HW 3

Tuesday, October 14, 2025 11:35

1) $\frac{\partial y}{\partial t} = -\alpha y$, $y(0)=1$, $\alpha>0$, $0 \le t \le T$

$\int -\frac{1}{\alpha y}\partial y = \int \partial t$

$-\frac{1}{\alpha}\ln y + C = t$

$-\frac{1}{\alpha}\ln(1) + C = 0$

$C = 0$

$-\frac{1}{\alpha}\ln y = t$

$\boxed{y = e^{-\alpha t}}$

- Forward Euler:

$\boxed{y(t+\Delta t) = y(t) + \Delta t(-\alpha y(t))}$

- Stability condition:

$y(t+\Delta t) = (1-\alpha\Delta t)y(t)$

$|1-\alpha\Delta t| \le 1$

$-1 \le 1-\alpha\Delta t \le 1$

$\boxed{\alpha\Delta t \le 2}$

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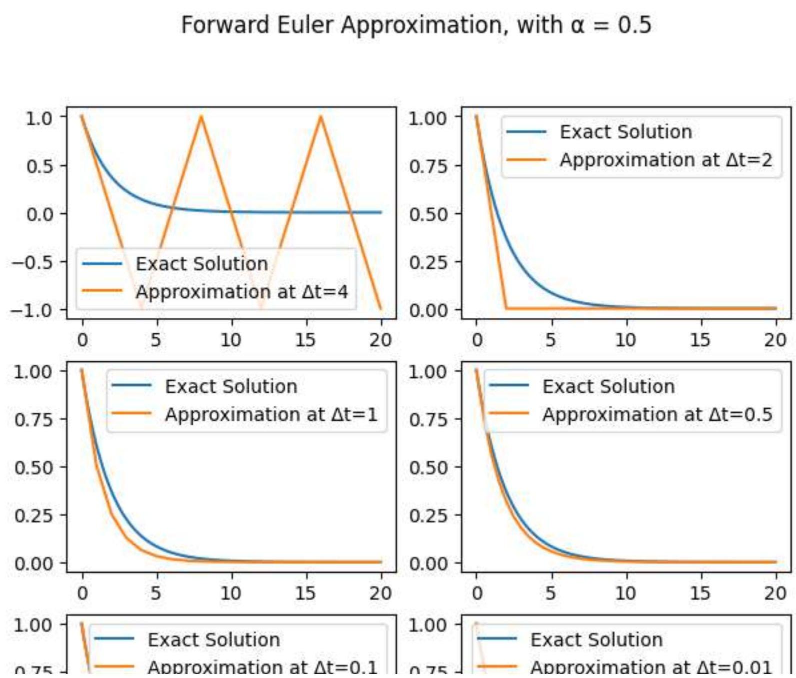
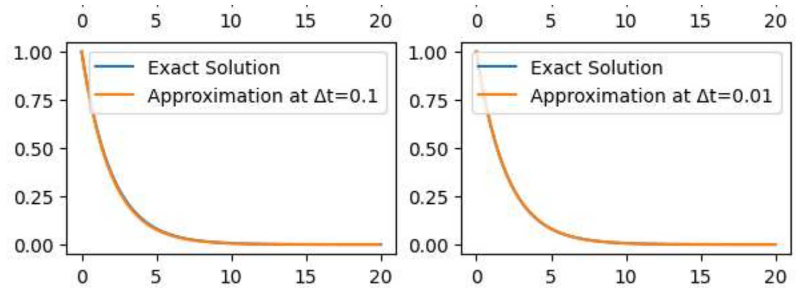




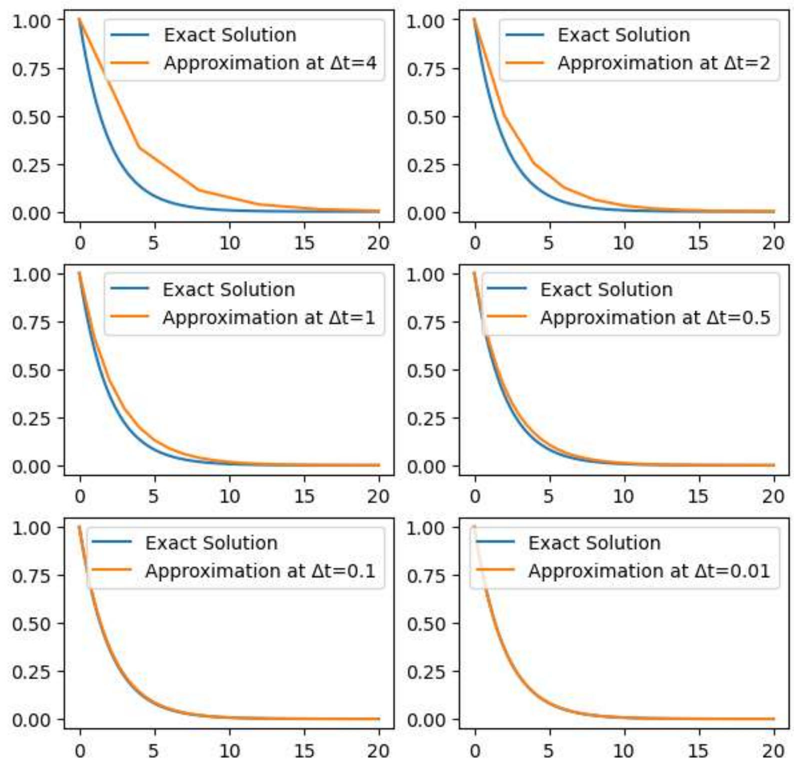
2) $y(t+\Delta t) = y(t) + (\Delta t)\left(\frac{\partial y}{\partial t}(t+\Delta t)\right)$

$$y(t+\Delta t) = y(t) + \Delta t\,(-\alpha y(t+\Delta t))$$

$$y(t+\Delta t)(1+\alpha\Delta t) = y(t)$$

$$\boxed{y(t+\Delta t) = \frac{1}{1+\alpha\Delta t}\,y(t)}$$

Backward Euler Approximation, with α = 0.5



3) Trapezoid Method:

$$y(t+\Delta t) = y(t) + \frac{\Delta t}{2}\left(\frac{\partial y}{\partial t}(t+\Delta t) + \frac{\partial y}{\partial t}(t)\right)$$

$$y(t+\Delta t) = y(t) + \frac{\Delta t}{2}\left(-\alpha y(t+\Delta t) - \alpha y(t)\right)$$

$$y(t+\Delta t)\left(1+\frac{\alpha\Delta t}{2}\right) = y(t)\left(1-\frac{\alpha\Delta t}{2}\right)$$

$$\boxed{y(t+\Delta t) = \frac{1-\frac{\alpha\Delta t}{2}}{1+\frac{\alpha\Delta t}{2}}\,y(t)}$$

$$\boxed{y(t+\Delta t) = \frac{1-\frac{a\Delta t}{2}}{1+\frac{a\Delta t}{2}}\, y(t)}$$

Trapezoid Method Approximation, with α = 0.5