# Tutoria

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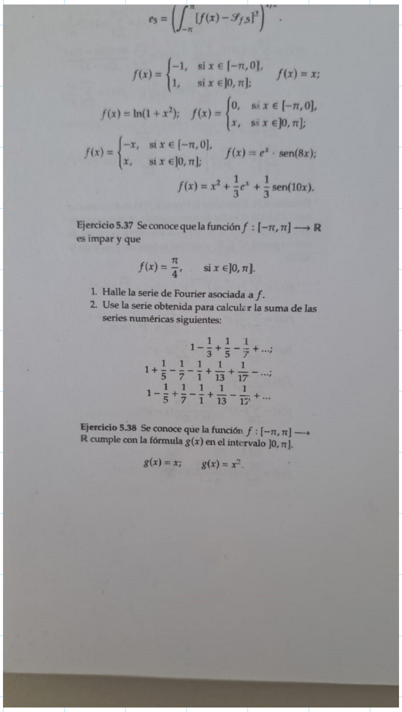

$$e_5 = \left(\int_{-\pi}^{\pi} [f(x) - \mathcal{S}_{f,5}]^2\right)^{1/2}.$$

$$f(x) = \begin{cases} -1, & \text{si } x \in [-\pi, 0], \\ 1, & \text{si } x \in ]0, \pi]; \end{cases} \qquad f(x) = x;$$

$$f(x) = \ln(1 + x^2); \quad f(x) = \begin{cases} 0, & \text{si } x \in [-\pi, 0], \\ x, & \text{si } x \in ]0, \pi]; \end{cases}$$

$$f(x) = \begin{cases} -x, & \text{si } x \in [-\pi, 0], \\ x, & \text{si } x \in ]0, \pi]; \end{cases} \qquad f(x) = e^x \cdot \text{sen}(8x);$$

$$f(x) = x^2 + \frac{1}{3}e^x + \frac{1}{3}\text{sen}(10x).$$

**Ejercicio 5.37** Se conoce que la función $f : [-\pi, \pi] \longrightarrow \mathbb{R}$ es impar y que

$$f(x) = \frac{\pi}{4}, \qquad \text{si } x \in ]0, \pi].$$

1. Halle la serie de Fourier asociada a $f$.
2. Use la serie obtenida para calcular la suma de las series numéricas siguientes:

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots;$$

$$1 + \frac{1}{5} - \frac{1}{7} - \frac{1}{1} + \frac{1}{13} + \frac{1}{17} - \ldots;$$

$$1 - \frac{1}{5} + \frac{1}{7} - \frac{1}{1} + \frac{1}{13} - \frac{1}{17} + \ldots$$

**Ejercicio 5.38** Se conoce que la función $f : [-\pi, \pi] \longrightarrow \mathbb{R}$ cumple con la fórmula $g(x)$ en el intervalo $]0, \pi]$.

$$g(x) = x; \qquad g(x) = x^2.$$

$$f(-x) = -f(x)$$

5.37)

$$f : [-\pi, \pi] \longrightarrow \mathbb{R}$$

* impar

$$f(x) = \frac{\pi}{4}, \quad \text{si } x \in ]0, \pi]$$

$$* \quad S_f(x) = \frac{a_0}{2} + \sum_{n=1}^{+\infty} a_n \cos(nx) + \sum_{n=1}^{+\infty} b_n \,\text{sen}(nx)$$

$$a_0 = 0$$

$$\forall n \in \mathbb{N}, \quad a_n = 0$$

$$S_f(x) = \sum_{n=1}^{+\infty} b_n \,\text{sen}(nx)$$

$$b_n \overset{*}{=} \frac{2}{\pi} \int_0^{\pi} f(x)\,\text{sen}(nx)\,dx \qquad b = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,\text{sen}(nx)\,dx$$

$$b_n = \frac{\cancel{2}}{\cancel{\pi}} \int_0^{\pi} \frac{\cancel{\pi}}{\cancel{4}_{\,2}}\,\text{sen}(nx)\,dx \qquad \frac{2}{\pi}\int_0^{\pi} f(x)\,\text{sen}(nx)\,dx$$

$$b_n = \frac{1}{2}\int_0^{\pi} \text{sen}(nx)\,dx \rightarrow \frac{1}{2} \cdot \int_0^{\pi} \text{sen}(u)\,\frac{du}{n}$$

$$\rightarrow \frac{1}{2n} \cdot \int_0^{\pi} \text{sen}(u)\,du$$

$$u = nx$$
$$du = n\,dx \rightarrow dx = \frac{du}{n}$$

$$= \frac{1}{2n} \cdot \Big[-\cos(nx)\Big]_0^{\pi}$$

$$\{ -\cos(n\pi) - (-\cos(n \cdot 0))$$
$$\{ -\cos(n\pi) + 1$$

$$= \frac{1}{2n}\left[-\cos(n\pi) + 1\right]$$

$$= \frac{1}{2n}\left[-(-1)^n + 1\right]$$

* im

* $f(x) = \frac{\pi}{4}$

$$* \quad S_f(x) = \sum_{n=1}^{+\infty} \frac{-(-1)^n + 1}{2n}\,\text{sen}(nx)$$

a) $* \quad \frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} \ldots$

$$1\,\frac{\pi}{2} = 1$$

$$2\,\frac{\pi}{2} = 0$$

a) 4 $\quad 3 \quad 5 \quad 7 \quad 9$

$x = \frac{\pi}{2} \rightarrow S_f(x) = \sum_{n=1}^{+\infty} -\frac{(-1)^n+1}{2n} \operatorname{sen}\left(n\frac{\pi}{2}\right)$ $\qquad 2\frac{\pi}{2}=0 \qquad [0,\pi]$ $\qquad = \frac{1}{2n}\left[-(-1)^n+1\right]$

$]0,\pi[ \qquad S_f\left(\frac{\pi}{2}\right) = \frac{\pi}{4}$ $\qquad f(x)=\frac{\pi}{4} \qquad f\left(\frac{\pi}{3}\right)=\frac{\pi}{4}$ $\qquad = \frac{-(-1)^n+1}{2n}$

$$\frac{\pi}{4} = \sum_{n=1}^{+\infty} -\frac{(-1)^n+1}{2n} \operatorname{sen}\left(n\frac{\pi}{2}\right)$$

$n=1$

$= -\frac{(-1)^1+1}{2\cdot 1}\cdot 1 = \frac{+1+1}{2}\cdot 1 = 1$ ✓

$n=2$

$= -\frac{(-1)^2+1}{2\cdot 2}\cdot 0 = 0$ $\qquad \operatorname{sen}\left(3\cdot\frac{\pi}{2}\right)=-1$

$n=3$

$= -\frac{(-1)^3+1}{2\cdot 3}\cdot -1 = \frac{+1+1}{6}\cdot -1 = -\frac{1}{3}$

$n=5$

$= -\frac{(-1)^5+1}{2\cdot 5}\cdot 1 = \frac{+1+1}{10}\cdot 1 = \frac{1}{5}$

$$* \quad 1-\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\dots = \sum_{n=1}^{+\infty} -\frac{(-1)^n+1}{2n}\cdot\operatorname{sen}\left(n\frac{\pi}{2}\right) = \frac{\pi}{4}$$

b) $\quad * \quad 1+\frac{1}{5}-\frac{1}{7}-\frac{1}{11}+\frac{1}{13}+\frac{1}{17}-\dots$

$$1-\frac{1}{3}+\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{9}-\frac{1}{11}+\frac{1}{13}-\frac{1}{15}+\frac{1}{15}+\frac{1}{17}\dots$$

$$\frac{\pi}{4}+\sum_{n=0}^{+\infty}\frac{(-1)^n}{3(2n+1)} = \sum_{n=1}^{+\infty} -\frac{(-1)^n+1}{2n}\cdot\operatorname{sen}\left(n\frac{\pi}{2}\right) + \sum_{n=0}^{+\infty}\frac{(-1)^n}{3(2n+1)} \quad (*)$$

$$\sum_{n=0}^{+\infty}\frac{(-1)^n}{3(2n+1)} = \underset{n=0}{\frac{1}{3}} - \underset{n=1}{\frac{1}{9}} + \underset{n=2}{\frac{1}{15}} - \underset{n=3}{\frac{1}{21}} \dots$$

Sabemos que

$$\sum_{n=0}^{+\infty}\frac{(-1)^n}{2n+1} = \frac{\pi}{4}$$

entonces

$$\frac{1}{3}\cdot\sum_{n=0}^{+\infty}\frac{(-1)^n}{2n+1} = \frac{1}{3}\frac{\pi}{4} \Rightarrow \sum_{n=0}^{+\infty}\frac{(-1)^n}{3(2n+1)} = \frac{\pi}{12} \quad (**)$$

Por $(*)$ y $(**)$, tenemos que

$$\frac{\pi}{4}+\frac{\pi}{12} = \sum^{+\infty} -\frac{(-1)^n+1}{2n}\operatorname{sen}\left(n\frac{\pi}{2}\right) + \sum^{+\infty}\frac{(-1)^n}{3(2n+1)} = 1+\frac{1}{5}-\frac{1}{7}-\frac{1}{11}+\frac{1}{13}+\frac{1}{17}-\dots$$

$$\frac{\pi}{4}+\frac{\pi}{12}=\sum_{n=1}^{+\infty}\frac{-(-1)^n+1}{2n}\operatorname{sen}\left(n\frac{\pi}{2}\right)+\sum_{n=0}^{+\infty}\frac{(-1)^n}{3(2n+1)}=1+\frac{1}{5}-\frac{1}{7}-\frac{1}{11}+\frac{1}{13}+\frac{1}{17}-\cdots$$

entonces

$$1+\frac{1}{5}-\frac{1}{7}-\frac{1}{11}+\frac{1}{13}+\frac{1}{17}-\cdots=\frac{\pi}{3}$$

c) $\overset{n=1}{1}-\frac{1}{5}+\frac{1}{7}-\frac{1}{11}+\frac{1}{13}-\frac{1}{17}+\frac{1}{19}\cdots$

Sabemos que:

$$\frac{\pi}{3}=\sum_{n=1}^{+\infty}\frac{-(-1)^n+1}{2n}\operatorname{sen}\left(n\frac{\pi}{2}\right)+$$

$$\rightarrow\sum_{n=0}^{+\infty}\frac{(-1)^n}{3(2n+1)}=1+\frac{1}{5}-\frac{1}{7}-\frac{1}{11}+\frac{1}{13}+\frac{1}{17}-\cdots$$

Usando las siguientes series:

$*\ \sum_{n=1}^{+\infty}-\frac{2}{4n^2+1}<\infty\rightarrow$ convergentes

$*\ \sum_{n=1}^{+\infty}+\frac{2}{4n^2+3}<\infty$

Obtenemos:

$$1+\frac{1}{5}-\frac{1}{7}-\frac{1}{11}+\frac{1}{13}+\frac{1}{17}-\cdots=\underbrace{\sum_{n=1}^{+\infty}\frac{-(-1)^n+1}{2n}\operatorname{sen}\left(n\frac{\pi}{2}\right)+\sum_{n=0}^{+\infty}\frac{(-1)^n}{3(2n+1)}}_{\frac{\pi}{3}}+$$

$$+\sum_{n=1}^{+\infty}\frac{2}{4n^2+3}-\frac{2}{4n^2+1}=\boxed{\frac{\pi}{3}+\sum_{n=1}^{+\infty}\frac{2}{4n^2+3}-\frac{2}{4n^2+1}}\ //$$