Assignment 2
Temporal Logic

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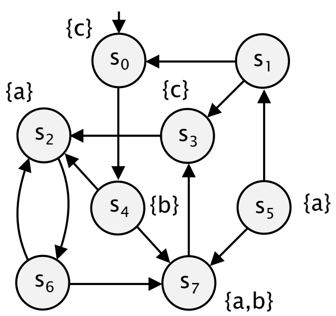
1. For which of the two logics LTL and CTL (if any) are these legal (i.e., syntactically correct) formulae? You can assume that a, b and c are atomic propositions.

   (a) $\Diamond\Box\Diamond b$
   (b) $\exists(c\,\mathtt{U}\,\forall\bigcirc a)$
   (c) $b \wedge c\,\Box(a \rightarrow \bigcirc\neg c)$
   (d) $\neg((\Box\Box a)\,\mathtt{U}\bigcirc(b \wedge c))$
   (e) $\exists\Box\Diamond a$

   [5 marks]

2. Consider the following LTS:



   and the temporal logic formulae below:

   (a) $c \wedge \bigcirc b \wedge \Diamond a$
   (b) $\forall\bigcirc a \wedge \exists\Diamond c$
   (c) $\exists\Diamond(\exists\bigcirc(a \wedge b) \wedge \exists\bigcirc(\neg a \wedge \neg b))$
   (d) $\Box\Diamond(a \wedge \neg c)$
   (e) $\Box\Diamond(b \vee c)$

   For each LTL formula, state whether the LTS satisfies it and, for each CTL formula, give the set of states of the LTS that satisfy it.

   [5 marks]

3. Translate the following informally described properties into the specified temporal logic, explaining the meaning of any atomic propositions that you use.

   (a) "servers 1 and 2 are never both down simultaneously" (in CTL)

   (b) "the robot eventually reaches room A and never passes through corridor B or C whilst doing so" (in LTL)

   (c) "program location l_7 is visited infinitely often and program locations l_2 and l_3 are visited only finitely often" (in LTL)

[6 marks]

4. We saw in lectures that, in addition to standard equivalences for propositional logic, various LTL equivalences exist for temporal operators, for example:

   - $\Box\psi \;\equiv\; \neg\Diamond\neg\psi$
   - $\Diamond\psi \;\equiv\; \psi \vee \bigcirc\Diamond\psi$

   Using these (where needed), either prove or disprove each of the following proposed LTL equivalences. You can assume that a, b and c are atomic propositions.

   (a) $\neg(\Box\Diamond a \rightarrow \Diamond\Box b) \;\equiv\; \Box\Diamond a \wedge \Box\Diamond\neg b$

   (b) $\Box(b \wedge c) \rightarrow \Diamond a \;\equiv\; \neg(\Box(b \wedge c) \wedge \Box a)$

[9 marks]