EDO

martes, 8 de agosto de 2023 23:57

x: función

$$P(1)\begin{cases} x'(t)+\alpha x(t)=0 & t\in[0,+\infty[\\ x(0)=x_0 \end{cases}$$

$$\frac{dx}{dt}+\alpha x=0$$

$$\frac{dx}{dt}=-\alpha x \qquad \frac{dx}{dt}=x$$

$$\int\frac{1}{x}\,dx=-\alpha\int dt \qquad \frac{1}{x}dx=dt$$

$$\ln(x)=-\alpha t+C_1$$

$$e^{\ln(x)}=e^{-\alpha t+C_1}$$

$$x(t)=e^{-\alpha t+C_1}=e^{-\alpha t}\cdot e^{C_1}=C_2$$

$$x(t)=C_2e^{-\alpha t}$$

$$\overset{t=0}{x(0)}=x_0 \qquad x(t)=C_2e^{-\alpha t}$$

$$x_0=x(0)=C_2e^{-\alpha\cdot 0} \qquad C_2=x_0$$

$$=C_2\cancel{e^{0}}^{\to 1}=C_2$$

$$*\ x(t)=x_0e^{-\alpha t},\ t\in[0,+\infty[$$

función

$$*\to x:[0,+\infty[\ \longrightarrow \mathbb{R}$$

$$t\longmapsto x(t):x_0e^{-\alpha t}$$

$$(P_2)\begin{cases} y'(t)+\beta y(t)=\alpha x(t) \\ y(0)=y_0 \end{cases} \qquad y''(t)+\beta y'(t)+\alpha y(t)=0$$

$$\begin{cases} y'(t)+\beta y(t)=\alpha x_0e^{-\alpha t} \\ y(0)=y_0 \end{cases}$$

$$\mu=e^{-\int p(t)}$$

$$\begin{cases} y'(t) + \beta y(t) = \alpha x_0 e \\ y(0) = y_0 \end{cases}$$

$$\boxed{y_h = e^{-\int p(t)}}$$

$$\int p(t) = \int \beta$$

$$y(t) = C y_h + y_p(t)$$

$$P(t) = \int p(t)$$

$$P(t) = \beta t$$

$$P(t) =$$

f ① f y

$$y'(t) + p(t) y(t) = f(t)$$

$$p(t) = \beta \qquad f(t) = \alpha x_0 e^{-\alpha t}$$

$$\boxed{y_h = e^{-P(t)}}$$

$$p(t) = 1$$

$$P(t) = t$$

$$y_h = e^{-t}$$

$$\boxed{y_h = e^{-\int p(t)}}$$

$$\int \beta \, dt = \beta t$$

$$P(t) = \int p(t)$$

$$y_h = e^{-\beta t}$$

$$y_p(t) = V(t)\, y_h(t), \quad t \in [0, +\infty[$$

$$V(t) = \int_0^t e^{P(z)} f(z)\, dz$$

$$V(t) = \int_0^t e^{\beta z} \cdot \alpha x_0 e^{-\alpha z} \, dz \qquad \int e^z dt = e^z$$

$$= \alpha x_0 \int_0^t e^{(\beta-\alpha)z} \, dt$$

$$u = (\beta - \alpha) z \qquad \int e^{(\beta-\alpha)z} dt$$

$$du = (\beta-\alpha) dz$$

$$\frac{du}{(\beta-\alpha)} = dz \qquad \to e^u$$

$$\int e^u \frac{du}{(\beta-\alpha)} = \frac{1}{(\beta-\alpha)} \int e^u du$$

$$= \frac{1}{\beta-\alpha} e^u$$

t

$$= \frac{1}{\beta-\alpha} e$$

$$= \alpha x_0 \frac{1}{\beta-\alpha} e^{(\beta-\alpha)z} \Big|_0^t$$

$$= \frac{\alpha x_0}{\beta-\alpha} \left[e^{(\beta-\alpha)t} - 1 \right]$$

$* \quad y_p(t) = V(t)\, y_h(t)$

$$y_p(t) = \frac{\alpha x_0}{\beta-\alpha} \left[e^{(\beta-\alpha)t} - 1 \right] \cdot e^{-\beta t}, \quad t \in [0,+\infty[$$

$$e^{\beta t - \beta t} = e^0$$

$$y_p(t) = \frac{\alpha x_0}{\beta-\alpha} \left[e^{\beta t} \cdot e^{-\alpha t} - 1 \right] \cdot e^{-\beta t}$$

$$\quad \text{''} \qquad \text{''} \qquad e^{\beta t} \cdot e^{-\alpha t} \cdot e^{-\beta t} - e^{-\beta t}$$

$$1 \cdot e^{-\alpha t} - e^{-\beta t}$$

$$= \frac{\alpha x_0}{\beta-\alpha} \left[e^{-\alpha t} - e^{-\beta t} \right], \quad t \in [0,+\infty[$$

$$y(t) = c\, y_h(t) + y_p(t)$$

$$* \quad y(t) = C e^{-\beta t} + \frac{\alpha x_0}{\beta-\alpha} \left[e^{-\alpha t} - e^{-\beta t} \right], \quad t \in [0,+\infty[$$

$y(0) = y_0$

$$y(0) = C \underbrace{e^{-\beta \cdot 0}}_{=1} + \frac{\alpha x_0}{\beta-\alpha} \Big[\underbrace{e^{-\alpha \cdot 0}}_{=1} - \underbrace{e^{-\beta \cdot 0}}_{=1} \Big] = 0$$

$$y_0 = y(0) = C \cdot 1 \qquad C = y_0$$

Función

$$y : [0,+\infty[\longrightarrow \mathbb{R}$$

$$t \longmapsto y(t) = y_0 e^{-\beta t} + \frac{\alpha x_0}{\beta-\alpha} \left[e^{-\alpha t} - e^{-\beta t} \right]$$

E.P.L.

$$(P1)\begin{cases} u'(x)+p(x)\,u(x)=4x & x\in[0,+\infty[\\ u(0)=3 \end{cases}$$

$$p=\begin{cases} 2 & 0< x\leq 1 \\ -2/x & x>1 \end{cases}$$

$$u(x)=\begin{cases} \boxed{1} & 0<x\leq 1 \\ \boxed{2} & x>1 \end{cases}$$

I

$$① \begin{cases} u'(x)+2u(x)=4x & x\in]0,1[\\ u(0)=3 \end{cases}$$

$$② \begin{cases} u'(x)-\frac{2}{x}u(x)=4x & x\in]1,+\infty[\\ u(0)=3 \end{cases}$$

$p(x)=2$ $\quad f(x)=4x$

$$P(x)=\int p(x)\,dx=\int 2\,dx=2x$$

* $u(x)=c\,u_h(x)+u_p(x)$

• $u_h(x)=e^{-P(x)}=e^{-2x}$

$$u_p(x)=V(x)\cdot u_h(x)$$

* $V(x)=\int_0^x e^{P(z)}f(z)\,dz$

* $V(x)=\int_0^x e^{2z}\,4z\,dz=$

$$u_p(x)=V(x)\cdot e^{-2x}$$

* $t=0$

$u(0)=3$

$u(1)=2$

$$\boxed{u(x)=c\,u_h(x)+u_p(x)},$$

$t=0$

$$③\ 3=u(0)=c\,u_h(0)+u_p(0)$$

* $$\boxed{c=\frac{3-u_p(0)}{u_h(0)}}$$

$$(1+x^2)\,u'(x) + 2x\,u(x) = f(x) \qquad (0,1)$$

$$u'(x) + \frac{2x}{1+x^2}\,u(x) = \frac{f(x)}{1+x^2}$$

$x > 1$

$$p(x) = \frac{2x}{1+x^2} \qquad g(x) = \frac{x}{1+x^2}$$

$$g(x) = \frac{x}{1+x^2}$$

$u: (\quad) \longrightarrow (\quad)$

$x \longrightarrow u(x)$

donde

$$u(x) = \begin{cases} \bigcirc & \text{—} \\ \bigcirc & \text{—} \end{cases}$$