# Computer Aided Verification

Assignment 2

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## 1 For which of the two logics LTL and CTL (if any) are these legal (i.e., syntactically correct) formulae? You can assume that a, b and c are atomic propositions.

| | | |
|---|---|---|
| (a) | $\Diamond\Box\Diamond b$ | LTL |
| (b) | $\exists(c \bigcup \forall\bigcirc a)$ | CTL |
| (c) | $b \wedge \Box\ (a \rightarrow \bigcirc \neg\ c)$ | Neither |
| (d) | $\neg\ ((\ \Box\Box a) \bigcup \bigcirc\ (b \wedge c))$ | LTL |
| (e) | $\exists\Box\Diamond a$ | Neither |

## 2 For each LTL formula, state whether the LTS satisfies it and, for each CTL formula, give the set of states of the LTS that satisfy it.

| | | |
|---|---|---|
| (a) | $c \wedge \bigcirc\ b \wedge \Diamond\ a$ | the LTS satisfies a |
| (b) | $\forall\ \bigcirc\ a \wedge \exists\ \Diamond\ c$ | $S_3$,$S_4$,$S_6$ |
| (c) | $\exists\Diamond\ (\exists\bigcirc\ (a \wedge b) \wedge \exists\bigcirc\ (\neg a \wedge \neg b))$ | $S_5$ |
| (d) | $\Box\Diamond\ (a \wedge \neg c)$ | the LTS satisfies d |
| (e) | $\Box\Diamond\ (b \vee c)$ | the LTS does not satisfy e |

## 3 Translate the following informally described properties into the specified temporal logic, explaining the meaning of any atomic propositions that you use.

### 3.a servers 1 and 2 are never both down simultaneously (in CTL)

$S_1$ = Server one is down

$S_2$ = Server 2 is down

$\neg\exists\Diamond(S_1 \wedge S_2)$

### 3.b the robot eventually reaches room A and never passes through corridor B or C whilst doing so (in LTL)

A = In room A

B = In corridor B

C = In corridor C

$\neg$(B $\vee$ C) $\bigcup$ A

### 3.c program location l 7 is visited infinitely often and program locations l 2 and l 3 are visited only finitely often (in LTL)

$l_7$ = at program location $l_7$

$l_2$ = at program location $l_2$

$l_3$ = at program location $l_3$

$(\Box\Diamond\ l_7) \wedge (\neg(\Box\Diamond\ (l_2 \vee l_3))$

## 4 Using these (where needed), either prove or disprove each of the following proposed LTL equivalences. You can assume that a, b and c are atomic propositions.

### 4.a $\neg\ (\Box\Diamond\ \mathbf{a} \rightarrow \Diamond\Box\mathbf{b}) \equiv \Box\Diamond\ \mathbf{a} \wedge \Box\Diamond\neg\mathbf{b}$

$\neg\ (\Box\Diamond\ \mathrm{a} \rightarrow \Diamond\Box\mathrm{b})$

$\neg\ (\neg\Box\Diamond\ \mathrm{a} \vee \Diamond\Box\mathrm{b})$ implication eq

$\neg\ (\neg\Box\Diamond\ \mathrm{a} \vee \Diamond\neg\Diamond\neg\mathrm{b})$ box eq

$\Box\Diamond\ \mathrm{a} \wedge \neg\Diamond\neg\Diamond\neg\mathrm{b}$ DeMorgan's Law

$\Box\Diamond$ a $\wedge$ $\Box\Diamond\neg$b Box eq

### 4.b $\Box$ (b $\wedge$ c) $\rightarrow$ $\Diamond$ a $\equiv$ $\neg$ ($\Box$ (b $\wedge$ c) $\wedge$ $\Box$ a)

The trace of only a e.g. a, a, a, ... satisfies the left side of the equality but not the right. A trace exists which satisfies one side but not the other, therefore they are not equivalent.