Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

    A. 0.3875
    B. 0.2676
    C. 0.5
    D. 0.6987

Ans: mean=45 minutes, standard deviation=8 minutes, so as per the given condition the work will start after 10 min so the mean is 45+10=55min
Then, Z=(x-mean)/std dev (since x=1hr=60min)
=(60-55)/8
=0.625
In Z table, Z value is 0.7327
Then the service manager cannot meet his condition is=1-0.7327=0.2676
Option b

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ=6. For each statement below, please specify True/False. If false, briefly explain why.

    A. More employees at the processing center are older than 44 than between 38 and 44.

    Ans: Probability of the employee greater than X>44=1-pr(x<=44)
    Z=(44-38)/6=1
    From Z table for 1
    Z=0.8413
    Probability for greater than 44 is=1-0.8413=0.1587
    For X>=38
    Z=(38-38)/6=0
    From z table for 0
    Z=0.5
    Probability between 44 and 38 is=0.8413-0.5
    =0.3413
    Therefore, the statement A of the question is also 'TRUE'.

    B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

    Ans:Probability of employees less than age of 30 = P (X<30)

Z = (X -38)/6

= (30 - 38)/6

P (X ≤ 30)= P (Z ≤ (30 - 38)/6)= P (Z ≤ -1.333)= 9.12%

So, the number of employees with probability 0.912 of them being under age30

= 0.0912*400= 36.48 (or 36 employees).

Therefore, the statement B of the question is also 'TRUE'.

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

   Ans:For 2X1

   Mean is 2 μ

   Variance is 4 σ^2

   For $X_1 + X_2$

   Mean is 2 μ

   Variance is 2 σ^2

Both the cases mean is same but variance is different.This is because 2*X*1 involves scaling, which increases the spread of the distribution, while *X*1+*X*2 involves adding, which increases the mean but does not increase the spread as much.

4. Let X ~ N(100, 20^2). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

   A. 90.5, 105.9
   B. 80.2, 119.8
   C. 22, 78
   D. 48.5, 151.5
   E. 90.1, 109.9

   Ans: the probability of getting in a and b is 0.99

   The probability of getting outside is =0.01

   Then the probability from right side b is 0.01/2=0.005

   The probability from left side a is -0.01/2=-0.005

   Z=(X-mu)/sigma

   X=Z*sigma+mu

   From Z table value of 0.005 is 2.575829

   Then X=-(2.575)*20+100

   =48.5

   X=(2.575)*20+100

   =151.5

   Option D

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: Profit= profit1+ profit2

Mean=5+7=12

Std=sqrt(9+16)

=5

Mean in rupees=12*45=540

Std in rupees=5*45=225

Range for 95% is

Stats.norm.interval(0.95,540,225)

Range is RS 99.0081034 , 980.991896

B. Specify the 5th percentile of profit (in Rupees) for the company

Ans:formula X=μ + Zσ; wherein from z table, 5 percentle = -1.645

X = 540(-1.645)*225

X = 169.875

C. Which of the two divisions has a larger probability of making a loss in a given year?

stats. norm. cdf (0, 5, 3)
0.0477903
Stats.norm. cdf(0, 7, 4) 0
0.040059