Set cover problem (II): scheduling problem

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Outline of Topics

Motivating example: A&E doctor scheduling

- A hospital A&E department needs to keep doctors on call, so that a qualified individual is available to perform every medical procedure that might be required
- For each of several doctors available for on-call duty, the additional salary they need to be paid, and which procedures they can perform, is known.
- The goal to choose doctors so that each procedure is covered, at a minimum cost.

	Doctor 1	Doctor 2	Doctor 3	Doctor 4	Doctor 5	Doctor 6
Procedure 1	✓			✓		
Procedure 2	✓				✓	
Procedure 3		✓	✓			
Procedure 4	✓					✓
Procedure 5		✓	✓			✓
Procedure 6		✓				✓
Salary (k)	180	160	50	30	30	70

Crew scheduling problem

- Crew scheduling: assigning a group of workers (a crew) to a set of tasks
  - The crews are typically interchangeable
  - In some cases different crews possess different characteristics that affect which subsets of tasks they can complete
- Appear in a number of transportation contexts, e.g., bus and rail transit, truck and rail freight transport, and freight and passenger air transportation
- **Main goal**: to cover all tasks while seeking to minimize labor costs, and a wide variety of constraints imposed by safety regulations and labor negotiations.
- Crew scheduling research focuses on a particular application, rather than the general case

Airline crew scheduling problem

- Airline crew scheduling problem: why it is important/interesting?
  - Airlines typically have a fixed schedule that changes **at most** monthly – A true planning/scheduling problem
  - They provide a context for examining many of the elements common to all crew scheduling problems such as safety regulations
  - Airline industry is highly regulated: more constraints $\longrightarrow$ more difficult to solve
  - Airline crews receive substantially higher salaries than equivalent personnel in other modes of transportation; the savings associated with an improved airline crew schedule can be quite significant

Airline crew scheduling problem

- An airline has $m = 5$ scheduled flight-legs connecting 6 cities per week in its current service
- A **flight-leg** is a single flight flown by a single crew e.g. London - Sheffield
- Each flight leg must be flown.
- For example:

Flight-leg	Origin	Destination
1	Newcastle	Bath
2	Sheffield	Edinburgh
3	Plymouth	London
4	Edinburgh	Newcastle
5	London	Sheffield

airline crew scheduling problem

- Round-trip rotation (pairing): a sequence of flight legs for a crew that begin and end at individual base locations
- Usually last for 2-5 days, must conform to all applicable regulations, work rules, restrictions and other factors
- Let $S_j (j = 1, \cdots, n)$ be all (in our example, $n = 7$) feasible round-trip rotations, each rotation is associated with a cost c_j

j	Round-trip Rotations S_j	cost c_j
1	Sheffield – Edinburgh – Newcastle – Sheffield	560
2	Sheffield – Edinburgh – London – Sheffield	335
3	Plymouth – London – Plymouth	420
4	Plymouth – London – Sheffield – Plymouth	470
5	Newcastle – Bath – Sheffield – Newcastle	545
6	Sheffield – Newcastle – Bath – London– Sheffield	660
7	Newcastle – Bath – Edinburgh – Newcastle	490

Motivating example: airline crew scheduling problem

- Let $x_j \in \{0,1\},\ j = 1, \cdots, n$ be the decision variable
  - $x_j = 1$: Rotations S_j is selected
  - $x_j = 0$: otherwise
- The objective: to find the optimal collection of rotations with minimal costs such that each flight leg is covered by exactly one rotation.
- The objective function:

$$\text{minimise} \sum_{j=1}^{n} c_j x_j$$

- How to represent constraints?

The constraints

- Constraints: **Each flight leg is covered by exactly one rotation.**
- We can construct a $m \times n$ constraint matrix $\mathbf{a}$ to represent the constraints, where $a_{ij} = 1$ indicates flight-leg i is covered by rotation j; $a_{ij} = 0$ otherwise. For example, we construct a 5×7 constraint matrix $\mathbf{a}$

Table : Constraint Matrix $\mathbf{a}$

Flight-leg	Rotation S_j						
	1	2	3	4	5	6	7
1 Newcastle to Bath	0	0	0	0	1	1	1
2 Sheffield to Edinburgh	1	1	0	0	0	0	0
3 Plymouth to London	0	0	1	1	0	0	0
4 Edinburgh to Newcastle	1	0	0	0	0	0	1
5 London to Sheffield	0	1	0	1	0	1	0

The constraints

- Constraints: **Each flight leg is covered by exactly one rotation.**
- The airline crew scheduling problem can be written as:

$$\text{minimise} \sum_{j=1}^{n} c_j x_j \tag{1}$$

$$\text{subject to} \sum_{j=1}^{n} a_{ij} x_j = 1, \qquad i = 1, \cdots, m, \tag{2}$$

$$x_j \in \{0, 1\}, \qquad j = 1, \cdots, n, \tag{3}$$

The constraints

- Constraints: **Each flight leg is covered by exactly one rotation.**
- Question: Why exactly one rotation?

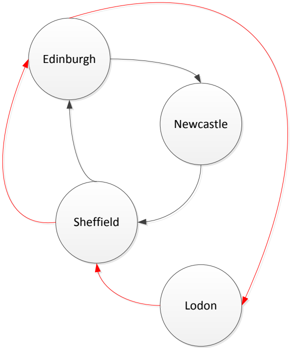
The constraints

- Constraints: **Each flight leg is covered by exactly one rotation.**
- Why exactly one rotation?
  - A flight leg must be covered
  - Flight leg is covered by more than one rotations
    - One rotation is executed by one crew
    - Only a single flight (plane) for a flight-lag executed by one crew, which means the other crew need to travel on a flight as passengers – Higher costs

The constraints

- Example: flight leg 2 (Sheffield to Edinburgh) is covered by rotations 1 and 2:

j	Round-trip Rotations S_j	cost c_j
1	Sheffield – Edinburgh – Newcastle – Sheffield	560
2	Sheffield – Edinburgh – London – Sheffield	335



Code Example 1: a real-world Boening 727 crew scheduling problem

- Open the B727 air crew scheduling problem (b727.dat), which has 157 rotations and 29 flight-legs
- Use ReadInData.m to read the data into Matlab

How to solve it: Stochastic local search algorithm

- The greedy algorithm you have implemented is a local search algorithm.
    - Good at exploitation: capable to find local optimum
    - Not good at exploration: gets stuck into local optimum
- Stochastic local search algorithm: escape from local optima by introducing randomness
- We will implement an algorithm in Balas and Ho [1980]

E. Balas and A. Ho. Set covering algorithms using cutting plans, heuristics and subgradient optimisation. A computational study. Mathematical Programming Study. 12:37-60, 1980

Set covering problem: stochastic local search algorithm

The stochastic local search for set cover problem

Let I represents which rows have been covered
Let $\mathcal{F}$ represents the solutions, i.e., columns have been selected by the algorithm
Initialise $I = \varnothing$
Initialise $\mathcal{F} = \varnothing$
while $|I| < m$ **do**
  Select a currently uncovered row j uniformly at random
  Select a column i that cover j with minimum cost
  Include column i as part of the solution : $\mathcal{F} \cup i$;
  Set I to include the rows covered by column i
end while

Exercise 1

- **Exercise 1 (30 mins)**: Using my source code template (StochasticSetCovering_outline.m), implement the stochastic local search algorithm for set covering problem
  - Run Example1.m to solve the toy problem of our example (page 6)
  - Run Example2.m to solve the B727 air crew scheduling problem (b727.dat). The best known solution is 94400 (see this paper)
  - There are multiple real-world air crew scheduling problems at this web page here. You can even try some very large-scale ones, the results from the stochastic local search is not too far way from the best known results.

Question

- Question: Are the solutions returned by the stochastic local search algorithm feasible?

Set Cover/Partitioning Problem

- Question: Are all the solutions returned by the stochastic local search algorithm feasible?
- Answer: No. Since some of the solutions cover a flight leg with more than one rotations, which violated the equality constraints:

$$\sum_{j=1}^{n} a_{ij}x_j = 1,\ i = 1, \cdots, m$$

- This is because the algorithm we implemented is used for solving **Set Cover Problem**, of which the constraints are inequality constraints:

$$\sum_{j=1}^{n} a_{ij}x_j \geq 1,\ i = 1, \cdots, m$$

- In fact, the crew scheduling problem is call **Set Partitioning Problem** because of Equations (2)

Let's solve the problem using GA

- Binary genetic algorithm:
  - Representation:

column j	1	2	3	4	$\cdots$	$n-1$	n
S_j	0	0	1	1	0	1	0

  - Constraint handling using penalty function:

$$h_i(\mathbf{x}) = 0, \ i = 1, \cdots, m$$

  - Note: each row (flight-leg) has a constraint, so in total there are m constraints
  - The new objective (fitness) function:

$$f'(\mathbf{x}) = f(\mathbf{x}) + \lambda \sum_{i=1}^{m} h_i(\mathbf{x})$$

  where $h_i(\mathbf{x})$ is the equality penalty function.

Let's solve the problem using GA

- Let's take a look at the equation

$$\sum_{j=1}^{n} a_{ij} x_j = 1,\ i = 1, \cdots, m,$$

- Note: each row (flight-leg) has a constraint, so in total there are m (number of legs) constraints (rows)
- We can write this as a matrix form:

$$\sum_{j=1}^{n} a_{ij} x_j = \mathbf{a}\mathbf{x} = 1,\ i = 1, \cdots, m,$$

where $\mathbf{x}$ is a binary vector $\mathbf{x} = [x_1, x_2, \cdots, x_n]^{\mathrm{T}}$

Let's solve the problem using GA

- For each solution $\mathbf{x}$, it must satisfy :

$$\mathbf{ax} - 1 = 0,$$

  where $\mathbf{x}$ is a binary vector $\mathbf{x} = [x_1, x_2, \cdots, x_n]^{\mathrm{T}}$
- The constraint violations are::

$$\mathbf{h} = (\mathbf{ax} - 1)^2$$

  , where $\mathbf{h}$ is is a positive vector $\mathbf{h} = [h_1, h_2, \cdots, h_m]^{\mathrm{T}}$
- The total degree of violation is:

$$\sum_{i=1}^{m} h_i$$

Code example 2: solving crew scheduling problem using BGA

- Open SimpleGASetPartition.m and I will explain the code.

CA2: Airline crew scheduling using GA

- CA2: Airline crew scheduling using GA.