An Online Method For Detecting Nonlinearity Within a Signal

Beth Jelfs[1], Phebe Vayanos[1], Mo Chen[1], Su Lee Goh[1], Christos Boukis[2], Temujin Gautama[3], Tomasz Rutkowski[4], Tony Kuh[5] and Danilo Mandic[1]

[1] Imperial College London, UK
[2] AIT, Greece
[3] Phillips Leuven, Belgium
[4] BSI RIKEN, Japan
[5] University of Hawaii

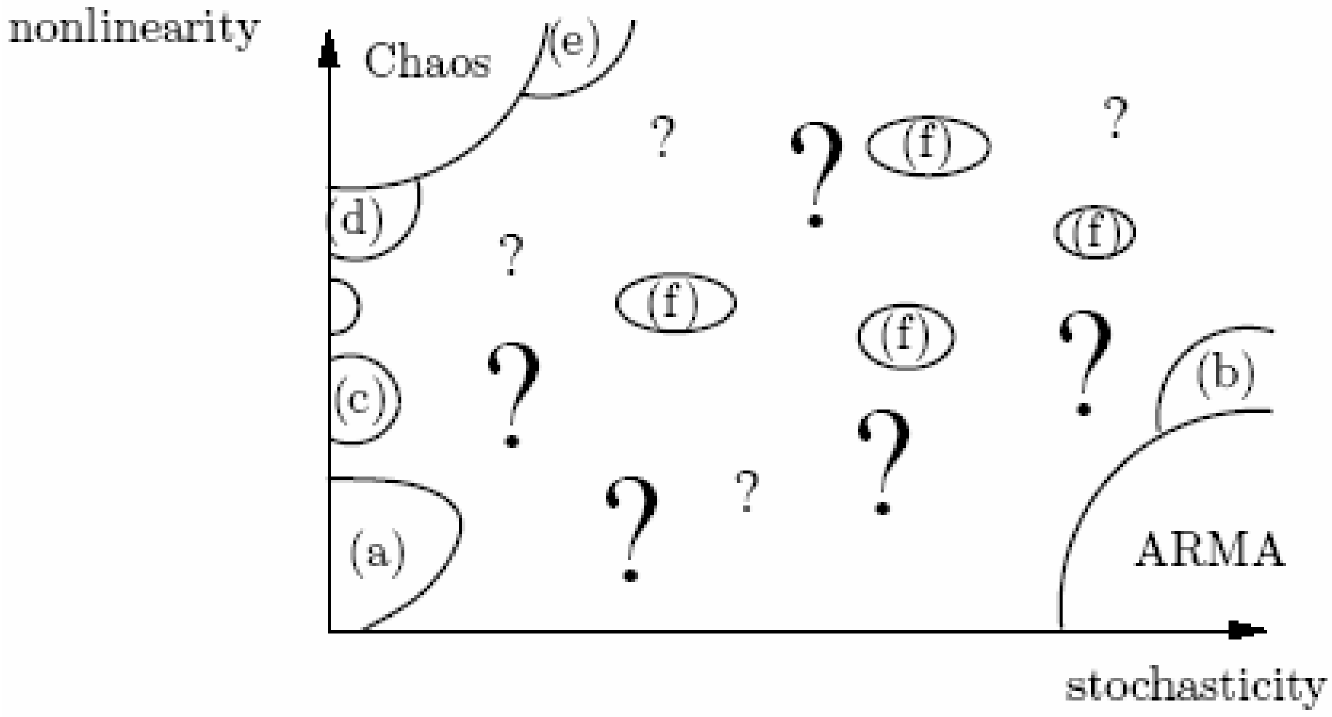
Outline

- General problem of **modality characterisation**
- Parametric methods (non–cooperative, batch based)
- Hybrid filters for **collaborative signal processing**
- On–line tracking of signal modality
- Verification of concept:- Simulation results on synthetic data
- Applications in real-world data (mental diseases)
- Conclusions

Signal Modality – General Perspective

Notice the difference between **Signal Nonlinearity and System Nonlinearity**

Deterministic vs. Stochatic nature or Linear vs. Nonlinear nature



Change in signal modality can indicate e.g. health hazard (fMRI, HRV)

Challenges in Signal Modality Characterisation

- Changes in the signal nature between (e.g. linear and nonlinear) can reveal **information** which is critical (e.g. health conditions);

- Existing algorithms based on **hypothesis testing** and operate in a batch manner;

- Other methods based on comparing outputs of **two adaptive filters of different kind** $\Rightarrow$ choice of many parameters **:-(**

- These filters **do not co–operate** $\Rightarrow$ simple test but *non–unique solution*.

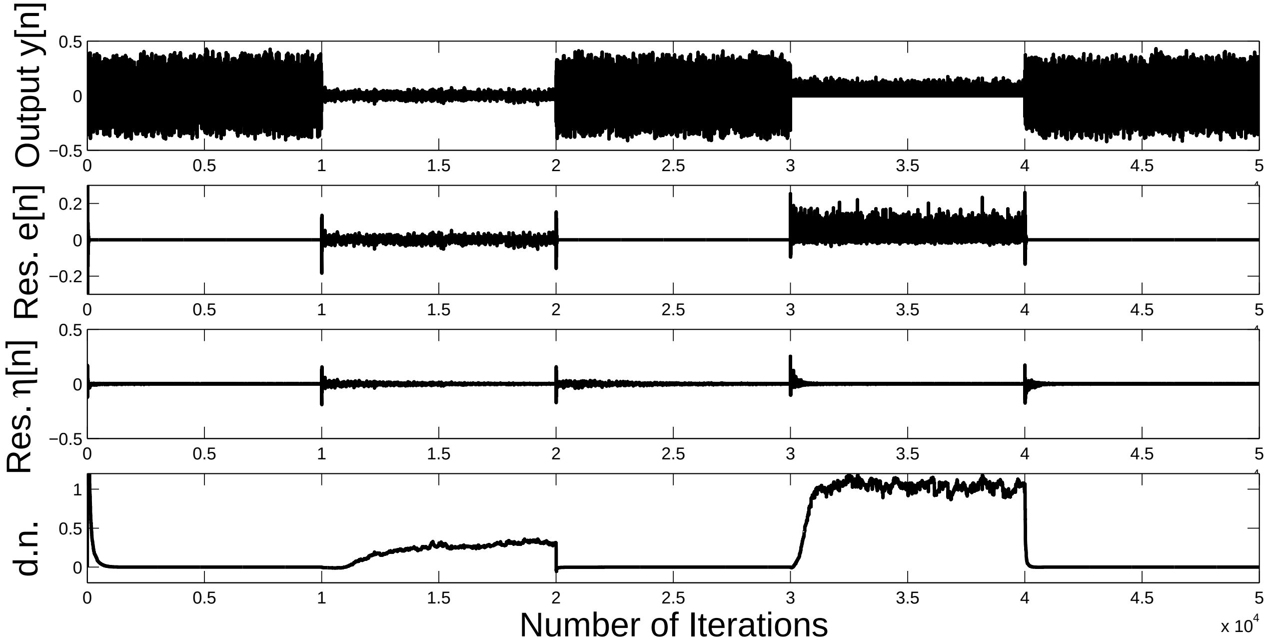
Our aim:- on–line signal modality characterisation for real–world problems

Benefits:- Synergy between the filters, existence and uniqueness of solution

Existing Parametric Methods

Illustration:- Run independently e.g. 3rd order Volterra and LMS FIR filter

Alternate 10,000 “linear” and 10,000 “nonlinear” samples



i) Relies on a parametric model to effectively model the system;

ii) **Slow response**;

ii) Ability to detect changes in nonlinearity particularly suited to the Volterra model.

Hybrid Filters

Key properties:-

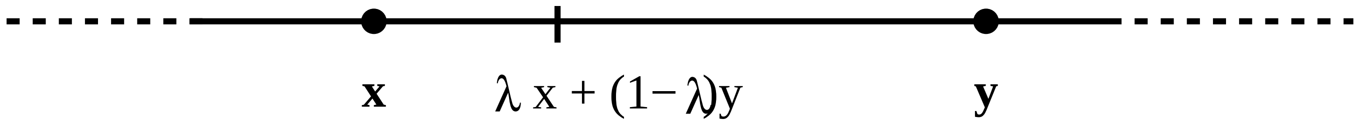
- Multiple individual adaptive subfilters operating in parallel;
- Subfilters feed into a mixing algorithm which produces the single output of the filter;
- Mixing algorithm is also adaptive and combines the outputs of the subfilters **(collaboration, synergy for two different filters**);

Advantages:-

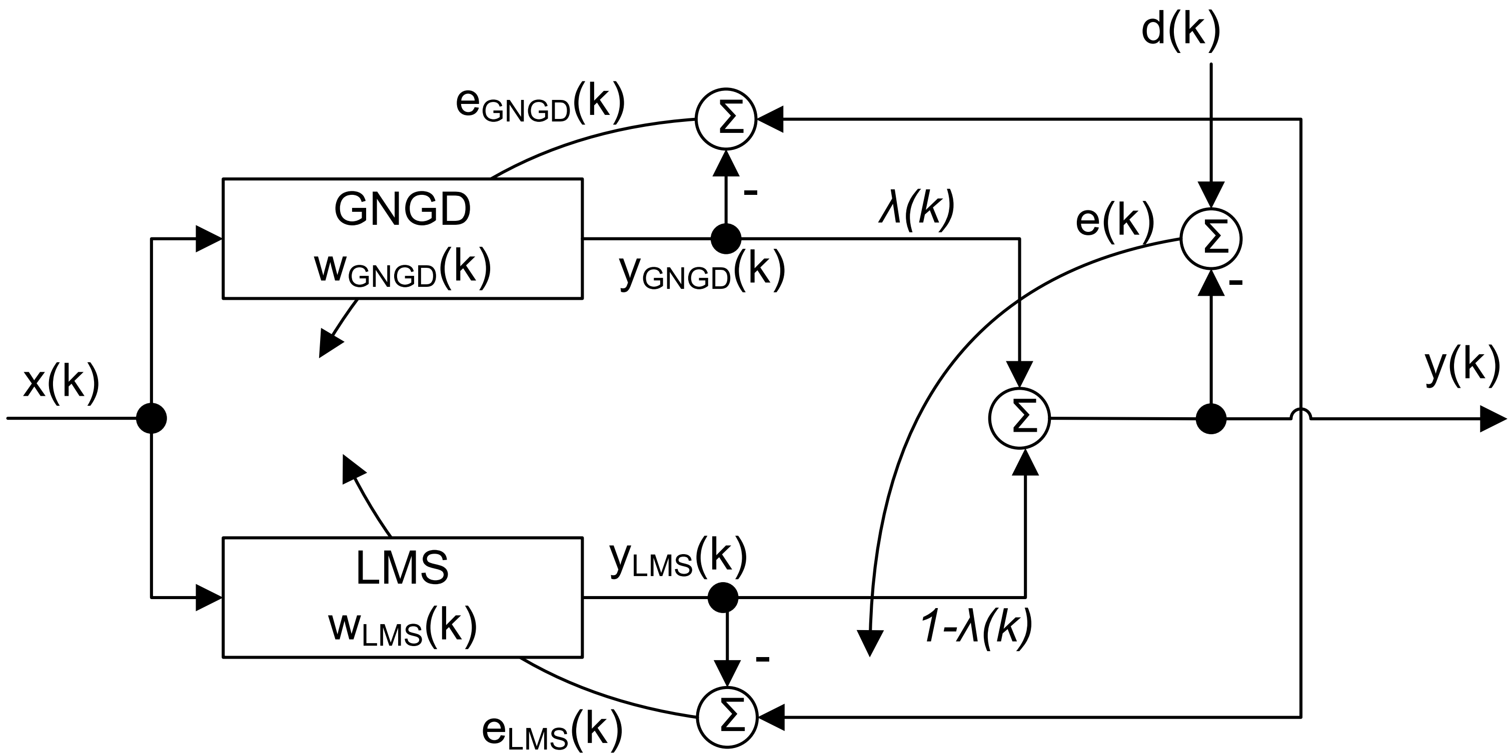
- When in "filtering mode", improved performance over the individual constituent filters;
- One effect of this mixing algorithm is that it can give an indication of which filter is currently responding to the input signal most effectively;
- By selecting algorithms which are suitable for either linear or nonlinear signals $\Rightarrow$ the mixing algorithm can adapt according to fundamental properties of the input signal.

Convex Hybrid Filtering Configuration

Virtues of Convex Combination ($\lambda \in [0, 1]$)



Convexity $\Rightarrow$ existence and uniqueness of solution :-)



LMS & GNGD

The LMS algorithm $\rightsquigarrow$ widely used, known for its robustness and excellent steady state properties

$$
\begin{aligned}
y_{\mathrm{LMS}}(k) &= \mathbf{x}^T(k)\mathbf{w}_{\mathrm{LMS}}(k) \\
e_{\mathrm{LMS}}(k) &= d(k) - y_{\mathrm{LMS}}(k) \\
\mathbf{w}_{\mathrm{LMS}}(k+1) &= \mathbf{w}_{\mathrm{LMS}}(k) + \mu_{\mathrm{LMS}} e_{\mathrm{LMS}}(k)\mathbf{x}(k)
\end{aligned}
$$

The GNGD algorithm $\rightsquigarrow$ faster convergence speed and much better tracking capabilities

$$
\begin{aligned}
y_{\mathrm{GNGD}}(k) &= \mathbf{x}^T(k)\mathbf{w}_{\mathrm{GNGD}}(k) \\
e_{\mathrm{GNGD}}(k) &= d(k) - y_{\mathrm{GNGD}}(k) \\
\mathbf{w}_{\mathrm{GNGD}}(k+1) &= \mathbf{w}_{\mathrm{GNGD}}(k) + \frac{\mu_{\mathrm{GNGD}}}{\|\mathbf{x}(k)\|_2^2 + \varepsilon(k)} e_{\mathrm{GNGD}}(k)\mathbf{x}(k) \\
\varepsilon(k+1) &= \varepsilon(k) - \rho\mu_{\mathrm{GNGD}} \frac{e_{\mathrm{GNGD}}(k)e_{\mathrm{GNGD}}(k-1)\mathbf{x}^T(k)\mathbf{x}(k-1)}{\left(\|\mathbf{x}(k-1)\|_2^2 + \varepsilon(k-1)\right)^2}
\end{aligned}
$$

$\looparrowright$ λ **adapts according to the dynamics of the input :-)**

Adaptation of Mixing Parameter λ (Modality Tracking)

To preserve the inherent characteristics of the subfilters, the constituent subfilters are each updated by their own errors $e_{\mathrm{LMS}}(k)$ and $e_{\mathrm{GNGD}}(k)$, whereas the parameter λ is updated based on the overall error $e(k)$.

The convex mixing parameter $\lambda(k)$ is updated using the following gradient adaptation

$$\lambda(k+1) = \lambda(k) - \mu_\lambda \nabla_\lambda E(k)_{|\lambda=\lambda(k)}$$

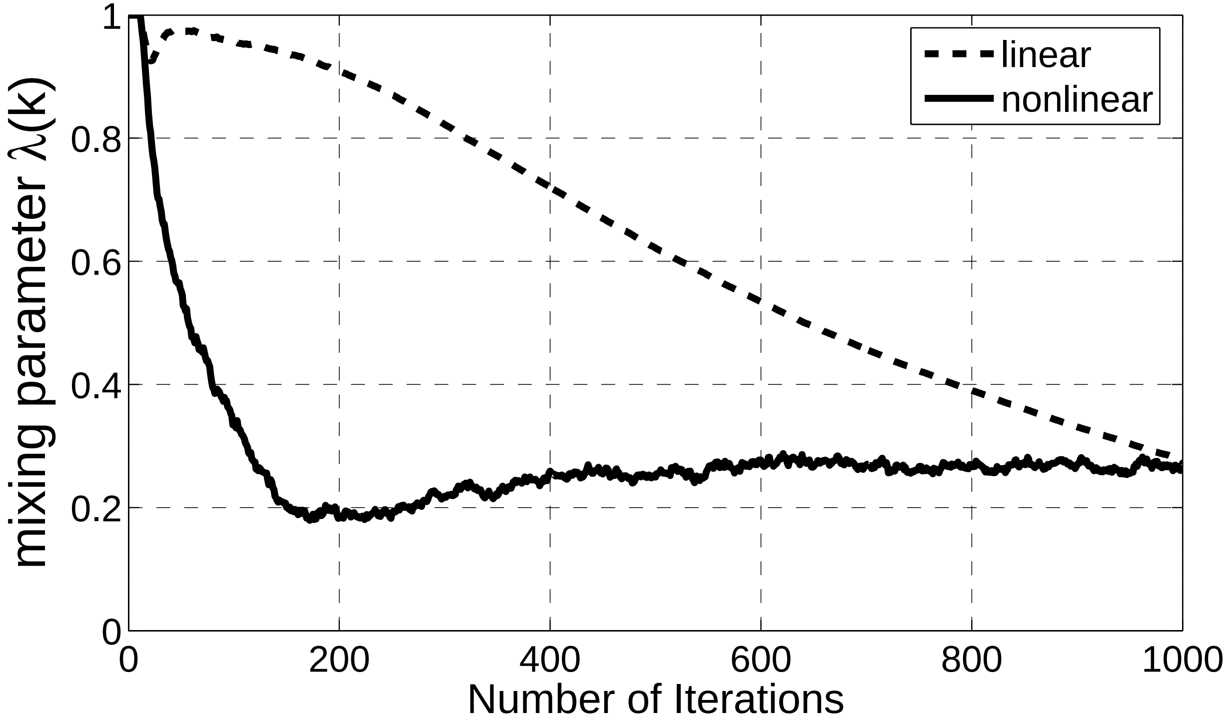
where μ_λ is the adaptation step-size. The λ update can be shown to be

$$\begin{aligned}\lambda(k+1) &= \lambda(k) - \frac{\mu_\lambda}{2}\frac{\partial e^2(k)}{\partial \lambda(k)} \\ &= \lambda(k) + \mu_\lambda e(k)(y_{GNGD}(k) - y_{LMS}(k))\end{aligned}$$

To ensure the combination of adaptive filters remains a convex function it is critical λ remains within the range $0 \le \lambda(k) \le 1$, a hard limit on the set of allowed values for $\lambda(k)$ was therefore implemented.

Standard Use of Hybrid Filters:- Synthetic Signals

Benchmark linear AR(4) and nonlinear (Narendra III)input

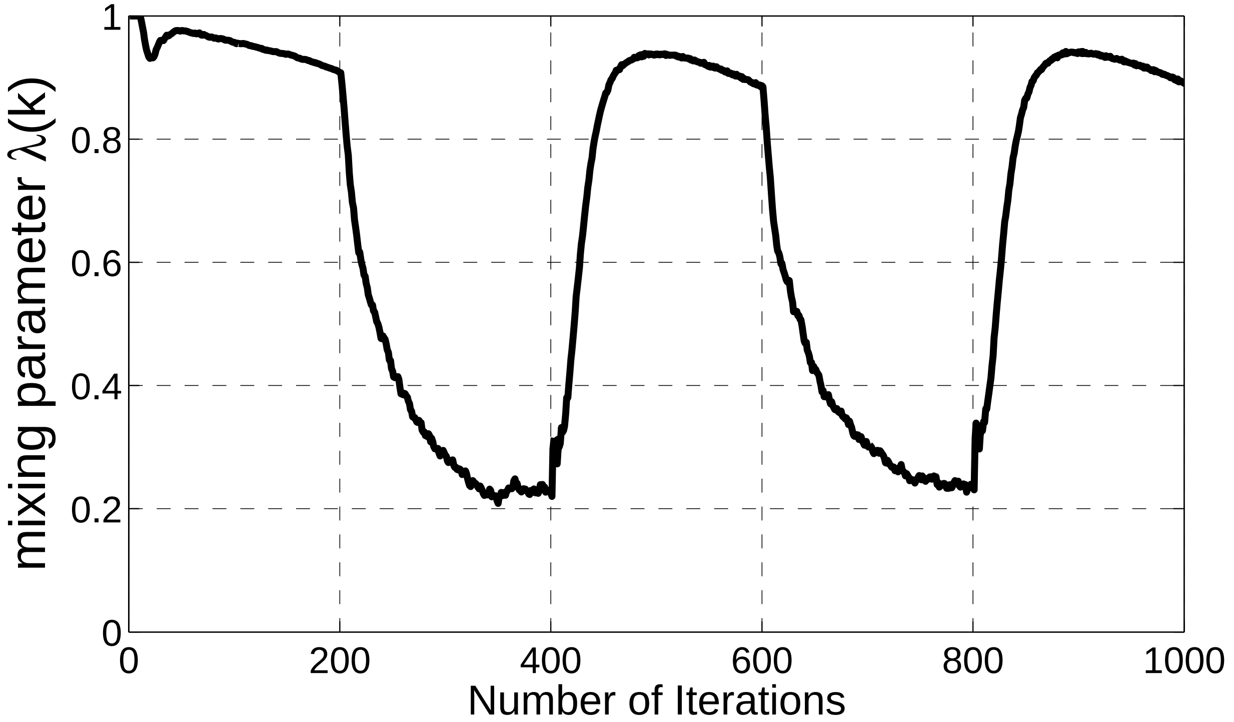


- When in **prediction mode** $\Rightarrow$ improved both speed of convergence (due to GNGD) and steady state properties (due to LMS);

For our application:- $\looparrowright$ Clear difference in the dynamics of λ!

Tracking Capability

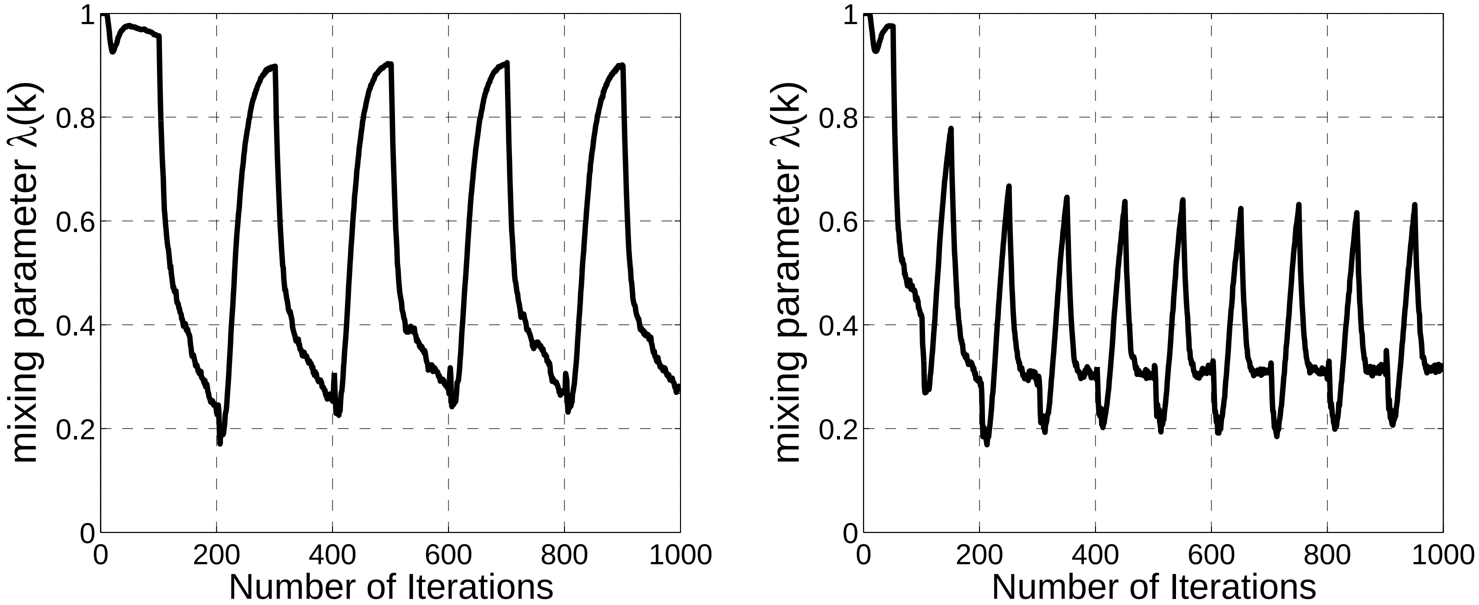
The convex combination of filters was presented with an input signal which alternated between linear and nonlinear every 200 samples



$\Rightarrow$ **output of the convex combination is dominated by the filter most appropriate for the input signal characteristics**

Tracking Capability cont.

The same experiment, with alternation every 100 and every 50 samples



- When alternating every 50 samples a small anomaly in the values of λ occurs immediately following the change in input signal from nonlinear to linear;
- **Not an issue for less regular alternations or if there is a more natural progression from "linear" to "nonlinear"** **:-)**

Real-World Applications:- Epileptic Seizure Data

To examine the usefulness of the proposed approach for the processing of real world signals a set of EEG signals has been analysed.

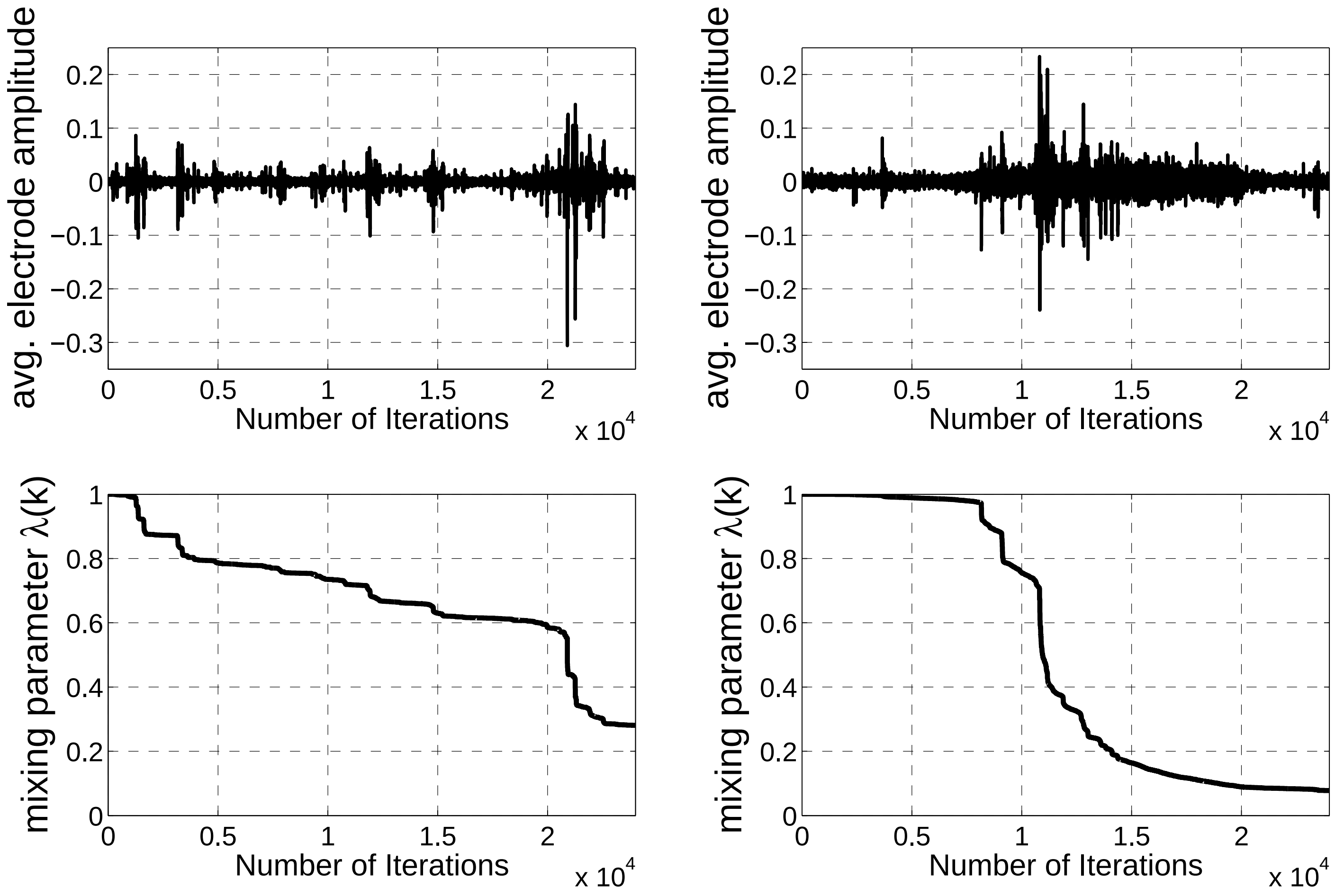
- The response of λ when applied to two different sets of EEG data from epileptic patients, both showing the onset of a seizure was observed;

- Following the standard practice, the EEG sensor signals were averaged across all the channels and any trends in the data were removed;

- These results show that the proposed approach can effectively detect changes in the nature of the EEG signals which can be very difficult to achieve otherwise;

- It would be interesting to ascertain whether by performing multiple step ahead prediction would help detect a change in signal nature before the actual onset of that change.

Real-World Applications:- Epileptic Seizure Data



Conclusions

- Novel approach to identify changes in the modality of a signal;
- Convex combination of two adaptive filters for which the transient responses are significantly different, in order to exploit the different performance capabilities of each;
- Collaborative adaptive signal processing approach, based on synergy between the constitutive filters;
- The evolution of the adaptive convex mixing parameter λ, helps determine which filter is more suited to the current input signal dynamics, and thereby gain information about the nature of the signal;
- The analysis and simulations illustrate that there is significant potential for the use of this method for online tracking of some fundamental properties of the input signal.

Acknowledgements:-

Udo Trutschell (Circadian Technologies, Boston) $\looparrowright$ for drawing our attention to this class of problems and numerous discussion

Martin Golz (Schmalkalden Germany) $\looparrowright$ fruitful discussion, provision of experimental data

Saied Sanei (Cardiff) $\looparrowright$ provision of data

Christopher James (Southampton) $\looparrowright$ provision of data

Mary Morrell (Royal Brompton Hospital) $\looparrowright$ general advice on how to deal with multimodal biomedical data

Andrzej Cichocki (RIKEN Japan) $\looparrowright$ for frequent discussion on blind approaches to this problem and general collaboration in multichannel and multimodal adaptive signal processing

Thank you

“A man who swings a cat by its tail learns things he can learn no other way”

Mark Twain

Literature

Some background from

- D. Mandic and J. Chambers, "*Recurrent Neural Networks for Prediction: Algorithms, Architectures, and Stability*", Wiley 2001.