

Database Management Systems

Lecture 9

Evaluating Relational Operators

Query Optimization

- running example - schema
 - Students (SID: integer, SName: string, Age: integer, RoundedGPA: integer)
 - Courses (CID: integer, CName: string, Description: string)
 - Exams (SID: integer, CID: integer, EDate: date, Grade: integer)
- Students
 - every record has 50 bytes
 - there are 80 records / page
 - 500 pages
- Courses
 - every record has 40 bytes
 - there are 100 records / page
 - 1 page
- Exams
 - every record has 40 bytes
 - there are 100 records / page
 - 1000 pages

IBM's System R Optimizer

- tremendous influence on subsequent relational optimizers
- design choices:
 - use statistics to estimate the costs of query evaluation plans
 - consider only plans with binary joins in which the inner relation is a base relation
 - focus optimization on SQL queries without nesting
 - don't eliminate duplicates when performing projections (unless DISTINCT is used)

Estimating the Cost of a Plan

- estimating the cost of an evaluation plan for a query block
 - for each node N in the tree:
 - estimate the cost of the corresponding operation (pipelining versus temporary relations)
 - estimate the size of N's result and whether it is sorted
 - N's result is the input of N's parent node
 - these estimates affect the estimation of cost, size, and sort order for N's parent

Estimating the Cost of a Plan

- estimating costs
 - use data about the input relations (such statistics are stored in the DBMS's system catalogs)
 - number of pages, existing indexes, etc.
 - obtained estimates are at best approximations to actual sizes and costs
- => one shouldn't expect the optimizer to find the best possible plan
- optimizer - goals:
 - avoid the worst plans
 - find a good plan

Statistics Maintained by the DBMS

- updated periodically, not every time the data is changed
 - relation R
 - cardinality - $NTuples(R)$
 - the number of tuples in R
 - size - $NPages(R)$
 - the number of pages in R
 - index I
 - cardinality - $NKeys(I)$
 - the number of distinct key values for I
 - size - $INPages(I)$
 - the number of pages for I
 - B+ tree index
 - number of leaf pages

Statistics Maintained by the DBMS

- index I
 - height - $IHeight(I)$
 - maintained for tree indexes
 - the number of nonleaf levels in I
 - range - $ILow(I), IHigh(I)$
 - the minimum / maximum key value in I

Estimating Result Sizes

- query Q
SELECT attribute list
FROM relation list
WHERE $term_1 \wedge \dots \wedge term_k$
- the maximum number of tuples in Q's result:
 - $\prod |R_i|$
where $R_i \in \text{relation list}$
- each $term_j$ in the WHERE clause eliminates some candidate tuples
 - associate a reduction factor RF_j with each term $term_j$
 - RF_j models the impact $term_j$ has on the result size
- estimate the actual size of the result:
 - $\prod |R_i| * \prod RF_j$
 - i.e., the maximum result size times the product of the reduction factors for the terms in the WHERE clause

Estimating Result Sizes

- query Q
SELECT attribute list
FROM relation list
WHERE $\text{term}_1 \wedge \dots \wedge \text{term}_k$
- assumption
 - the conditions tested by the terms in the WHERE clause are statistically independent

Estimating Result Sizes

- compute reduction factors for terms in the WHERE clause
- assumptions:
 - uniform distribution of values
 - independent distribution of values in different columns

SELECT attribute list

FROM relation list

WHERE term₁ AND ... AND term_k

- *column = value*
 - index I on *column*
=> RF approximated by $1/NKeys(I)$
 - no index on *column*
=> RF: $1/10$
 - maintain statistics on *column* (e.g., number of distinct values in *column*) to obtain a better value

Estimating Result Sizes

- *column1 = column2*
 - indexes *I1* on *column1*, *I2* on *column2*
=> RF: $1/\text{MAX}(\text{NKeys}(I1), \text{NKeys}(I2))$
 - only one index *I* (on one of the 2 columns)
=> RF: $1/\text{NKeys}(I)$
 - no indexes
=> RF: $1/10$
- *column > value*
 - index *I* on *column*
=> RF: $(\text{IHigh}(I) - \text{value}) / (\text{IHigh}(I) - \text{ILow}(I))$
 - no index on *column* or *column* not of an arithmetic type
=> a value less than 0.5 is arbitrarily chosen
 - similar formulas can be obtained for other range selections

Estimating Result Sizes

- *column IN (list of values)*
=> RF: (RF for *column = value*) * number of items in list (but at most 0.5)
- *NOT condition*
=> RF: 1 - RF for *condition*
- obtain better estimates
 - use more detailed statistics (e.g., histograms of the values in a column)

Relational Algebra Equivalences

- central role in generating alternative plans
- different join orders can be considered
- selections, projections can be pushed ahead of joins
- cross-products can be converted to joins
- selections
 - *cascading selections*
 - $\sigma_{c1 \wedge \dots \wedge cn}(R) \equiv \sigma_{c1}(\sigma_{c2}(\dots(\sigma_{cn}(R))\dots))$
 - *commutativity*
 - $\sigma_{c1}(\sigma_{c2}(R)) \equiv \sigma_{c2}(\sigma_{c1}(R))$
- projections
 - *cascading projections*
 - $\pi_{a1}(R) \equiv \pi_{a1}(\pi_{a2}(\dots(\pi_{an}(R))\dots))$
 - a_i – set of attributes in R
 - $a_i \subseteq a_{i+1}$, for $i = 1..n-1$

Relational Algebra Equivalences

- joins and cross-products
 - assumption
 - fields are identified by their name, not by their position
 - *associativity*
 - $R \times (S \times T) \equiv (R \times S) \times T$
 - $R * (S * T) \equiv (R * S) * T$
 - *commutativity*
 - $R \times S \equiv S \times R$
 - $R * S \equiv S * R$
 - can choose the inner / outer relation in a join

Relational Algebra Equivalences

- joins and cross-products
 - e.g., check that $R * (S * T) \equiv (T * R) * S$
 - commutativity
 - $R * (S * T) \equiv R * (T * S)$
 - associativity
 - $R * (T * S) \equiv (R * T) * S$
 - commutativity
 - $(R * T) * S \equiv (T * R) * S$

Relational Algebra Equivalences

- can commute σ with π if σ uses only attributes retained by π
 - $\pi_a(\sigma_c(R)) \equiv \sigma_c(\pi_a(R))$
- can combine σ with \times to form a join
 - $R \otimes_c S \equiv \sigma_c(R \times S)$
- can commute σ with \times or a join when the selection condition includes only fields of one of the arguments (to the cross-product or join)
 - for instance:
 - $\sigma_c(R * S) \equiv \sigma_c(R) * S$
 - $\sigma_c(R \times S) \equiv \sigma_c(R) \times S$
 - condition c must include only fields from R
- in general: $\sigma_c(R \times S) \equiv \sigma_{c_1}(\sigma_{c_2}(R) \times \sigma_{c_3}(S))$
 - c_1 – attributes of both R and S
 - c_2 – only attributes of R
 - c_3 – only attributes of S

Relational Algebra Equivalences

- can commute π with \times
 - $\pi_a(R \times S) \equiv \pi_{a_1}(R) \times \pi_{a_2}(S)$
 - a_1 – attributes in a that appear in R
 - a_2 – attributes in a that appear in S
- can commute π with join
 - $\pi_a(R \Join_c S) \equiv \pi_{a_1}(R) \Join_c \pi_{a_2}(S)$
 - every attribute in c must appear in a
 - a_1 – attributes in a that appear in R
 - a_2 – attributes in a that appear in S
 - a doesn't contain all the attributes in c – generalization
 - eliminate unwanted fields, compute join, eliminate fields not in a
 - $\pi_a(R \Join_c S) \equiv \pi_a(\pi_{a_1}(R) \Join_c \pi_{a_2}(S))$
 - a_1 – attributes of R that appear in either a or c
 - a_2 – attributes of S that appear in either a or c

Enumeration of Alternative Plans

- query Q
 - consider a certain set of plans
 - choose the plan with the least estimated cost
 - algebraic equivalences
 - implementation techniques for Q's operators
- not all algebraically equivalent plans are enumerated (optimization costs would be too high)
- two main cases:
 - queries with one relation in the FROM clause
 - queries with two or more relations in the FROM clause

Enumeration of Alternative Plans

- queries with one relation in the FROM clause
 - i.e., no joins; only σ , π , grouping, aggregate operations
 - if there is only one σ or π or aggregate operation: consider implementation techniques and cost estimates discussed in previous lectures
 - if there is a combination of operations:
 - plans with / without indexes
 - example query:

```
SELECT S.RoundedGPA, COUNT (*)  
FROM Students S  
WHERE S.RoundedGPA > 5 AND S.Age = 20  
GROUP BY S.RoundedGPA  
HAVING COUNT (DISTINCT S.SName) > 5
```

$\pi_{S.RoundedGPA, COUNT(*)}(\text{HAVING}_{COUNT\ DISTINCT\ (S.SName) > 5}(\text{GROUP BY}_{S.RoundedGPA}(\pi_{S.RoundedGPA, S.SName}(\sigma_{S.RoundedGPA > 5 \wedge S.Age = 20}(Students))))))$

Enumeration of Alternative Plans

* plans without indexes:

- apply σ, π while scanning Students
 - file scan
 - $\text{NPages}(\text{Students})$
 - 500 I/Os
 - write out tuples to a temporary relation T:
 - $\text{NPages}(\text{Students}) * \text{RF}(\text{RoundedGPA} > 5) * \text{RF}(\text{Age} = 20) *$
(size of a *pair* $\langle \text{RoundedGPA}, \text{SName} \rangle$ / size of a *Students tuple*)
 - RF for *RoundedGPA* > 5
 - 0.5
 - RF for *Age* = 20
 - 0.1
 - size of $\langle \text{RoundedGPA}, \text{SName} \rangle$
 - about $0.8 * \text{size of a Students tuple}$

Enumeration of Alternative Plans

* plans without indexes:

- apply σ, π while scanning Students
 - write out tuples to a temporary relation T:
 $\Rightarrow 500 * 0.5 * 0.1 * 0.8 = 20$ I/Os (temporary relation T)
- GROUP BY:
 - sort T in 2 passes
 - $4 * 20 = 80$ I/Os
- HAVING, aggregations
 - no additional I/O
- **total cost**
 - $500 + 20 + 80 = \mathbf{600}$ I/Os

Enumeration of Alternative Plans

* plans that use an index:

- available indexes on Students - a2
 - hash index on <Age>
 - B+ tree index on <RoundedGPA>
 - B+ tree index on <RoundedGPA, SName, Age>
- single-index access path:
 - choose the index that provides the most selective access path
 - apply π , nonprimary selection terms (i.e., that don't match the index)
 - compute grouping and aggregation operations
 - example:
 - use the hash index on Age to retrieve Students with Age = 20
 - cost: retrieve index entries and corresponding Students tuples
 - apply condition *RoundedGPA* > 5 to each retrieved tuple
 - retain RoundedGPA and SName

Enumeration of Alternative Plans

* plans that use an index:

- single-index access path – example:
 - write out tuples to a temporary relation
 - sort the temporary relation by RoundedGPA to identify groups
 - apply the HAVING condition (to eliminate some groups)
- multiple-index access path
 - several indexes using a2 / a3 match the selection condition, e.g., I1, I2
 - retrieve $Rids_{I1}$, $Rids_{I2}$ using I1, I2
 - get tuples with rids in $Rids_{I1} \cap Rids_{I2}$ (tuples satisfying the primary selection terms of I1 and I2)
 - apply π , nonprimary selection terms
 - compute grouping and aggregation operations
 - example:
 - use the index on Age \Rightarrow rids of tuples with Age = 20 (R1)

Enumeration of Alternative Plans

* plans that use an index:

- multiple-index access path – example:
 - index on RoundedGPA \Rightarrow rids of tuples with RoundedGPA > 5 (R2)
 - retrieve tuples with rids in $R1 \cap R2$
 - keep only RoundedGPA and SName
 - write out tuples to a temporary relation
 - sort the temporary relation by RoundedGPA to identify groups
 - apply the HAVING condition (to eliminate some groups)
- sorted index access path:
 - works well when the index is clustered
 - B+ tree index I with search key K
 - GROUP BY attributes – prefix of K
 - use the index to retrieve tuples in the order required by the GROUP BY clause

Enumeration of Alternative Plans

* plans that use an index:

- apply σ , π
 - compute aggregation operations
 - sorted index access path – example:
 - use the B+ tree index on RoundedGPA to retrieve Students tuples with $\text{RoundedGPA} > 5$, ordered by RoundedGPA
 - aggregations in HAVING, SELECT - computed on-the-fly
 - index-only access path:
 - index I with search key K
 - all the attributes in the query are included in K
- => index-only scan, don't need to retrieve tuples from the relation
- data entries: apply σ , perform π , sort the result (to identify groups), compute aggregate operations
- * obs. index I doesn't have to match the selections in the WHERE clause

Enumeration of Alternative Plans

- * plans that use an index:

- index-only access path

- * obs. 1 – tree index, GROUP BY attributes – prefix of K

- => can avoid sorting

- example:

- use the B+ tree index on <RoundedGPA, SName, Age> to retrieve entries with RoundedGPA > 5, ordered by RoundedGPA
 - select entries with Age = 20
 - aggregation operations in the HAVING and SELECT clauses - computed on-the-fly

Enumeration of Alternative Plans

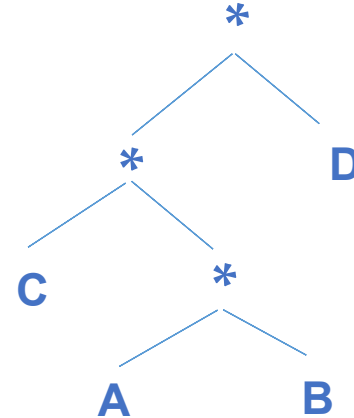
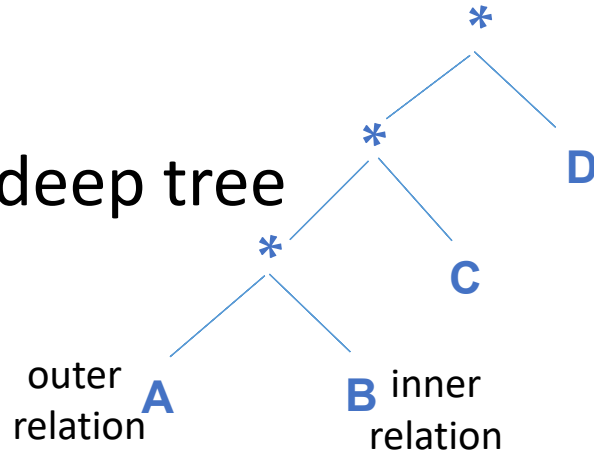
- queries with several relations in the FROM clause:
 - joins, cross-products => queries can be quite expensive
 - different join orders => intermediate relations of widely varying sizes => plans with very different costs
- class of plans considered by the optimizer
- plan enumeration

Enumeration of Alternative Plans

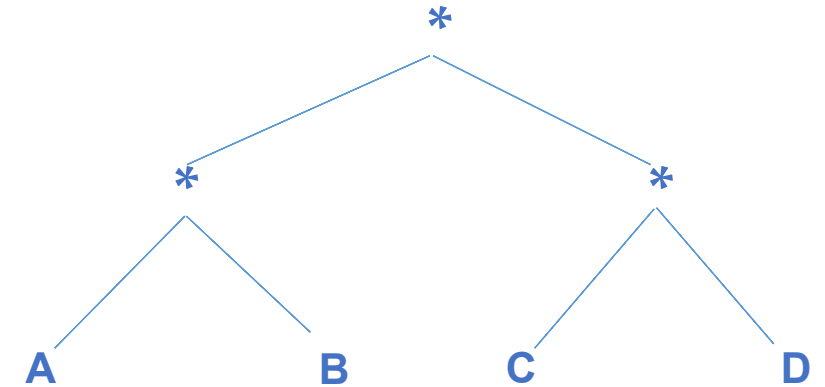
- queries with several relations in the FROM clause

$A * B * C * D$

left-deep tree



linear trees

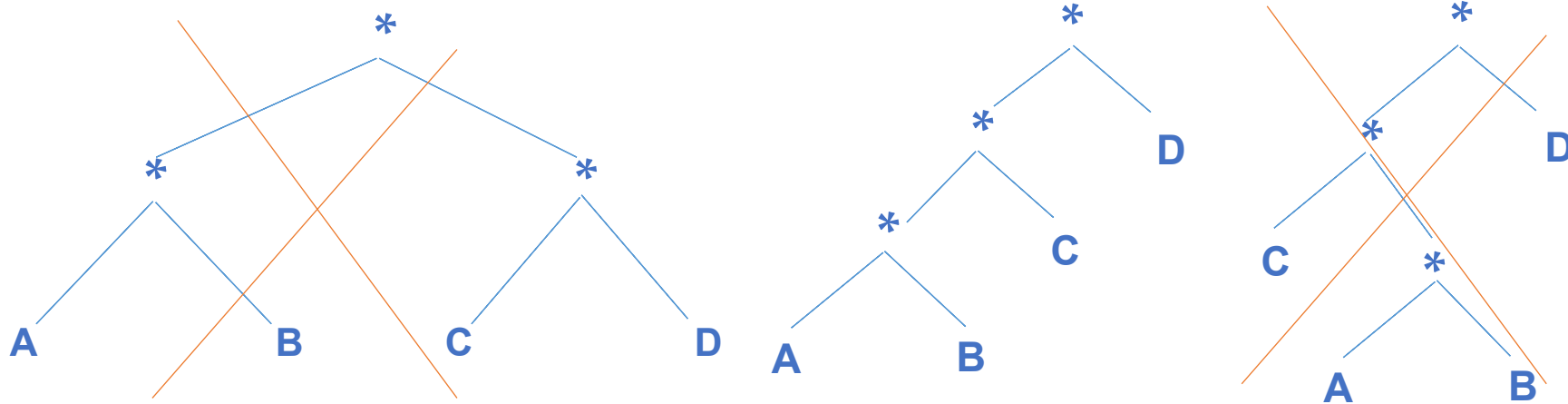


bushy tree

- linear trees:
 - at least one child of a join node is a base relation
- left-deep trees:
 - the right child of each join node is a base relation
- bushy tree: not linear

Enumeration of Alternative Plans

- queries with several relations in the FROM clause
- fundamental decision in System R:
 - only *left-deep trees* are considered



- motivation:
 - number of joins increases => number of alternative plans increases quickly => must prune the search space
 - left-deep trees generate all fully pipelined plans (all joins are evaluated using pipelining)

Enumeration of Alternative Plans

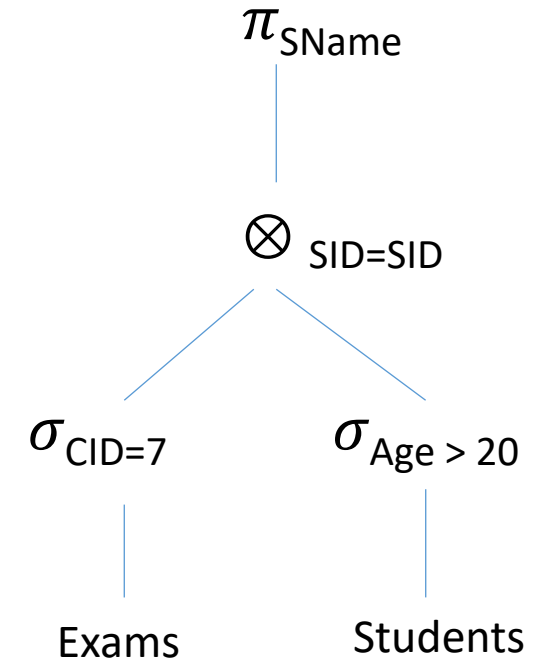
- queries with several relations in the FROM clause
- multiple passes:
 - pass 1:
 - find best 1-relation plan for each relation
 - pass 2:
 - find best way to join the result of each 1-relation plan (as the outer argument) with another relation (all 2-relation plans)
 - ...
 - repeat until the obtained plans contain all the relations in the query
- for each subset of relations, retain: the cheapest plan overall & the cheapest plan for each interesting ordering of tuples

Enumeration of Alternative Plans

- queries with several relations in the FROM clause
 - GROUP BY, aggregates are handled as a final step:
 - use a plan with an interesting ordering of tuples
 - use an additional sorting operator
- obs. avoid cross-products if possible

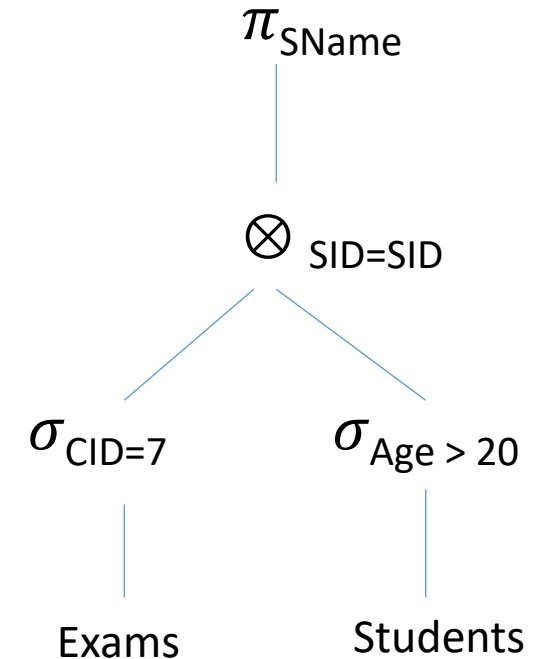
Example

- unclustered indexes using a2
- Students: B+ tree index on Age, hash index on SID
- Exams: B+ tree index on CID
- pass 1:
 - Students – 3 possible access paths:
B+ tree index, hash index, file scan
 - *Age > 20* matches the B+ tree index on Age
 - hash index / file scan => probably much higher cost
 - keep the plan using the B+ tree index => retrieved tuples are ordered by Age
 - Exams – 2 possible access paths
 - *CID = 7* matches the B+ tree index on CID
 - better than file scan => keep the plan using the B+ tree index



Example

- Students: B+ tree index on Age, hash index on SID
- Exams: B+ tree index on CID
- pass 2:
 - consider (the result of) each plan from pass 1 as the outer argument and analyze how to join it with the other relation
 - e.g., Exams – outer argument
 - examine alternative access methods / join methods
 - access methods:
 - need Students tuples s.t. $SID = value$ from outer tuple and $Age > 20$
 - hash index \Rightarrow Students tuples s.t. $SID = value$ from outer tuple
 - B+ tree index \Rightarrow Students tuples s.t. $Age > 20$
 - join methods:
 - consider all available methods



Example

- Students: B+ tree index on Age, hash index on SID
- Exams: B+ tree index on CID
- pass 2:
 - consider (the result of) each plan from pass 1 as the outer argument and analyze how to join it with the other relation
 - e.g., Students – outer argument
 - access / join methods
 - access methods:
 - need Exams tuples s.t. $SID = value$ from outer tuple and $CID = 7$
 - join methods:
 - consider all available methods
 - retain cheapest plan overall!

Nested Queries - optional

- usually handled using some form of nested loops evaluation
- correlated query - typical evaluation strategy:
 - the inner block is evaluated for each tuple of the outer block
- some strategies are not considered
 - e.g., index on SID in Students
 - best plan could be INLJ with Exams as the outer argument, and Students as the inner one; such a plan is never considered by the optimizer
- the unnested version of the query is typically optimized better

```
SELECT S.SName  
FROM Students S  
WHERE EXISTS  
  (SELECT *  
   FROM Exams E  
   WHERE E.CID=7  
        AND E.SID=S.SID)
```

Equivalent unnested query:
SELECT S.SName
FROM Students S, Exams E
WHERE E.SID=S.SID AND E.CID = 7

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