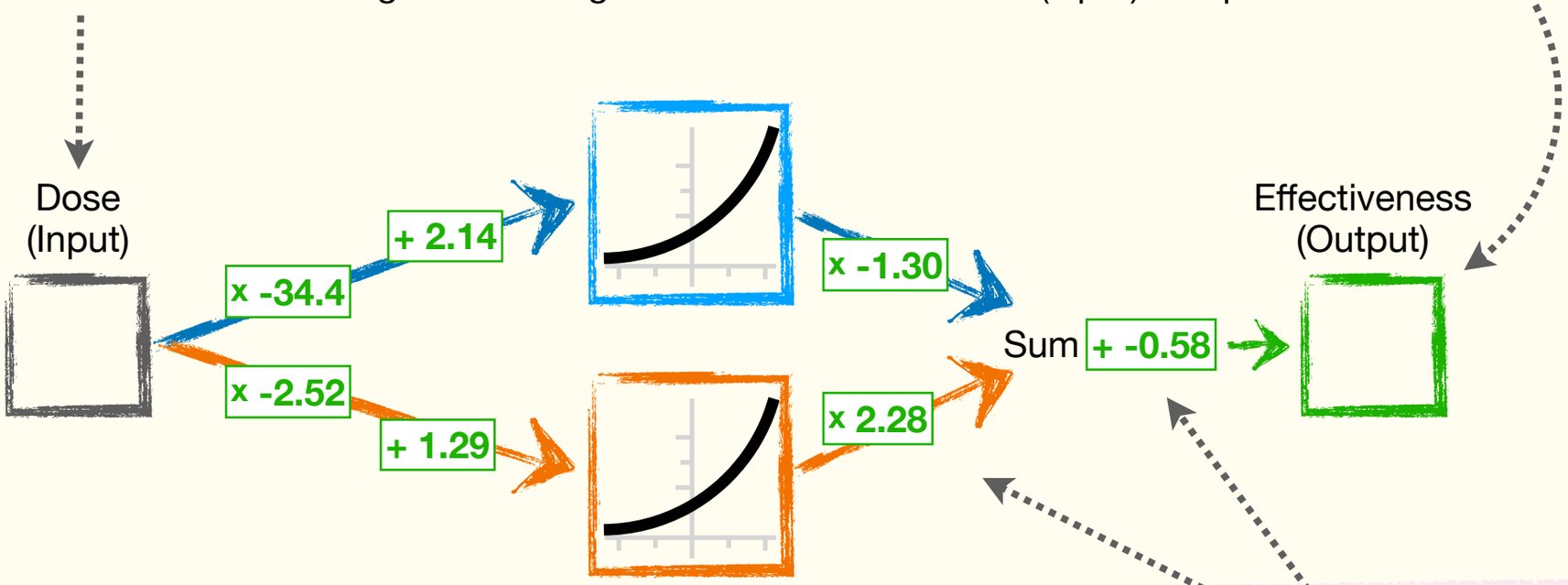
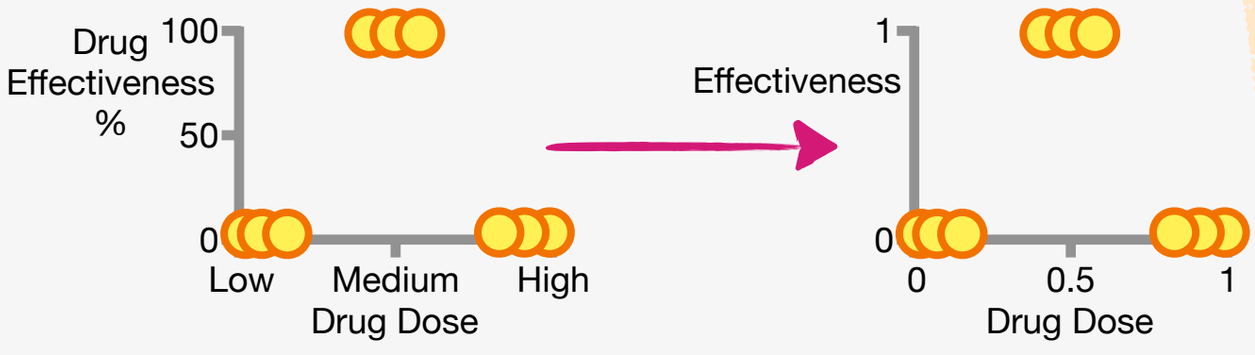
A Neural Network in Action: Step-by-Step

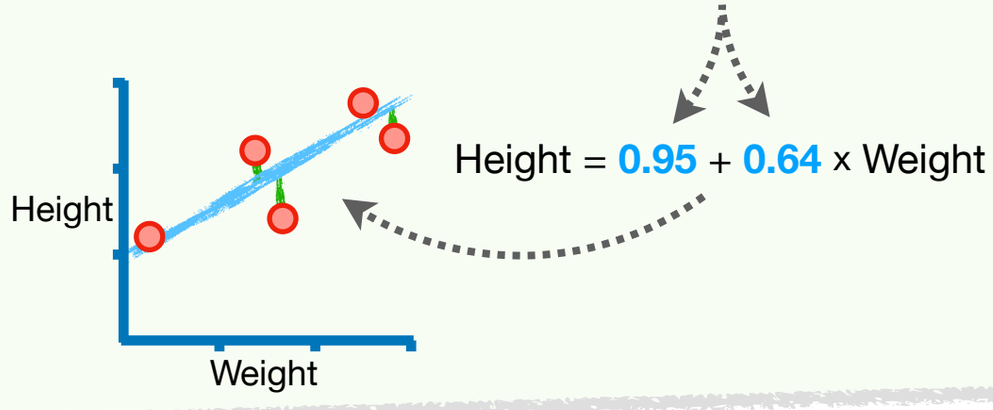
1 A lot of people say that **Neural Networks** are black boxes and that it's difficult to understand what they're doing. Unfortunately, this is true for big, fancy **Neural Networks**. However, the good news is that it's *not* true for simple ones. So, let's walk through how this simple **Neural Network** works, one step at a time, by plugging in Dose values from low to high and seeing how it converts the Dose (input) into predicted Effectiveness (output).



2 **NOTE:** To keep the math in this section simple, let's scale both the x- and y-axes so that they go from **0**, for *low* values, to **1**, for *high* values.

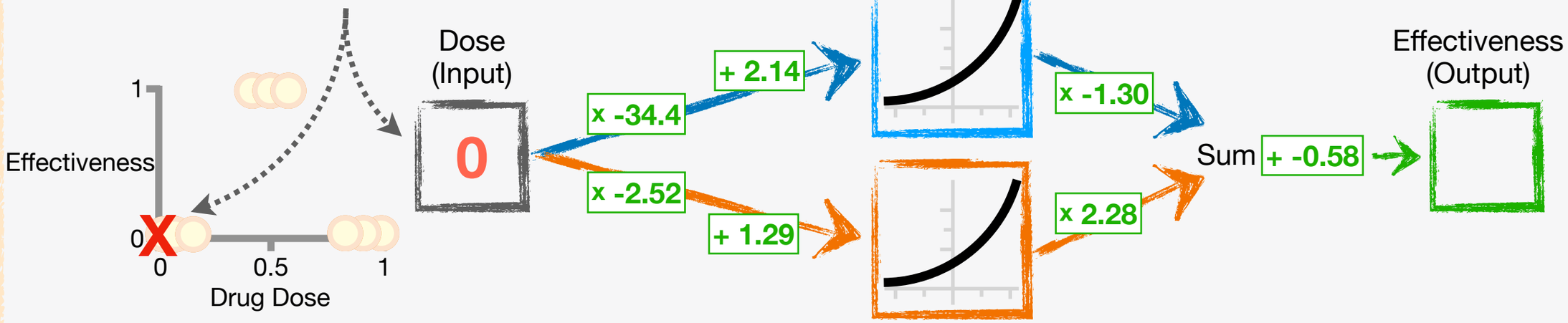


3 **ALSO NOTE:** These numbers are *parameters* that are estimated using a method called **Backpropagation**, and we'll talk about how that works, step-by-step, later. For now, just assume that these values have already been optimized, much like the **slope** and **intercept** are optimized when we fit a line to data.



A Neural Network in Action: Step-by-Step

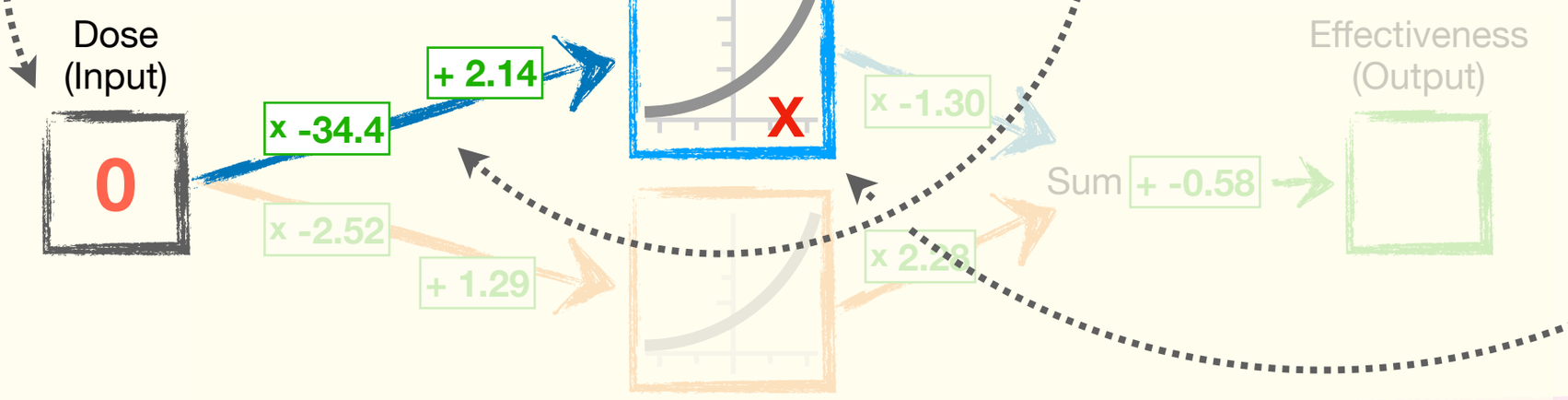
4 The first thing we do is plug the lowest Dose, **0**, into the **Neural Network**.



5 Now, the connection from the **Input Node**… …to the *top* **Node** in the **Hidden Layer**… …multiplies the Dose by **-34.4** and then adds **2.14** … …to get a new x-axis coordinate for the **Activation Function**, **2.14**.

(**Dose** x -34.4) + 2.14 = x-axis coordinate

(**0** x -34.4) + 2.14 = x-axis coordinate = 2.14



A Neural Network in Action: Step-by-Step

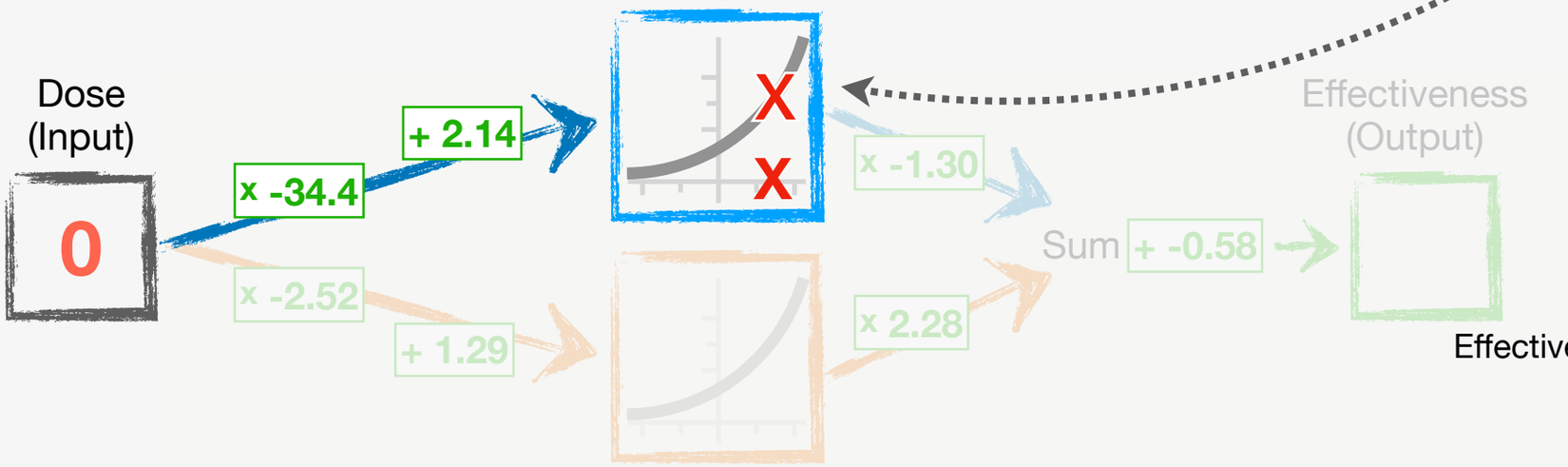
6 Then we plug the x-axis coordinate, **2.14** into the **SoftPlus Activation Function**…

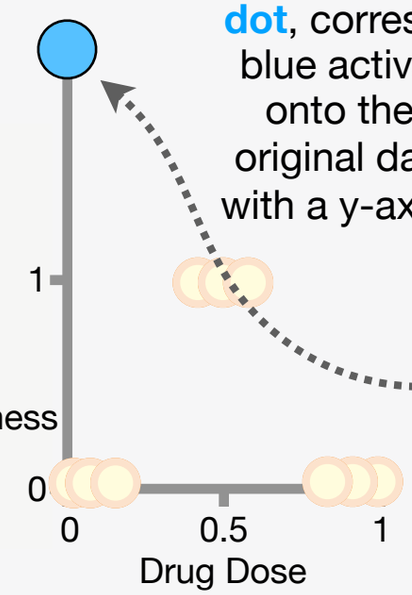
…and do the math, just like we saw earlier…

…and we get **2.25**. So, when the x-axis coordinate is **2.14**, the corresponding y-axis value is **2.25**…

…thus, when Dose **= 0**, the output for the **SoftPlus** is **2.25**. So, let's put a **blue dot**, corresponding to the blue activation function, onto the graph of the original data at Dose **= 0** with a y-axis value = **2.25**.

SoftPlus(x) = **SoftPlus**(2.14) = **log**(1 + $e^{2.14}$) = y-axis coordinate = **2.25**





7 Now let's increase the Dose to **0.2**…

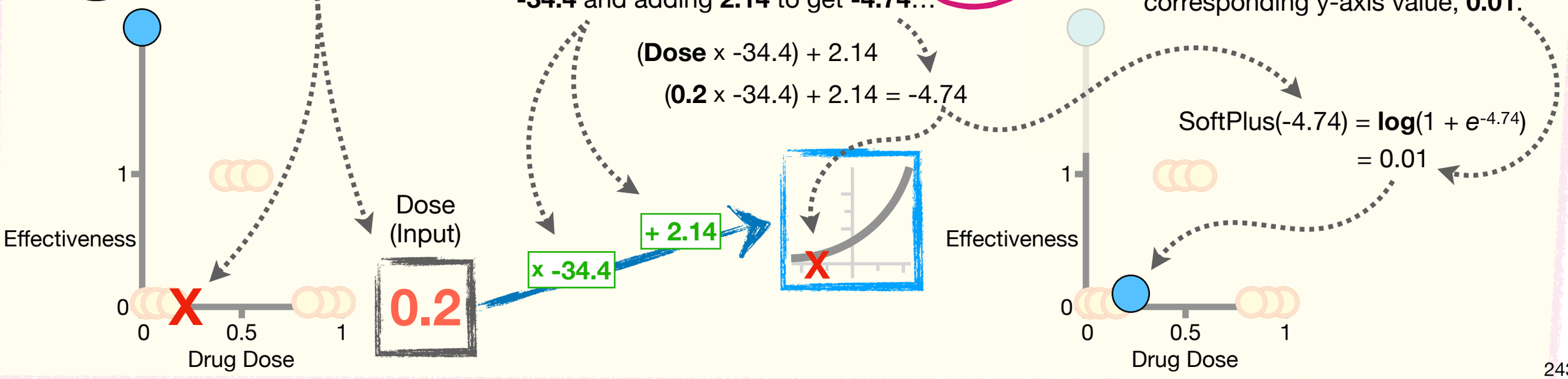
…and calculate the new x-axis coordinate by multiplying the Dose by **-34.4** and adding **2.14** to get **-4.74**…

(**Dose** x -34.4) + 2.14

(**0.2** x -34.4) + 2.14 = -4.74

…and plug **-4.74** into the **SoftPlus Activation Function** to get the corresponding y-axis value, **0.01**.

SoftPlus(-4.74) = **log**(1 + $e^{-4.74}$) = 0.01



A Neural Network in Action: Step-by-Step

8 Now let's increase the Dose to **0.4**…

…and calculate the new x-axis coordinate by multiplying the Dose by **-34.4** and adding **2.14** to get **-11.6**…

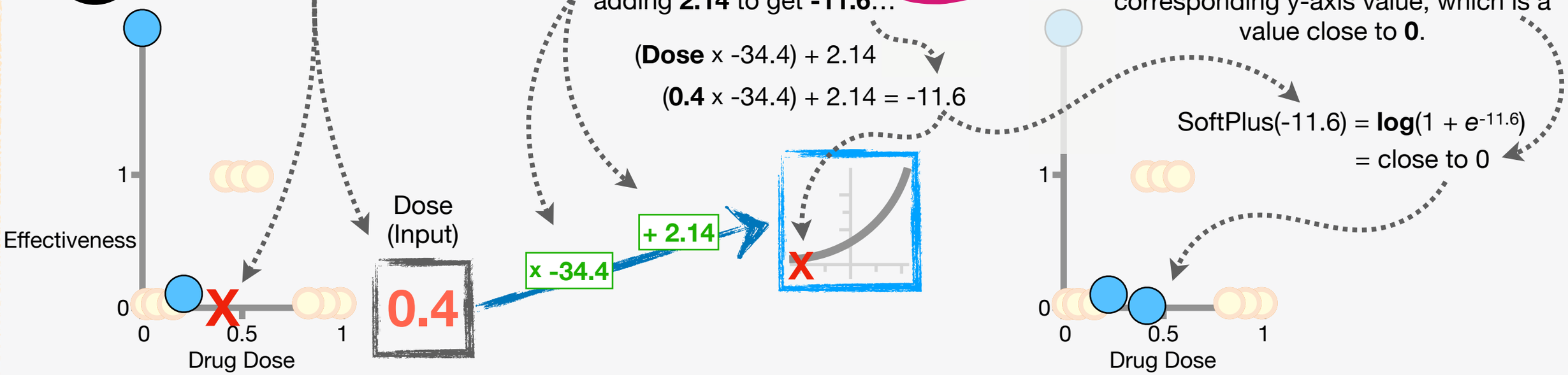
(**Dose** x -34.4) + 2.14

(**0.4** x -34.4) + 2.14 = -11.6

…and plug **-11.6** into the **SoftPlus Activation Function** to get the corresponding y-axis value, which is a value close to **0**.

SoftPlus(-11.6) = **log**(1 + $e^{-11.6}$)

= close to 0



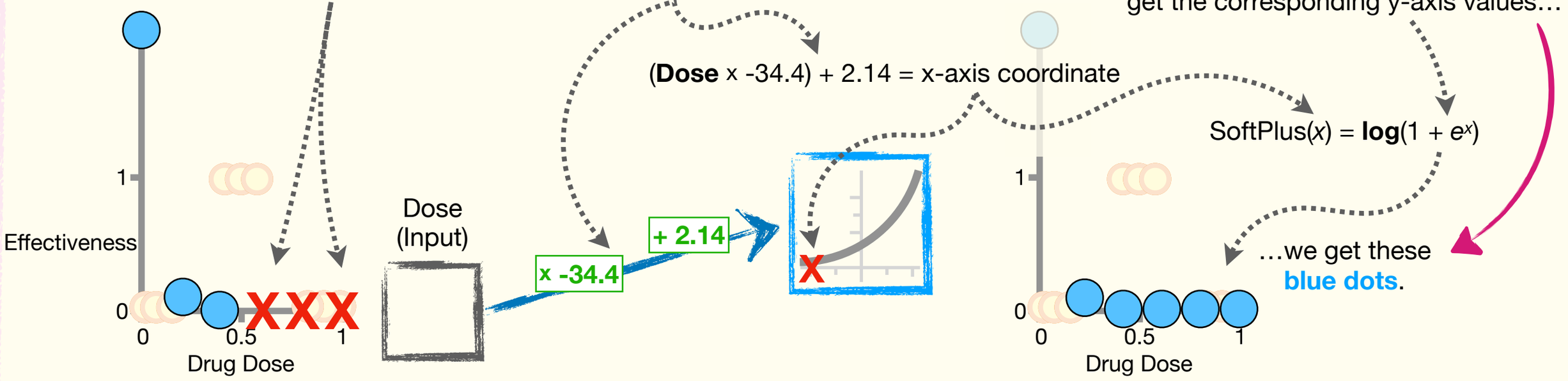
9 And if we continue to increase the Dose all the way to **1** (the maximum Dose)…

…and calculate the new x-axis coordinates by multiplying the Dose by **-34.4** and adding **2.14**…

…and plug the x-axis coordinates into the **SoftPlus Activation Function** to get the corresponding y-axis values…

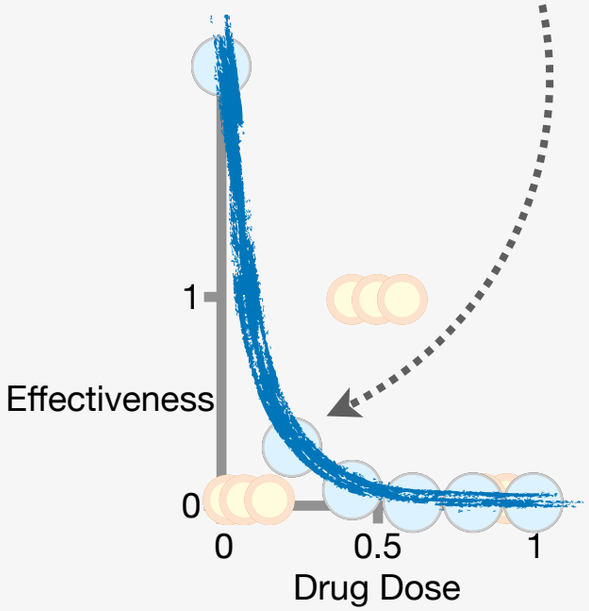
(**Dose** x -34.4) + 2.14 = x-axis coordinate

SoftPlus(x) = **log**(1 + e^{x})

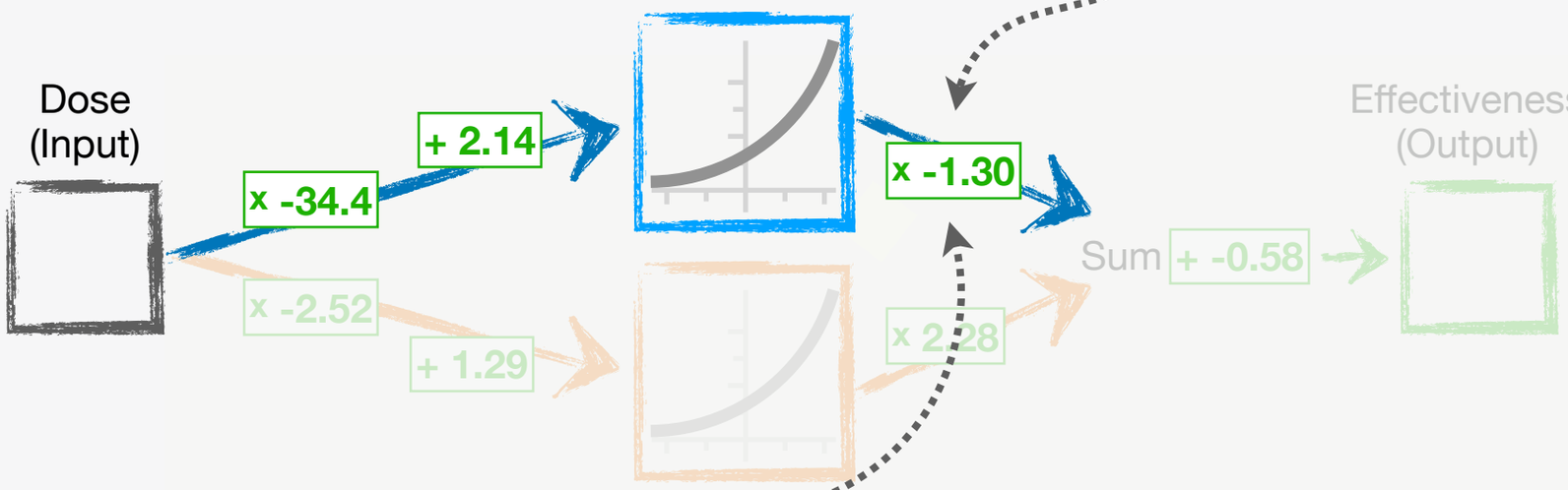


A Neural Network in Action: Step-by-Step

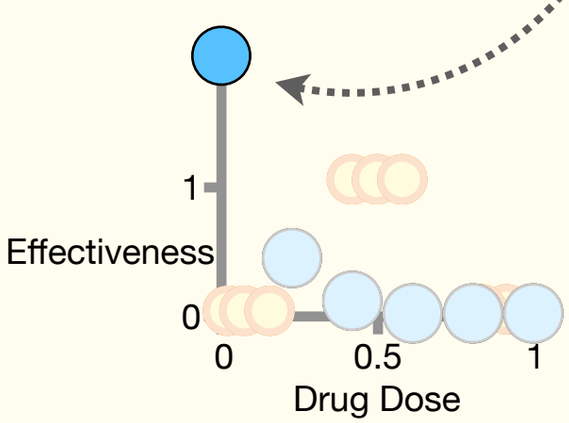
10 Ultimately, the **blue dots** result in this **blue curve**…



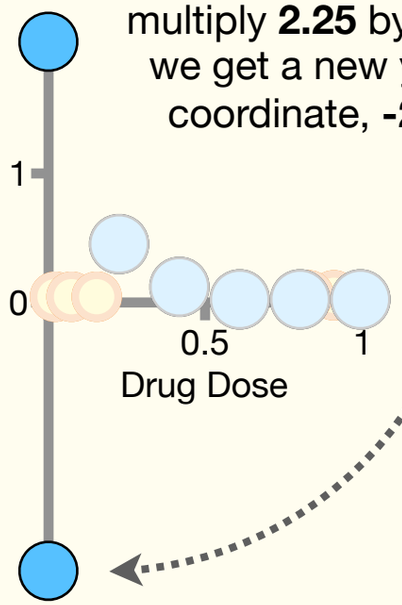
…and the next step in the **Neural Network** tells us to multiply the y-axis coordinates on the **blue curve** by **-1.30**.



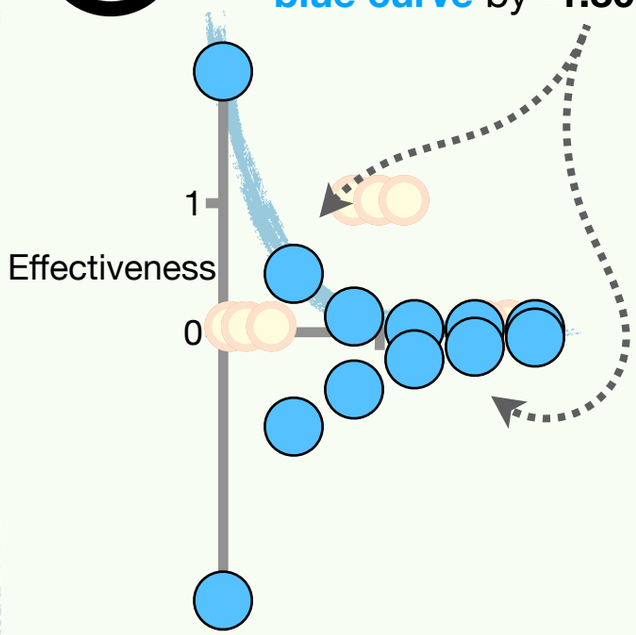
11 For example, when Dose = **0**, the current y-axis coordinate on the **blue curve** is **2.25**…



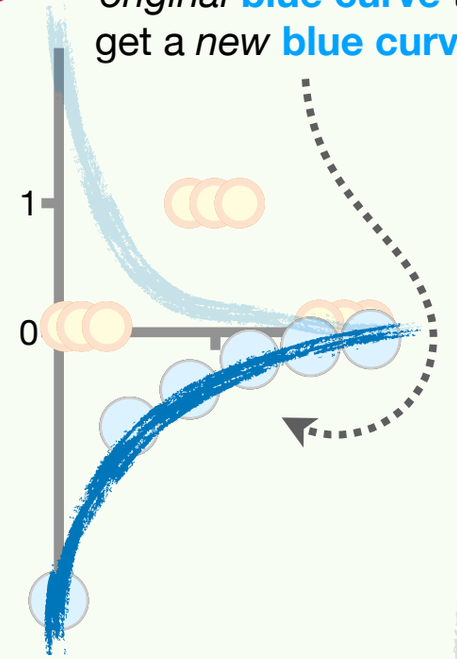
…and when we multiply **2.25** by **-1.30**, we get a new y-axis coordinate, **-2.93**.



12 Likewise, when we multiply all of the y-axis coordinates on the **blue curve** by **-1.30**…



…we end up flipping and stretching the *original* **blue curve** to get a *new* **blue curve**.



A Neural Network in Action: Step-by-Step

13 Okay, we just finished a major step, so this is a good time to review what we've done so far.

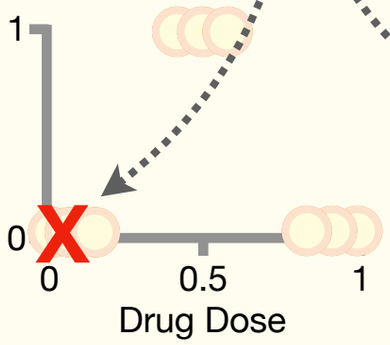
First, we plugged Dose values, ranging from **0** to **1**, into the input…

…then we transformed the Doses by multiplying them by **-34.4** and adding **2.14**…

…then the **SoftPlus Activation Function** converted the transformed Doses into y-axis coordinates…

…and lastly, we transformed the y-axis coordinates by multiplying them by **-1.30**.

BAM!!!



Dose (Input)

x -34.4

+ 2.14

x -2.52

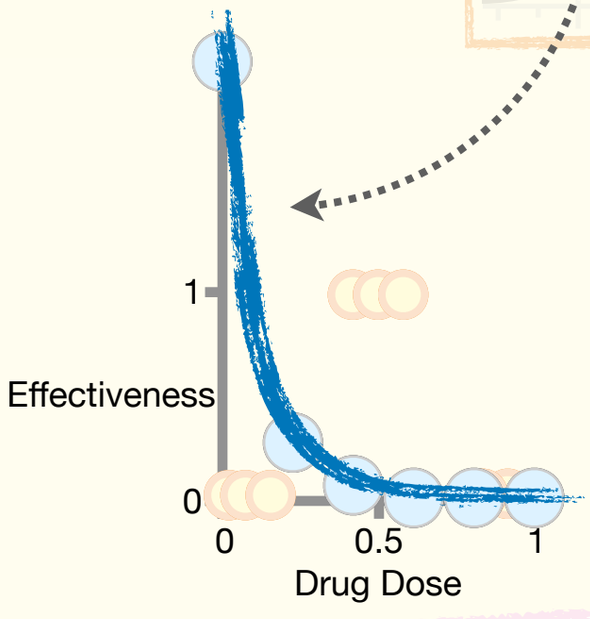
+ 1.29

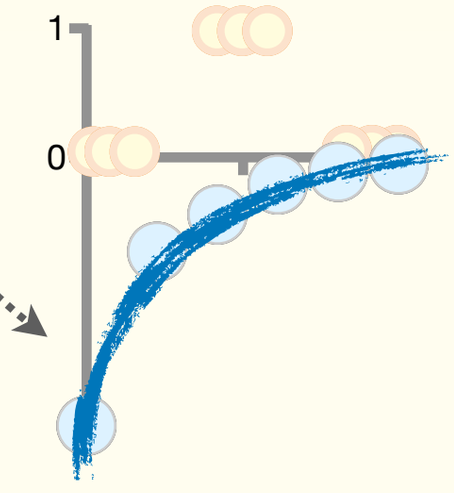
x -1.30

x 2.28

Sum + -0.58

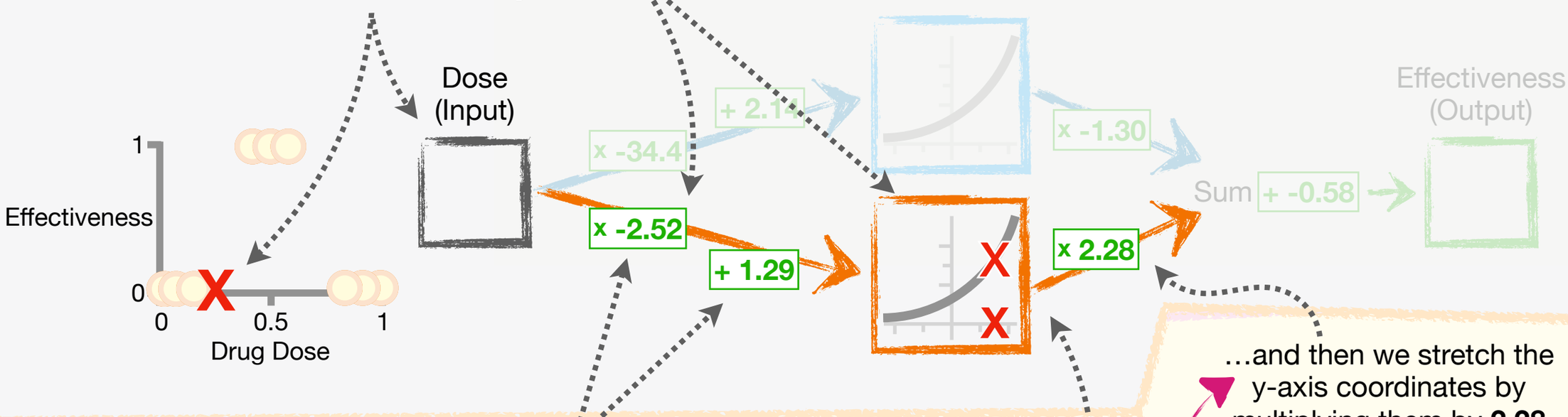
Effectiveness (Output)



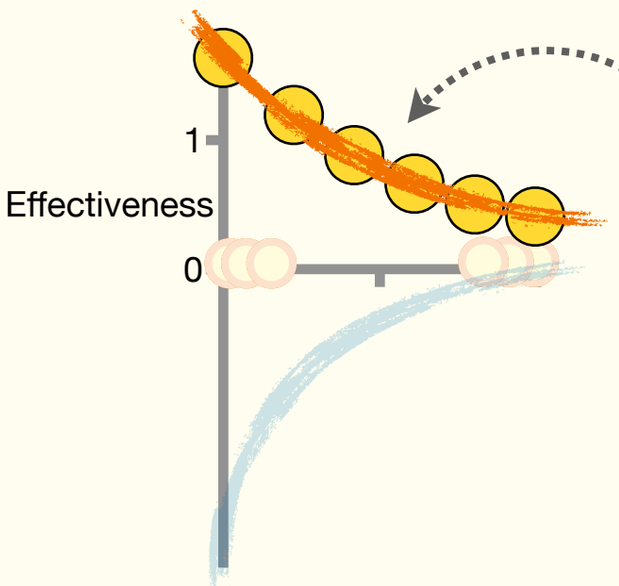


A Neural Network in Action: Step-by-Step

14 Now we run Dose values through the connection to the *bottom* **Node** in the **Hidden Layer**.

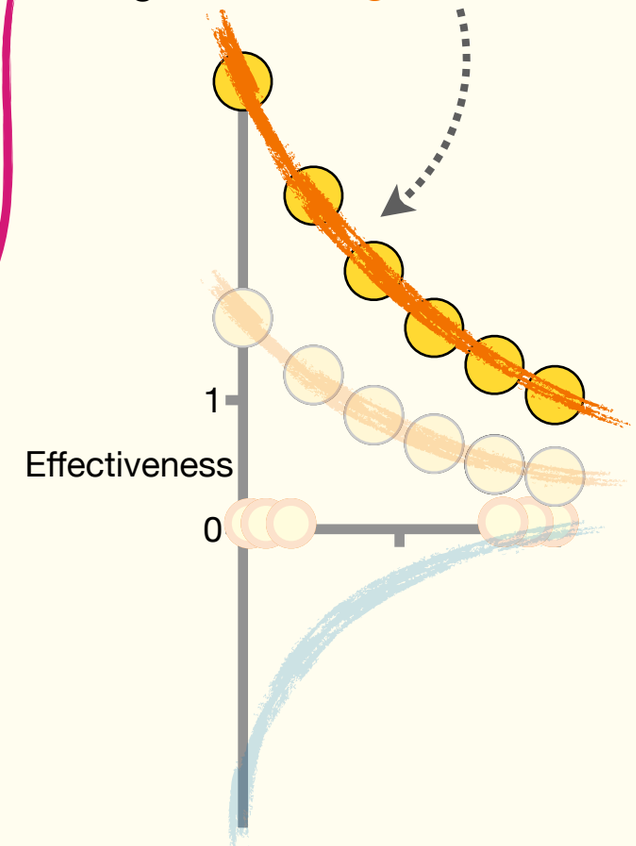


15 The good news is that the only difference between what we just did and what we're doing now is that now we multiply the Doses by **-2.52** and add **1.29**…



…before using the **SoftPlus Activation Function** to convert the transformed Doses into y-axis coordinates…

…and then we stretch the y-axis coordinates by multiplying them by **2.28** to get this **orange curve**.

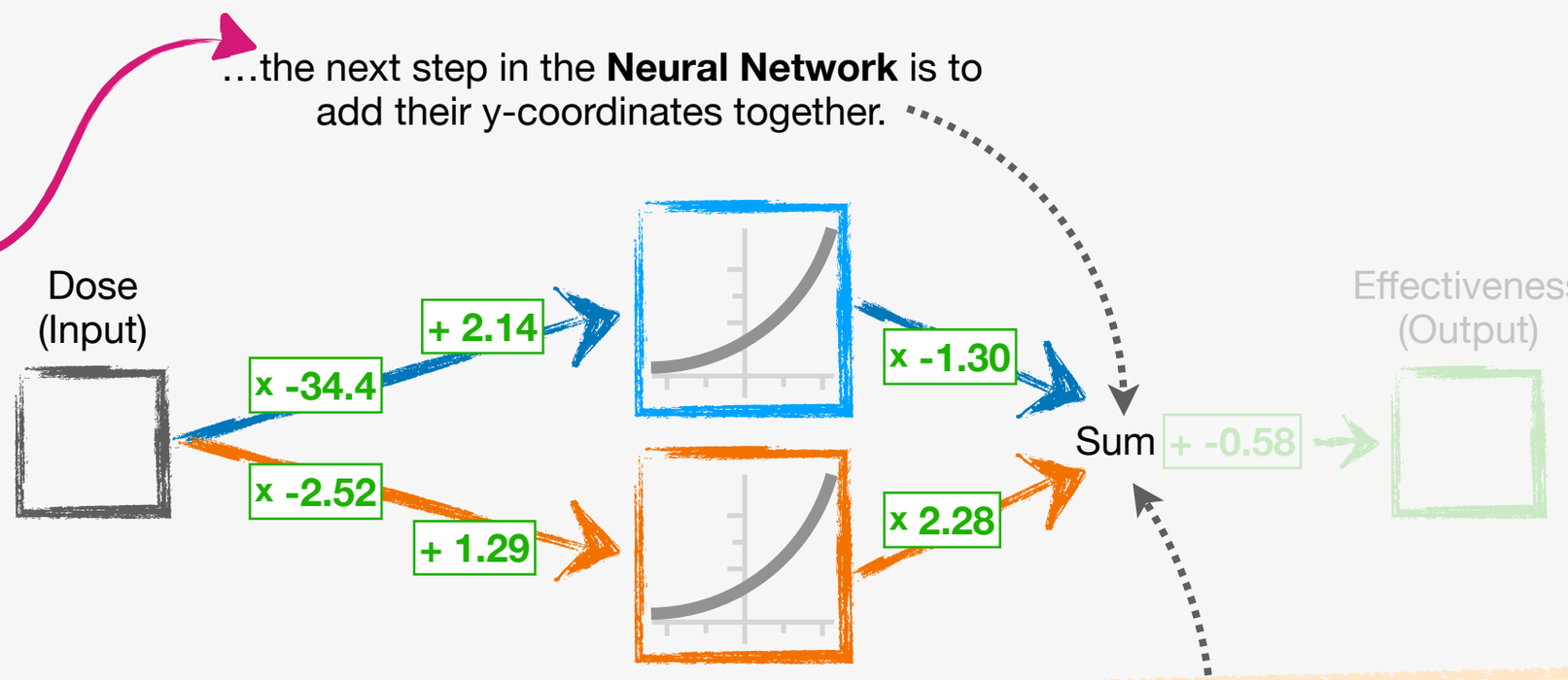


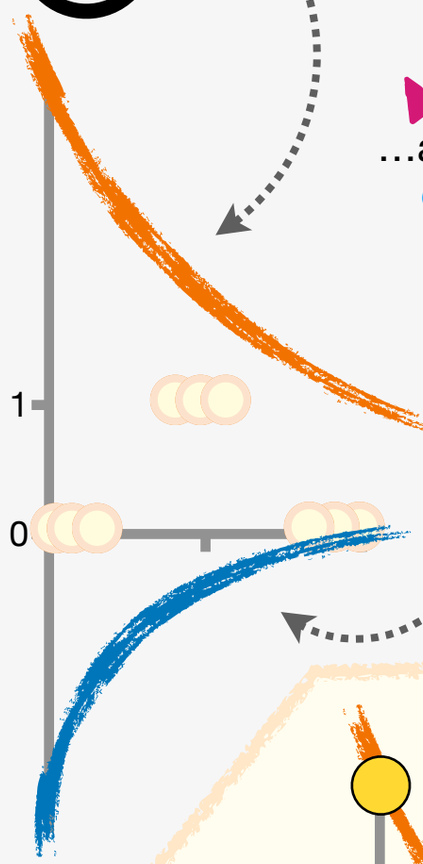
A Neural Network in Action: Step-by-Step

16 So, now that we have an **orange curve**…

…and a **blue curve**…

…the next step in the **Neural Network** is to add their y-coordinates together.

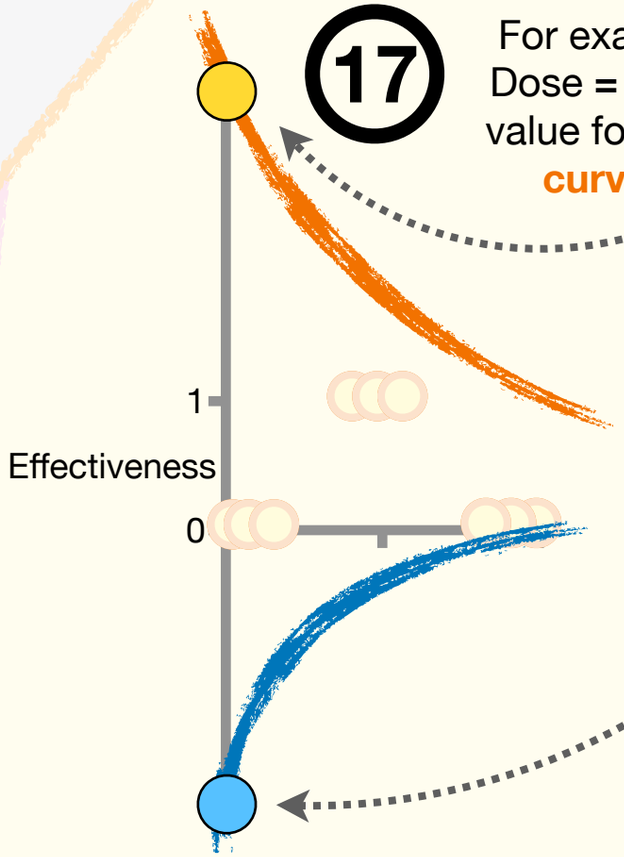


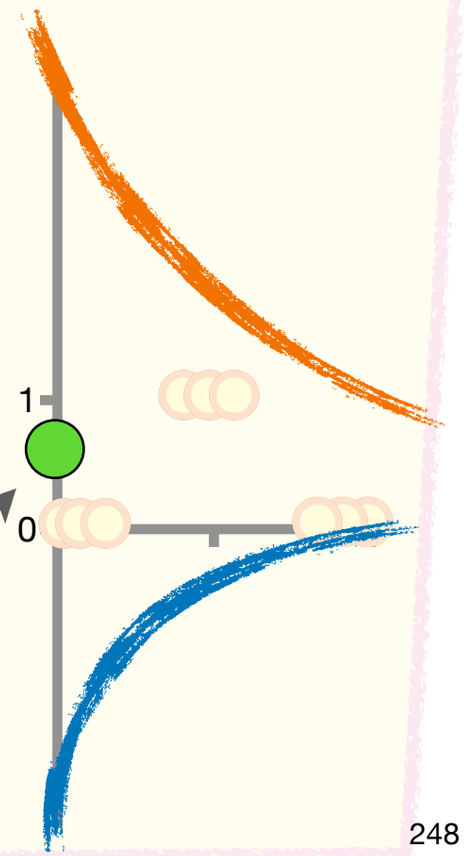


17 For example, when Dose = **0**, the y-axis value for the **orange curve** is **3.5**…

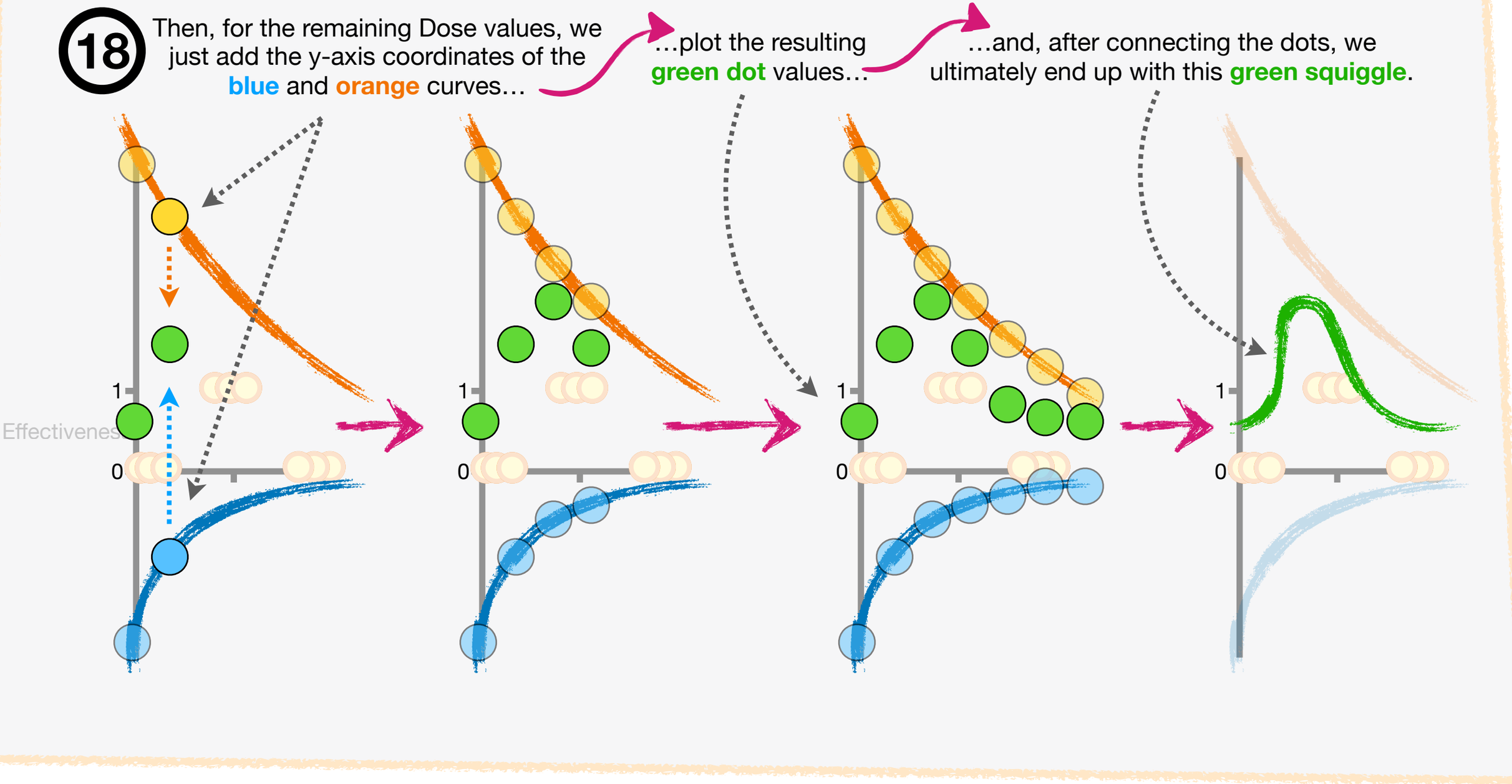
…and the y-axis value for **blue curve** is **-2.9**…

…thus, when Dose = **0**, the new y-axis value is the sum of the **orange** and **blue** y-axis values, **3.5 + -2.9 = 0.6**. We'll keep track of that **0.6** value by putting a **green dot** here.



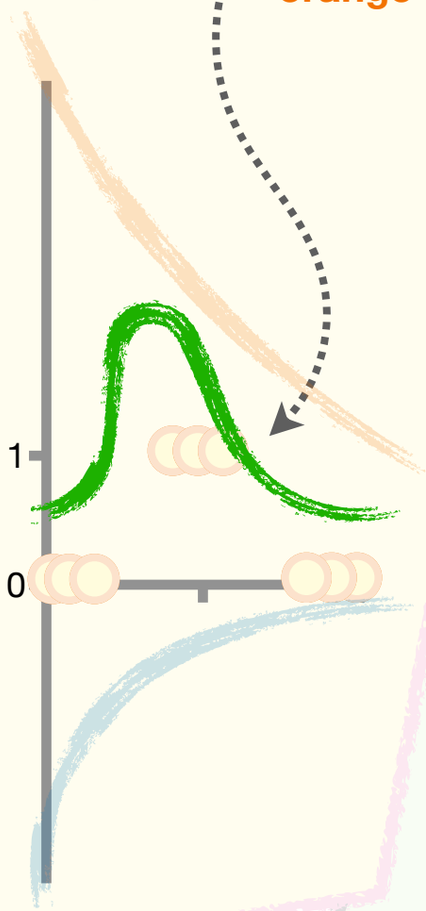


A Neural Network in Action: Step-by-Step

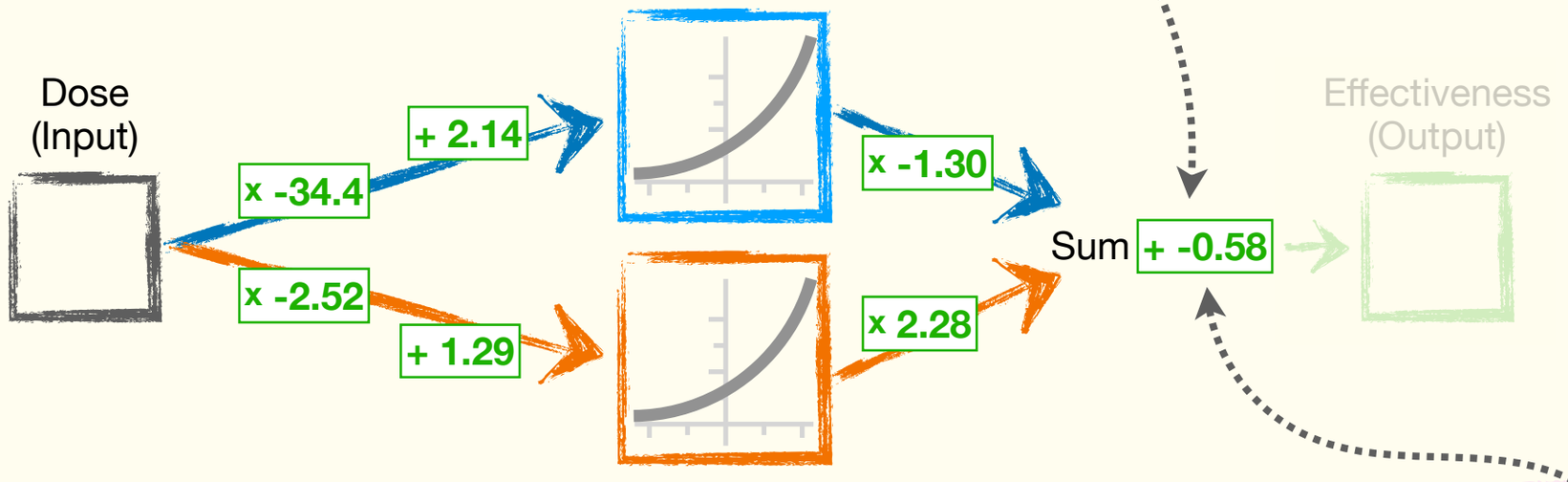


A Neural Network in Action: Step-by-Step

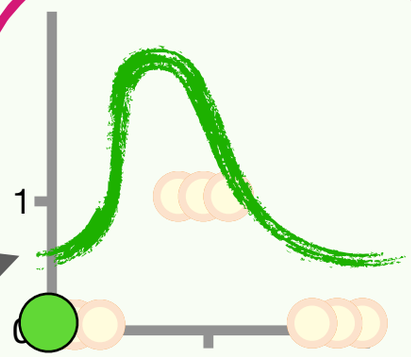
19 Now that we have this **green squiggle** from the **blue** and **orange** curves…



…we're ready for the final step, which is to add **-0.58** to each y-axis value on the **green squiggle**.

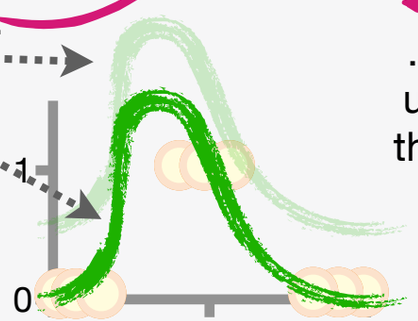


20 For example, when Dose **= 0**, the y-axis coordinate on the **green squiggle** starts out at **0.6**…

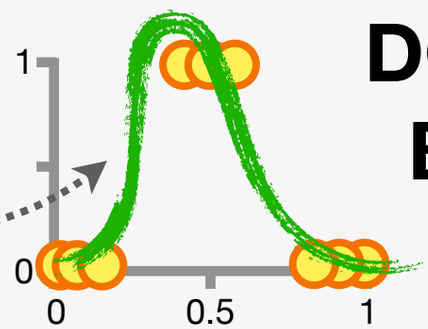


…but after subtracting **0.58**, the new y-axis coordinate for when Dose **= 0** is **0.0** (rounded to the nearest **tenth**).

21 Likewise, subtracting **0.58** from all of the other y-axis coordinates on the **green squiggle** shifts it down…



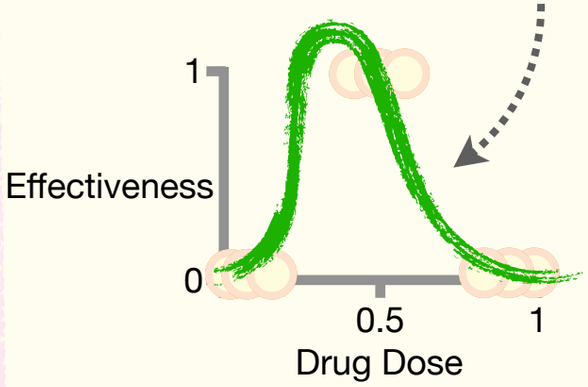
…and, ultimately, we end up with a **green squiggle** that fits the **Training Data**.



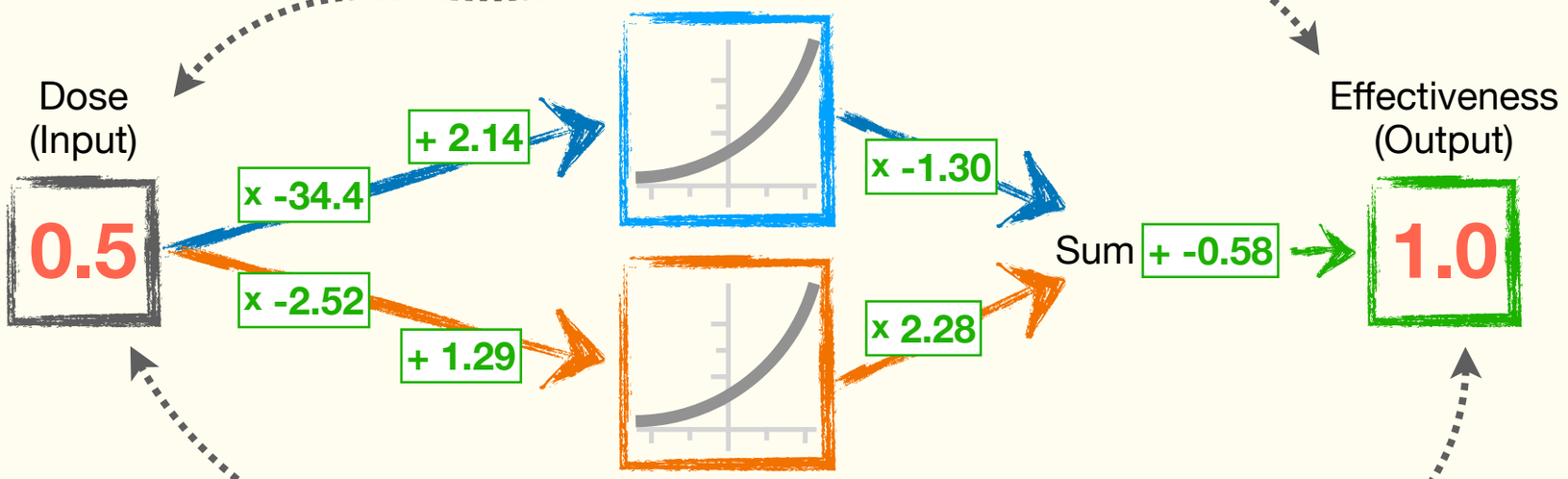
DOUBLE BAM!!!

A Neural Network in Action: Step-by-Step

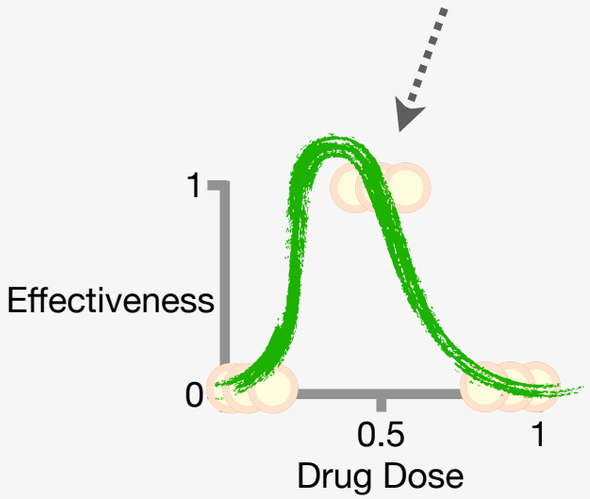
22 Hooray!!! At long last, we see the final **green squiggle**…



…that this **Neural Network** uses to predict **Effectiveness** from Doses.

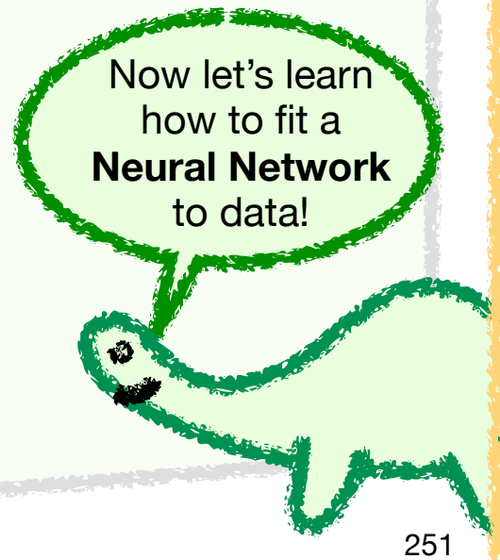


23 Now, if we're wondering if a medium Dose of **0.5** will be effective, we can look at the **green squiggle** and see that the output from the **Neural Network** will be **1**, and thus, Dose **= 0.5** will be effective.



24 Alternatively, if we plug **Dose = 0.5** into the **Neural Network** and do the math, we get **1**, and thus, Dose **= 0.5** will be effective.

TRIPLE BAM!!!



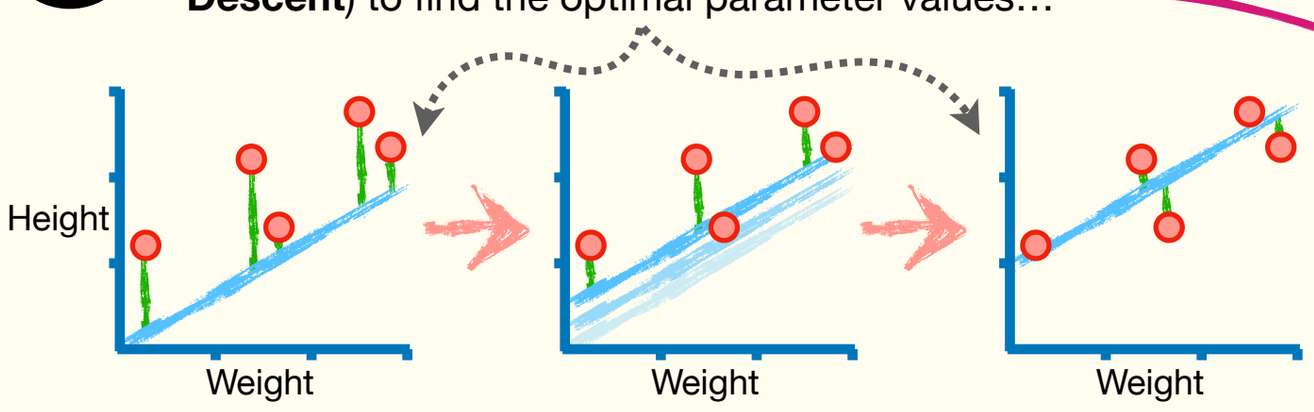
Neural Networks
Part Deux:

Fitting a Neural Network to Data with Backpropagation

Backpropagation: Main Ideas

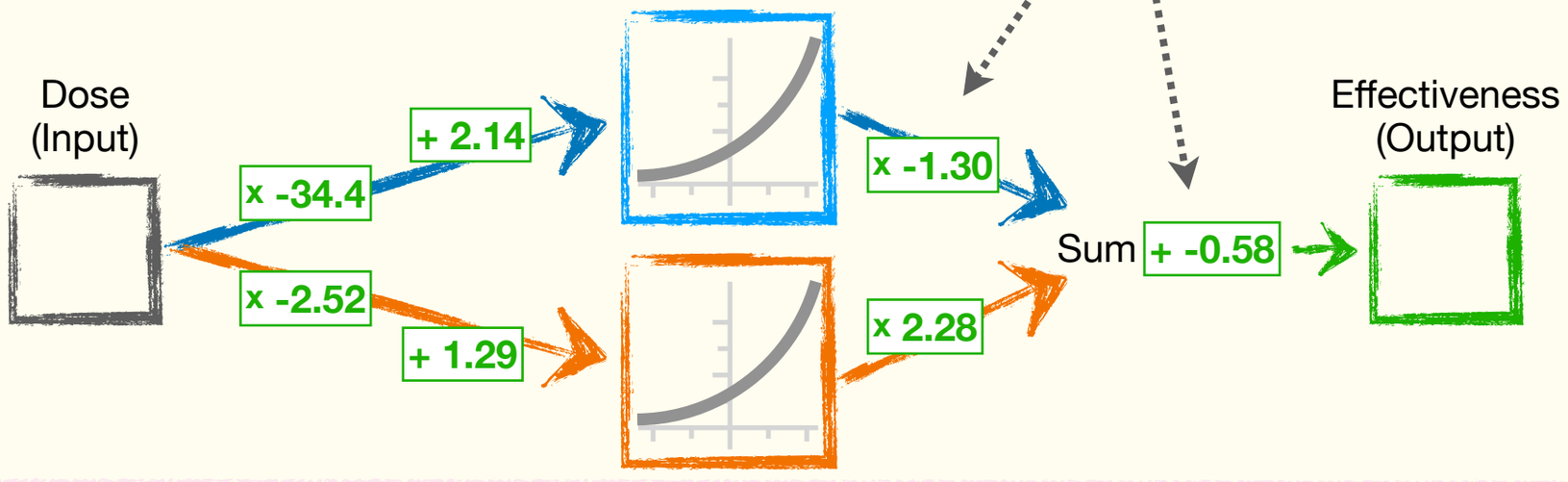
1 **The Problem:** Just like for **Linear Regression**, **Neural Networks** have parameters that we need to optimize to fit a squiggle or bent shape to data. How do we find the optimal values for these parameters?

2 **A Solution:** Just like for **Linear Regression**, we can use **Gradient Descent** (or **Stochastic Gradient Descent**) to find the optimal parameter values…

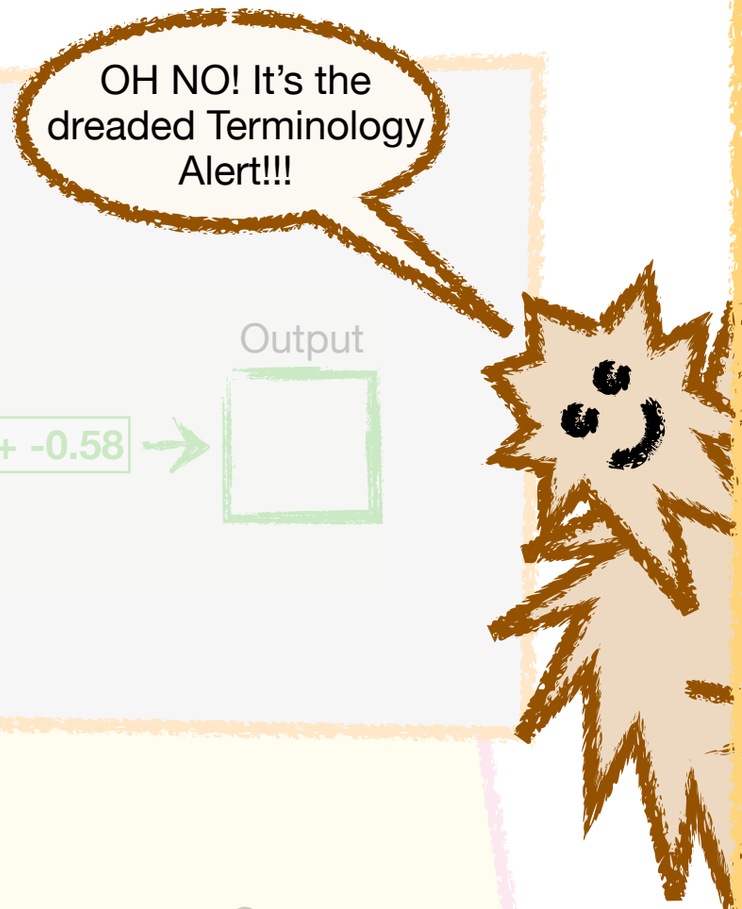


…however, we don't call it **Gradient Descent**. That would be too easy. Instead, because of how the derivatives are found for each parameter in a **Neural Network** (from the back to the front), we call it **Backpropagation**.

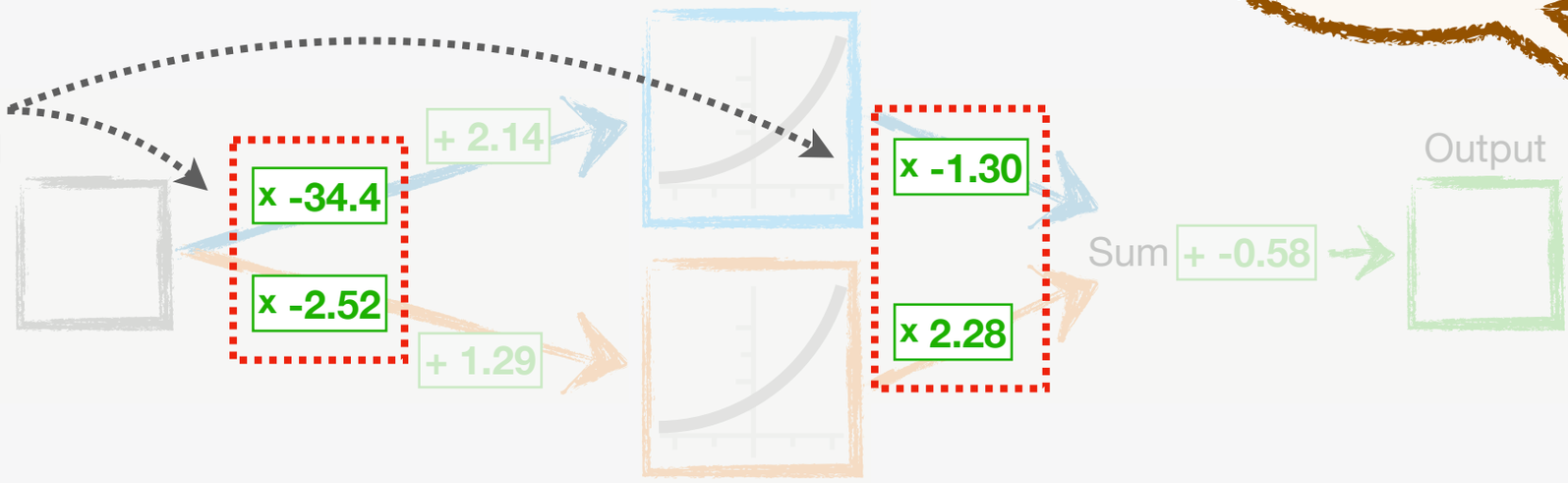
BAM!!!



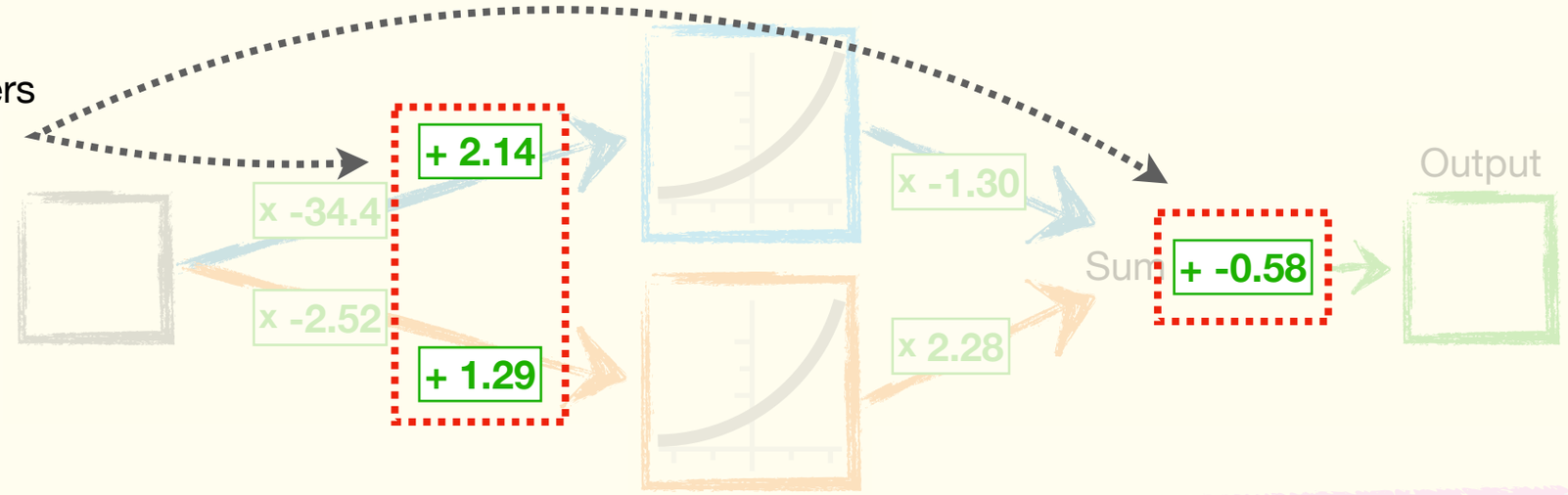
Terminology Alert!!! Weights and Biases

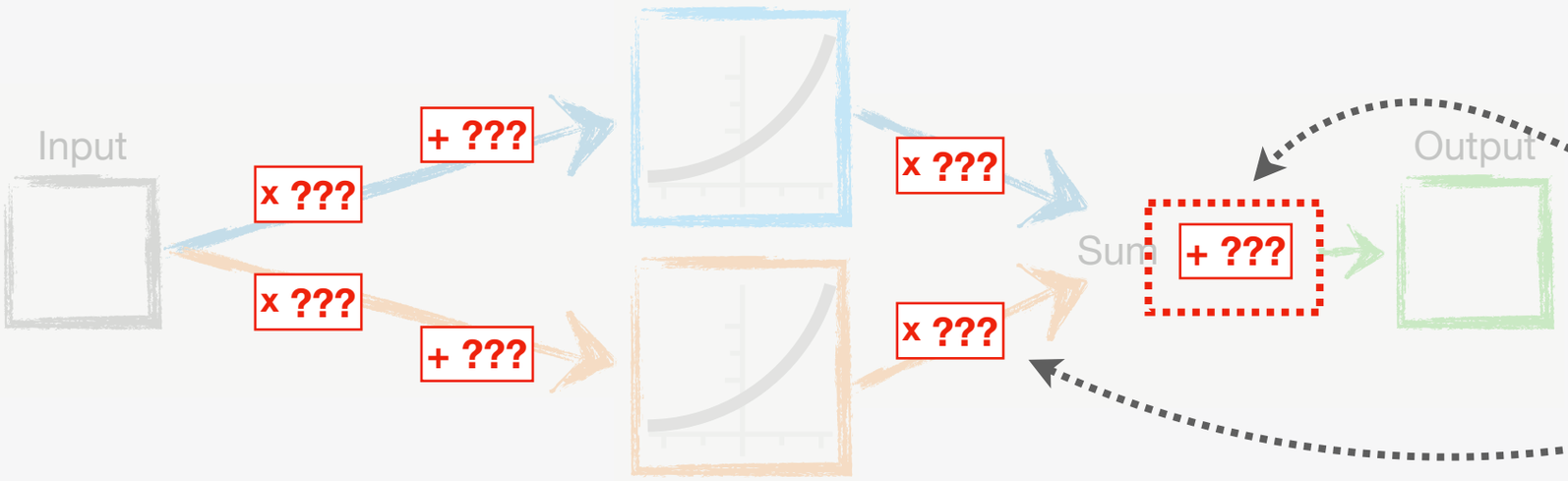


1 In **Neural Networks**, the parameters that we multiply are called **Weights**…



2 …and the parameters we add are called **Biases**.



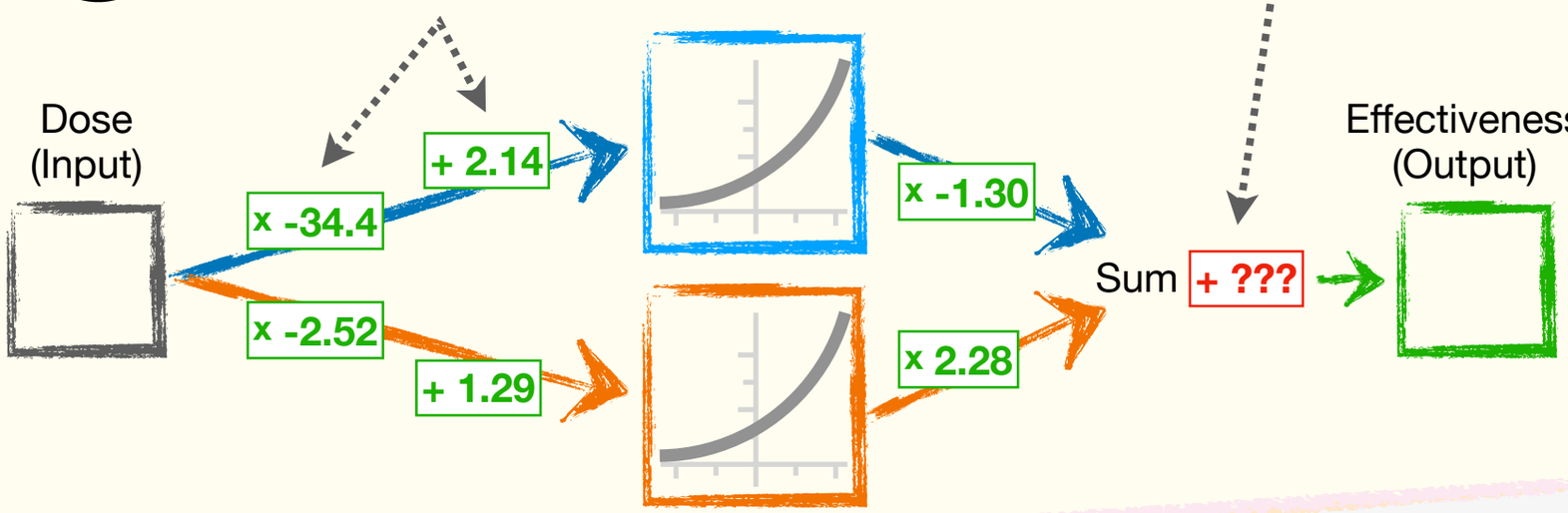


3 In the example that follows, we'll use **Backpropagation** to optimize the **Final Bias**. However, the same process and ideas apply to optimizing all of the parameters.

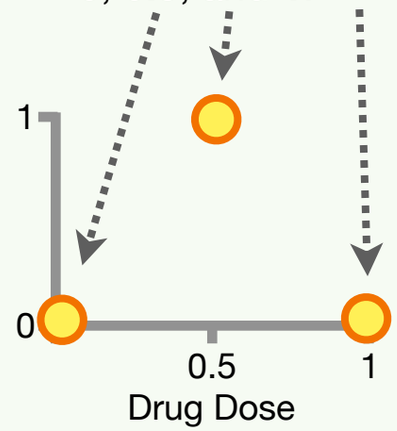
Backpropagation: Details Part 1

1 In this example, let's assume that we already have the optimal values for all of the **Weights** and **Biases**…

…except for the **Final Bias**. So the goal is to use **Backpropagation** to optimize this **Final Bias**.



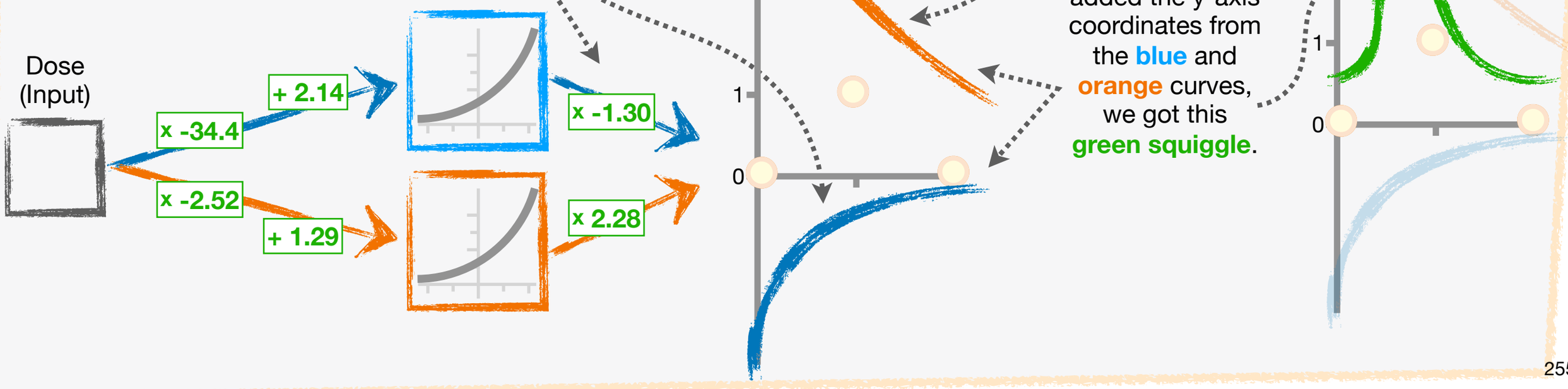
NOTE: To keep the math relatively simple, from now on, the **Training Data** will have only **3 Dose** values, **0**, **0.5**, and **1**.



2 Now let's remember that when we plugged Doses into the input, the top of the **Hidden Layer** gave us this **blue curve**…

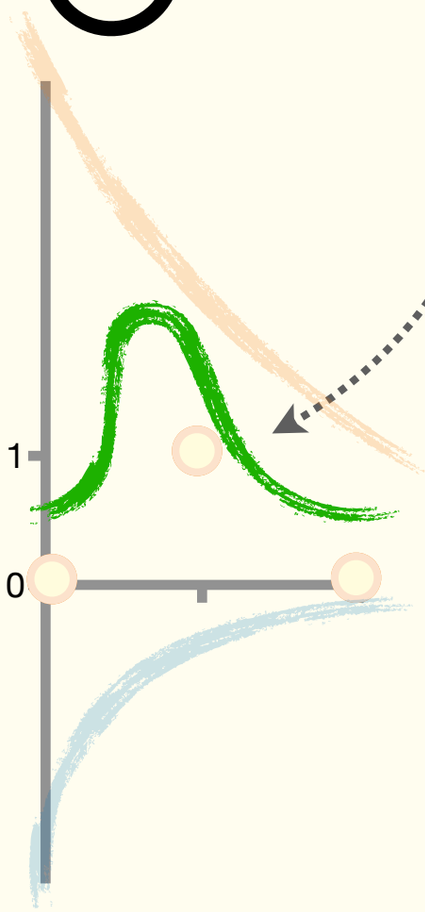
…and the bottom of the **Hidden Layer** gave us this **orange curve**…

…and when we added the y-axis coordinates from the **blue** and **orange** curves, we got this **green squiggle**.

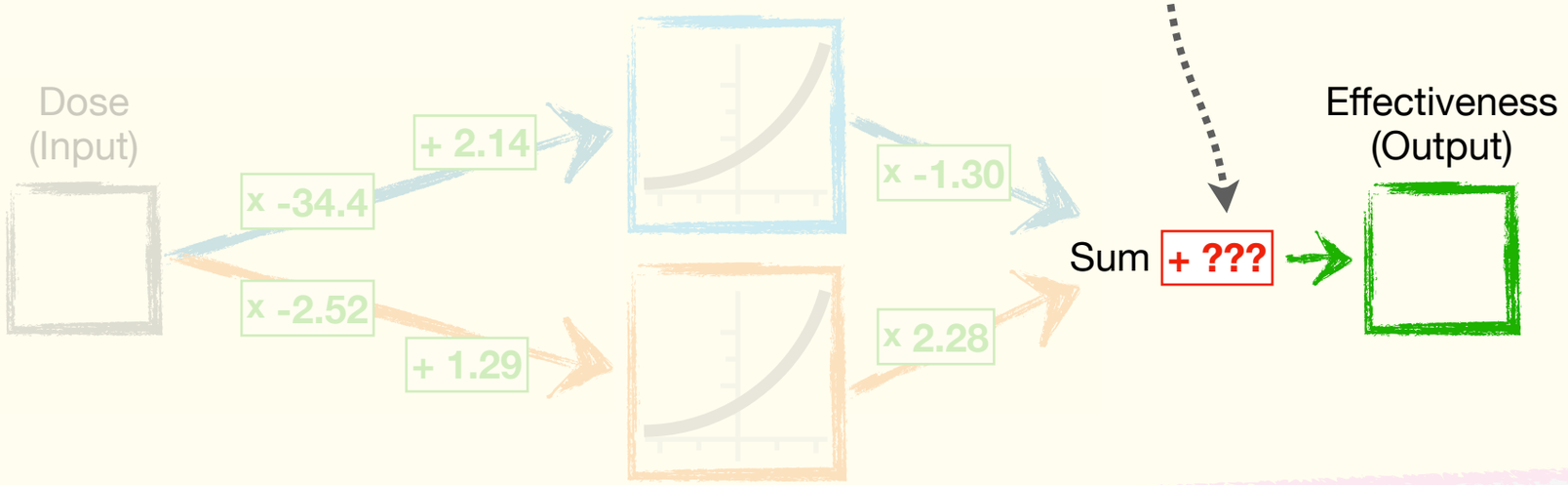


Backpropagation: Details Part 2

3 Now that the **Neural Network** has created the **green squiggle**…

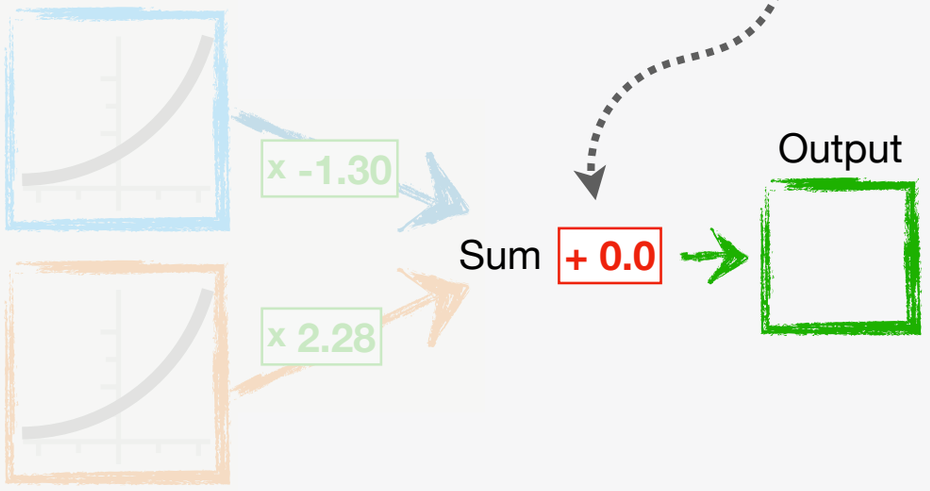


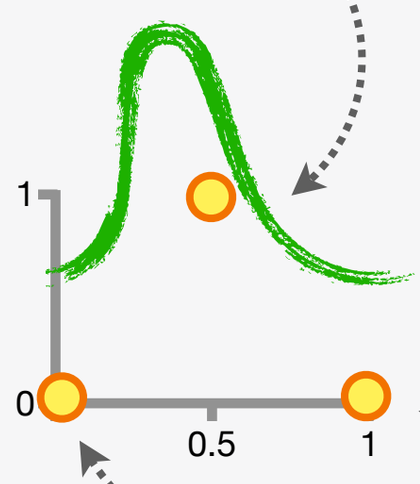
…we're ready to add the **Final Bias** to its y-axis coordinates.



4 However, because we don't yet know the optimal value for the **Final Bias**, we have to give it an initial value. Because **Bias** terms are frequently initialized to **0**, we'll set it to **0**…

…and adding **0** to all of the y-axis coordinates on the **green squiggle** leaves it right where it is…



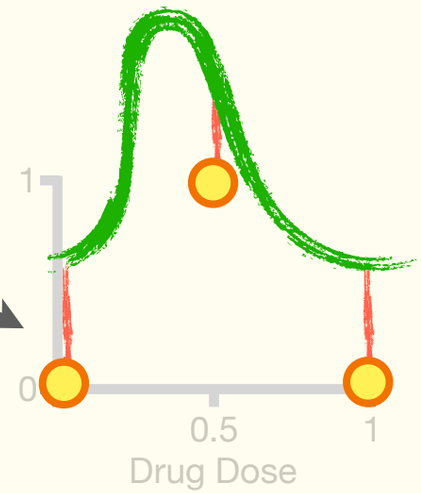


…which means the **green squiggle** doesn't fit the **Training Data** very well.

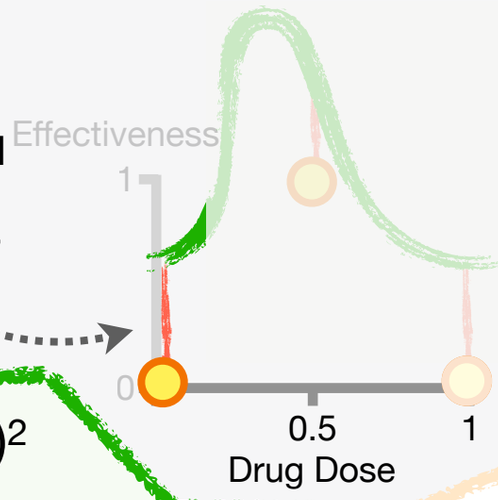
Backpropagation: Details Part 3

5 Now, just like we did for R^2, **Linear Regression**, and **Regression Trees**, we can quantify how well the **green squiggle** fits all of the **Training Data** by calculating the **Sum of the Squared Residuals** (**SSR**).

$$\text{SSR} = \sum_{i=1}^{n} (\text{Observed}_i - \text{Predicted}_i)^2$$

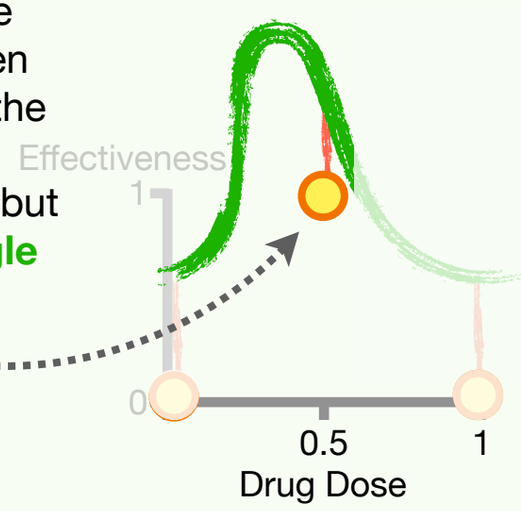


6 For example, for the first Dose, **0**, the **Observed** Effectiveness is **0** and the **green squiggle** created by the **Neural Network** predicts **0.57**, so we plug in **0** for the **Observed** value and **0.57** for the **Predicted** value into the equation for the **SSR**.



$$\text{SSR} = (0 - 0.57)^2$$

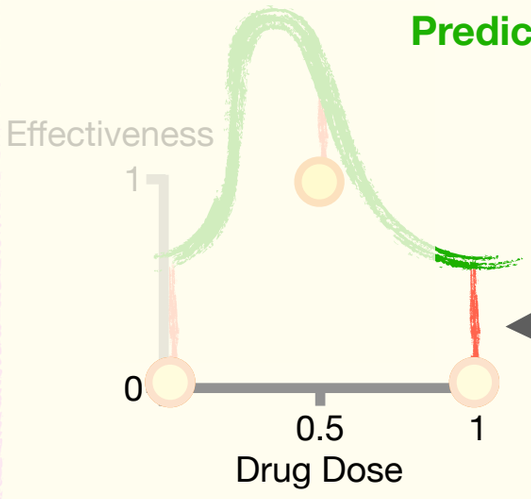
7 Then we add the **Residual** for when Dose **= 0.5**. Now the **Observed** Effectiveness is **1**, but the **green squiggle** predicts **1.61**.



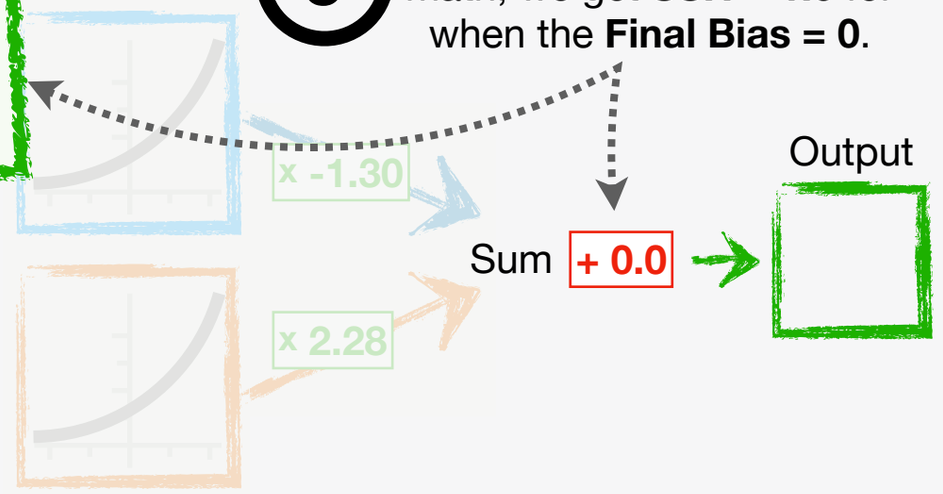
$$+ (1 - 1.61)^2$$

8 Then we add the **Residual** when Dose **= 1**, where the **Observed** value is **0** and **Predicted** value is **0.58**.

$$+ (0 - 0.58)^2 = 1.0$$

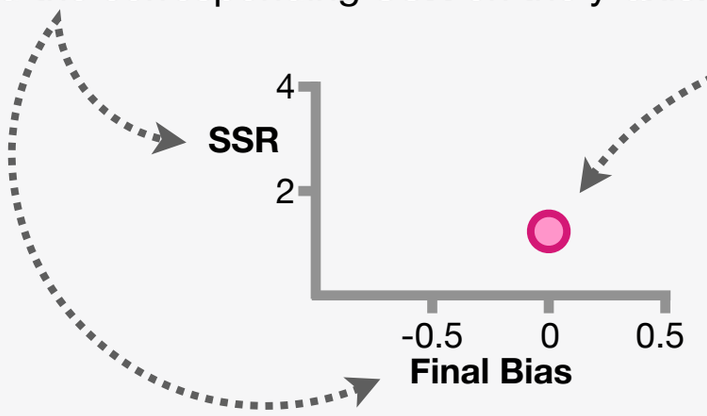


9 Lastly, when we do the math, we get **SSR = 1.0** for when the **Final Bias = 0**.

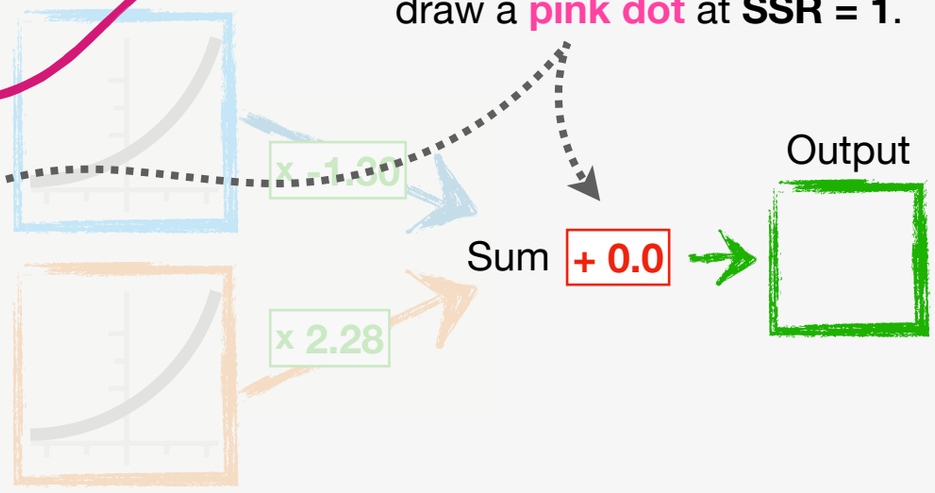


Backpropagation: Details Part 4

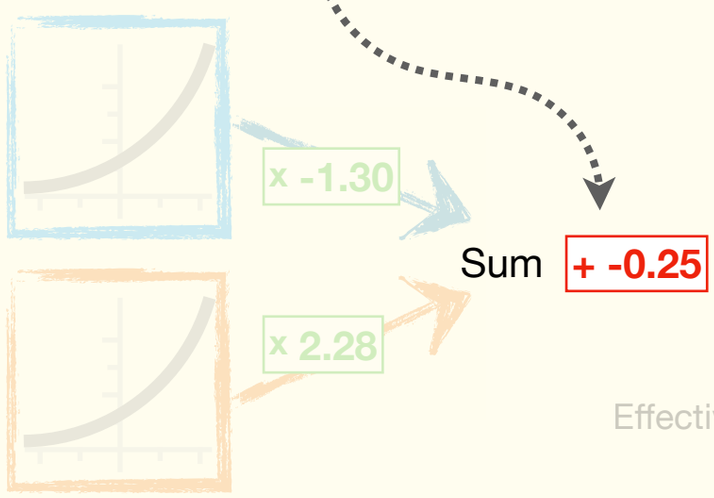
10 Now we can compare the **SSR** for different values for the **Final Bias** by plotting them on this graph that has values for the **Final Bias** on the x-axis and the corresponding **SSR** on the y-axis.



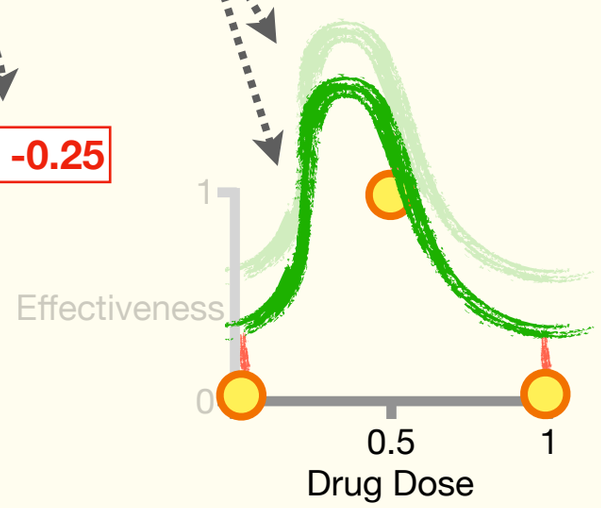
Thus, when the **Final Bias = 0**, we can draw a **pink dot** at **SSR = 1**.



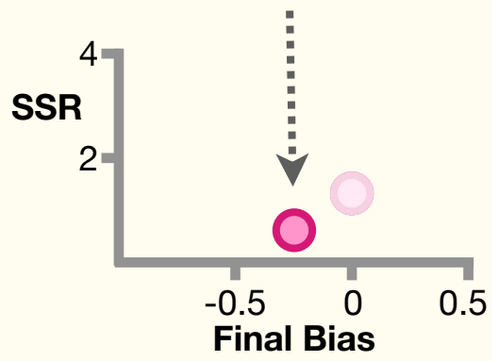
11 If we set the **Final Bias** to **-0.25**…



…then we shift the **green squiggle** down a little bit…

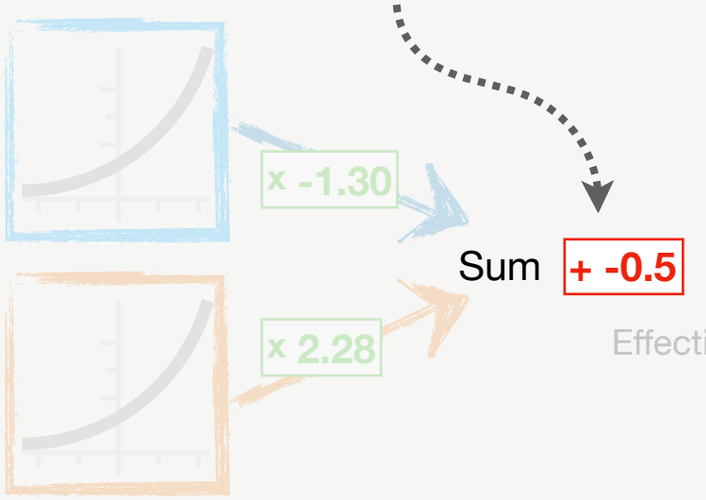


…and we can calculate the **SSR** and plot the value on our graph.

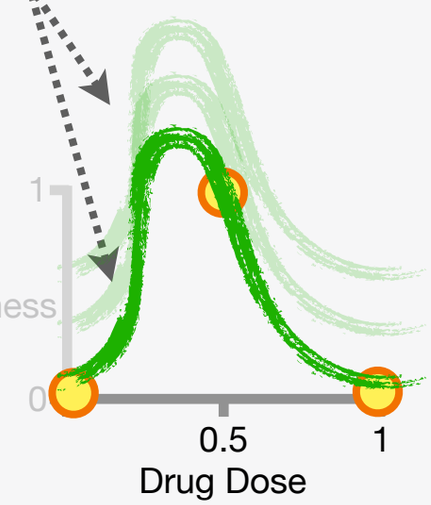


Backpropagation: Details Part 5

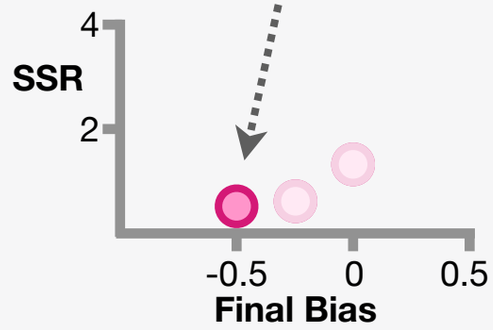
12 Setting the **Final Bias** to **-0.5**…



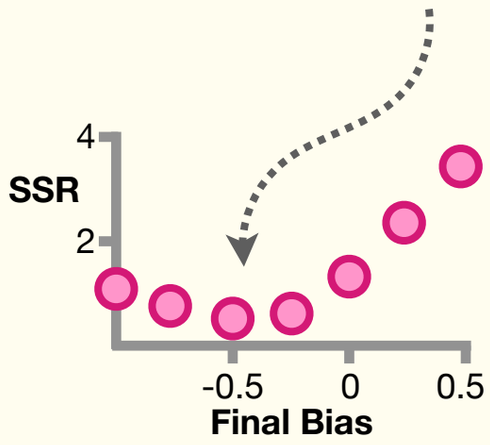
…shifts the **green squiggle** down a little bit more…



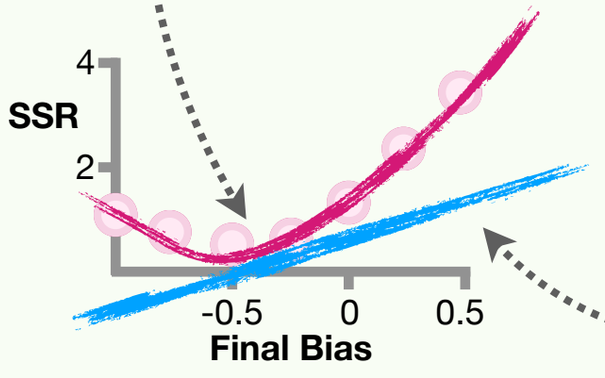
…and results in a slightly lower **SSR**.



13 And if we try a bunch of different values for the **Final Bias**, we can see that the lowest **SSR** occurs when the **Final Bias** is close to **0.5**…



14 …however, instead of just plugging in a bunch of numbers at random, we'll use **Gradient Descent** to quickly find the lowest point in the **pink curve**, which corresponds to the **Final Bias** value that gives us the minimum **SSR**…



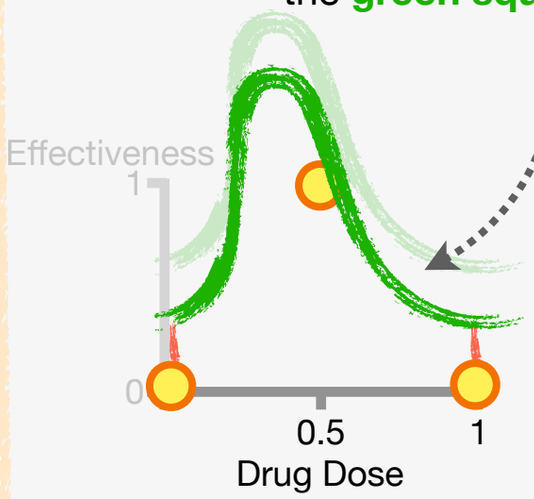
…and to use **Gradient Descent**, we need the derivative of the **SSR** with respect to the **Final Bias**.

$$\frac{d\,SSR}{d\,Final\,Bias}$$

Backpropagation: Details Part 6

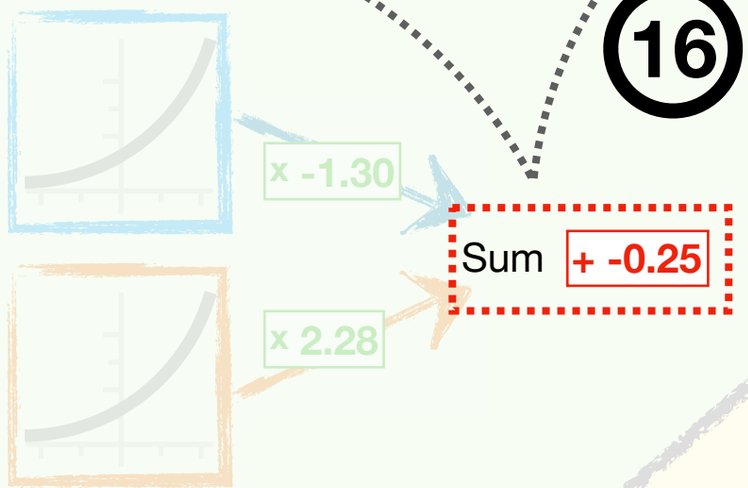
15 Remember, each **Predicted** value in the **SSR** comes from the **green squiggle**…

$$\textbf{SSR} = \sum_{i=1}^{n} (\text{Observed}_i - \text{Predicted}_i)^2$$



Predicted = **green squiggle** = **blue curve** + **orange curve** + **Final Bias**

16 …and the **green squiggle** comes from the last part of the **Neural Network**, when we add the y-axis values from the **blue** and **orange** curves to the **Final Bias**.



17 Now, because the **Predicted** values link the **SSR**…

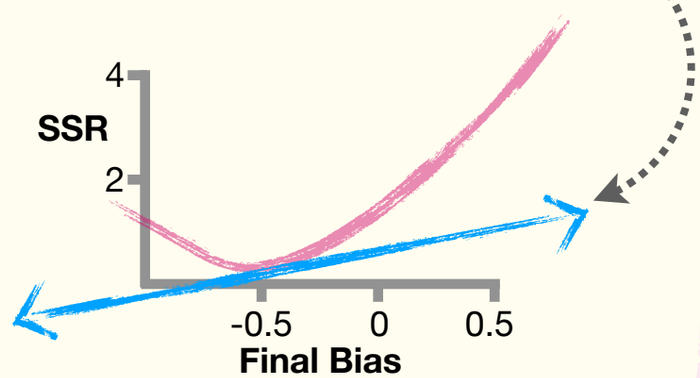
…to the **Final Bias**…

…we can use **The Chain Rule** to solve for the derivative of the **SSR** with respect to the **Final Bias**.

$$\textbf{SSR} = \sum_{i=1}^{n} (\text{Observed}_i - \text{Predicted}_i)^2$$

Predicted = **green squiggle** = **blue curve** + **orange curve** + **Final Bias**

$$\frac{d\,SSR}{d\,Final\,Bias}$$



Backpropagation: Details Part 7

18 **The Chain Rule** says that the derivative of the **SSR** with respect to the **Final Bias**…

$$\frac{d\,SSR}{d\,Final\,Bias} = \frac{d\,SSR}{d\,Predicted} \times \frac{d\,Predicted}{d\,Final\,Bias}$$

…is the derivative of the **SSR** with respect to the **Predicted** values…

$$SSR = \sum_{i=1}^{n} (Observed_i - Predicted_i)^2$$

…multiplied by the derivative of the **Predicted** values with respect to the **Final Bias**.

Predicted = green squiggle = blue curve + orange curve + **Final Bias**

BAM!!!

Psst!
If this doesn't make any sense and you need help with **The Chain Rule**, see **Appendix F**.

Backpropagation: Details Part 8

19 Now that we see that the derivative of the **SSR** with respect to the **Final Bias**… …is the derivative of the **SSR** with respect to the **Predicted** values… …multiplied by the derivative of the **Predicted** values with respect to the **Final Bias**…

$$\frac{d\,SSR}{d\,Final\,Bias} = \frac{d\,SSR}{d\,Predicted} \times \frac{d\,Predicted}{d\,Final\,Bias}$$

20 …we can solve for the *first* part, the derivative of the **SSR** with respect to the **Predicted** values… …which, in turn, can be solved using **The Chain Rule**… …by moving the square to the front… …and multiplying everything by the derivative of the stuff inside the parentheses, which is **-1**…

$$\frac{d\,SSR}{d\,Predicted} = \frac{d}{d\,Predicted} \sum_{i=1}^{n} (Observed_i - Predicted_i)^2 = \sum_{i=1}^{n} 2 \times (Observed_i - Predicted_i) \times -1$$

$$\frac{d\,SSR}{d\,Predicted} = \sum_{i=1}^{n} -2 \times (Observed_i - Predicted_i)$$

…and, lastly, we simplify by multiplying **2** by **-1**.

BAM!!!

We solved for the *first* part of the derivative. Now let's solve for the *second* part.

NOTE: For more details on how to solve for this derivative, see **Chapter 5**.

Backpropagation: Details Part 9

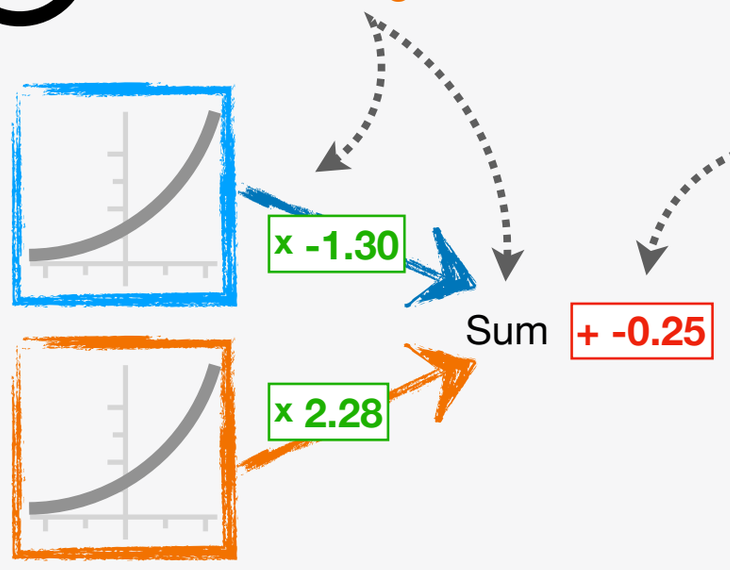
21 The *second* part, the derivative of the **Predicted** values with respect to the **Final Bias**…

…is the derivative of the **green squiggle** with respect to the **Final Bias**…

…which, in turn, is the derivative of the sum of the **blue** and **orange** curves and the **Final Bias**.

$$\frac{d\,Predicted}{d\,Final\,Bias} = \frac{d}{d\,Final\,Bias}\,\text{green squiggle} = \frac{d}{d\,Final\,Bias}(\text{blue curve} + \text{orange curve} + \text{Final Bias})$$

22 Now, remember that the **blue** and **orange** curves…



…were created before we got to the **Final Bias**…

…thus, the derivatives of the **blue** and **orange** curves with respect to the **Final Bias** are both **0** because they do not depend on the **Final Bias**…

$$\frac{d}{d\,Final\,Bias}(\text{blue curve} + \text{orange curve} + \text{Final Bias}) = 0 + 0 + 1$$

…and the derivative of the **Final Bias** with respect to the **Final Bias** is **1**. So, when we do the math, the derivative of the Predicted values with respect to the **Final Bias** is **1**. **DOUBLE BAM!!!**

$$\frac{d\,Predicted}{d\,Final\,Bias} = 1$$

We solved for the *second* part of the derivative.

Backpropagation: Details Part 10

23 Now, to get the derivative of the **SSR** with respect to the **Final Bias**, we simply plug in… …the derivative of the **SSR** with respect to the **Predicted** values… …and the derivative of the **Predicted** values with respect to the **Final Bias**.

$$\frac{d\,SSR}{d\,Predicted} = \sum_{i=1}^{n} -2 \times (Observed_i - Predicted_i)$$

$$\frac{d\,Predicted}{d\,Final\,Bias} = 1$$

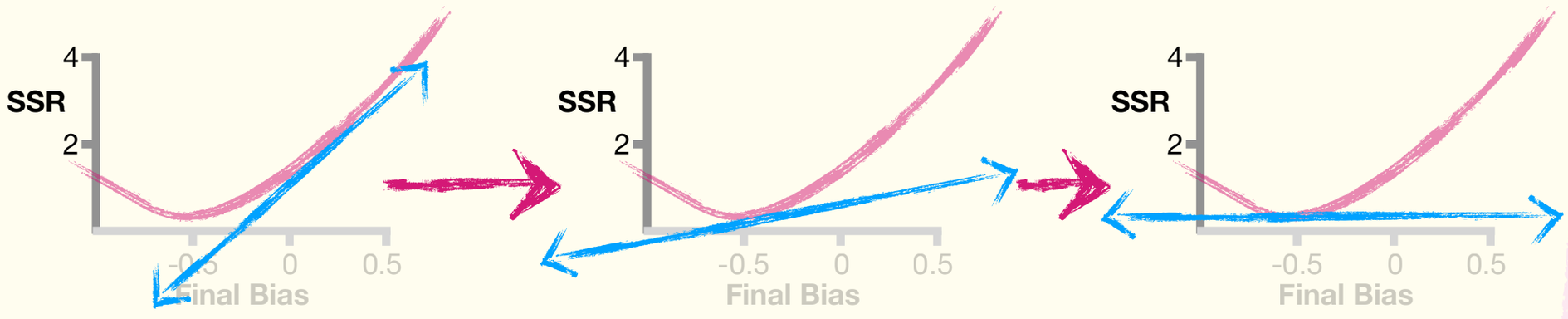
$$\frac{d\,SSR}{d\,Final\,Bias} = \frac{d\,SSR}{d\,Predicted} \times \frac{d\,Predicted}{d\,Final\,Bias}$$

$$\frac{d\,SSR}{d\,Final\,Bias} = \sum_{i=1}^{n} -2 \times (Observed_i - Predicted_i) \times 1$$

24 At long last, we have the derivative of the **SSR** with respect to the **Final Bias!!!**

TRIPLE BAM!!!

25 In the next section, we'll plug the derivative into **Gradient Descent** and solve for the optimal value for the **Final Bias**.



Backpropagation: Step-by-Step

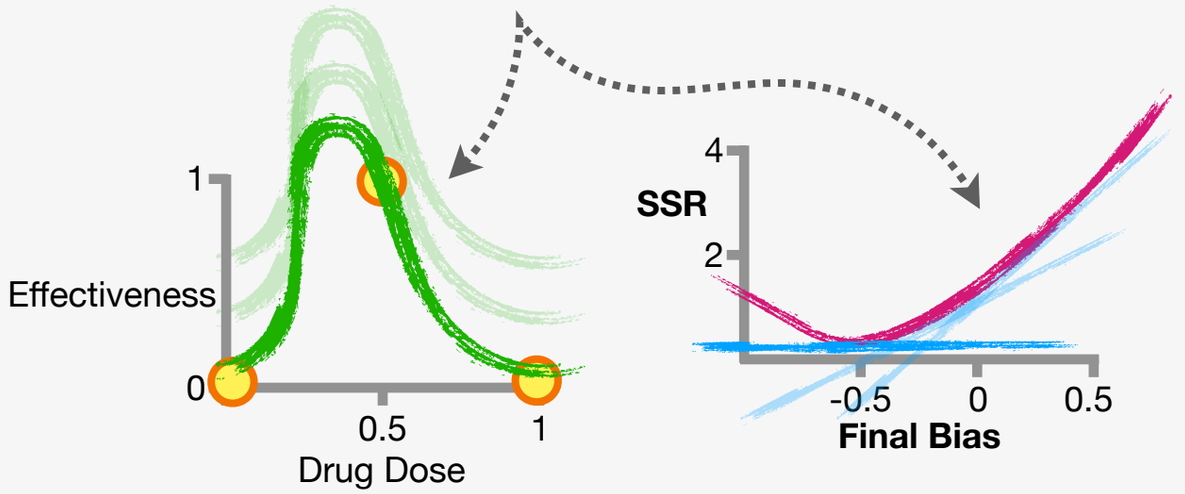
1 Now that we have the derivative of the **SSR** with respect to the **Final Bias**…

…which tells us how the **SSR** changes when we change the **Final Bias**…

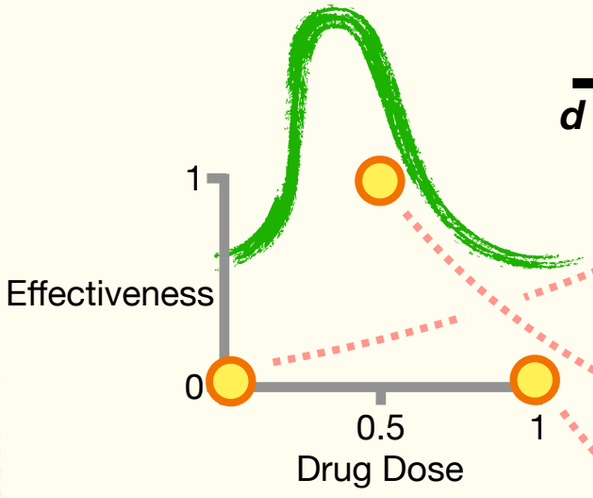
…we can optimize the **Final Bias** with **Gradient Descent**.

$$\frac{d\,SSR}{d\,Final\,Bias} = \sum_{i=1}^{n} -2 \times (Observed_i - Predicted_i) \times 1$$

NOTE: We're leaving the **"x 1"** term in the derivative to remind us that it comes from **The Chain Rule**. However, multiplying by **1** doesn't do anything, and you can omit it.



2 First, we plug in the **Observed** values from the **Training Dataset** into the derivative of the **SSR** with respect to the **Final Bias**.



$$\frac{d\,SSR}{d\,Final\,Bias} = \sum_{i=1}^{n} -2 \times (Observed_i - Predicted_i)^2 \times 1$$

$$= -2 \times (\,Observed_1 - Predicted_1\,) \times 1$$

$$+ -2 \times (\,Observed_2 - Predicted_2\,) \times 1$$

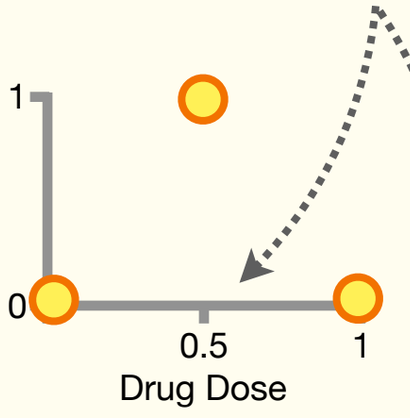
$$+ -2 \times (\,Observed_3 - Predicted_3\,) \times 1$$

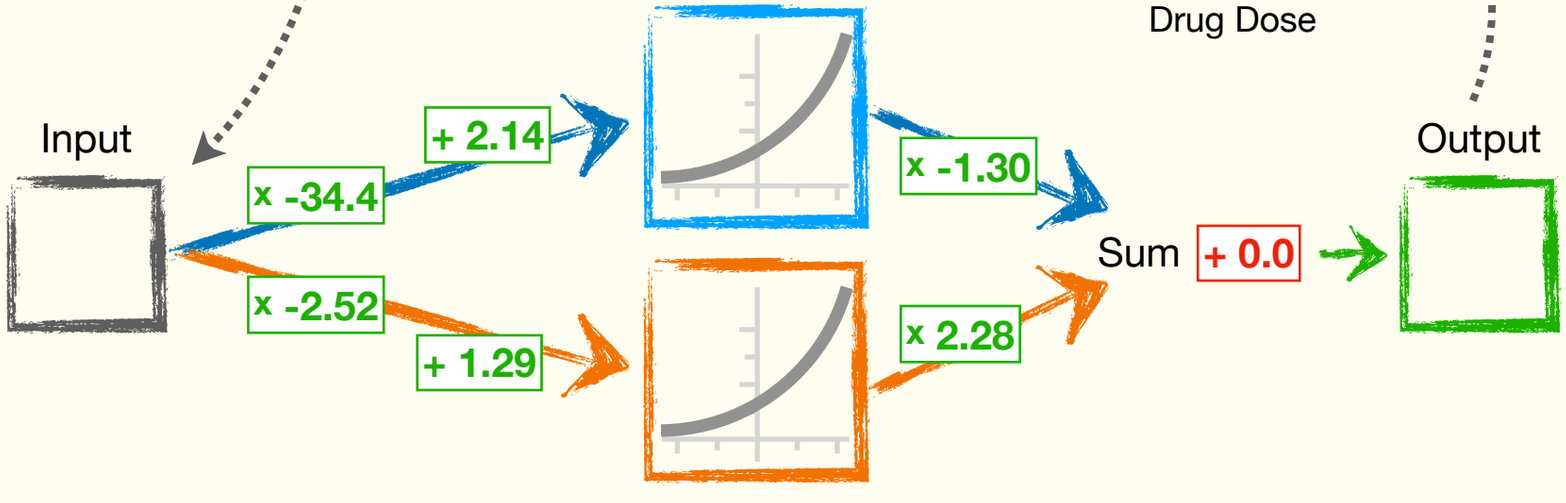
$$\frac{d\,SSR}{d\,Final\,Bias} = -2 \times (\,0 - Predicted_1\,) \times 1$$
$$+ -2 \times (\,1 - Predicted_2\,) \times 1$$
$$+ -2 \times (\,0 - Predicted_3\,) \times 1$$

Backpropagation: Step-by-Step

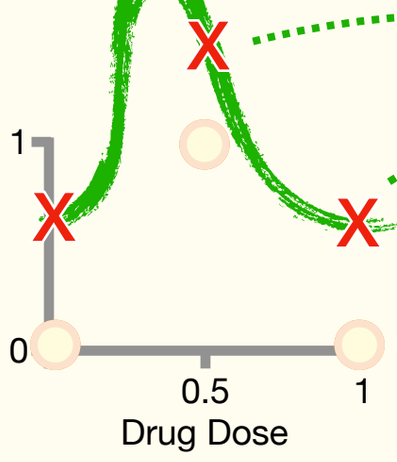
3 Then we initialize the **Final Bias** to a random value. In this case, we'll set the **Final Bias** to **0.0**.

4 Then we run the **3 Doses** from the **Training Dataset**, **0**, **0.5** and **1**, through the **Neural Network** to get the **Predicted** values….





…and we plug the **Predicted** values into the derivative.



$$\frac{d\,SSR}{d\,Final\,Bias} = -2 \times (0 - Predicted) \times 1 + -2 \times (1 - Predicted) \times 1 + -2 \times (0 - Predicted) \times 1$$

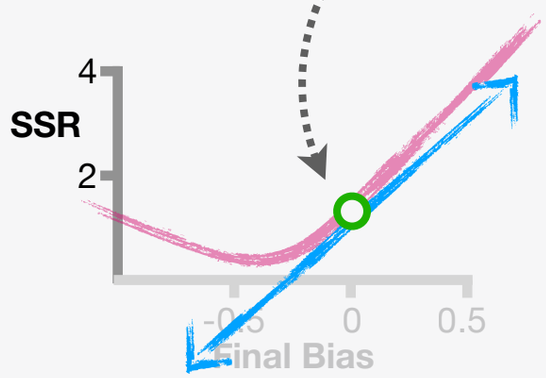
$$= -2 \times (0 - 0.57) \times 1 + -2 \times (1 - 1.61) \times 1 + -2 \times (0 - 0.58) \times 1$$

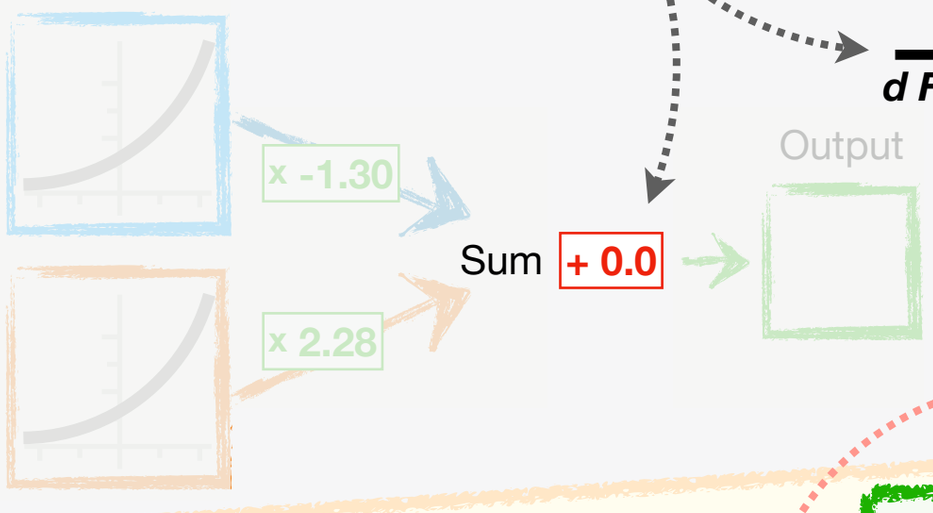
Backpropagation: Step-by-Step

5 Now we evaluate the derivative at the current value for the **Final Bias**, which, at this point, is **0.0**.

When we do the math, we get **3.5**…

…thus, when the **Final Bias = 0**, the slope of this **tangent line** is **3.5**.





$$\frac{d\,SSR}{d\,Final\,Bias} = -2 \times (0 - 0.57) \times 1 + -2 \times (1 - 1.61) \times 1 + -2 \times (0 - 0.58) \times 1 = 3.5$$

6 Then we calculate the **Step Size** with the standard equation for **Gradient Descent** and get **0.35**.

NOTE: In this example, we've set the **Learning Rate** to **0.1**.

Step Size = **Derivative** x **Learning Rate**

= **3.5** x **0.1**

= **0.35**

Gentle Reminder: The magnitude of the derivative is proportional to how big of a step we should take toward the minimum. The sign (+/-) tells us in what direction.

7 Lastly, we calculate a *new* value for the **Final Bias** from the *current* **Final Bias**…

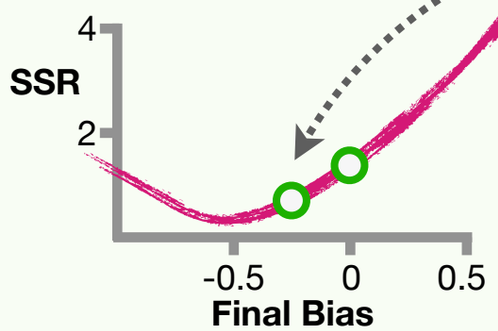
New Bias = **Current Bias** - **Step Size**

= **0.0** - **0.35**

= **-0.35**

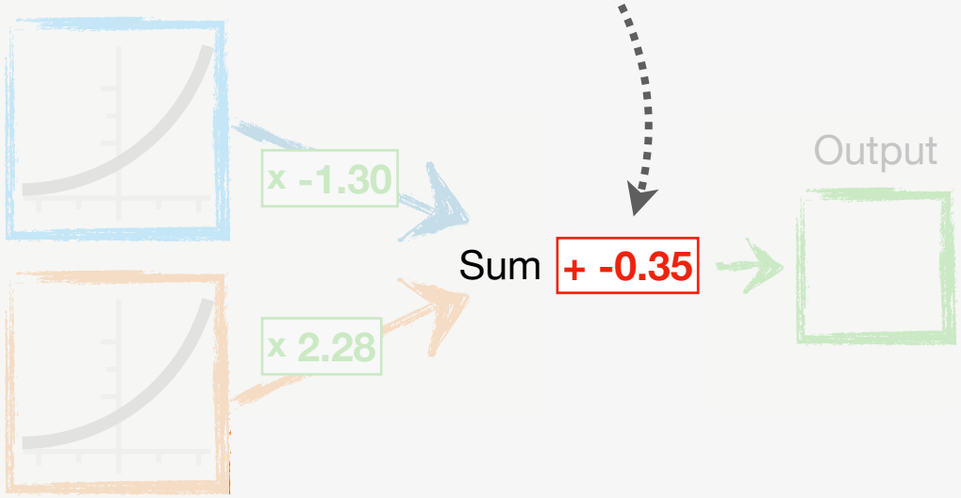
Remember, we initialized the **Final Bias** to **0.0**.

…and the *new* **Final Bias** is **-0.35**, which results in a lower **SSR**.

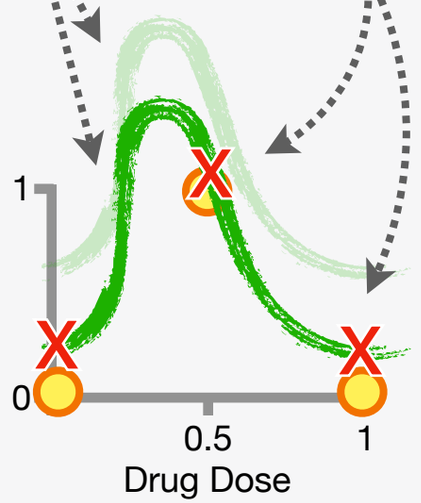


Backpropagation: Step-by-Step

8 With the new value for the **Final Bias**, **-0.35**…



…we shift the **green squiggle** down a bit, and the **Predicted** values are closer to the **Observed** values.



BAM!!!

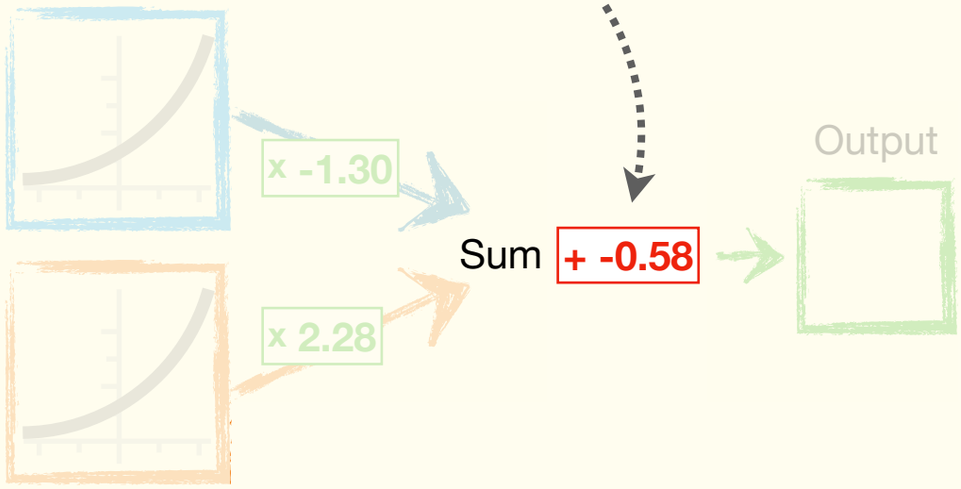
9 Now **Gradient Descent** iterates over the past three steps…

a Evaluate the derivative at the current value…

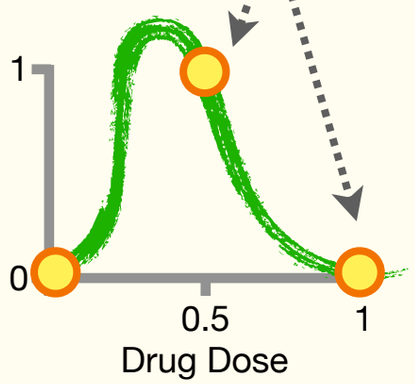
b Calculate the **Step Size**…

c Calculate the new value…

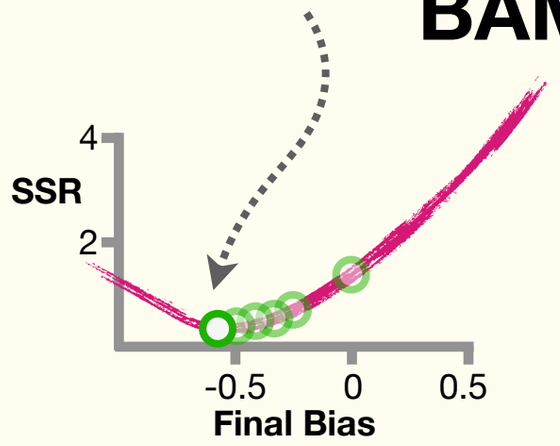
…and after **7** iterations, the **Final Bias** = **-0.58**…



…and the **green squiggle** fits the **Training Data** really well…



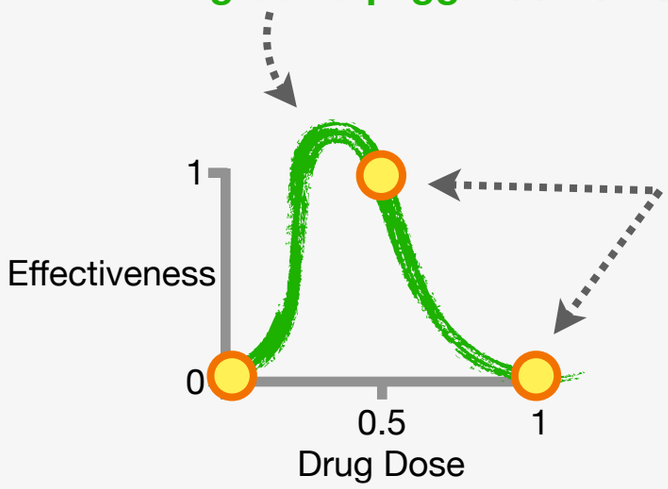
…and we've made it to the lowest **SSR**.



BAM!!!

Neural Networks: FAQ

Where the heck did this bump in the green squiggle come from?

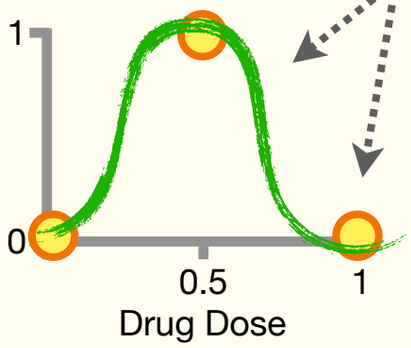


When we used **Backpropagation** to estimate the **Weights** and **Biases** in the **Neural Network**, we only calculated the **SSR** for the original **3** Doses, **0**, **0.5**, and **1**.

That means we only judge the **green squiggle** by how well it predicts Effectiveness at the original **3** Doses, **0**, **0.5**, and **1**, *and no other* Doses.

And that means the **green squiggle** can do whatever it wants in between the original **3** Doses, including making a strange bump that may or may not make good predictions. This is something I think about when people talk about using **Neural Networks** to drive cars. The **Neural Network** probably fits the **Training Data** really well, but there's no telling what it's doing between points, and that means it will be hard to predict what a self-driving car will do in new situations.

Wouldn't it be better if the green squiggle fit a bell-shaped curve to the Training Data?

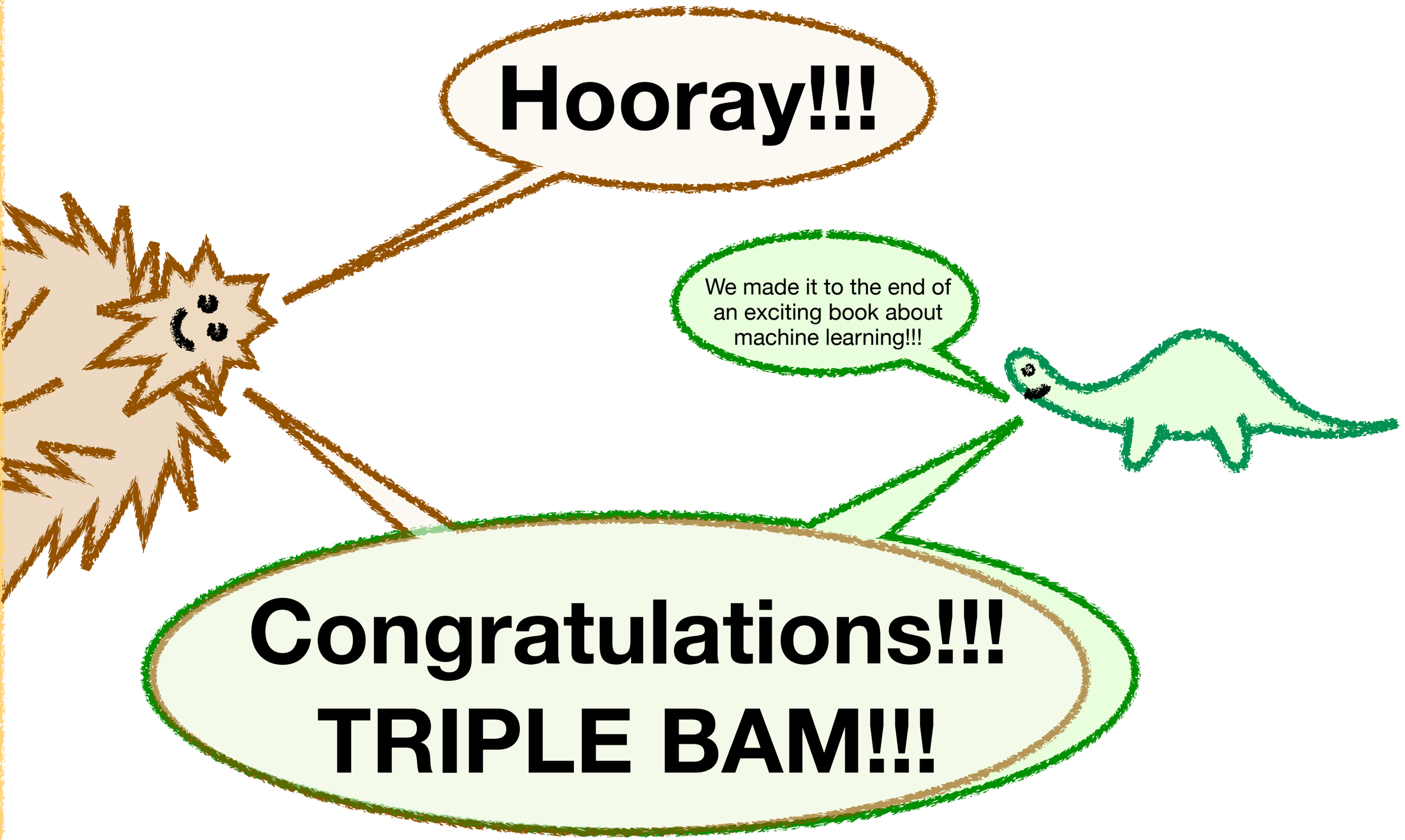


Maybe. Because we don't have any data between the **3** Doses in the **Training Data**, it's hard to say what the best fit would be.

When we have *Neural Networks*, which are cool and super flexible, why would we ever want to use *Logistic Regression*, which is a lot less flexible?

Neural Networks are cool, but deciding how many **Hidden Layers** to use and how many **Nodes** to put in each **Hidden Layer** and even picking the best **Activation Function** is a bit of an art form. In contrast, creating a model with **Logistic Regression** is a science, and there's no guesswork involved. This difference means that it can sometimes be easier to get **Logistic Regression** to make good predictions than a **Neural Network**, which might require a lot of tweaking before it performs well.

Furthermore, when we use a lot of variables to make predictions, it can be much easier to interpret a **Logistic Regression** model than a **Neural Network**. In other words, it's easy to know how **Logistic Regression** makes predictions. In contrast, it's much more difficult to understand how a **Neural Network** makes predictions.


Hooray!!!
We made it to the end of an exciting book about machine learning!!!
Congratulations!!!
TRIPLE BAM!!!

Appendices!!!

Stuff you might have learned in school but forgot

Appendix A:

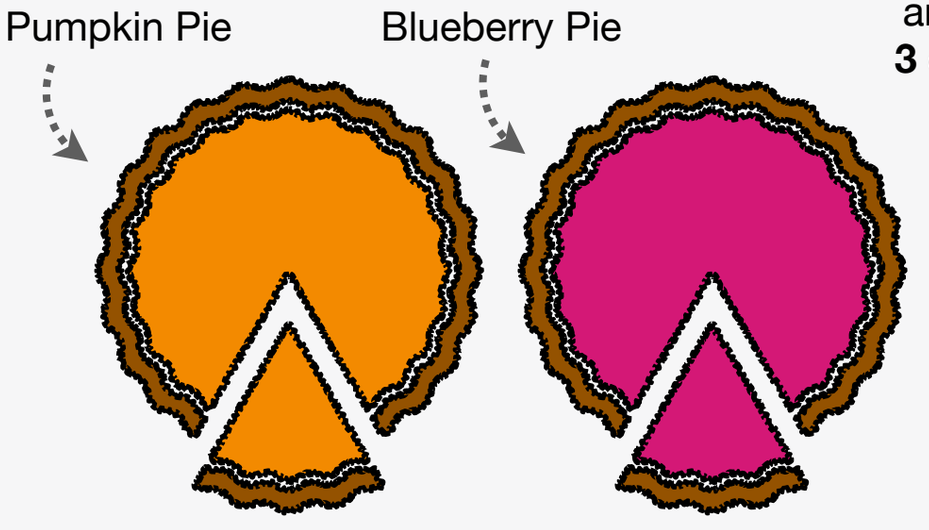
Pie Probabilities

Appendix A: Pie Probabilities

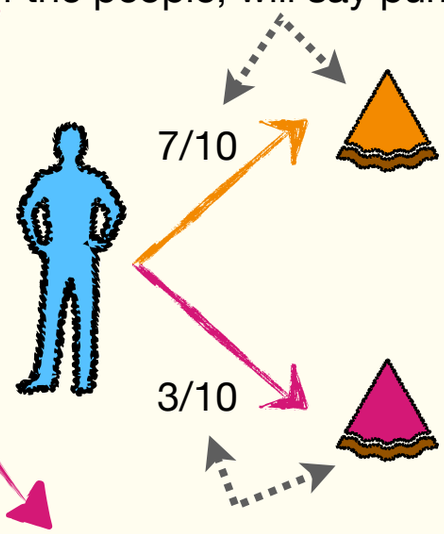
NOTE: In this case, randomly meeting one person who prefers pumpkin or blueberry pie does not affect the next person's pie preference. In probability lingo, we say that these two events, discovering the pie preferences from two randomly selected people, are **independent**. If, for some strange reason, the second person's preference was influenced by the first, the events would be **dependent**, and, unfortunately, the math ends up being different from what we show here.

1 We're in **StatLand**, where **70%** of the people prefer pumpkin pie and **30%** of the people prefer blueberry pie.

In other words, **7** out of **10** people in **StatLand**, or **7/10**, prefer pumpkin pie, and the remaining **3** out of **10** people, or **3/10**, prefer blueberry pie.



2 Now, if we just randomly ask someone which type of pie they prefer, **7** out of **10** people, or **7/10^{ths}** of the people, will say pumpkin…

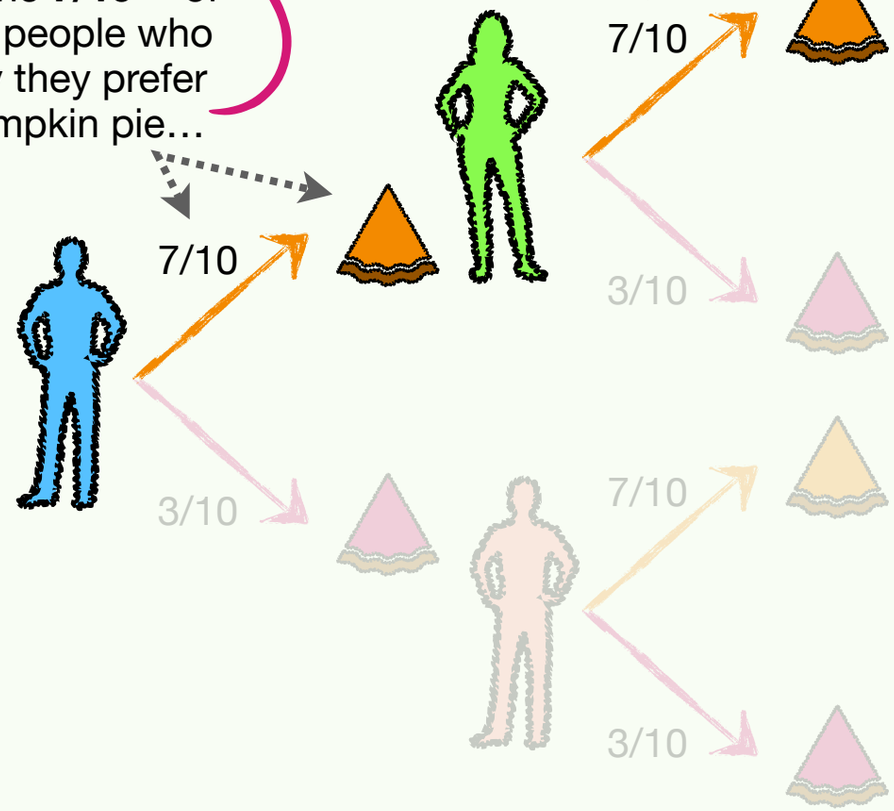


…and the remaining **3** out of **10** people, or **3/10^{ths}**, will say blueberry.

3 Of the **7/10^{ths}** of the people who say they prefer pumpkin pie…

…only **7/10^{ths}** will be followed by people who also prefer pumpkin pie.

Thus, the probability that two people in a row will prefer pumpkin pie is **7/10^{ths}** of the original **7/10^{ths}** who preferred pumpkin pie, which is **7/10** x **7/10 = 49/100**, or **49%**.



0.7 x **0.7** = **0.49**

BAM!!!

Appendix A: Pie Probabilities

4 Now let's talk about what happens when we ask a third person which pie they prefer.

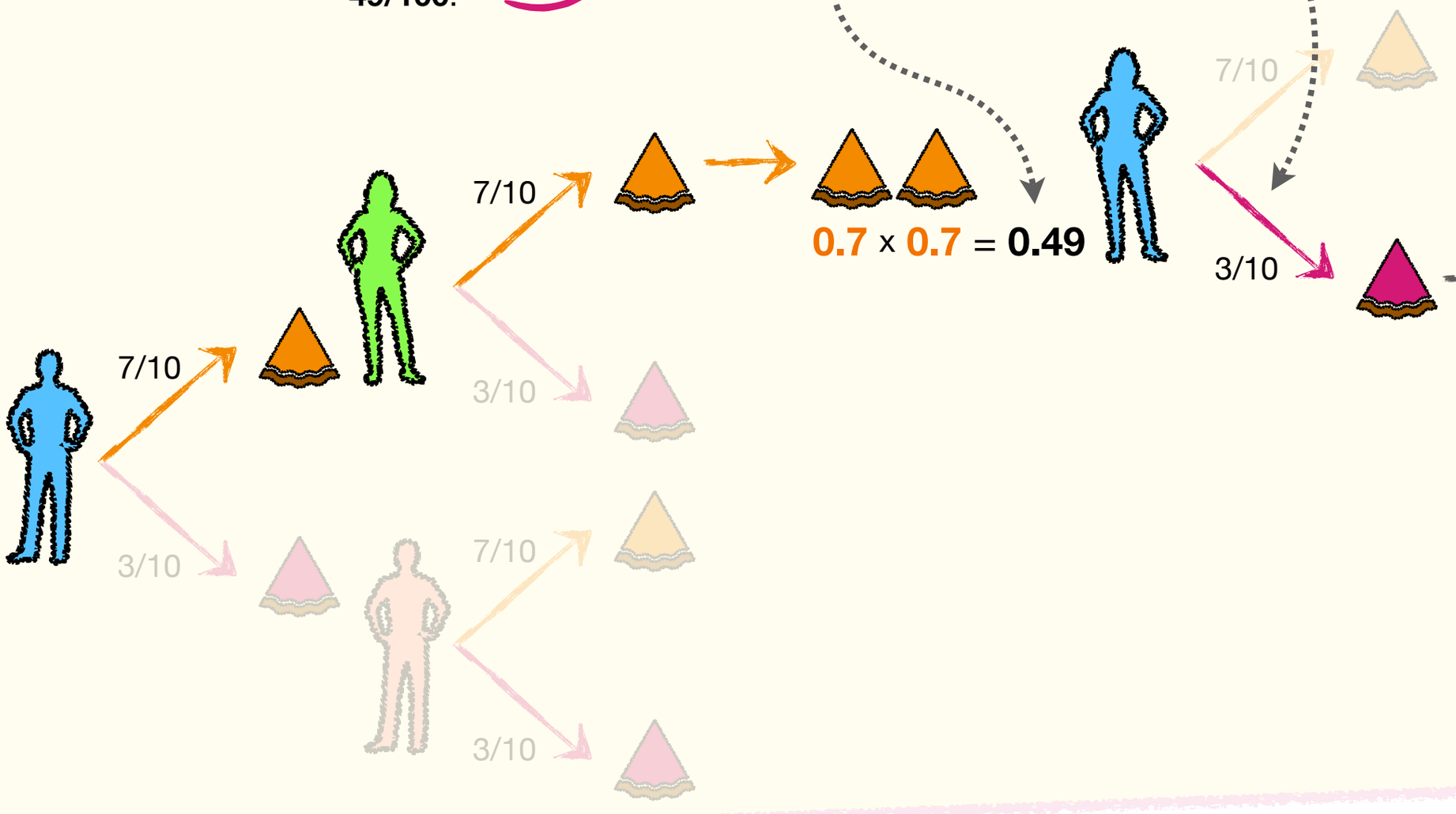
Specifically, we'll talk about the probability that the first two people prefer pumpkin pie and the third person prefers blueberry.

5 First, we just saw on the previous page that we will meet two people who both prefer pumpkin pie only **49%** of the time, or **49/100**.

Now, of that **49/100**…

…only **3/10ths** will be followed by people who prefer blueberry pie.

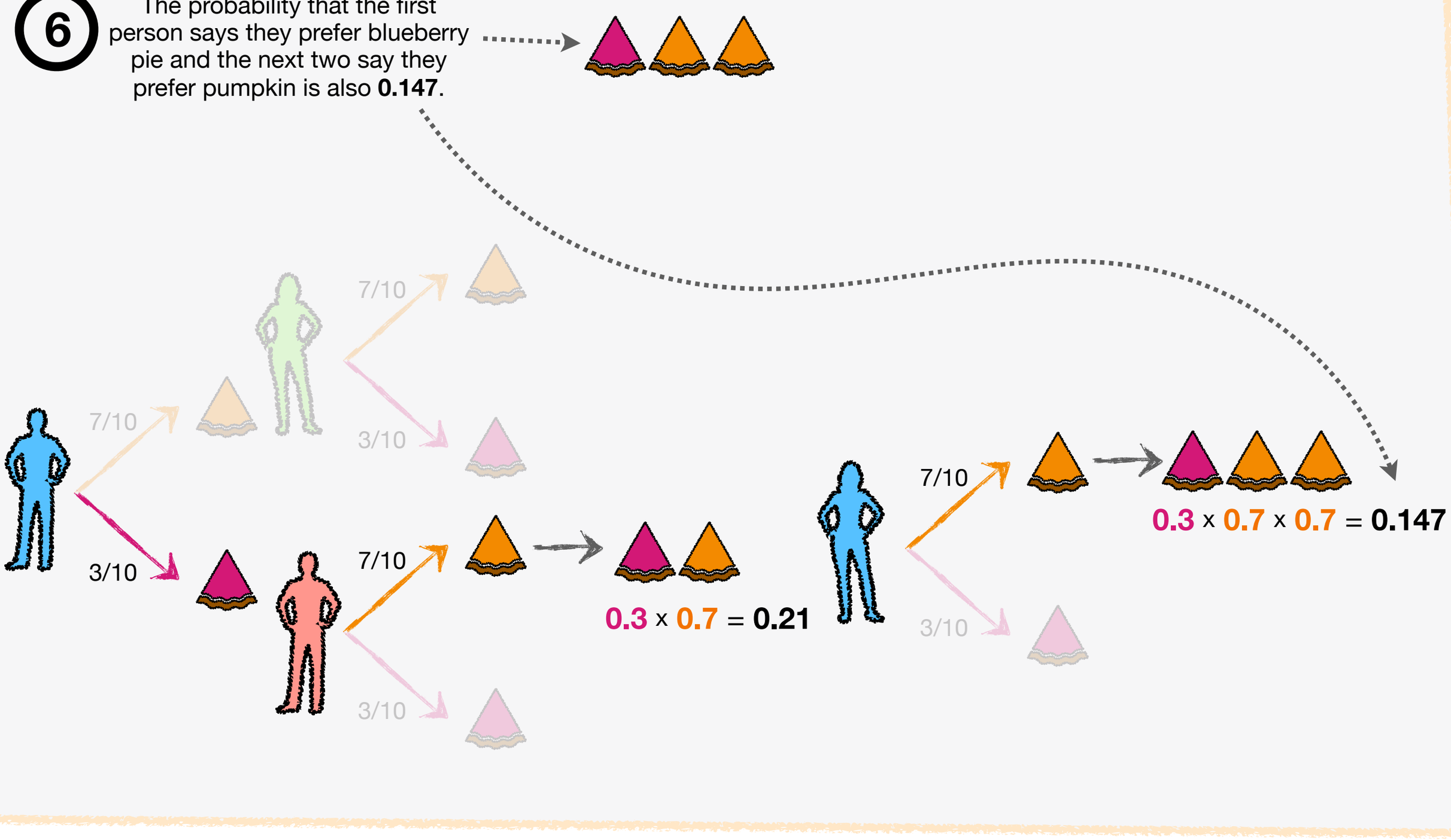
And to find **3/10ths** of the **49/100ths** who preferred pumpkin pie, we multiply: **3/10** x **49/100 = 147/1,000**. In other words, when we ask three random people their pie preferences, **14.7%** of the time, the first two will prefer pumpkin pie and the third will prefer blueberry.



DOUBLE BAM!!!

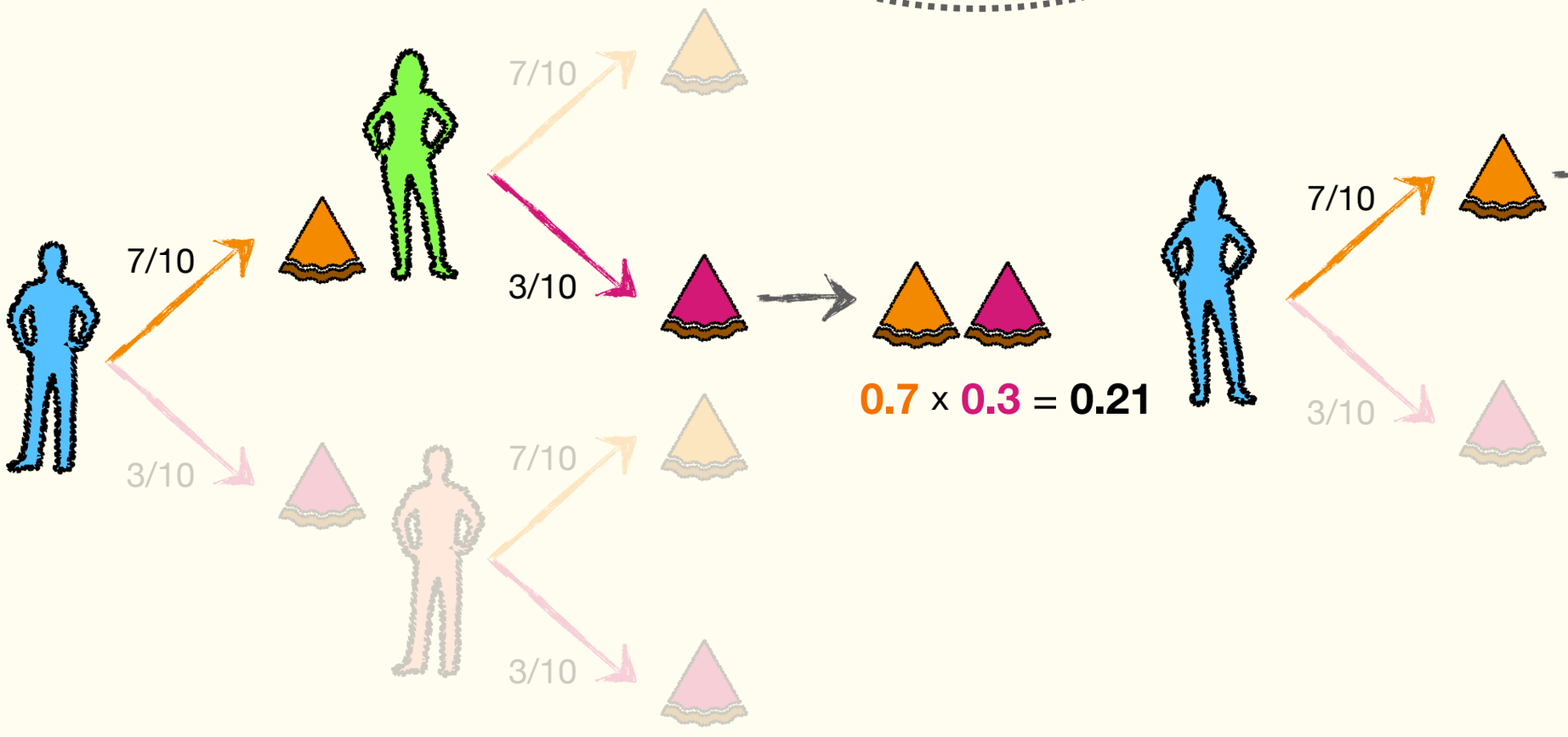
Appendix A: Pie Probabilities

6 The probability that the first person says they prefer blueberry pie and the next two say they prefer pumpkin is also **0.147**.



Appendix A: Pie Probabilities

7 Lastly, the probability that the first person prefers pumpkin pie, the second person prefers blueberry, and the third person prefers pumpkin is also **0.147**.



0.7 x **0.3** = **0.21**

0.7 x **0.3** x **0.7** = **0.147**

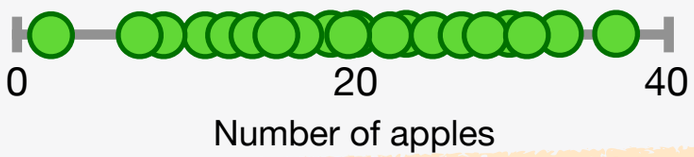
TRIPLE BAM!!!

Appendix B:

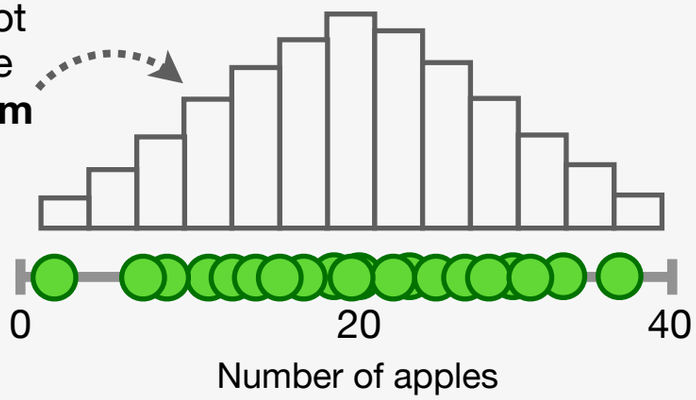
The Mean, Variance, and Standard Deviation

Appendix B: The Mean, Variance, and Standard Deviation

1 Imagine we went to *all* **5,132 Spend-n-Save** food stores and counted the number of **green apples** that were for sale. We could plot the number of **green apples** at each store on this number line…

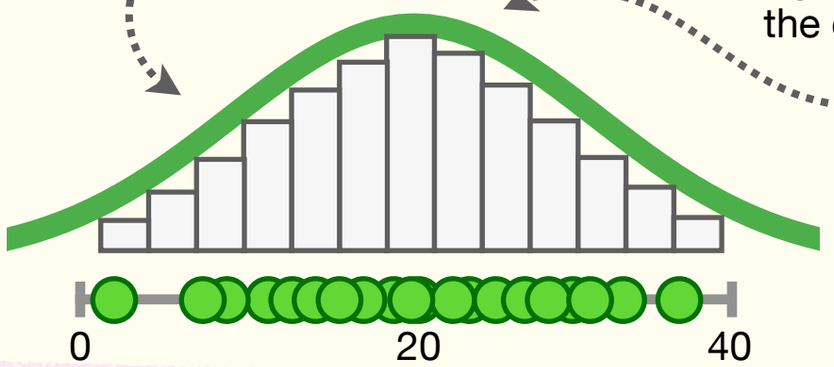


…but because there's a lot of overlap in the data, we can also draw a **Histogram** of the measurements.



2 If we wanted to fit a **Normal Curve** to the data like this…

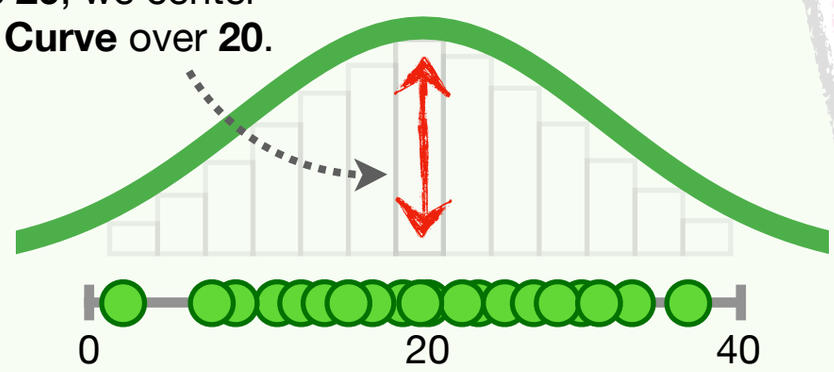
…then, first, we need to calculate the **Population Mean** to figure out where to put the center of the curve.



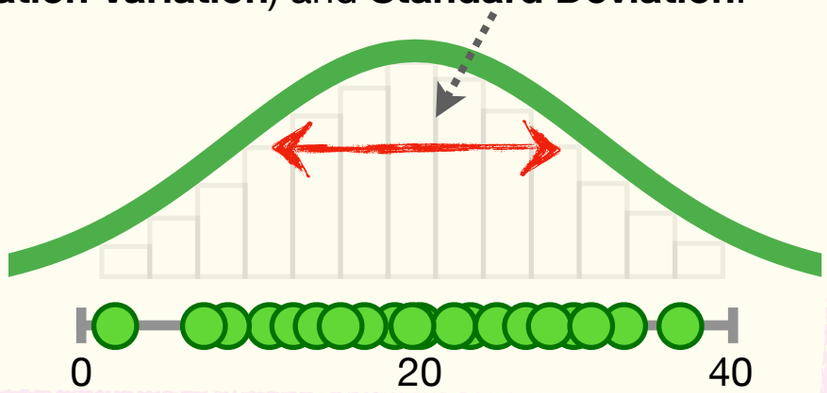
3 Because we counted the number of **green apples** in *all* **5,132 Spend-n-Save** stores, calculating the **Population Mean**, which is frequently denoted with the Greek character $\boldsymbol{\mu}$ (***mu***), is relatively straightforward: we simply calculate the average of all of the measurements, which, in this case, is **20**.

$$\textbf{Population Mean} = \boldsymbol{\mu} = \frac{\text{Sum of Measurements}}{\text{Number of Measurements}} = \frac{2 + 8 + \ldots. + 37}{5132} = 20$$

4 Because the **Population Mean**, $\boldsymbol{\mu}$, is **20**, we center the **Normal Curve** over **20**.

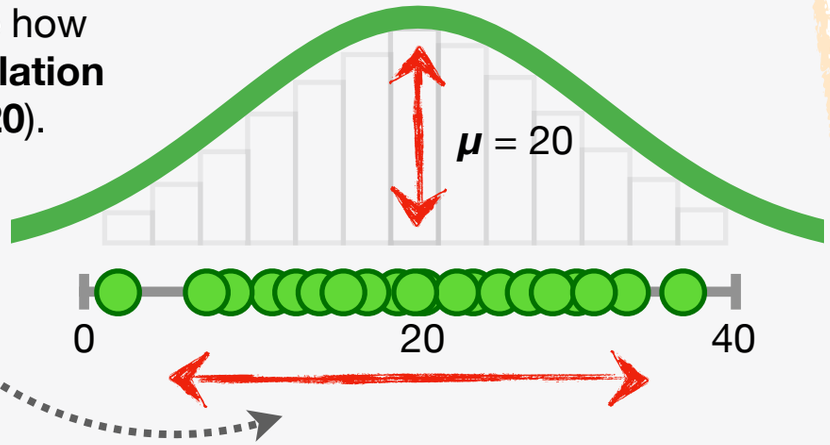


5 Now we need to determine with width of the curve by calculating the **Population Variance** (also called the **Population Variation**) and **Standard Deviation**.



Appendix B: The Mean, Variance, and Standard Deviation

6 In other words, we want to calculate how the data are spread around the **Population Mean** (which, in this example, is **20**).



7 The formula for calculating the **Population Variance** is…

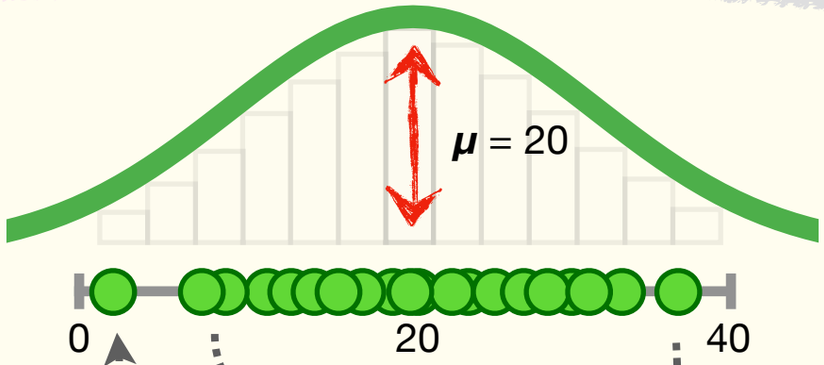
$$\text{Population Variance} = \frac{\sum (x - \mu)^2}{n}$$

…which is a pretty fancy-looking formula, so let's go through it one piece at a time.

8 The part in the parentheses, $x - \mu$, means we subtract the **Population Mean**, μ, from each measurement, x.

$$\text{Population Variance} = \frac{\sum (x - \mu)^2}{n}$$

For example, the first measurement is **2**, so we subtract μ, which is **20**, from **2**…



$(2 - 20)$ $(8 - 20)$ $(28 - 20)$

…then the square tells us to square each term…

$(2 - 20)^2$ $(8 - 20)^2$ $(28 - 20)^2$

…and the Greek character **Σ** (**Sigma**) tells us to add up all of the terms…

$$\frac{(2 - 20)^2 + (8 - 20)^2 + \ldots + (28 - 20)^2}{\text{Number of Measurements}}$$

…and lastly, we want the average of the squared differences, so we divide by the total number of measurements, ***n***, which, in this case, is *all* **Spend-n-Save** food stores, **5,132**.

$$\text{Population Variance} = \frac{\sum (x - \mu)^2}{n}$$

Appendix B: The Mean, Variance, and Standard Deviation

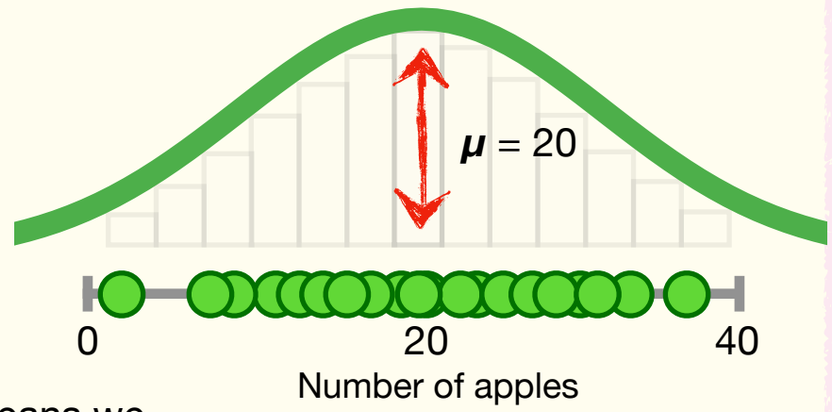
9 Now that we know how to calculate the **Population Variance**…

…when we do the math, we get **100**.

BAM?

Nope, not yet.

$$\text{Population Variance} = \frac{\sum (x - \mu)^2}{n} = \frac{(2 - 20)^2 + (8 - 20)^2 + \ldots + (28 - 20)^2}{5132} = 100$$



10 Because each term in the equation for **Population Variance** is squared…

…the units for the result, **100**, are **Number of Apples Squared**…

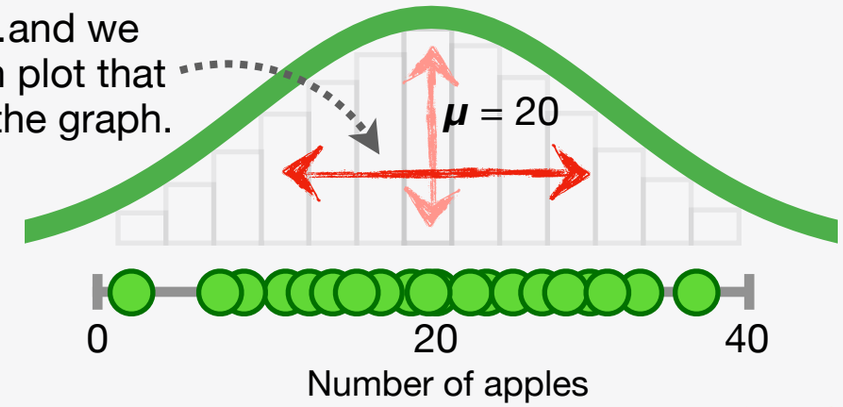
…and that means we can't plot the **Population Variance** on the graph, since the units on the x-axis are not squared.

11 To solve this problem, we take the *square root* of the **Population Variance** to get the **Population Standard Deviation**…

…and because the **Population Variance** is **100**, the **Population Standard Deviation** is **10**…

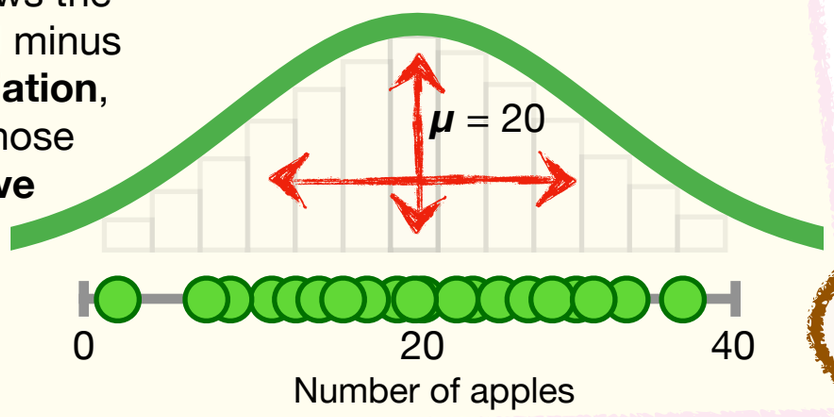
…and we can plot that on the graph.

$$\text{Population Standard Deviation} = \sqrt{\frac{\sum (x - \mu)^2}{n}} = \sqrt{\text{Population Variance}} = \sqrt{100} = 10$$



12 Now we have a graph that shows the **Population Mean**, **20**, plus and minus the **Population Standard Deviation**, **10** apples, and we can use those values to fit a **Normal Curve** to the data.

BAM!!!



13 **NOTE:** Before we move on, I want to emphasize that we almost never have the population data, so we almost never calculate the **Population Mean**, **Variance**, or **Standard Deviation**.

If we don't usually calculate **Population Parameters**, what do we do???