 Full Matrix

This next example uses a matrix defined in equation **??**, and designated B. Unlike the other matrix, this matrix is filled with nothing but $\frac{1}{2}$ except on the diagonal. One nice feature about this example are the huge levels of sparsity that emerge as the matrix is transformed. This example only uses the conventional multiplication algorithm as the mean of calculating B^2 and $W(B)^2$). Strassen and Winograd may used for future work.

$$\frac{1}{256} \begin{matrix} 64 & 128 & 128 & 128 & 128 & 128 & 128 & 128 \\ 128 & 64 & 128 & 128 & 128 & 128 & 128 & 128 \\ 128 & 128 & 64 & 128 & 128 & 128 & 128 & 128 \\ 128 & 128 & 128 & 64 & 128 & 128 & 128 & 128 \\ 128 & 128 & 128 & 128 & 64 & 128 & 128 & 128 \\ 128 & 128 & 128 & 128 & 128 & 64 & 128 & 128 \\ 128 & 128 & 128 & 128 & 128 & 128 & 64 & 128 \\ 128 & 128 & 128 & 128 & 128 & 128 & 128 & 64 \end{matrix} = \begin{matrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} \end{matrix}$$

2.1 One Resolution of Wavelet Transform

The first resolution produces a huge level of sparseness in the vertical section and the horizontal section has most of its energy concentrated in the lower and upper rows. Furthermore the diagonal component as expected has its energy concentrated in the diagonal. Twenty values out sixty-four have been reduced to less than $\frac{1}{512}$. Another seventeen has been reduced to less than $\frac{3}{512}$. The rest of the values, hold significance and may need to transformed again to remove some redundancy.

First step is to show the wavelet transform, and displayed in equation **??**.

$$\frac{1}{256}\begin{matrix} 893 & 1021 & 1021 & 1021 & 128 & 0 & 0 & 0 \\ 893 & 893 & 1021 & 1021 & -127 & 128 & 0 & 0 \\ 1021 & 893 & 893 & 1021 & 0 & -127 & 128 & 0 \\ 1021 & 1021 & 893 & 766 & 0 & 0 & -127 & 0 \\ -127 & 1 & 1 & 1 & 128 & 0 & 0 & 0 \\ 128 & -127 & 1 & 1 & 128 & 128 & 0 & 0 \\ 1 & 128 & -127 & 1 & 0 & 128 & 128 & 0 \\ 1 & 1 & 128 & -255 & 0 & 0 & 128 & 0 \end{matrix} = \begin{matrix} \frac{893}{256} & \frac{1021}{256} & \frac{1021}{256} & \frac{1021}{256} & \frac{1}{2} & 0 \\ \frac{893}{256} & \frac{893}{256} & \frac{1021}{256} & \frac{1021}{256} & -\frac{127}{256} & \frac{1}{2} \\ \frac{1021}{256} & \frac{893}{256} & \frac{893}{256} & \frac{1021}{256} & 0 & -\frac{127}{256} \\ \frac{1021}{256} & \frac{1021}{256} & \frac{893}{256} & \frac{383}{128} & 0 & 0 \\ -\frac{127}{256} & \frac{1}{256} & \frac{1}{256} & \frac{1}{256} & \frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{127}{256} & \frac{1}{256} & \frac{1}{256} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{256} & \frac{1}{2} & -\frac{127}{256} & \frac{1}{256} & 0 & \frac{1}{2} \\ \frac{1}{256} & \frac{1}{256} & \frac{1}{2} & -\frac{255}{256} & 0 & 0 \end{matrix}$$

$$= \begin{matrix} 3.4883 & 3.9883 & 3.9883 & 3.9883 & .5 & 0 & 0 & 0 \\ 3.4883 & 3.4883 & 3.9883 & 3.9883 & -.49609 & .5 & 0 & 0 \\ 3.9883 & 3.4883 & 3.4883 & 3.9883 & 0 & -.49609 & .5 & 0 \\ 3.9883 & 3.9883 & 3.4883 & 2.9922 & 0 & 0 & -.49609 & 0 \\ -.49609 & 3.9063\times10^{-3} & 3.9063\times10^{-3} & 3.9063\times10^{-3} & .5 & 0 & 0 & 0 \\ .5 & -.49609 & 3.9063\times10^{-3} & 3.9063\times10^{-3} & .5 & .5 & 0 & 0 \\ 3.9063\times10^{-3} & .5 & -.49609 & 3.9063\times10^{-3} & 0 & .5 & .5 & 0 \\ 3.9063\times10^{-3} & 3.9063\times10^{-3} & .5 & -.99609 & 0 & 0 & .5 & 0 \end{matrix}$$

Next step is to compute the value of $W(B)^2$, and is shown in equation **??**.

$$\frac{1}{256}\begin{matrix} 14791 & 14791 & 14791 & 14791 & 0 & 0 & 0 & 0 \\ 14536 & 14281 & 14281 & 14281 & 1 & 0 & 0 & 0 \\ 14281 & 14536 & 14281 & 14281 & 0 & 1 & 0 & 0 \\ 13771 & 13771 & 14026 & 14026 & 0 & 0 & 1 & 0 \\ -509 & -509 & -509 & -509 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 256 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 256 & 0 & 0 \\ -509 & -509 & -509 & -254 & 1 & 1 & 256 & 0 \end{matrix} = \begin{matrix} \frac{14791}{256} & \frac{14791}{256} & \frac{14791}{256} & \frac{14791}{256} & 0 & 0 & 0 \\ \frac{1817}{32} & \frac{14281}{256} & \frac{14281}{256} & \frac{14281}{256} & \frac{1}{256} & 0 & 0 \\ \frac{14281}{256} & \frac{1817}{32} & \frac{14281}{256} & \frac{14281}{256} & 0 & \frac{1}{256} & 0 \\ \frac{13771}{256} & \frac{13771}{256} & \frac{7013}{128} & \frac{7013}{128} & 0 & 0 & \frac{1}{256} \\ -\frac{509}{256} & -\frac{509}{256} & -\frac{509}{256} & -\frac{509}{256} & 0 & \frac{1}{256} & \frac{1}{256} \\ \frac{1}{256} & \frac{1}{256} & \frac{1}{256} & \frac{1}{256} & 1 & 0 & \frac{1}{256} \\ \frac{1}{256} & \frac{1}{256} & \frac{1}{256} & \frac{1}{256} & \frac{1}{256} & 1 & 0 \\ -\frac{509}{256} & -\frac{509}{256} & -\frac{509}{256} & -\frac{127}{128} & \frac{1}{256} & \frac{1}{256} & 1 \end{matrix}$$

$$= \begin{matrix} 57.777 & 57.777 & 57.777 & 57.777 & 0 & 0 \\ 56.781 & 55.785 & 55.785 & 55.785 & 3.9063\times10^{-3} & 0 \\ 55.785 & 56.781 & 55.785 & 55.785 & 0 & 3.9063\times10^{-3} \\ 53.793 & 53.793 & 54.789 & 54.789 & 0 & 0 \\ -1.9883 & -1.9883 & -1.9883 & -1.9883 & 0 & 3.9063\times10^{-3} \\ 3.9063\times10^{-3} & 3.9063\times10^{-3} & 3.9063\times10^{-3} & 3.9063\times10^{-3} & 1.0 & 0 \\ 3.9063\times10^{-3} & 3.9063\times10^{-3} & 3.9063\times10^{-3} & 3.9063\times10^{-3} & 3.9063\times10^{-3} & 1.0 \\ -1.9883 & -1.9883 & -1.9883 & -.99219 & 3.9063\times10^{-3} & 3.9063\times10^{-3} \end{matrix}$$

Finally, the inverse transform is applied to matrix $W(B)^2$, shown in equation **??**.

$$
\frac{1}{256}\begin{matrix}
7651 & 7651 & 7651 & 7651 & 7651 & 7651 & 7651 & 7651 \\
7141 & 7141 & 7141 & 7141 & 7141 & 7141 & 7141 & 7141 \\
7396 & 7141 & 7141 & 7141 & 7141 & 7141 & 7141 & 7141 \\
7141 & 7396 & 7141 & 7141 & 7141 & 7141 & 7141 & 7141 \\
7141 & 7141 & 7396 & 7141 & 7141 & 7141 & 7141 & 7141 \\
7141 & 7141 & 7141 & 7396 & 7141 & 7141 & 7141 & 7141 \\
7141 & 7141 & 7141 & 7141 & 7396 & 7141 & 7141 & 7141 \\
6631 & 6631 & 6631 & 6631 & 6631 & 6886 & 6886 & 6886
\end{matrix}
=
\begin{matrix}
\frac{7651}{256} & \frac{7651}{256} & \frac{7651}{256} & \frac{7651}{256} & \frac{7651}{256} & \frac{7651}{256} & \frac{7651}{256} \\
\frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} \\
\frac{1849}{64} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} \\
\frac{7141}{256} & \frac{1849}{64} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} \\
\frac{7141}{256} & \frac{7141}{256} & \frac{1849}{64} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} \\
\frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{1849}{64} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} \\
\frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{1849}{64} & \frac{7141}{256} & \frac{7141}{256} \\
\frac{6631}{256} & \frac{6631}{256} & \frac{6631}{256} & \frac{6631}{256} & \frac{6631}{256} & \frac{3443}{128} & \frac{3443}{128}
\end{matrix}
$$

$$
=\begin{matrix}
29.887 & 29.887 & 29.887 & 29.887 & 29.887 & 29.887 & 29.887 & 29.887 \\
27.895 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 \\
28.891 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 \\
27.895 & 28.891 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 \\
27.895 & 27.895 & 28.891 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 \\
27.895 & 27.895 & 27.895 & 28.891 & 27.895 & 27.895 & 27.895 & 27.895 \\
27.895 & 27.895 & 27.895 & 27.895 & 28.891 & 27.895 & 27.895 & 27.895 \\
25.902 & 25.902 & 25.902 & 25.902 & 25.902 & 26.898 & 26.898 & 26.898
\end{matrix}
$$

2.2 Matrix Multiplication 8×8 on dense matrix with 2 resolutions

Next step, the square of matrix $W^2(B)$ is $(W^2(B))^2$ and shown in equation **??**.

$$
\frac{1}{256}\begin{matrix}
1849 & 2041 & 64 & 0 & 64 & 0 & 64 & 0 \\
1977 & 1786 & -63 & 1 & -63 & 0 & -63 & 0 \\
-63 & 1 & 192 & 0 & -63 & 1 & -63 & 0 \\
64 & -127 & 64 & 128 & 64 & -127 & 64 & -127 \\
-63 & 0 & -63 & 0 & -63 & 1 & 192 & 0 \\
64 & -127 & 64 & -127 & 64 & -127 & 64 & 128 \\
64 & 1 & 64 & 0 & -191 & 1 & 64 & 0 \\
-63 & 0 & -63 & 0 & -63 & -255 & -63 & 1
\end{matrix}
=
$$

$$
\begin{matrix}
\frac{1849}{256} & \frac{2041}{256} & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\
\frac{1977}{256} & \frac{893}{128} & -\frac{63}{256} & \frac{1}{256} & -\frac{63}{256} & 0 & -\frac{63}{256} & 0 \\
-\frac{63}{256} & \frac{1}{256} & \frac{3}{4} & 0 & -\frac{63}{256} & \frac{1}{256} & -\frac{63}{256} & 0 \\
\frac{1}{4} & -\frac{127}{256} & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & -\frac{127}{256} & \frac{1}{4} & -\frac{127}{256} \\
-\frac{63}{256} & 0 & -\frac{63}{256} & 0 & -\frac{63}{256} & \frac{1}{256} & \frac{3}{4} & 0 \\
\frac{1}{4} & -\frac{127}{256} & \frac{1}{4} & -\frac{127}{256} & \frac{1}{4} & -\frac{127}{256} & \frac{1}{4} & \frac{1}{2} \\
\frac{1}{4} & \frac{1}{256} & \frac{1}{4} & 0 & -\frac{191}{256} & \frac{1}{256} & \frac{1}{4} & 0 \\
-\frac{63}{256} & 0 & -\frac{63}{256} & 0 & -\frac{63}{256} & -\frac{255}{256} & -\frac{63}{256} & \frac{1}{256}
\end{matrix}
=
$$

$$\begin{matrix} 7.2227 & 7.9727 & .25 & 0 & .25 & 0 & .25 & 0 \\ 7.7227 & 6.9766 & -.24609 & 3.9063\times10^{-3} & -.24609 & 0 & -.24609 & 0 \\ -.24609 & 3.9063\times10^{-3} & .75 & 0 & -.24609 & 3.9063\times10^{-3} & -.24609 & 0 \\ .25 & -.49609 & .25 & .5 & .25 & -.49609 & .25 & -.496 \\ -.24609 & 0 & -.24609 & 0 & -.24609 & 3.9063\times10^{-3} & .75 & 0 \\ .25 & -.49609 & .25 & -.49609 & .25 & -.49609 & .25 & .5 \\ .25 & 3.9063\times10^{-3} & .25 & 0 & -.74609 & 3.9063\times10^{-3} & .25 & 0 \\ -.24609 & 0 & -.24609 & 0 & -.24609 & -.99609 & -.24609 & 3.9063\times \end{matrix}$$

Matrix Multiply by Wavelet transform after 2 resolutions ψ^n expansion is expressed in equation **??**.

$$\frac{1}{256}\begin{matrix} 7651 & 7651 & 7651 & 7651 & 7651 & 7651 & 7651 & 7651 \\ 7141 & 7141 & 7141 & 7141 & 7141 & 7141 & 7141 & 7141 \\ 7396 & 7141 & 7141 & 7141 & 7141 & 7141 & 7141 & 7141 \\ 7141 & 7396 & 7141 & 7141 & 7141 & 7141 & 7141 & 7141 \\ 7141 & 7141 & 7396 & 7141 & 7141 & 7141 & 7141 & 7141 \\ 7141 & 7141 & 7141 & 7396 & 7141 & 7141 & 7141 & 7141 \\ 7141 & 7141 & 7141 & 7141 & 7396 & 7141 & 7141 & 7141 \\ 6631 & 6631 & 6631 & 6631 & 6631 & 6886 & 6886 & 6886 \end{matrix} =$$

$$\begin{matrix} \frac{7651}{256} & \frac{7651}{256} & \frac{7651}{256} & \frac{7651}{256} & \frac{7651}{256} & \frac{7651}{256} & \frac{7651}{256} & \frac{7651}{256} \\ \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} \\ \frac{1849}{64} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} \\ \frac{7141}{256} & \frac{1849}{64} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} \\ \frac{7141}{256} & \frac{7141}{256} & \frac{1849}{64} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} \\ \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{1849}{64} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} \\ \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{1849}{64} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} \\ \frac{6631}{256} & \frac{6631}{256} & \frac{6631}{256} & \frac{6631}{256} & \frac{6631}{256} & \frac{3443}{128} & \frac{3443}{128} & \frac{3443}{128} \end{matrix} =$$

$$\begin{matrix} 29.887 & 29.887 & 29.887 & 29.887 & 29.887 & 29.887 & 29.887 & 29.887 \\ 27.895 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 \\ 28.891 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 \\ 27.895 & 28.891 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 \\ 27.895 & 27.895 & 28.891 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 \\ 27.895 & 27.895 & 27.895 & 28.891 & 27.895 & 27.895 & 27.895 & 27.895 \\ 27.895 & 27.895 & 27.895 & 27.895 & 28.891 & 27.895 & 27.895 & 27.895 \\ 25.902 & 25.902 & 25.902 & 25.902 & 25.902 & 26.898 & 26.898 & 26.898 \end{matrix}$$

2.3 Matrix Multiplication on 8×8 matrix with 3 resolutions

$$\frac{1}{256}\begin{matrix} 64 & 128 & 128 & 128 & 128 & 128 & 128 & 128 \\ 128 & 64 & 128 & 128 & 128 & 128 & 128 & 128 \\ 128 & 128 & 64 & 128 & 128 & 128 & 128 & 128 \\ 128 & 128 & 128 & 64 & 128 & 128 & 128 & 128 \\ 128 & 128 & 128 & 128 & 64 & 128 & 128 & 128 \\ 128 & 128 & 128 & 128 & 128 & 64 & 128 & 128 \\ 128 & 128 & 128 & 128 & 128 & 128 & 64 & 128 \\ 128 & 128 & 128 & 128 & 128 & 128 & 128 & 64 \end{matrix} = \begin{matrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} \end{matrix}$$

After three resolution of wavelet transforms, matrix $W^3(B)$ has fourteen of its elements with an epsilon of $\frac{1}{512}$. Another, seven within an epsilon of $\frac{3}{512}$. Most of the energy is in the second, third, fifth, and last rows. The remaining energy is located in the diagonal. Again nearly, $\frac{7}{32}$ of elements are not likely to contribute anything significant to this multiplication. $W^3(B)$ is shown in equation **??**.

Next step, the square of matrix $W^3(B)$ is $(W^3(B))^2$ and shown in equation **??**.

$$\frac{1}{256}\begin{matrix} 3826 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -63 & 192 & -63 & -63 & -63 & -63 & -63 & -63 \\ -63 & -63 & -63 & 192 & -63 & -63 & -63 & -63 \\ 0 & 0 & -255 & 1 & 0 & 0 & 0 & 1 \\ -63 & -63 & -63 & -63 & -191 & 64 & 64 & 64 \\ 0 & 1 & 0 & 1 & -127 & 128 & -127 & -127 \\ 0 & 1 & 0 & 1 & -127 & -127 & -127 & 128 \\ -63 & -63 & -63 & -63 & 64 & 64 & -191 & 64 \end{matrix}$$

Matrix Multiply by Conventional Method

$$\frac{1}{256}\begin{matrix} 7650 & 7650 & 7650 & 7650 & 7650 & 7650 & 7650 & 7650 \\ 7140 & 7140 & 7140 & 7140 & 7140 & 7140 & 7140 & 7140 \\ 7395 & 7140 & 7140 & 7140 & 7140 & 7140 & 7140 & 7140 \\ 7140 & 7395 & 7140 & 7140 & 7140 & 7140 & 7140 & 7140 \\ 7140 & 7140 & 7395 & 7140 & 7140 & 7140 & 7140 & 7140 \\ 7140 & 7140 & 7140 & 7395 & 7140 & 7140 & 7140 & 7140 \\ 7140 & 7140 & 7140 & 7140 & 7395 & 7140 & 7140 & 7140 \\ 6630 & 6630 & 6630 & 6630 & 6630 & 6885 & 6885 & 6885 \end{matrix} = \begin{matrix} \frac{3825}{128} & \frac{3825}{128} & \frac{3825}{128} & \frac{3825}{128} & \frac{3825}{128} & \frac{3825}{128} & \frac{3825}{128} & \frac{3825}{128} \\ \frac{1785}{64} & \frac{1785}{64} & \frac{1785}{64} & \frac{1785}{64} & \frac{1785}{64} & \frac{1785}{64} & \frac{1785}{64} & \frac{1785}{64} \\ \frac{7395}{256} & \frac{1785}{64} & \frac{1785}{64} & \frac{1785}{64} & \frac{1785}{64} & \frac{1785}{64} & \frac{1785}{64} & \frac{1785}{64} \\ \frac{1785}{64} & \frac{7395}{256} & \frac{1785}{64} & \frac{1785}{64} & \frac{1785}{64} & \frac{1785}{64} & \frac{1785}{64} & \frac{1785}{64} \\ \frac{1785}{64} & \frac{1785}{64} & \frac{7395}{256} & \frac{1785}{64} & \frac{1785}{64} & \frac{1785}{64} & \frac{1785}{64} & \frac{1785}{64} \\ \frac{1785}{64} & \frac{1785}{64} & \frac{1785}{64} & \frac{7395}{256} & \frac{1785}{64} & \frac{1785}{64} & \frac{1785}{64} & \frac{1785}{64} \\ \frac{1785}{64} & \frac{1785}{64} & \frac{1785}{64} & \frac{1785}{64} & \frac{7395}{256} & \frac{1785}{64} & \frac{1785}{64} & \frac{1785}{64} \\ \frac{3315}{128} & \frac{3315}{128} & \frac{3315}{128} & \frac{3315}{128} & \frac{3315}{128} & \frac{6885}{256} & \frac{6885}{256} & \frac{6885}{256} \end{matrix}$$

$$= \begin{matrix} 29.883 & 29.883 & 29.883 & 29.883 & 29.883 & 29.883 & 29.883 & 29.883 \\ 27.891 & 27.891 & 27.891 & 27.891 & 27.891 & 27.891 & 27.891 & 27.891 \\ 28.887 & 27.891 & 27.891 & 27.891 & 27.891 & 27.891 & 27.891 & 27.891 \\ 27.891 & 28.887 & 27.891 & 27.891 & 27.891 & 27.891 & 27.891 & 27.891 \\ 27.891 & 27.891 & 28.887 & 27.891 & 27.891 & 27.891 & 27.891 & 27.891 \\ 27.891 & 27.891 & 27.891 & 28.887 & 27.891 & 27.891 & 27.891 & 27.891 \\ 27.891 & 27.891 & 27.891 & 27.891 & 28.887 & 27.891 & 27.891 & 27.891 \\ 25.898 & 25.898 & 25.898 & 25.898 & 25.898 & 26.895 & 26.895 & 26.895 \end{matrix}$$

The inverse of $(W^3(B))^2$ is very close to that of B^2 and is shown in equation **??**.

$$\frac{1}{256}\begin{matrix} 7651 & 7651 & 7651 & 7651 & 7651 & 7651 & 7651 & 7651 \\ 7141 & 7141 & 7141 & 7141 & 7141 & 7141 & 7141 & 7141 \\ 7396 & 7141 & 7141 & 7141 & 7141 & 7141 & 7141 & 7141 \\ 7141 & 7396 & 7141 & 7141 & 7141 & 7141 & 7141 & 7141 \\ 7141 & 7141 & 7396 & 7141 & 7141 & 7141 & 7141 & 7141 \\ 7141 & 7141 & 7141 & 7396 & 7141 & 7141 & 7141 & 7141 \\ 7141 & 7141 & 7141 & 7141 & 7396 & 7141 & 7141 & 7141 \\ 6631 & 6631 & 6631 & 6631 & 6631 & 6886 & 6886 & 6886 \end{matrix} =$$

$$\begin{matrix} \frac{7651}{256} & \frac{7651}{256} & \frac{7651}{256} & \frac{7651}{256} & \frac{7651}{256} & \frac{7651}{256} & \frac{7651}{256} & \frac{7651}{256} \\ \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} \\ \frac{1849}{64} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} \\ \frac{7141}{256} & \frac{1849}{64} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} \\ \frac{7141}{256} & \frac{7141}{256} & \frac{1849}{64} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} \\ \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{1849}{64} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} \\ \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} & \frac{1849}{64} & \frac{7141}{256} & \frac{7141}{256} & \frac{7141}{256} \\ \frac{6631}{256} & \frac{6631}{256} & \frac{6631}{256} & \frac{6631}{256} & \frac{6631}{256} & \frac{3443}{128} & \frac{3443}{128} & \frac{3443}{128} \end{matrix} =$$

$$\begin{matrix} 29.887 & 29.887 & 29.887 & 29.887 & 29.887 & 29.887 & 29.887 & 29.887 \\ 27.895 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 \\ 28.891 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 \\ 27.895 & 28.891 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 \\ 27.895 & 27.895 & 28.891 & 27.895 & 27.895 & 27.895 & 27.895 & 27.895 \\ 27.895 & 27.895 & 27.895 & 28.891 & 27.895 & 27.895 & 27.895 & 27.895 \\ 27.895 & 27.895 & 27.895 & 27.895 & 28.891 & 27.895 & 27.895 & 27.895 \\ 25.902 & 25.902 & 25.902 & 25.902 & 25.902 & 26.898 & 26.898 & 26.898 \end{matrix}$$

3 Matrix Multiplication between Upper Triangular and Full Matrix

In this section, an upper triangular matrix A and a full matrix B are multiplied together both in conventional space and in wavelet space. Their results are compared for fidelity, and any notable discrepancies. The upper triangular matrix is the same A used in section **??** . The full matrix is the same B used in section **??**. There wavelet transforms are the same as in those sections as well. Here the results for each resolution is described for completeness. The matrix C is the product of A and B in conventional matrix multiplication and is provided for comparison in equation **??**.

$$A \cdot B = \begin{matrix} 6630 & 6630 & 6630 & 6630 & 6630 & 6630 & 7140 & 7140 \\ 6120 & 6120 & 6120 & 6120 & 6120 & 6120 & 6630 & 6630 \\ 5355 & 5100 & 5100 & 5100 & 5100 & 5100 & 5610 & 5610 \\ 4590 & 4335 & 4080 & 4080 & 4080 & 4080 & 4590 & 4590 \\ 3570 & 3570 & 3315 & 3060 & 3060 & 3060 & 3570 & 3570 \\ 2550 & 2550 & 2550 & 2295 & 2040 & 2040 & 2550 & 2550 \\ 1530 & 1530 & 1530 & 1530 & 1275 & 1020 & 1530 & 1530 \\ 1020 & 1020 & 1020 & 1020 & 1020 & 765 & 765 & 765 \end{matrix}$$

3.1 One resolution case

With one resolution, relative fidelity is retained within $9.8 \cdot 10^{-15}$. The result of $W(A) \cdot W(B)$ is shown in equation **??**.

$$W(A) \cdot W(B) = \begin{matrix} 12751 & 12751 & 12751 & 13771 & 0 & 0 & 0 & 0 \\ 9691 & 9181 & 9181 & 10201 & -255 & 0 & 0 & 0 \\ 6121 & 5611 & 5101 & 6121 & 1 & -255 & 0 & 0 \\ 2551 & 2551 & 2041 & 2296 & 0 & 1 & -255 & 0 \\ -509 & -509 & -509 & -509 & 0 & 1 & 1 & 0 \\ -764 & -1020 & -1020 & -1020 & 0 & 0 & 1 & 0 \\ -1020 & -765 & -1020 & -1020 & 0 & 0 & 0 & 0 \\ -510 & -510 & -255 & -765 & 0 & 0 & 0 & 0 \end{matrix}$$

The result is shown in equation **??** .

$$W^{-1}(W(A) \cdot W(B)) = \begin{matrix} 6631 & 6631 & 6631 & 6631 & 6631 & 6631 & 7141 & 7141 \\ 6121 & 6121 & 6121 & 6121 & 6121 & 6121 & 6631 & 6631 \\ 5356 & 5101 & 5101 & 5101 & 5101 & 5101 & 5611 & 5611 \\ 4591 & 4336 & 4081 & 4081 & 4081 & 4081 & 4591 & 4591 \\ 3571 & 3571 & 3316 & 3061 & 3061 & 3061 & 3571 & 3571 \\ 2551 & 2551 & 2551 & 2296 & 2041 & 2041 & 2551 & 2551 \\ 1531 & 1531 & 1531 & 1531 & 1276 & 1021 & 1531 & 1531 \\ 1021 & 1021 & 1021 & 1021 & 1021 & 766 & 766 & 766 \end{matrix}$$

3.2 Two resolution case

With two resolutions, relative fidelity stays at $1.4 \cdot 10^{-14}$. The result of $W(A) \cdot W(B)$ is shown in equation **??**.

$$W(A) \cdot W(B) = \begin{matrix} 2218622951 - 1271 - 25510201280 \\ 84167778 - 127 - 127 - 255638 - 127128 \\ -1402 - 153000 - 127111 \\ -1402 - 153000128 - 25501 \\ -3315 - 3570 - 1271 - 25511280 \\ -3315 - 3442128 - 127256 - 382128128 \\ -382 - 51000 - 127101 \\ 38351100 - 127 - 25501 \end{matrix}$$

The result is shown in equation **??** .

$$W^{-1}(W(A) \cdot W(B)) = \begin{matrix} 6631 & 6631 & 6631 & 6631 & 6631 & 6631 & 7141 & 7141 \\ 6121 & 6121 & 6121 & 6121 & 6121 & 6121 & 6631 & 6631 \\ 5356 & 5101 & 5101 & 5101 & 5101 & 5101 & 5611 & 5611 \\ 4591 & 4336 & 4081 & 4081 & 4081 & 4081 & 4591 & 4591 \\ 3571 & 3571 & 3316 & 3061 & 3061 & 3061 & 3571 & 3571 \\ 2551 & 2551 & 2551 & 2296 & 2041 & 2041 & 2551 & 2551 \\ 1531 & 1531 & 1531 & 1531 & 1276 & 1021 & 1531 & 1531 \\ 1021 & 1021 & 1021 & 1021 & 1021 & 766 & 766 & 766 \end{matrix}$$

3.3 Three resolution case

With three resolutions, relative fidelity stays at $2.3 \cdot 10^{-14}$. The result of $W(A) \cdot W(B)$ is shown in equation **??**.

$$W(A) \cdot W(B) = \begin{array}{cccccccc} 30664 & -191 & 574 & 64 & 64 & 64 & 1084 & 64 \\ -2932 & 0 & -127 & 1 & -127 & 0 & -127 & 1 \\ -6821 & -63 & -191 & 192 & -191 & -63 & -191 & -63 \\ 0 & 0 & -255 & 1 & 0 & 1 & 0 & 1 \\ -14471 & -63 & -191 & -63 & -701 & -63 & -191 & 192 \\ 0 & 0 & 0 & 1 & 1 & 0 & -255 & 1 \\ 64 & 64 & 64 & 64 & 64 & -191 & -446 & 64 \\ 893 & 0 & -127 & 1 & 128 & 0 & -127 & 1 \end{array}$$

The result is shown in equation **??** .

$$W^{-1}(W(A) \cdot W(B)) = \begin{array}{cccccccc} 6631 & 6631 & 6631 & 6631 & 6631 & 6631 & 7141 & 7141 \\ 6121 & 6121 & 6121 & 6121 & 6121 & 6121 & 6631 & 6631 \\ 5356 & 5101 & 5101 & 5101 & 5101 & 5101 & 5611 & 5611 \\ 4591 & 4336 & 4081 & 4081 & 4081 & 4081 & 4591 & 4591 \\ 3571 & 3571 & 3316 & 3061 & 3061 & 3061 & 3571 & 3571 \\ 2551 & 2551 & 2551 & 2296 & 2041 & 2041 & 2551 & 2551 \\ 1531 & 1531 & 1531 & 1531 & 1276 & 1021 & 1531 & 1531 \\ 1021 & 1021 & 1021 & 1021 & 1021 & 766 & 766 & 766 \end{array}$$

4 Conclusion

For these 8×8 examples, the product matrix in either conventional space or recovered from wavelet space are very close. In all cases, relative fidelity is maintained within the order of 10^{-14}. More examples can be obtained at larger levels. Examples for this are images at 512 and 768×768 which are also matrices.