An Online Method For Detecting Nonlinearity Within a Signal

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Abstract. A novel method for online analysis of the changes in signal modality is proposed. This is achieved by tracking the dynamics of the mixing parameter within a hybrid filter rather than the actual filter performance. An implementation of the proposed hybrid filter using a combination of the Least Mean Square (LMS) and the Generalised Normalised Gradient Descent (GNGD) algorithms is analysed and the potential of such a scheme for tracking signal nonlinearity is highlighted. Simulations on linear and nonlinear signals in a prediction configuration support the analysis. Biological applications of the approach have been illustrated on EEG data of epileptic patients.

1 Introduction

Signal modality characterisation is becoming an increasingly important area of multi-disciplinary research and large effort has been put into devising efficient algorithms for this purpose. The idea is that the changes in the signal nature between linear and nonlinear and deterministic and stochastic can reveal information (knowledge) which is critical in certain applications (e.g. health conditions). Our focus is on the detection of linear/nonlinear changes in signal modality. The existing algorithms in this area are based on hypothesis testing [1–3] and describe the signal changes in a statistical manner. However, there are very few online algorithms which are suitable for this purpose. The purpose of the new approach described in this paper is to show the possibility of an online algorithm which can be used not only to identify the nature of the signal, but also to track changes in the nature of the signal (signal modality detection).

One intuitive method to determine the nature of a signal has been to present the signal as input to two adaptive filters with different characteristics, one nonlinear and the other linear. By comparing the responses of each filter this can be used to identify whether the input signal is linear or not. Whilst this is a very useful simple test for signal nonlinearity, it does not provide an online solution. There are ambiguities due to the need to choose many parameters of the corresponding filters and this approach does not rely on the "synergy" between the filters considered.

1.1 Existing Approaches

In [4] an online approach is considered which successfully tracks the degree of nonlinearity of a signal using adaptive algorithms, but relies on a parametric model to effectively model the system in order to provide a true indication of the degree of nonlinearity. Figure 1 shows an implementation of this method using a third order Volterra system expansion as the system model and the normalised LMS (NLMS) algorithm with a step size $\mu = 0.008$ to update the system parameters. The system input and output can be described by

$$u[n] = \sum_{i=0}^{I} a_i x[n-i] \text{ where } I = 2 \text{ and } a_0 = 0.5, a_1 = 0.25, a_2 = 0.125 \quad (1)$$

$$y[n] = F(u[n]; n) + \eta[n] \quad (2)$$

where $x[n]$ are i.i.d uniformly distributed over the range $[-0.5, 0.5]$ and $\eta[n] \sim \mathcal{N}(0, 0.0026)$. The function $F(u[n]; n)$ varies with n, $F(u[n]; n) = u^3[n]$ for $10000 < n \leq 20000$, $F(u[n]; n) = u^2[n]$ for $30000 < n \leq 40000$ and $F(u[n]; n) = u[n]$ at all other times. The output $y[n]$ can be seen in the first trace of Fig. 1 the second and third traces show the residual estimation errors of the optimal linear system and Volterra system respectively, the final trace is the estimated degree of system nonlinearity. Whilst these results show that this approach can detect changes in nonlinearity and is not affected by the presence of noise this may be largely due to nature of the input signal being particularly suited to the Volterra model.

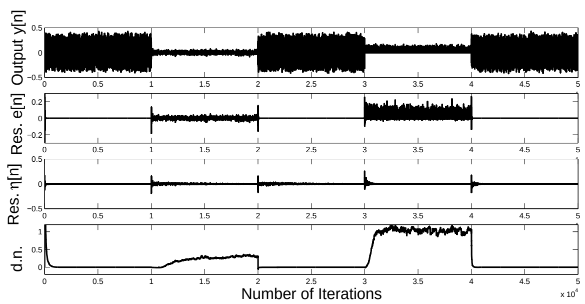

Fig. 1. NLMS with Volterra series

1.2 The Proposed Approach

The proposed approach develops on the tracking capabilities of adaptive filters by using combinations of adaptive filters in a more natural way to produce a single hybrid filter without the need for any underlying signal generation models.

Hybrid filters consist of multiple individual adaptive subfilters operating in parallel and all feeding into a mixing algorithm which produces the single output of the filter [5, 6]. This mixing algorithm is also adaptive and combines the outputs of the subfilters based on their current performance on the input signal. Previous applications of hybrid filters have focused mainly on the improved performance they can offer over the individual constituent filters. One effect of this mixing algorithm is that it can give an indication of which filter is currently responding to the input signal most effectively. Therefore by again selecting algorithms which are suitable for either linear or nonlinear signals, it is possible to cause this mixing algorithm to adapt according to fundamental properties of the input signal. A simple form of mixing algorithm for two adaptive filters is a convex combination. Convexity can be described as [7]

$$\lambda x + (1 - \lambda)y \text{ where } \lambda \in [0, 1] \tag{3}$$

and x and y are two points on a line, the resultant of this will lie on the same line between x and y. By observing the mixing parameter λ and which of the subfilters is dominating, conclusions can then be drawn about the nature of the input signal.

The proposed approach uses the least mean square (LMS) algorithm [8] to train one of the subfilters and the generalised normalised gradient descent (GNGD) algorithm [9] for the other, with the purpose of distinguishing the linearity/nonlinearity of a signal and is illustrated in Fig. 2. The LMS algorithm was chosen as it is widely used, known for its robustness and excellent steady state properties whereas the GNGD algorithm has a faster convergence speed and much better tracking capabilities. We aim at exploiting these properties and set out to show that due to the synergy and simultaneous mode of operation, our proposed hybrid filter has excellent tracking capabilities for signals with extrema in their inherent linearity and nonlinearity characteristics.

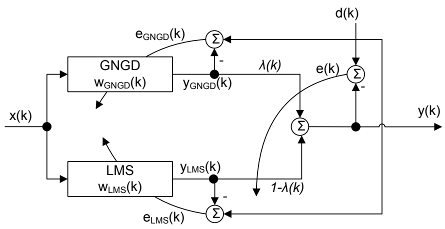

Fig. 2. Convex combination of adaptive filters

2 Derivation of The Proposed Approach

Unlike the existing approaches to hybrid adaptive filters which focus on the quantitive performance of such filters, our aim is to design a hybrid filter which combines the characteristics of two adaptive filters with the aim of making the value of the "mixing" parameter λ adapt according to the dynamics of the input signal. This is achieved following the approach from Fig. 2 where two adaptive filters are combined, one trained by the GNGD algorithm and the other by the LMS algorithm, in a convex manner. The output of the hybrid filter from Fig. 2 $y(k)$ is an adaptive convex combination of the output of the LMS trained subfilter y_{LMS} and the output of the GNGD trained subfilter y_{GNGD} and is given by

$$y(k) = \lambda(k) y_{GNGD}(k) + (1 - \lambda(k))\, y_{LMS}(k) \tag{4}$$

where y_{LMS} is generated from [8]

$$\begin{aligned} y_{LMS}(k) &= \mathbf{x}^T(k)\mathbf{w}_{LMS}(k) \\ e_{LMS}(k) &= d(k) - y_{LMS}(k) \\ \mathbf{w}_{LMS}(k+1) &= \mathbf{w}_{LMS}(k) + \mu_{LMS} e_{LMS}(k)\mathbf{x}(k) \end{aligned} \tag{5}$$

and y_{GNGD} is the corresponding output of the GNGD trained subfilter given by [9]

$$\begin{aligned} y_{GNGD}(k) &= \mathbf{x}^T(k)\mathbf{w}_{GNGD}(k) \\ e_{GNGD}(k) &= d(k) - y_{GNGD}(k) \\ \mathbf{w}_{GNGD}(k+1) &= \mathbf{w}_{GNGD}(k) + \frac{\mu_{GNGD}}{\|\mathbf{x}(k)\|_2^2 + \varepsilon(k)} e_{GNGD}(k)\mathbf{x}(k) \\ \varepsilon(k+1) &= \varepsilon(k) - \rho\mu_{GNGD} \frac{e_{GNGD}(k) e_{GNGD}(k-1)\mathbf{x}^T(k)\mathbf{x}(k-1)}{\left(\|\mathbf{x}(k-1)\|_2^2 + \varepsilon(k-1)\right)^2} \end{aligned} \tag{6}$$

where $e_{GNGD}(k)$ and $e_{LMS}(k)$ are the individual output errors of the subfilters at time instant k, $d(k)$ is the desired signal, $\mathbf{x}(k) = [x(k-1), \ldots, x(k-N)]^T$ is the input signal vector, N is the length of the filter and $\mathbf{w}_{LMS}(k) = [w_{LMS_1}(k), \ldots, w_{LMS_N}(k)]^T$ and $\mathbf{w}_{GNGD}$ are the filter weight vectors corresponding to the LMS and GNGD trained subfilters. The step-size parameters of the filters are μ_{LMS} and μ_{GNGD}, and in the case of the GNGD ρ is the step-size adaptation parameter and ε the regularisation term.

To preserve the inherent characteristics of the subfilters, which are the basis of our approach, the constituent subfilters are each updated by their own errors $e_{LMS}(k)$ and $e_{GNGD}(k)$, whereas the parameter λ is updated based on the overall error $e(k)$. The convex mixing parameter $\lambda(k)$ is updated using the following gradient adaptation

$$\lambda(k+1) = \lambda(k) - \mu_\lambda \nabla_\lambda E(k)_{|\lambda=\lambda(k)} \tag{7}$$

where μ_λ is the adaptation step-size. From (4) and (7), the λ update can be shown to be

$$\lambda(k+1) = \lambda(k) - \frac{\mu_\lambda}{2}\frac{\partial e^2(k)}{\partial \lambda(k)} = \lambda(k) + \mu_\lambda e(k)(y_{GNGD}(k) - y_{LMS}(k)) \tag{8}$$

To ensure the combination of adaptive filters remains a convex function it is critical λ remains within the range $0 \leq \lambda(k) \leq 1$. To ensure this, in [5] the authors used a sigmoid function as a post-nonlinearity to bound $\lambda(k)$. Since, in order to determine the changes in the modality of a signal (linear, nonlinear) we are not interested in the overall performance of the filter but in the variable λ the use of a sigmoid function would interfere with true values of $\lambda(k)$ and was therefore not appropriate. A hard limit on the set of allowed values for $\lambda(k)$ was therefore implemented.

3 Simulations

For all simulations the proposed approach was evaluated in an adaptive one step ahead prediction setting with the length of the adaptive filters set to $N = 10$.

Several experiments were conducted in order to illustrate the ability of the proposed approach to track the modality changes within a signal of interest. In the first experiment the behaviour of λ was investigated for benchmark linear and nonlinear inputs. Values of λ were averaged over a set of 1000 independent simulation runs. The inputs used in the simulations were a stable linear AR(4) process given by:

$$x(k) = 1.79x(k-1) - 1.85x(k-2) + 1.27x(k-3) - 0.41x(k-4) + n(k) \quad (9)$$

and a benchmark nonlinear signal [10] given by:

$$x(k+1) = \frac{x(k)}{1 + x^2(k)} + n^3(k) \quad (10)$$

where $n(k)$ is a zero mean, unit variance white Gaussian process.

The values of the step-sizes used were $\mu_{LMS} = 0.01$ and $\mu_{GNGD} = 0.6$. For the GNGD filter $\rho = 0.15$ and the initial value of the regularisation parameter was $\varepsilon(0) = 0.1$. Within the convex combination of the filters the step-size for the adaptation of $\lambda(k)$ was $\mu_\lambda = 0.05$ and the initial value of $\lambda(0) = 1$[1]. From the curves shown in Fig. 3 the value of $\lambda(k)$ for both inputs move towards zero as the adaptation progresses. As expected the output of the proposed convex combination of adaptive filters approaches the output of the LMS filter y_{LMS} predominately due to the better steady state properties of the LMS filter when compared to the GNGD filter. In the early stages of adaptation the nonlinear input (10) adapts to be dominated by the LMS filter much faster than the linear input and quickly converges whereas the linear input (9) has a much more gradual change between the two filters[2].

In the second experiment, we investigate whether we can use the changes in λ along the adaptation to track the changes in signal modality. Since the behaviour of λ as a response to the different inputs is clearly distinct, especially in the

[1] Since GNGD exhibits much faster convergence than LMS it is natural to start the adaptation with $\lambda(0) = 1$ this way, we avoid possible artefacts that may arise due to the slow initial response to the changes in signal modality.

[2] Both filters perform well on a linear input and are competing along the adaptation.

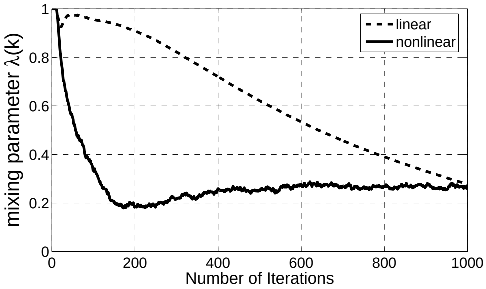

Fig. 3. Comparison of λ for linear and nonlinear inputs

earliest stages of adaptation, the proposed convex combination was presented with an input signal which alternated between linear and nonlinear. The input signal was alternated every 200 samples and the dynamics of the mixing parameter $\lambda(k)$ are shown in Fig. 4. Figure 4 shows how the value of $\lambda(k)$ adapts in a way which ensures that the output of the convex combination is dominated by the filter most appropriate for the input signal characteristics.

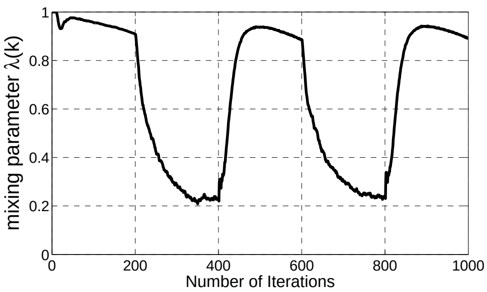

Fig. 4. Mixing parameter λ for input nature alternating every 200 samples

The third experiment was to investigate the fastest speed with which the proposed approach can adapt to the alternating signal. Figure 5 shows the response of $\lambda(k)$ to the input signal alternating every 100 and 50 samples respectively. There is a small anomaly in the values of λ immediately following the change in input signal from nonlinear to linear, which can be clearly seen in Fig. 5b around sample numbers $100i,\ i = 1, 2, \ldots$ where the value of λ exhibits a small dip before it increases. This is due to the fact that the input to both the current AR process 9 and the tap inputs to both filters use previous nonlinear samples where we are in fact predicting the first few "linear" samples. This does not become an issue when alternations between the input signals occur less regularly or if there is a more natural progression from "linear" to "nonlinear" in the the input signal.

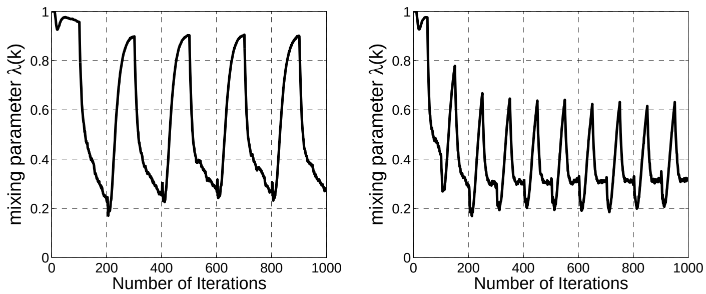

Fig. 5. Mixing parameter λ for input nature alternating every a)100 b)50 samples

Finally, to examine the usefulness of the proposed approach for the processing of real world signals a set of EEG signals has been analysed. Following the standard practice, the EEG sensor signals were averaged across all the channels and any trends in the data were removed. Figure 6 shows the response of λ when applied to two different sets of EEG data from epileptic patients, both showing the onset of a seizure. These results show that the proposed approach can effectively detect changes in the nature of the EEG signals which can be very difficult to achieve otherwise. It would be interesting to ascertain whether by performing multiple step ahead prediction would help detect a change in signal nature before the actual onset of that change.

4 Conclusions

We have proposed a novel approach to identify changes in the modality of a signal. This is achieved by a convex combination of two adaptive filters for which the transient responses are significantly different. By training the two filters with different algorithms, in this case the least mean square (LMS) and generalised normalised gradient descent (GNGD) algorithms, it is possible to exploit the different performance capabilities of each. The evolution of the adaptive convex mixing parameter λ, helps determine which filter is more suited to the current input signal dynamics, and thereby gain information about the nature of the signal. The analysis and simulations illustrate that there is significant potential for the use of this method for online tracking of some fundamental properties of the input signal.

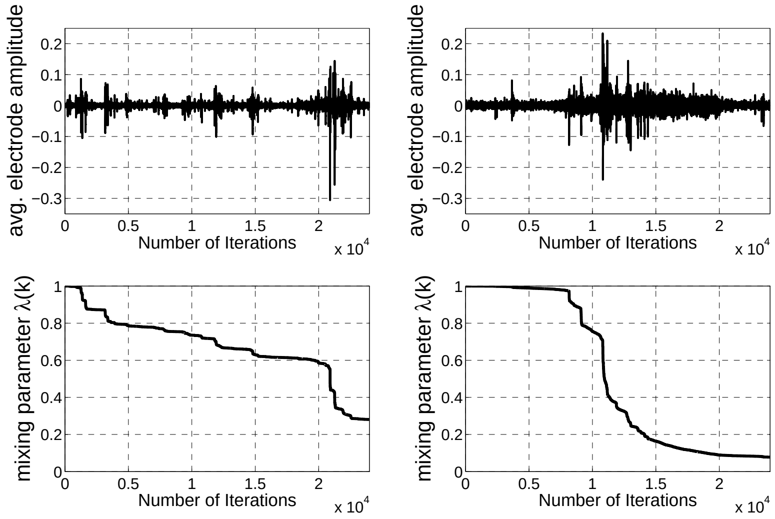

Fig. 6. EEG signals for two patients showing epileptic seizures and corresponding λ

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