

(a) Laplacian

- Description:
利用 kernel_1 : $\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, kernel_2 : $\frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ 來 Laplacian detect edge 。
- Algorithm:
先建立 convolution function 和 zero_crossing_edge function 。
 - convolution function with threshold
針對 kernel 大小進行對應的 padding , 上下左右 padding $\text{kernel.shape}[0]//2$, 接著從左到右從上到下拜訪 padding_img 的每個 pixel , 再針對以此 pixel 為中心擴到對應的 kernel 大小再與 kernel 相乘 , 若此數值 \geq 閾值 , 則輸出的該 pixel 值為 1 , \leq 負的閾值則為 -1 , 都不符合則為 0 。
 - zero_crossing_edge function
先針對資料進行上下左右各 1 的 padding , 接著從左到右從上到下拜訪每個 pixel , 若該數值為 1 時 , 八鄰域中有 -1 , 則該 pixel 會輸出 0 , 其餘都皆為 255 。

利用 kernel_1 、 kernel_2 , 閾值設 15 , 先進行 convolution function with threshold , 然後再進行 zero_crossing_edge function , 即可得到對應的 output 。

- Principal code fragment:

```
def convolution(img, kernel, threshold):
    h, w = img.shape
    output = np.array(img).astype(np.float)
    padding = int(kernel.shape[0]//2)
    padding_img = cv2.copyMakeBorder(img, padding, padding, padding, padding, cv2.BORDER_REPLICATE)
    for r in range(h):
        for c in range(w):
            x = r + padding; y = c + padding
            tmp = np.sum(padding_img[x-padding:x+padding+1, y-padding:y+padding+1] * kernel)
            if (tmp >= threshold):
                output[r, c] = 1
            elif (tmp <= -threshold):
                output[r, c] = -1
            else:
                output[r, c] = 0
    return output
```

```
def zero_crossing_edge(img):
    h, w = img.shape
    output = np.zeros(img.shape).astype(np.uint8)
    padding_img = cv2.copyMakeBorder(img, 1, 1, 1, 1, cv2.BORDER_REPLICATE)
    for r in range(h):
        for c in range(w):
            x = r + 1; y = c + 1
            output[r, c] = 255
            if (img[r, c] == 1):
                n = np.sum(padding_img[x-1:x+1+1, y-1:y+1+1] == -1)
                if (n > 0):
                    output[r, c] = 0
    return output
```

```
def Laplacian(img, kernel, threshold=15):
    output = convolution(img, kernel, threshold=threshold)
    output = zero_crossing_edge(output)
    return output
```

- Result:

kernel_1 : $\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$



kernel_2 : $\frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$



(b) minimum_variance_Laplacian

- Description:
利用 minimum_variance_Laplacian 做 edge detect
- Algorithm:
先建立 kernel: $\frac{1}{3} \begin{bmatrix} 2 & -1 & 2 \\ -1 & -4 & -1 \\ 2 & -1 & 2 \end{bmatrix}$ ，閾值設 20，先進行 convolution function with threshold，然後再進行 zero_crossing_edge function，即可得到對應的 output。
- Principal code fragment:

```
def minimum_variance_Laplacian(img, threshold=20):  
    kernel = 1/3*np.array([[2, -1, 2], [-1, -4, -1], [2, -1, 2]])  
    output = convolution(img, kernel, threshold=threshold)  
    output = zero_crossing_edge(output)  
    return output
```

- Result:



(c) Laplacian_of_Gaussian

- Description:
利用 Laplacian of Gaussian 找 edge
- Algorithm:

先建立 kernel = $\text{np.array}(\begin{bmatrix} 0 & 0 & 0 & -1 & -1 & -2 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -2 & -4 & -8 & -9 & -8 & -4 & -2 & 0 & 0 \\ 0 & -2 & -7 & -15 & -22 & -23 & -22 & -15 & -7 & -2 & 0 \\ -1 & -4 & -15 & -24 & -14 & -1 & -14 & -24 & -15 & -4 & -1 \\ -1 & -8 & -22 & -14 & 52 & 103 & 52 & -14 & -22 & -8 & -1 \\ -2 & -9 & -23 & -1 & 103 & 178 & 103 & -1 & -23 & -9 & -2 \\ -1 & -8 & -22 & -14 & 52 & 103 & 52 & -14 & -22 & -8 & -1 \\ -1 & -4 & -15 & -24 & -14 & -1 & -14 & -24 & -15 & -4 & -1 \\ 0 & -2 & -7 & -15 & -22 & -23 & -22 & -15 & -7 & -2 & 0 \end{bmatrix})$

[0,0,-2,-4,-8,-9,-8,-4,-2,0,0],
[0,0,0,-1,-1,-2,-1,-1,0,0,0]])，閾值設 3000，先進行
convolution function with threshold，然後再進行 zero_crossing_edge
function，即可得到對應的 output。

- Principal code fragment:

```
def Laplacian_of_Gaussian(img, threshold=3000):  
    kernel = np.array([[0,0,0,-1,-1,-2,-1,-1,0,0,0],  
                       [0,0,-2,-4,-8,-9,-8,-4,-2,0,0],  
                       [0,-2,-7,-15,-22,-23,-22,-15,-7,-2,0],  
                       [-1,-4,-15,-24,-14,-1,-14,-24,-15,-4,-1],  
                       [-1,-8,-22,-14,52,103,52,-14,-22,-8,-1],  
                       [-2,-9,-23,-1,103,178,103,-1,-23,-9,-2],  
                       [-1,-8,-22,-14,52,103,52,-14,-22,-8,-1],  
                       [-1,-4,-15,-24,-14,-1,-14,-24,-15,-4,-1],  
                       [0,-2,-7,-15,-22,-23,-22,-15,-7,-2,0],  
                       [0,0,-2,-4,-8,-9,-8,-4,-2,0,0],  
                       [0,0,0,-1,-1,-2,-1,-1,0,0,0]])  
    output = convolution(img, kernel, threshold=threshold)  
    output = zero_crossing_edge(output)  
    return output
```

- Result:



(d) Difference_of_Gaussian

- Description:

利用 Difference of Gaussian 做 edge detect

Algorithm:

```
先建立 kernel = np.array([[ -1,-3,-4,-6,-7,-8,-7,-6,-4,-3,-1],  
                             [-3,-5,-8,-11,-13,-13,-13,-11,-8,-5,-3],  
                             [-4,-8,-12,-16,-17,-17,-17,-16,-12,-8,-4],  
                             [-6,-11,-16,-16,0,15,0,-16,-16,-11,-6],  
                             [-7,-13,-17,0,85,160,85,0,-17,-13,-7],  
                             [-8,-13,-17,15,160,283,160,15,-17,-13,-8],  
                             [-7,-13,-17,0,85,160,85,0,-17,-13,-7],  
                             [-6,-11,-16,-16,0,15,0,-16,-16,-11,-6],  
                             [-4,-8,-12,-16,-17,-17,-17,-16,-12,-8,-4],  
                             [-3,-5,-8,-11,-13,-13,-13,-11,-8,-5,-3],  
                             [-1,-3,-4,-6,-7,-8,-7,-6,-4,-3,-1]])，閾值設 1，
```

先進行 convolution function with threshold，然後再進行 zero_crossing_edge function，即可得到對應的 output。

- Principal code fragment:

```
def Difference_of_Gaussian(img, threshold=1):
    kernel = np.array([[-1,-3,-4,-6,-7,-8,-7,-6,-4,-3,-1],
                        [-3,-5,-8,-11,-13,-13,-13,-11,-8,-5,-3],
                        [-4,-8,-12,-16,-17,-17,-17,-16,-12,-8,-4],
                        [-6,-11,-16,-16,0,15,0,-16,-16,-11,-6],
                        [-7,-13,-17,0,85,160,85,0,-17,-13,-7],
                        [-8,-13,-17,15,160,283,160,15,-17,-13,-8],
                        [-7,-13,-17,0,85,160,85,0,-17,-13,-7],
                        [-6,-11,-16,-16,0,15,0,-16,-16,-11,-6],
                        [-4,-8,-12,-16,-17,-17,-17,-16,-12,-8,-4],
                        [-3,-5,-8,-11,-13,-13,-13,-11,-8,-5,-3],
                        [-1,-3,-4,-6,-7,-8,-7,-6,-4,-3,-1]])
    output = convolution(img, kernel, threshold=threshold)
    output = zero_crossing_edge(output)
    return output
```

- Result:

