Nature Inspired Search and Optimisation
Exercise 2

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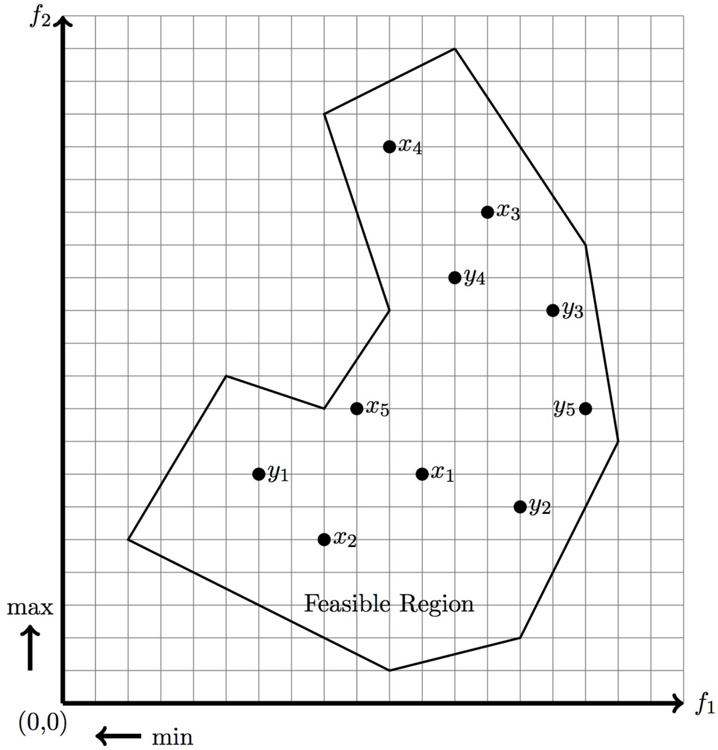
1 For a multi-objective optimisation problem, what is the difference between Pareto-optimal solutions and Pareto-optimal front?

Pareto-optimal front is a curve of non-dominated points, with respect to the objectives, on the boundary of the feasible region. There are infinite points on this front of optimal solutions and so infinite solutions.

Pareto-optimal solutions are specified points on the Pareto-optimal front. These solutions maximise the objectives.

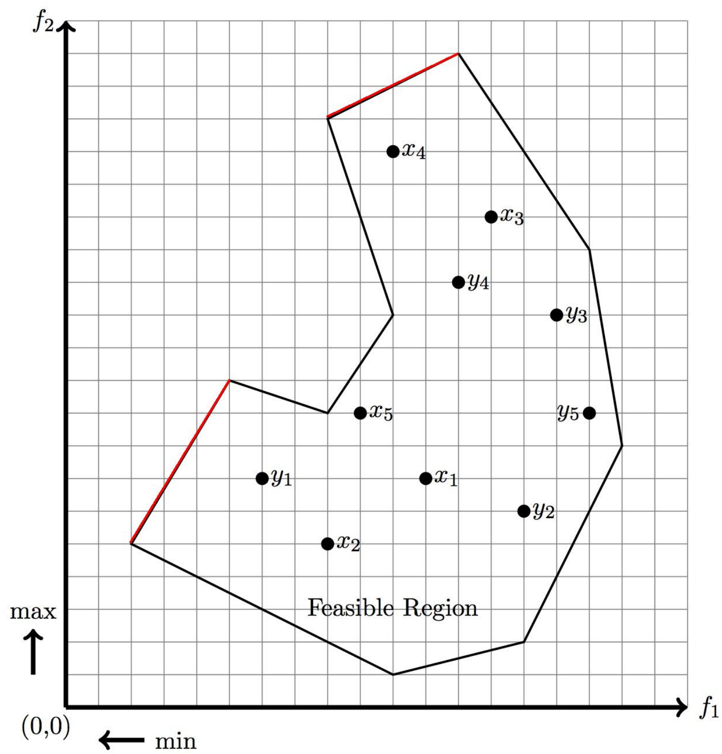
2

x_1	x_2	x_3	x_4	x_5	y_1	y_2	y_3	y_4	y_5
(11,7)	(8,5)	(13,15)	(10,17)	(9,9)	(6,7)	(14,6)	(15,12)	(12,13)	(16,9)



f_2
x_4
x_3
y_4
y_3
x_5
y_5
y_1
x_1
y_2
x_2
Feasible Region
max
f_1
(0,0)
min

2.1 Identify the Pareto-optimal front of the above bi-objective problem by drawing it on the figure



2.2 Consider search point y_4. For each of the other search points, state if y_4 dominates the point, is dominated by the point or is incomparable to the point

x_1	Incomparable
x_2	Incomparable
x_3	Incomparable
x_4	Dominates y_4
x_5	Incomparable
y_1	Incomparable
y_2	Dominated by y_4
y_3	Dominated by y_4
y_5	Dominated by y_4

2.3 NSGA-II [1] uses non-dominated sorting to divide the population into different non-dominated classes of solutions. Perform this step of NSGA-II for the given individuals. For each of the individuals indicate its class number, where the best class has number 0

Implementing the fast-non-dominated-sort algorithm on page 184 of [1].

For each individual, find the items that it dominates and is dominated by.

$\mathcal{F}_1 = \emptyset$

For x_1
$S_{x_1} = \{y_2\}$ Set dominated by x_1.
$n_{x_1} = 3$ Domination Counter - number of solutions x_1 is dominated by.

$S_{x_2} = \{\}$
$n_{x_2} = 1$

$S_{x_3} = \{y_2, y_3, y_5\}$
$n_{x_3} = 1$

$S_{x_4} = \{x_1, x_3, y_2, y_3, y_4, y_5\}$
$n_{x_4} = 0$

$x_{4_{rank}} = 1$
$\mathcal{F}_1 = \{x_4\}$

$S_{x_5} = \{x_1, y_2, y_5\}$
$n_{x_5} = 0$

$x_{5_{rank}} = 1$
$\mathcal{F}_1 = \{x_4, x_5\}$

$S_{y_1} = \{x_1, x_2, y_2\}$
$n_{y_1} = 0$

$y_{1_{rank}} = 1$
$\mathcal{F}_1 = \{x_4, x_5, y_1\}$

$S_{y_2} = \{\}$
$n_{y_2} = 6$

$S_{y_3} = \{y_5\}$
$n_{y_3} = 3$

$S_{y_4} = \{y_2, y_3, y_5\}$
$n_{y_4} = 1$

$S_{y_5} = \{\}$
$n_{y_5} = 5$

Remove the current non-dominated set and select the new one.

$\mathcal{F}_1 = \{x_4, x_5, y_1\}$
$n_{x_1} = 0$
$n_{x_3} = 0$
$n_{y_2} = 3$
$n_{y_3} = 2$
$n_{y_4} = 0$
$n_{y_5} = 3$
$n_{x_2} = 0$

$\mathcal{F}_2 = \{x_1, x_3\ y_4, x_2\}$
$n_{y_2} = 0$
$n_{y_3} = 0$
$n_{y_5} = 1$

$\mathcal{F}_3 = \{y_2, y_3\}$
$n_{y_5} = 0$

$\mathcal{F}_4 = \{y_5\}$

The classes are as follows:

Class 0 = $\{x_4, x_5, y_1\}$
Class 1 = $\{x_1, x_3\ y_4, x_2\}$
Class 2 = $\{y_2, y_3\}$
Class 3 = $\{y_5\}$

2.4 Using your result from c), which individuals does NSGA-II choose for the next iteration?

Five individuals should be chosen for the next iteration. Because of this, all of class 0 should be carried forward. This means that 2 individuals are required from class 1 where there are 4 individuals. NSGA-II performs Crowding Distance Sorting on class 1 in order to find a diverse population for the next iteration. This algorithm is on page 185 [1].

Class 1 has fitness of 5.

Using Manhattan distance function for crowding distance.

Individual	Crowding Distance	Niching Fitness
x_1	12	2.5
x_2	∞	5
x_3	∞	5
y_4	10	2.5

So the two individuals to be chosen for the next iteration from class 1 are x_3 and x_2.

Therefore the set of individuals to choose for the next iteration are: $\{x_4$, x_5, y_1, x_3, $x_2\}$.