Chapter 2
Dynamic Modeling of Induction Machines

The cage rotor induction machine is widely used in industrial applications. Indeed, due to its conception, it has quite a low cost compared to the cost of other machines. It has a great electromechanical sturdiness and there is a good standardization between the diverse producers. Nevertheless, the relative simplicity of conception of the machine hides quite a great functional complexity, as soon as it is aimed at controlling the performed electromechanical conversion. The goal of this chapter is to establish various mathematical models, which allow understanding the machine's functioning, so as to establish its control in the next chapters.

In the previous chapter, the fundamental concepts of the electromechanical conversion have been recalled and applied to the modeling of an elementary machine, composed of a stator coil and a rotor coil. In this chapter, the physical principles are extended to the study of a three-phase induction motor. The mathematical models are established, regardless of the type of rotor technology (machines with coiled rotor that can be connected to an extern supply or squirrel-cage machines whose rotor circuit is short-circuited).

First the model of this machine is clarified in the three-axe frame linked to its supply, making the most of the matrix formalism. Then, several mathematical transformations are presented and used so as to substitute components from electrical quantity. The components will make the calculations easier and will simplify the representations. A general modeling of this machine is then presented along with some models, which are more suitable to the control design.

2.1 Presentation of the Three-Phase Induction Machine

2.1.1 Constitution and Structure

The stator of an induction machine is composed of three windings, coupled in star or in triangle. The rotor of the machine will be considered to support a winding, similar to the one of the stator, that is to say a three-phase winding with the same

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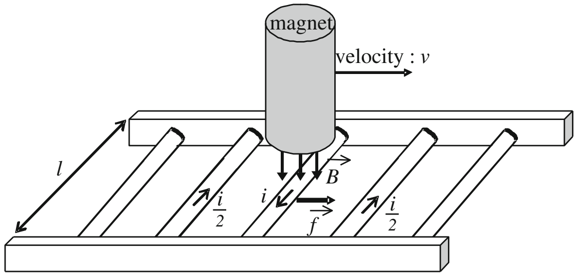

Fig. 2.1 Effect of the magnetic field's move on short-circuited conductors

amount of poles as in the stator. These three windings are coupled in star and two machine technologies can be distinguished according to this rotor constitution.

For the first technology, the coiled rotor includes a three-phase winding, similar to the one of the stator connected in star and for which each winding free bound is linked to a rotating ring by the rotor tree. Three brushes establish contacts on this ring and allow an external connection to the rotor. This connection can be a short-circuit or a link to resistors—so that it modifies the machine's characteristics in some of the functioning areas. This connection can also be an extern supply that allows controlling the rotor quantities. This second technology is still known as "Doubly-Fed Induction Machine".

For the second technology, the rotor windings are short-circuited on themselves. The rotor construction is then broadly simplified. Industrial realizations generally use a rotor, which is composed of conducting copper or aluminum bars that are short-circuited by a conductor ring at each bound. The result appears to be a squirrel cage, which explains the current name of "cage machine". This cage rotor works the same way as a coiled rotor.

The model developed in this chapter works for both technologies, depending on the voltages across the winding terminals. If they are null, it is a cage machine, and if they are imposed by an extern supply, it is a doubly feed induction machine.

2.1.2 Working Principle

In order to understand the functioning of the induction machine, the experiment of the cut flow in generator functioning (Chap. 1, paragraph 1.2.6.1) is modified as follows. This time, the l length conductors are fixed between themselves and short-circuited by extreme conductor bars forming thus a rail.

Moreover, the magnetic field (similar to a magnet, for instance) moves rapidly above this group (Fig. 2.1).

According to Faraday's law, a voltage is induced across each conductor cut by the magnetic field (expression 1.23):

$$e = -B\,l\,v \tag{2.1}$$

A first consequence appears. As each conductor is short-circuited, an eddy current i starts to circulate into the conductor, which is momentarily under the magnetic field (or the magnet). As this current crosses the magnetic field, according to Laplace's law, a magnetic force (Eq. 1.5) is carried out on this conductor. This force leads the conductor who follows the magnetic field's movement. If those conductors are mobile they speed up and while they are gaining speed, the speed in which the magnetic field is cut by those conductors slows down and the induced voltage decreases, as well as current i. This effect of Lenz's law results in decreasing Laplace's force. Thus if the conductors moved at the same speed as the magnetic field, the induced voltages, the current i and the force would be canceled out. The rotor speed is then slightly inferior to the magnetic field's speed.

In a cage induction machine, the rail considered in the example of the Fig. 2.1 is curved to form the squirrel cage and the magnetic field's movement becomes a rotating field, created by three stator windings. The stator windings are supplied by a three-phase system of balanced voltages with the pulsation ω_s, crossed by a three-phase current system with the same pulsation and generate stator fluxes. We define the pulsation as the electrical speed $\omega_s = 2\pi f_s$ with f_s the frequency of the stator voltage in Hz. According to Ferraris's theorem (Caron and Hautier 1995), a rotating magnetic field is created in the air gap (and curls itself in the rotor and stator framework). Its angular speed, which is called synchronous speed, equals the pulsation of the three-phase system balanced with currents that run through these windings. For the stator of the studied machine, the magnetic field generated by the stator rotates by one spin during an electrical quantity period. In the case where the stator is composed of a number p of pole pairs by phase, the stator magnetic field is rotating at the synchronous speed (in rad/s): $\Omega_s = \omega_s/p$.

As previously explained, the speed of the rotor is inferior to the synchronous speed in permanent operating: $\Omega \neq \Omega_s$. The rotor conductors are then submitted to a variable magnetic field that rotates in relation to themselves at relative speed $\Omega_r = \Omega_s - \Omega$.

This relative speed induces the voltage (Faraday's law) in each closed loop of the rotor conductors with pulsation: $\omega_r = p\Omega_r$.

The stator windings are in short circuits for cage machines or are connected to an external supply circuit for Doubly-Fed Induction Machine. Hence, the induced e.m.f. will generate rotor currents of the same pulsation in the closed rotor circuit. These currents then create a rotor magnetic field that rotates, in relation to the rotor, at speed $\Omega_r = \omega_r/p$.

Given that the rotor turns at speed Ω, the rotor field speed in relation to the stator is $\Omega_r + \Omega = \Omega_s$.

The magnetic field generated by the rotor windings and the magnetic field generated by the stator windings then rotate at the same synchronous speed and are combined to create a "resulting magnetic field" in the air gap. Therefore, the physical phenomena (in physics) generated by the stator circuit, including those induced by the rotor circuit will generate electrical quantities at the stator circuit at ω_s pulsation level. The electrical quantities specific to rotor circuit will all be at ω_r pulsation.

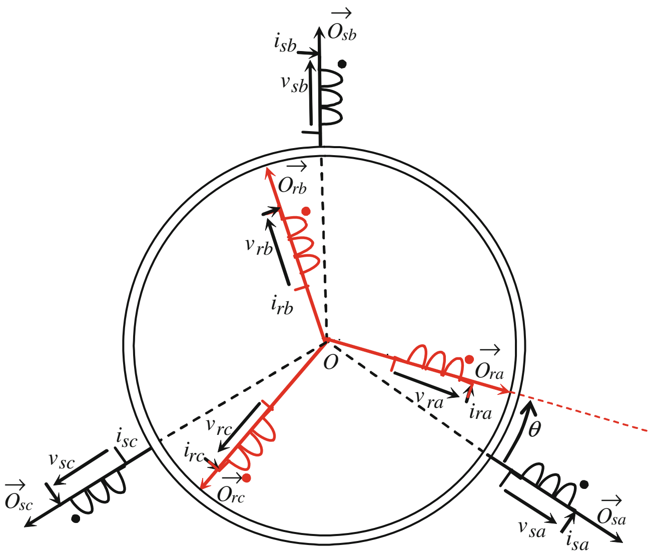

Fig. 2.2 Three-phase windings in the "natural frame"

2.1.3 Equivalent Representation and Vector Formulation

The studied machine is described in the electrical space by:

- Three identical windings for the stator phases set on the stator length and whose axis are distant two by two of an electrical angle equal to $2\pi/3$.
- Three rotor windings whose axis are equally distant between themselves of an electrical angle equal to $2\pi/3$, rotating at the mechanic speed Ω. In the case of a short-circuited rotor machine, those windings strictly account for the cage behavior.

The Fig. 2.2 represents the axis position of the stator $(\vec{O}_{\mathrm{sa}},\ \vec{O}_{\mathrm{sb}},\ \vec{O}_{\mathrm{sc}})$ and rotor $(\vec{O}_{\mathrm{ra}},\ \vec{O}_{\mathrm{rb}},\ \vec{O}_{\mathrm{rc}})$ phases in the electrical space (the electrical angle θ equals to the mechanical angle multiplied by the number p of pole pairs per phase). The stator quantities are respectively written under vector form:

$$[V_s] = \begin{bmatrix} v_{\mathrm{sa}} \\ v_{\mathrm{sb}} \\ v_{\mathrm{sc}} \end{bmatrix},\ [I_s] = \begin{bmatrix} i_{\mathrm{sa}} \\ i_{\mathrm{sb}} \\ i_{\mathrm{sc}} \end{bmatrix},\ [\phi_s] = \begin{bmatrix} \phi_{\mathrm{sa}} \\ \phi_{\mathrm{sb}} \\ \phi_{\mathrm{sc}} \end{bmatrix} \tag{2.2}$$

For the rotor windings, the following voltage, current and flux vectors will be taken into account:

$$[V_r] = \begin{bmatrix} v_{\mathrm{ra}} \\ v_{\mathrm{rb}} \\ v_{\mathrm{rc}} \end{bmatrix},\ [I_r] = \begin{bmatrix} i_{\mathrm{ra}} \\ i_{\mathrm{rb}} \\ i_{\mathrm{rc}} \end{bmatrix},\ [\phi_r] = \begin{bmatrix} \phi_{\mathrm{ra}} \\ \phi_{\mathrm{rb}} \\ \phi_{\mathrm{rc}} \end{bmatrix} \tag{2.3}$$

If the considered machine is a cage machine, the rotor voltages' vector is null. The only power supply of the stator windings creates simultaneously the rotating magnetic field and the induced currents into the rotor windings.

2.2 Dynamic Modeling in a Three-Axe Frame

2.2.1 Hypotheses

The modeling of the three-phase induction machine generally retained, relies on several hypotheses which are now recalled [Caron and Hautier 1995].

The first hypothesis consists in assuming that the magnetomotive forces created by different stator and rotor phases are spread in a sinusoidal way along the air gap, when those windings are crossed by a constant current. An appropriate dispersion of the windings in space allows reaching this aim.

The machine air gap is also supposed to be constantly thick: the notching effects, generating space harmonic, are ignored. These hypotheses will allow restricting the modeling of fundamental components (low frequency) of alternative quantities.

For this modeling, hypotheses about the physical behavior of the materials are expressed:

- The magnetic fields are not saturated, are not submitted to the hysteresis phenomenon and are not the center of Foucault's currents (for all practical purposes, the magnetic circuit is leafed through to limit the effects). This allows defining linear inductions.
- The skin effect is not taken into account.
- The temperature in the motor stays constant whatever the operating point is, which leads to constant parameters in mathematical models (stationarity).

These hypotheses will allow adding the associated fluxes to the different currents, using proper constant inductions, characterizing couplings by sinusoidal variations of the mutual inductions and representing induction flows by a spatial vector. They allow the development of modeling with a limited complexity, and thus the development of control strategies which can be implemented in practice.

In fact, the parameters of these models vary in saturation, skin effect and temperature. These variations' influences on the different control strategies' performances are analyzed in Chap. 4.

2.2.2 Study of the Electromechanical Conversion

2.2.2.1 Electrical Conversions

Finally, the hypotheses allow the electrical and magnetic behaviors to be considered as totally linear. The total variation of the energy exchanged with the electrical supply can be divided in two terms linked with stator and rotor circuits (paragraph 1.4.1):

$$\mathrm{d}W_{\mathrm{et}} = \mathrm{d}W_{\mathrm{et\_s}} + \mathrm{d}W_{\mathrm{et\_r}} \tag{2.4}$$

Those terms are expressed in matrix form as follows:

$$\mathrm{d}W_{\mathrm{et\_s}} = [I_s]^T [V_s]\, \mathrm{d}t \tag{2.5}$$

$$\mathrm{d}W_{\mathrm{et\_r}} = [I_r]^T [V_r]\, \mathrm{d}t \tag{2.6}$$

It must be retained that, in order to facilitate the understanding, all the matrix and vector quantities will be written between brackets. The total variation of energy dispersed in Joule losses is written:

$$dW_{\mathrm{pertesJoule}} = [I_s]^T [R_s][I_s]\mathrm{d}t + [I_r]^T [R_r][I_r]\mathrm{d}t \tag{2.7}$$

with

$$[R_s] = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \text{ and } [R_r] = \begin{bmatrix} R_r & 0 & 0 \\ 0 & R_r & 0 \\ 0 & 0 & R_r \end{bmatrix} \tag{2.8}$$

where:

R_s is a stator winding resistor,
R_r is a rotor winding resistor.

The iron losses being ignored, the electromechanical energy variation stored up in the machine is then:

$$\mathrm{d}W_e = \mathrm{d}W_{\mathrm{et}} - \mathrm{d}W_{\mathrm{Joulelosses}} \tag{2.9}$$

The variation of the electromagnetic energy is then written:

$$\mathrm{d}W_e = [I_s]^T \mathrm{d}[\phi_s] + [I_r]^T \mathrm{d}[\phi_r] \tag{2.10}$$

By identifying the Eq. 2.9 and integrating (2.7), (2.4), (2.5) and (2.6), Faraday's law is to be found again, which expresses the relation between voltages across the windings resistors' terminals, the currents and the total flow variations in their respective frames:

$$\left\{ \frac{\mathrm{d}[\phi_s]}{\mathrm{d}t} = [V_s] - [R_s][I_s] \right\}_{\mathrm{stator}} \tag{2.11}$$

$$\left\{ \frac{\mathrm{d}[\phi_r]}{\mathrm{d}t} = [V_r] - [R_r][I_r] \right\}_{\mathrm{rotor}} \tag{2.12}$$

where:

$[\phi_s]$ is the vector gathering the magnetic flows captured by the stator windings,
$[\phi_r]$ is the vector gathering the magnetic flows captured by the rotor windings.

Both equations are a matrix generalization of the Eq. 1.59. After integrating each differential equation, the expression of the flux is obtained in its own frame (stator or rotor).

The induction of each winding is divided into a main induction (or proper, written p) that participates in the common flux and a leak induction (written σ). We then define:

- the proper induction of a stator winding:

$$l_s = l_{\mathrm{sp}} + l_{s\sigma} \tag{2.13}$$

- the proper induction of a rotor winding:

$$l_r = l_{\mathrm{rp}} + l_{r\sigma} \tag{2.14}$$

Without saturation, the fluxes are supposed to be dependent of the currents in a linear way. It must be kept in mind that there are six windings magnetically coupled, three of which are mobile. The total flux in each winding is given by the sum of its proper flux (linked by the induction l_s, for a stator flux) with three rotor-coupling fluxes (linked by mutual inductions variable according to the rotor position). For a stator flux, we then obtain:

$$\begin{aligned}\phi_{\mathrm{sa}} &= l_s i_{\mathrm{sa}} + M_s i_{\mathrm{sb}} + M_s i_{\mathrm{sc}} \\ &\quad + M_{\mathrm{sr}}\left(i_{\mathrm{ra}}\cos(\theta) + i_{\mathrm{rb}}\cos\left(\theta - \frac{2\pi}{3}\right) + i_{\mathrm{rc}}\cos\left(\theta - \frac{4\pi}{3}\right)\right)\end{aligned} \tag{2.15}$$

The mutual induction between two distinct stator phases is written as follows:

$$M_s = l_{\mathrm{sp}}\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}l_{\mathrm{sp}} \tag{2.16}$$

M_{sr} is the maximal mutual induction between a stator phase and a rotor phase when their axes are collinear. Given the spatial structure of the machine, the mutual inductions between the stator phase and the rotor phase are written as follows:

$$\begin{aligned}
M_{\mathrm{sr}}\cos\left(\vec{O}_{\mathrm{sa}}, \vec{O}_{\mathrm{ra}}\right) &= M_{\mathrm{sr}}\cos(\theta) \\
M_{\mathrm{sr}}\cos\left(\vec{O}_{\mathrm{sa}}, \vec{O}_{\mathrm{rb}}\right) &= M_{\mathrm{sr}}\cos\left(\theta - \frac{2\pi}{3}\right) = M_{\mathrm{sr}}\cos\left(\theta + \frac{4\pi}{3}\right) \\
M_{\mathrm{sr}}\cos\left(\vec{O}_{\mathrm{sa}}, \vec{O}_{\mathrm{rc}}\right) &= M_{\mathrm{sr}}\cos\left(\theta - \frac{4\pi}{3}\right) = M_{\mathrm{sr}}\cos\left(\theta + \frac{2\pi}{3}\right)
\end{aligned}$$

where θ is the electrical angle between a rotor phase and a stator phase of the same name and defined as such in Fig. 2.2.

The other stator fluxes' expressions are obtained by circularly permuting the Eq. 2.15. Thus, under vector form, a generalization at the three-phase case of the Eq. 1.51 of the elementary machine described in Chap. 1 is obtained:

$$\left\{ [\phi_s] = \begin{bmatrix} \phi_{sa} \\ \phi_{sb} \\ \phi_{sc} \end{bmatrix} = \begin{bmatrix} l_s & M_s & M_s \\ M_s & l_s & M_s \\ M_s & M_s & l_s \end{bmatrix} \begin{bmatrix} i_{sa} \\ i_{sb} \\ i_{sc} \end{bmatrix} + M_{sr}[R(\theta)] \begin{bmatrix} i_{ra} \\ i_{rb} \\ i_{rc} \end{bmatrix} \right\}_{\text{stator}} \tag{2.17}$$

$[R(\theta)]$ is a rotational matrix that allows projecting rotor quantities in the stator frame according to the rotor position in relation to the stator:

$$[R(\theta)] = \begin{bmatrix} \cos(\theta) & \cos\left(\theta - \frac{4}{3}\pi\right) & \cos\left(\theta - \frac{2}{3}\pi\right) \\ \cos\left(\theta - \frac{2}{3}\pi\right) & \cos(\theta) & \cos\left(\theta - \frac{4}{3}\pi\right) \\ \cos\left(\theta - \frac{4}{3}\pi\right) & \cos\left(\theta - \frac{2}{3}\pi\right) & \cos(\theta) \end{bmatrix} \tag{2.18}$$

To determine the equations of the rotor circuit, the mutual induction between two distinct rotor phases is defined:

$$M_r = l_{rp} \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2} l_{rp} \tag{2.19}$$

The rotor fluxes are obtained by keeping the same reasoning:

$$\left\{ [\phi_r] = \begin{bmatrix} \phi_{ra} \\ \phi_{rb} \\ \phi_{rc} \end{bmatrix} = \begin{bmatrix} l_r & M_r & M_r \\ M_r & l_r & M_r \\ M_r & M_r & l_r \end{bmatrix} \begin{bmatrix} i_{ra} \\ i_{rb} \\ i_{rc} \end{bmatrix} + M_{sr}[R(\theta)]^{\mathrm{T}} \begin{bmatrix} i_{sa} \\ i_{sb} \\ i_{sc} \end{bmatrix} \right\}_{\text{rotor}} \tag{2.20}$$

Each relation is given again in its respective frame. The windings being coupled in star, the three-phase current systems are balanced, and it results in:

$$i_{sa} + i_{sb} + i_{sc} = 0 \tag{2.21}$$

$$i_{ra} + i_{rb} + i_{rc} = 0 \tag{2.22}$$

By expressing the winding current c depending on the two others, each phase's flux is simplified. For instance, for the stator flux of phase a, the flux is given by its own flux linked by $(l_s + M_s)$ and the coupling fluxes:

$$\phi_{sa} = (l_s - M_s) i_{sa} + M_{sr}\left(i_{ra}\cos(\theta) + i_{rb}\cos\left(\theta - \frac{2\pi}{3}\right) + i_{rc}\cos\left(\theta - \frac{4\pi}{3}\right)\right) \tag{2.23}$$

By taking the other fluxes into account, the following matrix form is obtained:

$$\left\{ [\phi_s] = \begin{bmatrix} \phi_{sa} \\ \phi_{sb} \\ \phi_{sc} \end{bmatrix} = \begin{bmatrix} L_s & 0 & 0 \\ 0 & L_s & 0 \\ 0 & 0 & L_s \end{bmatrix} \begin{bmatrix} i_{sa} \\ i_{sb} \\ i_{sc} \end{bmatrix} + M_{sr}[R(\theta)] \begin{bmatrix} i_{ra} \\ i_{rb} \\ i_{rc} \end{bmatrix} \right\}_{\text{stator}} \tag{2.24}$$

$$\left\{ [\phi_r] = \begin{bmatrix} \phi_{ra} \\ \phi_{rb} \\ \phi_{rc} \end{bmatrix} = \begin{bmatrix} L_r & 0 & 0 \\ 0 & L_r & 0 \\ 0 & 0 & L_r \end{bmatrix} \begin{bmatrix} i_{ra} \\ i_{rb} \\ i_{rc} \end{bmatrix} + M_{sr}[R(\theta)]^T \begin{bmatrix} i_{sa} \\ i_{sb} \\ i_{sc} \end{bmatrix} \right\}_{\text{rotor}} \tag{2.25}$$

The cyclic stator inductance per phase, taking into account the contribution of the three stator windings (at the creation of a stator flux), is written as:

$$L_s = l_s - M_s = \frac{3}{2} l_{sp} + l_{s\sigma} \tag{2.26}$$

The inner cyclic inductance, l_{sp}, is still called magnetizing inductance since it creates the flux into the air gap without current in the rotor circuit.

The rotor cyclic inductance per phase, taking into account the contribution of the three rotor windings (at the creation of a rotor flux), is written as follows:

$$L_r = l_r - M_r = \frac{3}{2} l_{rp} + l_{r\sigma} \tag{2.27}$$

The relations between flux and currents can be condensed using particular matrices:

$$\begin{bmatrix} [\phi_s] \\ [\phi_r] \end{bmatrix} = \begin{bmatrix} [L_s] & [M_{sr}(\theta)] \\ [M_{sr}(\theta)]^{\mathrm{T}} & [L_r] \end{bmatrix} \begin{bmatrix} [I_s] \\ [I_r] \end{bmatrix} \tag{2.28}$$

with the matrices defined as follows:

$$[L_s] = \begin{bmatrix} L_s & 0 & 0 \\ 0 & L_s & 0 \\ 0 & 0 & L_s \end{bmatrix}, \; [L_r] = \begin{bmatrix} L_r & 0 & 0 \\ 0 & L_r & 0 \\ 0 & 0 & L_r \end{bmatrix}, \; [M_{sr}(\theta)] = M_{sr}[R(\theta)]$$

2.2.2.2 Electromagnetic Transformation

By replacing flux expressions (2.24) and (2.25) in the relation of electromagnetic energy variation (2.10), the following formula is obtained:

$$\begin{aligned} \mathrm{d}W_e = {} & [I_s]^T [L_s]\mathrm{d}[I_s] + [I_s]^T M_{sr}\mathrm{d}([R(\theta)]\,[I_r]) \\ & + [I_r]^T [L_r]\mathrm{d}[I_r] + [I_r]^T M_{sr}\mathrm{d}\big([R(\theta)]^T\,[I_s]\big) \end{aligned} \tag{2.29}$$

By developing the product's derivatives, the following formula is obtained:

$$\begin{aligned} \mathrm{d}W_e = {} & [I_s]^T [L_s]\mathrm{d}[I_s] + [I_s]^T M_{sr}\mathrm{d}[R(\theta)]\,[I_r] + M_{sr}[I_s]^T [R(\theta)]\,\mathrm{d}[I_r] \\ & + [I_r]^T [L_r]\mathrm{d}[I_r] + [I_r]^T M_{sr}\mathrm{d}[R(\theta)]\,[I_s] + M_{sr}[I_r]^T [R(\theta)]\,\mathrm{d}\,[I_s] \end{aligned} \tag{2.30}$$

$$\begin{aligned} \mathrm{d}W_e \; = \; & [I_s]^T\,[L_s]\;\mathrm{d}[I_s] \;+\; [I_r]^T\,[L_r]\;\mathrm{d}[I_r] \;+\; [I_s]^T 2M_{\mathrm{sr}}\mathrm{d}[R(\theta)]\,[I_r] \\ & + [I_s]^T M_{\mathrm{sr}}[R(\theta)]\;\mathrm{d}[I_r] + \; [I_r]^T\,M_{\mathrm{sr}}[R(\theta)]\;\mathrm{d}\;[I_s] \end{aligned} \tag{2.31}$$

along with

$$\mathrm{d}[R(\theta)] = -\begin{bmatrix} \sin(\theta) & \sin\left(\theta - \frac{4}{3}\pi\right) & \sin\left(\theta - \frac{2}{3}\pi\right) \\ \sin\left(\theta - \frac{2}{3}\pi\right) & \sin(\theta) & \sin\left(\theta - \frac{4}{3}\pi\right) \\ \sin\left(\theta - \frac{4}{3}\pi\right) & \sin\left(\theta - \frac{2}{3}\pi\right) & \sin(\theta) \end{bmatrix}\mathrm{d}\theta \tag{2.32}$$

Further information on matrix computation can be found in [Rotella and Borne 1995].

When the angular variation is null ($\theta = 0$), the accumulated magnetic energy is immediately induced as follows:

$$W_{\mathrm{mag}} = \frac{1}{2}\,[I_s]^T\,[L_s]\,[I_s] + \frac{1}{2} \times \,[I_r]^T\,[L_r]\,[I_r] \,+\, [I_s]^T M_{\mathrm{sr}}\mathrm{d}[R(\theta)]^T\,[I_r] \tag{2.33}$$

A generalized expression of Eq. 1.54 is found again. Therefore, the total variation of the magnetic energy is expressed as follows:

$$\begin{aligned} \mathrm{d}W_{\mathrm{mag}} \; = \; & [I_s]^T\,[L_s]\;\mathrm{d}[I_s] \; + [I_r]^T\,[L_r]\;\mathrm{d}[I_r] \\ + \;\mathrm{d}[I_s]^T M_{\mathrm{sr}}[R(\theta)]\,[I_r] + \; & [I_s]^T M_{\mathrm{sr}}\mathrm{d}[R(\theta)] \times [I_r] + \; [I_s]^T M_{\mathrm{sr}}[R(\theta)]\;\mathrm{d}[I_r] \end{aligned} \tag{2.34}$$

When ignoring the iron losses, the generalization of relation (1.47) results in:

$$\mathrm{d}W_m = \; \mathrm{d}W_e - \mathrm{d}W_{\mathrm{mag}} \tag{2.35}$$

That is, by calculating the difference between (2.31) and (2.34) for a pair of poles:

$$\mathrm{d}W_m = \; [I_s]^T M_{\mathrm{sr}}\mathrm{d}[R(\theta)]\,[I_r] \tag{2.36}$$

The electrical energy variation corresponds to the mechanical energy variation:

$$\mathrm{d}W_m = \; T\mathrm{d}\theta \tag{2.37}$$

Therefore, for a number p of pole pairs, the torque is expressed by integrating (2.36) into (2.37):

$$T = p[I_s]^T M_{\mathrm{sr}}\frac{\mathrm{d}[R(\theta)]}{\mathrm{d}\theta}\,[I_r] \tag{2.38}$$

or the following transposed expression:

$$T = p[I_r]^T M_{\mathrm{sr}}\frac{\mathrm{d}[R(\theta)]}{\mathrm{d}\theta}\,[I_s] \tag{2.39}$$

Table 2.1 Causal organization of the mathematical equations for the induction machine

Electrical relation:	
$R1_s$: $\{[V_{Rs}] = [R_s][I_s]\}_{stator}$	$R1_r$:$\{[V_{Rr}] = [R_r][I_r]\}_{rotor}$
$R2_s$: $\{[e_s] = [V_s] - [V_{Rs}]\}_{stator}$	$R2_r$: $\{[e_r] = [V_r] - [V_{Rr}]\}_{rotor}$
$R3_s$: $\left\{[\phi_s] = \int_{t0}^{t0+\Delta t} [e_s]\,dt + [e_s](t_0)\right\}_{stator}$	$R3_r$: $\left\{[\phi_r] = \int_{t0}^{t0+\Delta t} [e_r]\,dt + [e_r](t_0)\right\}_{rotor}$
Flow/current coupling: *R4:* (2.28) inverse relation	
Electromechanical conversion: $R5$: $T = p[I_s]^T M_{sr} \frac{d[R(\theta)]}{d\theta}[I_r]$	

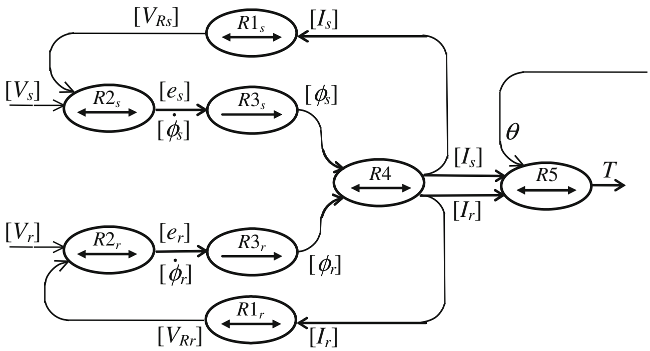

Fig. 2.3 COG of the three-phase model for the doubly-fed induction machine

or any linear combination of these two expressions, for instance [Grenier et al. 2001]:

$$T = \frac{1}{2} p M_{sr} \left([I_s]^T \frac{d[R(\theta)]}{d\theta} [I_r] + [I_r]^T \frac{d[R(\theta)]}{d\theta} [I_s] \right) \tag{2.40}$$

2.2.2.3 Causal Ordering Graph of the Model

Stator fluxes are obtained through the integration of matrix Eq. 2.11. In this equation, Joule effect losses are modeled by a voltage drop at the terminals of a resistor, for which the relation is $R1_s$ (Table 2.1). The mesh law, $R2_s$, allows determining the e.m.f. depending on the voltage drops and on the voltages applied on the stator terminals. Integrating this equation, $R3_s$, allows calculating the stator fluxes. The same can be done for the rotor equations. Currents can subsequently be determined by the inverse equations in the coupling with the fluxes (Eq. 2.28) $R4$.

Expressions for the torque (2.38), for the rotation matrix and its derivative are directly useable: relation $R5$. Figure 2.3 represents the causal ordering graph for the model of the coiled rotor induction machine. The mechanical part, which highly depends on the load, is not represented. The induction cage machine model

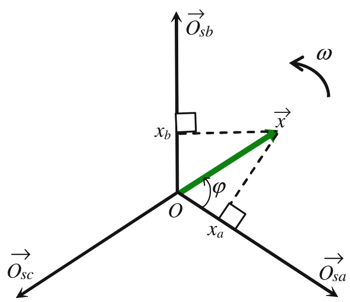

Fig. 2.4 Representation of a rotating phasor in a fixed three-axe frame

is obtained by fixing$[V_r] = 0$. Under these conditions, since$[e_r] = [V_{\mathrm{Rr}}]$, there is no need to use relation $R2_{\mathrm{r.}}$

2.3 Dynamic Model in a Two-Axe Frame

2.3.1 Phasor in a Three-Axe Frame

One way to make the established mathematical model less complex is to describe the induction machine by taking into account two (equivalent) windings rather than three. This method relies on working out the equation of a phasor and will be explained in the following paragraphs.

A three-phase winding supplied by a three-phase currents system creates three-phase pulsating magnetic fields. According to the Ferraris theorem [Caron and Hautier 1995], a rotating magnetic field appears in the air gap of the machine and results from the spatial combination of the fields. Voltages, currents and stator, as well as rotor three-phase flows create as many phasors in the three-phase space defined in Fig. 2.2. Therefore, a three-phase system is represented with the following vector form

$$[X] = \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix}.$$

It includes three variables x_a, x_b, x_c, which are made to correspond to projections of a spatial vector $\vec{x}$ on the three axes directed by unit vectors $\vec{O}_{\mathrm{sa}}$, $\vec{O}_{\mathrm{sb}}$, $\vec{O}_{\mathrm{sc}}$ out of phase with $2/3\,\pi$ (Fig. 2.4).

$$\vec{x} = K\left(x_a\,\vec{O}_{\mathrm{sa}} + x_b\,\vec{O}_{\mathrm{sb}} + x_c\,\vec{O}_{\mathrm{sc}}\right) \tag{2.41}$$

where K is a constant.

The phasor corresponds to a Fresnel vector rotating at angular speed ω and can thus be represented with a complex number:

$$\underline{x} = K\begin{bmatrix} 1 & e^{j\frac{2}{3}\pi} & e^{j\frac{4}{3}\pi} \end{bmatrix}\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} \tag{2.42}$$

To enhance readability, each complex value is underlined.

We can for instance consider a three-phase system with a balanced current, a pulsation equal to ω and a root-mean-square-value equal to I:

$$[I] = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = I\sqrt{2}\begin{bmatrix} \sin(\omega t) \\ \sin\left(\omega t - \frac{2\pi}{3}\right) \\ \sin\left(\omega t - \frac{4\pi}{3}\right) \end{bmatrix} \tag{2.43}$$

The associated complex number is:

$$\underline{i} = I\sqrt{3}\mathrm{e}^{j\omega t} \tag{2.44}$$

The inverse relations are subsequently expressed according to:

$$[X] = \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} = \frac{2}{3K}\begin{bmatrix} \mathrm{Real}(x) \\ \mathrm{Real}\left(x\mathrm{e}^{-j\frac{2\pi}{3}}\right) \\ \mathrm{Real}\left(x\mathrm{e}^{-j\frac{4\pi}{3}}\right) \end{bmatrix} \tag{2.45}$$

where Real(…) represents the real part of the expression between brackets.

When $K = 2/3$, a vector representation that preserves the amplitudes is obtained. When $K = \sqrt{2/3}$, the obtained representation preserves the power. This value is the chosen one for the modeling presentation in this chapter.

2.3.2 Phasor in a Fixed Two-Axe Frame

In an orthogonal frame $\left(\vec{O}_\alpha, \vec{O}_\beta\right)$, aligned on axe $\left(\vec{O}_{\mathrm{sa}}\right)$, this same vector will be expressed by means of (Fig. 2.5):

$$\vec{x} = x_\alpha \vec{O}_{s\alpha} + x_\beta \vec{O}_{s\beta} \tag{2.46}$$

Those coordinates may be obtained directly from the coordinates in the three-axe frame by using a rotation matrix:

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = \sqrt{\frac{2}{3}}\begin{bmatrix} \cos(0) & \cos\left(\frac{2\pi}{3}\right) & \cos\left(\frac{4\pi}{3}\right) \\ \sin(0) & \sin\left(\frac{2\pi}{3}\right) & \sin\left(\frac{4\pi}{3}\right) \end{bmatrix}\begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix}$$

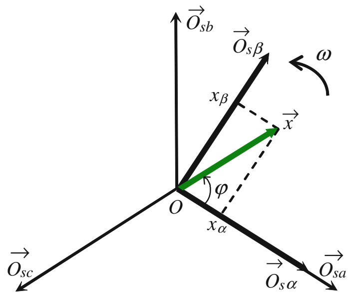

Fig. 2.5 Representation of a phasor in a two-axe frame

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} \tag{2.47}$$

If this same principle is applied to the magnetic field created by three-phase windings (Chap. 2, paragraph 2.1.3), this same field can also be obtained with two coils, the axes of which are perpendicular and are supplied by a two-phase electric system.

If an angle θs exists between the frame and the synchronous frame, it is necessary to take this angle into account:

$$\vec{x} = \underline{x}\, e^{j(\theta s)}\, \vec{O}_{s\alpha} \tag{2.48}$$

2.3.3 Transformation Matrices

An equivalent rotating magnetic field can also be created by taking into account only two phases. The Concordia transformation makes it possible to obtain an equivalent system formed by three orthogonal windings (axes). Two of these windings are located in the same plan as the three-phase windings. The third winding is located in the orthogonal plan formed by the three-phase windings. It represents the homopolar component, which characterizes the balance of the studied system:

$$x_0 = \frac{1}{\sqrt{3}} (x_a + x_b + x_c) \tag{2.49}$$

This homopolar component is null when the system is balanced. The passage between coordinates in the three-axe frame, as well as two-axe and homopolar coordinates is defined in terms of:

$$\begin{bmatrix} x_\alpha \\ x_\beta \\ x_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix} \tag{2.50}$$

which can also be written as follows:

$$\left[X_{\alpha,\beta,0}\right] = [C]\,[X] \tag{2.51}$$

along with:

$$[C] = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & \frac{-1}{2} & \frac{-1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{2.52}$$

This transformation allows going from a three-axe base to a two-axe orthogonal and orthonormal base. In literature, this is generally called Concordia inverse transform and it is written as follows: $[T_3]^{-1}(=[C])$. For instance, a three-phase current-balanced system, with a pulsation equal to $\omega = 2\pi 50$ rad/s and with a root-mean-square-value equal to $I = 8$A:

$$[I] = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = I\sqrt{2} \begin{bmatrix} \sin(\omega\,t) \\ \sin\left(\omega\,t - \frac{2\pi}{3}\right) \\ \sin\left(\omega\,t - \frac{4\pi}{3}\right) \end{bmatrix} \tag{2.53}$$

The transformation leads to:

$$\left[I_{\alpha\beta 0}\right] = \begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = [C] \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \sqrt{\frac{2}{3}} I\sqrt{2} \begin{bmatrix} \sin(\omega\,t) \\ \sin\left(\omega\,t - \frac{\pi}{2}\right) \\ 0 \end{bmatrix} \tag{2.54}$$

The timing evolution of these values is represented on Fig. 2.6.

By means of the coordinates in the two-axe frame, the coordinates of a vector are found again in the three-axe frame by using the inverse matrix:

$$[X] = [C]^{-1}\left[X_{\alpha,\beta,0}\right] \tag{2.55}$$

along with:

$$[C]^{-1} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \end{bmatrix} = [C]^{\mathrm{T}} \tag{2.56}$$

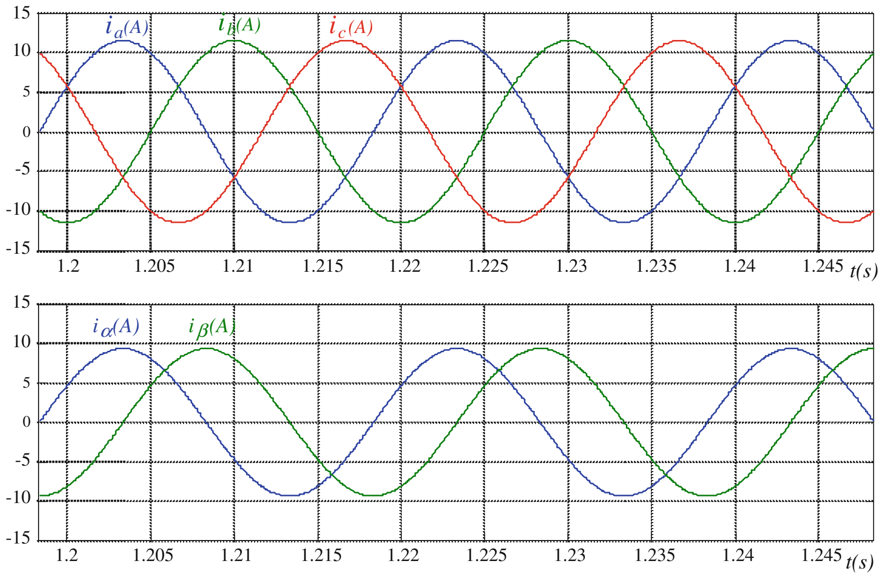

Fig. 2.6 Timing evolution of sinusoidal values in a three-axe frame of reference (*top* figure) and in a two-axe frame (*bottom* figure)

The Concordia inverse matrix is orthogonal and thus equals its transpose. In literature, this is generally called Condordia direct transform and it is written as follows: $[T_3]\left(=[C]^{-1}\right)$.

The instantaneous electrical power is expressed with:

$$p_{\text{et}}(t) = v_a i_a + v_b i_b + v_c i_c = [V]^T [I] \tag{2.57}$$

With the Concordia transposed, the power is expressed with:

$$p_{\text{et}}(t) = \left([C]^{-1}\left[V_{\alpha\beta 0}\right]\right)^T [C]^{-1}\left[I_{\alpha\beta 0}\right] = \left[V_{\alpha\beta 0}\right]^T [C]\,[C]^{-1}\left[I_{\alpha\beta 0}\right] = \left[V_{\alpha\beta 0}\right]^T\left[I_{\alpha\beta 0}\right] \tag{2.58}$$

The instantaneous electrical power is found again but is expressed with the Concordia components.

2.3.4 Vector Model in a Two-Axe Frame

2.3.4.1 Principle

At the stator, the three-phase voltages, the three-phase currents and the three-phase fluxes form three vectors rotating at the angular speed ω_s, depending on the fix stator: $\vec{V}_s, \vec{I}_s, \vec{\phi}_s$. At the rotor, the three-phase voltages, the three-phase currents and the three-phase fluxes, that rotate at the angular speed ω_r, depending on the rotor: $\vec{V}_r, \vec{I}_r, \vec{\phi}_r$. If the rotor spins at speed Ω, then these vectors rotate at the speed $p\Omega + \omega_r = \omega_s$ depending on the fix stator.

The use of the vector notations will allow the generalization of the obtained results with scalar values when modeling the elementary machine in paragraph 1.4.2.2.

By bringing the variables of three-axe frame (a,b,c) back on the axis of a two-axe frame (α, β), an equivalent two-axe machine can be taken into account. It is physically possible to create this two-phase machine.

2.3.4.2 Application to the Expressions of Fluxes

Working with the model of the induction machine on the three-axe frame, the aim is to determine the equivalent model on a two-axe frame. Flows in the three-axe frame (Eq. 2.28) are expressed according to the coordinates in the two-axe frame by applying Concordia inverse transformation (2.50) [Lesenne et al. 1995]:

$$[C]^{-1}\begin{bmatrix}[\phi_{s\_\alpha\beta0}] \\ [\phi_{r\_\alpha\beta0}]\end{bmatrix} = \begin{bmatrix}[L_s] & [M_{sr}(\theta)] \\ [M_{sr}(\theta)]^T & [L_r]\end{bmatrix}[C]^{-1}\begin{bmatrix}[I_{s\_\alpha\beta0}] \\ [I_{r\_\alpha\beta0}]\end{bmatrix} \tag{2.59}$$

Bringing all the Concordia transformations back into the right hand side, results in:

$$\begin{bmatrix}[\phi_{s\_\alpha\beta0}] \\ [\phi_{r\_\alpha\beta0}]\end{bmatrix} = \begin{bmatrix}[C]\,[L_s][C]^T & [C]\,[M_{sr}(\theta)][C]^T \\ [C]\,[M_{sr}(\theta)]^T[C]^T & [C]\,[L_r][C]^T\end{bmatrix}\begin{bmatrix}[I_{s\_\alpha\beta0}] \\ [I_{r\_\alpha\beta0}]\end{bmatrix} \tag{2.60}$$

The development of the previous formula leads to the expression of four submatrices, which define the coupling of the equivalent model in the two-axe frame.

$$\begin{bmatrix}[\phi_{s\_\alpha\beta0}] \\ [\phi_{r\_\alpha\beta0}]\end{bmatrix} = \begin{bmatrix}[L_{cs}] & [M_{csr}(\theta)] \\ [M_{csr}(\theta)]^T & [L_{cr}]\end{bmatrix}\begin{bmatrix}[I_{s\_\alpha\beta0}] \\ [I_{r\_\alpha\beta0}]\end{bmatrix} \tag{2.61}$$

along with:

$$[L_{cs}] = \begin{bmatrix} l_s - M_s & 0 & 0 \\ 0 & l_s - M_s & 0 \\ 0 & 0 & l_s + 2M_s \end{bmatrix} = \begin{bmatrix} L_s & 0 & 0 \\ 0 & L_s & 0 \\ 0 & 0 & L_{s0} \end{bmatrix} \tag{2.62}$$

$$[L_{cr}] = \begin{bmatrix} l_r - M_r & 0 & 0 \\ 0 & l_r - M_r & 0 \\ 0 & 0 & l_r + 2M_r \end{bmatrix} = \begin{bmatrix} L_r & 0 & 0 \\ 0 & L_r & 0 \\ 0 & 0 & L_{r0} \end{bmatrix} \tag{2.63}$$

$$[M_{csr}(\theta)] = \frac{3}{2}M_{sr}\,[R_c(\theta)] \tag{2.64}$$

along with:

$$[R_c(\theta)] = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (2.65)$$

The formula uses several expressions with the following meanings:

- $L_s = l_s - M_s$ is the stator cyclic inductance,
- $L_r = l_r - M_r$ is the rotor cyclic inductance,
- $M = 3/2M_{\mathrm{sr}}$ is the mutual cyclic inductance between a stator phase and a rotor phase when their axes are collinear (in the equivalent machine)
- $L_{\mathrm{so}} = l_s + 2M_s$ and $L_{\mathrm{ro}} = l_r + 2M_r$ correspond to homopolar inductances specific to each armature. If the magnetomotive forces have a sinusoidal repartition in space and if the rotor is smooth then the homopolar inductances L_{s0} and L_{r0} are null.

It can be noticed that the obtained model in the two-phase frame uses diagonal inductance matrices ($[L_{\mathrm{cs}}]$ and $[L_{\mathrm{cr}}]$). Therefore, the mutual inductances are null. This will bring important simplifications in the future calculations.

2.3.4.3 Application to Differential Equations

By including the Concordia transformation (Eq. 2.55) in the differential equations (Eqs. 2.1 and 2.12), results in:

$$\left\{ [C]^{-1}\frac{\mathrm{d}\left[\phi_{s\_\alpha\beta 0}\right]}{\mathrm{d}t} = [C]^{-1}\left[V_{s\_\alpha\beta 0}\right] - [R_s]\,[C]^{-1}\left[I_{s\_\alpha\beta 0}\right] \right\}_{\mathrm{stator}} \quad (2.66)$$

$$\left\{ [C]^{-1}\frac{\mathrm{d}\left[\phi_{r\_\alpha\beta 0}\right]}{\mathrm{d}t} = [C]^{-1}\left[V_{r\_\alpha\beta 0}\right] - [R_r]\,[C]^{-1}\left[I_{r\_\alpha\beta 0}\right] \right\}_{\mathrm{rotor}} \quad (2.67)$$

By multiplying on the left by using Concordia transformation, the expressions are simplified:

$$\left\{ \begin{array}{l} \frac{\mathrm{d}\phi_{s\alpha}}{\mathrm{d}t} = v_{s\alpha} - R_s i_{s\alpha} \\ \frac{\mathrm{d}\phi_{s\beta}}{\mathrm{d}t} = v_{s\beta} - R_s i_{s\beta} \\ \\ \frac{\mathrm{d}\phi_{s0}}{\mathrm{d}t} = v_{s0} - R_s i_{s0} \end{array} \right\}_{\mathrm{stator}} \quad (2.68)$$

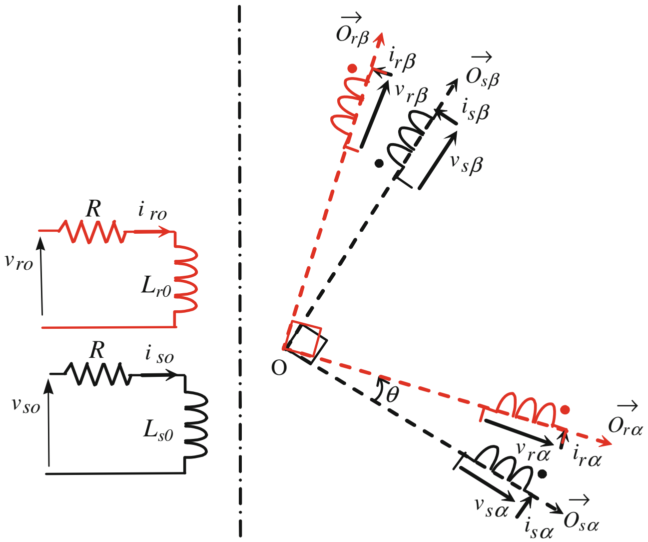

Fig. 2.7 Equivalent two-phase windings in an orthogonal frame

$$\left\{\begin{array}{l}\frac{\mathrm{d}\phi_{r\alpha}}{\mathrm{d}t}=v_{r\alpha}-R_r i_{r\alpha}\\[2ex]\frac{\mathrm{d}\phi_{r\beta}}{\mathrm{d}t}=v_{r\beta}-R_r i_{r\beta}\\[2ex]\frac{\mathrm{d}\phi_{r0}}{\mathrm{d}t}=v_{r0}-R_r i_{r0}\end{array}\right\}_{\text{rotor}} \tag{2.69}$$

When the sum of the three-phase components (a, b, c) is null, the third equation corresponding to the homopolar component is null (Eq. 2.49) and becomes void.

2.3.4.4 Equivalent Two-Phase Machine

The obtained electrical equations correspond to the equations of an equivalent two-phase induction machine. They are linked to two orthogonal two-axe frames as well as to the equations of two electrical circuits R-L connected to two homopolar machines (orthogonal to both two-axe frames). Figure 2.7 shows the position of the two-phase windings of the equivalent machine in both orthogonal frames (α, β). Axes $\vec{O}_{s\alpha}$ and $\vec{O}_{s\beta}$ are collinear because of the specific value of the Concordia matrix. Both homopolar circuits R-L are located orthogonally to these two two-phase windings.

2.3.4.5 Application to the Expression of the Torque

Starting from/working with the expression of the torque (2.38) in the three-axe frame, by making Concordia transformation (2.55) appear in the stator and rotor currents, results in:

$$T = p\left([C]^{-1}\left[I_{s\_\alpha\beta 0}\right]\right)^{\mathrm{T}}\frac{\mathrm{d}[M_{\mathrm{sr}}(\theta)]}{\mathrm{d}\theta}[C]^{-1}\left[I_{r\_\alpha\beta 0}\right] \tag{2.70}$$

$$T = p\,\frac{3}{2}\left[I_{s\_\alpha\beta 0}\right]^{\mathrm{T}}M_{\mathrm{sr}}\frac{\mathrm{d}[R_c(\theta)]}{\mathrm{d}\theta}\left[I_{r\_\alpha\beta 0}\right] \tag{2.71}$$

along with

$$\frac{d[R_c(\theta)]}{d\theta} = \begin{bmatrix} \sin(\theta) & \cos(\theta) & 0 \\ -\cos(\theta) & \sin(\theta) & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{2.72}$$

By writing:

$$M = \frac{3}{2}M_{\mathrm{sr}} \tag{2.73}$$

the scalar expression of the torque is obtained:

$$T = pM\left[\left(i_{\mathrm{r}\beta}i_{\mathrm{s}\alpha} - i_{\mathrm{r}\alpha}i_{\mathrm{s}\beta}\right)\cos(\theta) + \left(i_{\mathrm{r}\alpha}i_{\mathrm{s}\alpha} + i_{\mathrm{r}\beta}i_{\mathrm{s}\beta}\right)\sin(\theta)\right] \tag{2.74}$$

It is notable that only the mutual inductance contributes to the creation of the torque. Therefore, the homopolar components are only sources of losses.

2.3.4.6 Working With Balanced Currents

To cancel the homopolar components, the three-phase current system has to be balanced (Eqs. 2.21 and 2.22).

To impose a current-balanced system to the stator of a three-phase machine, it is possible to power the windings with only three phases, therefore without the connection of the neutral. The instantaneous sum of the stator currents is then null as well as the homopolar component of the current. For a balanced supply in terms of stator voltages, the homopolar component of the stator voltages is null too. If the homopolar component becomes void, all vectors will be reduced to two components.

$$\left\{\frac{\mathrm{d}\left[\phi_{s\_\alpha\beta}\right]}{\mathrm{d}t} = \left[V_{s\_\alpha\beta}\right] - [R_s]\left[I_{s\_\alpha\beta}\right]\right\}_{\text{stator}} \tag{2.75}$$

$$\left\{\frac{\mathrm{d}\left[\phi_{r\_\alpha\beta}\right]}{\mathrm{dt}} = \left[V_{r\_\alpha\beta}\right] - [R_r]\left[I_{r\_\alpha\beta}\right]\right\}_{\text{rotor}} \tag{2.76}$$

with $[R_s] = R_s[I]$, $[R_r] = R_r[I]$ where $[I]$ is the identity matrix with a dimension of 2×2.

$$\begin{bmatrix} \left[\phi_{s\_\alpha\beta}\right] \\ \left[\phi_{r\_\alpha\beta}\right] \end{bmatrix} = \begin{bmatrix} [L_{\mathrm{cs}}] & [M_{\mathrm{csr}}(\theta)] \\ [M_{\mathrm{csr}}(\theta)]^{\mathrm{T}} & [L_{\mathrm{cr}}] \end{bmatrix} \begin{bmatrix} \left[I_{s\_\alpha\beta}\right] \\ \left[I_{r\_\alpha\beta}\right] \end{bmatrix} \tag{2.77}$$

along with $[L_{\mathrm{cs}}] = L_s[I]$, $[L_{\mathrm{cr}}] = L_r[I]$ and

$$[R_c(\theta)] = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}.$$

2.4 Vector Model in a Two-Axe Rotating Frame

2.4.1 The Park Transformation

The complex number associated to a balanced current system with root-mean-square value I and pulsation ω have been determined beforehand (Eq. 2.44). The same mathematical result can be obtained when supplying a coil with direct current and making it rotate at speed ω. This operation amounts to defining a rotating frame for axes d and q where the pulsation of every quantity is null.

$$\underline{i} = \underline{i}_{\mathrm{dq}} \mathrm{e}^{j\omega.t} \tag{2.78}$$

Under steady conditions, the electrical quantities shown in this frame are constant. The Concordia transformation and a rotation matrix will be used to establish this new frame. Rotation matrix $[R(\psi)]$ allows to bring the variables of (α, β, o) frame back to the axes of a (d,q,o) frame whose angle ψ may vary.

$$[R_P(\psi)] = \begin{bmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2.79}$$

If quantity ψ is time dependent, the frame will rotate. The product of the two frame changes (Concordia, rotation) defines the Park transformation of which basic property is to bring the stator and rotor quantities back to the same frame of reference. The matrix product that defines the Park transform is determined from:

$$\left[X_{\mathrm{dqo}}\right] = [R_P(\psi)]\left[X_{\alpha\beta o}\right] \text{ et } \left[X_{\alpha\beta o}\right] = [C]\ [X_{\mathrm{abc}}] \tag{2.80}$$

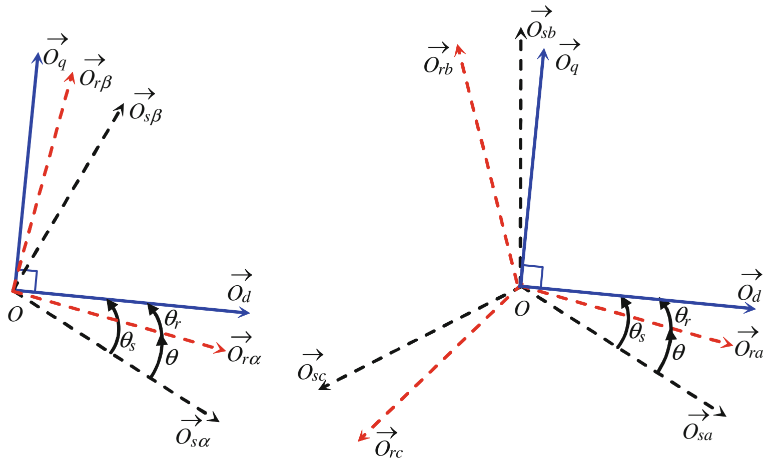

Fig. 2.8 Angular representation of the axis systems in the electrical space

The Park transformation is directly expressed by the following matrix product:

$$\left[X_{\mathrm{dqo}}\right] = [P(\psi)]\;[X_{\mathrm{abc}}] \tag{2.81}$$

along with:

$$[P(\psi)]=[R_P(\psi)]\,[C] \tag{2.82}$$

$$[P(\psi)] = \sqrt{\frac{2}{3}}\begin{bmatrix} \cos(\psi) & \cos\left(\psi - \frac{2\pi}{3}\right) & \cos\left(\psi - \frac{4\pi}{3}\right) \\ -\sin(\psi) & -\sin\left(\psi - \frac{2\pi}{3}\right) & -\sin\left(\psi - \frac{4\pi}{3}\right) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{2.83}$$

As matrices $\left[R_p(\psi)\right]$and [C] are orthogonal, matrix $[P(\psi)]$ is orthogonal, too. Consequently, the inverse Park transformation is equal to its transposed and the power is retained [Caron and Hautier 1995]. It is relevant to note that the matrix defined as $[P(\psi)]$ is generally called “the inverse Park transformation”. Consequently, its inverse is called “the direct Park transformation”.

As an example, a current-balanced three-phase system with a pulsation $\omega = 2\pi 50$ rad/s and a root-mean-square-value $I = 8$A is studied again. When $\psi = \omega t$, applying the Park transformation leads to constant quantities depending on time of which the amplitude is proportional to the root-mean-square value:

$$\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = \sqrt{3}\,I\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{2.84}$$

2.4.2 Induction Machine Model in the Park Reference Frame

2.4.2.1 Principle

In paragraph 2.3.4, the induction machine was modeled using two separate frames. The first one is used to express stator quantities; the second one is used to express rotor quantities. Since these two frames are linked with angle θ, a model of the machine in a common frame named *d, q* can be obtained using the two rotation matrices.

Figure 2.8 shows the disposition of the two-phase or three-phase axis systems in the electrical space. At a certain point, the position of the magnetic field rotating in the air gap (paragraph 2.1.2) is pinpointed by angle θ_s, in relation to stationary axis $\vec{O}_{\mathrm{sa}}$. For the development of the machine model, a Park reference frame is assumed to be lined up with this magnetic field and to rotate at the same speed (ω_s). Angle θ_s corresponds to the angle of axes $\vec{O}_{\mathrm{s}\alpha}$ and θ_r; angle θ_r corresponds to the angle of axes $\vec{O}_{\mathrm{r}\alpha}$ and $\vec{O}_d$. Transforming angle θ_s is necessary to bring the stator quantities back to the Park rotating reference frame. Transforming angle θ_r is necessary to bring the rotor quantities back. The figure indicates that the angles are linked by a relation in order to express the rotor and stator quantities in the same Park reference frame $(O, \vec{O}_d, \vec{O}_q)$. This relation is:

$$\theta_s = \theta + \theta_r \tag{2.85}$$

The same situation happens between the frame speeds in each frame and the mechanical speed, that is:

$$\omega_s = \omega + \omega_r, \tag{2.86}$$

with

$$\omega_s = \frac{\mathrm{d}\theta_s}{\mathrm{d}t}, \quad \omega_r = \frac{\mathrm{d}\theta_r}{\mathrm{d}t}, \quad \omega = p\Omega = \frac{\mathrm{d}\theta}{\mathrm{d}t} \tag{2.87}$$

Where Ω is the mechanical speed and ω this very speed viewed in the electrical space.

The speed of the rotor quantities is ω_r in relation to rotor speed ω. In relation to the stator frame, the rotor quantities consequently rotate at the same speed ω_s as the stator quantities. Using the Park transform will allow the conception of an induction machine model independent from the rotor position.

Two transformations are used. One $[P(\theta_s)]$ is applied to the stator quantities; the other $[P(\theta_r)]$ is applied to the rotor quantities.

$$\left[X_{\mathrm{s\_dqo}}\right] = [P(\theta_s)]\ [X_{\mathrm{s\_abc}}] \text{ et } \left[X_{\mathrm{r\_dqo}}\right] = [P(\theta_r)]\,[X_{\mathrm{r\_abc}}] \tag{2.88}$$

Direct and squared components x_d, x_q represent coordinates x_a, x_b, x_c in an orthogonal frame of reference rotating in the same plane. Term x_o represents the homopolar component, which is orthogonal to the plane constituted by the system x_a, x_b, x_c.

2.4.2.2 Determination of Differential Equations

Revealing the Park reference frame coordinates in the differential equations which are specific to the three-axe frame (Eqs. 2.11 and 2.12)—and applying the inverse Park transformation to the variables (flux, voltage and current)-, results in:

$$\frac{\mathrm{d}\left([P(\theta_s)]^{-1}\left[\phi_{\mathrm{s\_dq0}}\right]\right)}{\mathrm{dt}} = [P(\theta_s)]^{-1}\left[V_{\mathrm{s\_dq0}}\right] - [R_s][P(\theta_s)]^{-1}\left[I_{\mathrm{s\_dq0}}\right] \quad (2.89)$$

$$\frac{\mathrm{d}\left([P(\theta_r)]^{-1}\left[\phi_{\mathrm{r\_dq0}}\right]\right)}{\mathrm{dt}} = [P(\theta_r)]^{-1}\left[V_{\mathrm{r\_dq0}}\right] - [R_r][P(\theta_r)]^{-1}\left[I_{\mathrm{r\_dq0}}\right] \quad (2.90)$$

Multiplying on the left by the Park transformation, results in:

$$[P(\theta_s)]\frac{\mathrm{d}\left([P(\theta_s)]^{-1}\right)}{\mathrm{dt}}\left[\phi_{\mathrm{s\_dq0}}\right] + \frac{\mathrm{d}\left[\phi_{\mathrm{s\_dq0}}\right]}{\mathrm{dt}} = \left[V_{\mathrm{s\_dq0}}\right] - [R_s]\left[I_{\mathrm{s\_dq0}}\right] \quad (2.91)$$

$$[P(\theta_r)]\frac{\mathrm{d}\left([P(\theta_r)]^{-1}\right)}{\mathrm{dt}}\left[\phi_{\mathrm{r\_dq0}}\right] + \frac{\mathrm{d}\left[\phi_{\mathrm{r\_dq0}}\right]}{\mathrm{dt}} = \left[V_{\mathrm{r\_dq0}}\right] - [R_r]\left[I_{\mathrm{r\_dq0}}\right] \quad (2.92)$$

The product of the Park transformation with its derivative is developed as follows:

$$[P(\psi)]\frac{\mathrm{d}\left([P(\psi)]^{-1}\right)}{\mathrm{d}t} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\frac{\mathrm{d}\psi}{\mathrm{d}t} \quad (2.93)$$

The formula of the differential equations is:

$$\left\{\begin{array}{c} \frac{\mathrm{d}\phi_{\mathrm{sd}}}{\mathrm{d}t} = v_{\mathrm{sd}} - R_s i_{\mathrm{sd}} + \omega_s \phi_{\mathrm{sq}} \\ \frac{\mathrm{d}\phi_{\mathrm{sq}}}{\mathrm{d}t} = v_{\mathrm{sq}} - R_s i_{\mathrm{sq}} - \omega_s \phi_{\mathrm{sd}} \\ \frac{\mathrm{d}\phi_{s0}}{\mathrm{d}t} = v_{s0} - R_s i_{s0} \end{array}\right\} \quad (2.94)$$

$$\left\{\begin{array}{c} \frac{\mathrm{d}\phi_{\mathrm{rd}}}{\mathrm{d}t} = v_{\mathrm{rd}} - R_r i_{\mathrm{rd}} + \omega_r \phi_{\mathrm{rq}} \\ \frac{\mathrm{d}\phi_{\mathrm{rq}}}{\mathrm{d}t} = v_{\mathrm{rq}} - R_r i_{\mathrm{rq}} - \omega_r \phi_{\mathrm{rd}} \\ \frac{\mathrm{d}\phi_{r0}}{\mathrm{d}t} = v_{r0} - R_r i_{r0} \end{array}\right\} \quad (2.95)$$

2.4.2.3 Determination of Equations Between Flux and Currents

According to the relations existing between flux and currents (Eq. 2.28), the Park reference frame coordinates are expressed with the inverse Park transformation:

$$\begin{bmatrix} [P(\theta_s)]^{-1}[\phi_{s\_dq0}] \\ [P(\theta_r)]^{-1}[\phi_{r\_dq0}] \end{bmatrix} = \begin{bmatrix} [L_s] & [M_{sr}(\theta)] \\ [M_{sr}(\theta)] & [L_r] \end{bmatrix} \begin{bmatrix} [P(\theta_s)]^{-1}[I_{s\_dq0}] \\ [P(\theta_r)]^{-1}[I_{r\_dq0}] \end{bmatrix} \tag{2.96}$$

Multiplying on the left by the Park transformation, results in:

$$\begin{bmatrix} [\phi_{s\_dq0}] \\ [\phi_{r\_dq0}] \end{bmatrix} = \begin{bmatrix} [P(\theta_s)][L_s][P(\theta_s)]^T & [P(\theta_s)][M_{sr}(\theta)][P(\theta_r)]^T \\ [P(\theta_r)][M_{sr}(\theta)]^T[P(\theta_s)]^T & [P(\theta_r)][L_r][P(\theta_r)]^T \end{bmatrix} \begin{bmatrix} [I_{s\_dq0}] \\ [I_{r\_dq0}] \end{bmatrix} \tag{2.97}$$

Developing the previous formula leads to the expression of four submatrices that define the couplings of the equivalent model in the two-axe frame:

$$\begin{bmatrix} [\phi_{s\_dq0}] \\ [\phi_{r\_dq0}] \end{bmatrix} = \begin{bmatrix} [L_{ps}] & [M_{psr}] \\ [M_{psr}]^{\mathrm{T}} & [L_{pr}] \end{bmatrix} \begin{bmatrix} [I_{s\_dq0}] \\ [I_{r\_dq0}] \end{bmatrix} \tag{2.98}$$

with

$$[L_{ps}] = \begin{bmatrix} L_s & 0 & 0 \\ 0 & L_s & 0 \\ 0 & 0 & L_{s0} \end{bmatrix} \tag{2.99}$$

$$[L_{pr}] = \begin{bmatrix} L_r & 0 & 0 \\ 0 & L_r & 0 \\ 0 & 0 & L_{r0} \end{bmatrix} \tag{2.100}$$

$$[M_{psr}] = [P(\theta_s)]^{-1}[M_{sr}(\theta)][P(\theta_r)] = \frac{3}{2}\, M_{sr}[R_r] \tag{2.101}$$

$$[R_r] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{2.102}$$

Using the Park transformation, it can be noted that the mutual inductances of Eq. 2.98 do not vary with the rotor position. A model with constant coefficients is then obtained. The number of parameters to use is significantly less. Furthermore, as the matrices are now diagonal, the equations referring respectively to axes d and q are not coupled.

2.4.2.4 Torque Calculation

Starting from the torque expression in the three-axe frame (2.38), applying the Park transform to the current results in:

$$T = p\left([P(\theta)]^{-1}[I_{s\_dq0}]\right)^{\mathrm{T}} \frac{d[M_{sr}(\theta)]}{d\theta}[P(\theta)]^{-1}[I_{r\_dq0}] \tag{2.103}$$

Table 2.2 Causal organization of the mathematical equations in the Park reference frame

Electromagnetic part	
$R1_{\text{sd}}$: $e_{\phi_{\text{sd}}} = v_{\text{sd}} - v_{\text{Rsd}} - e_{\text{sd}}$	$R1_{\text{rd}}$: $e_{\phi_{\text{rd}}} = v_{\text{rd}} - v_{\text{Rrd}} - e_{\text{rd}}$
$R1_{\text{sq}}$: $e_{\phi\text{sq}} = v_{\text{sq}} - v_{\text{Rsq}} + e_{\text{sq}}$	$R1_{\text{rq}}$: $e_{\phi\text{rq}} = v_{\text{rq}} - v_{\text{Rrq}} + e_{\text{rq}}$
$R2_{\text{sd}}$: $\phi_{\text{sd}} = \int_{t_0}^{t_0+\Delta t} e_{\phi_{\text{sd}}}\,\text{dt} + \phi_{\text{sd}}(t_0)$	$R2_{\text{rd}}$: $\phi_{\text{rd}} = \int_{t_0}^{t_0+\Delta t} e_{\phi_{\text{rd}}}\,\text{dt} + \phi_{\text{rd}}(t_0)$
$R2_{\text{sq}}$: $\phi_{\text{sq}} = \int_{t_0}^{t_0+\Delta t} e_{\phi_{\text{sq}}}\,\text{dt} + \phi_{\text{sq}}(t_0)$	$R2_{\text{rq}}$: $\phi_{\text{rq}} = \int_{t_0}^{t_0+\Delta t} e_{\phi_{\text{rq}}}\,\text{dt} + \phi_{\text{rq}}(t_0)$
$R3_{\text{sd}}$: $i_{\text{sd}} = \frac{1}{L_s}(\phi_{\text{sd}} - Mi_{\text{rd}})$	$R3_{\text{rd}}$: $i_{\text{rd}} = \frac{1}{L_r}(\phi_{\text{rd}} - Mi_{\text{sd}})$
$R3_{\text{sq}}$: $i_{\text{sq}} = \frac{1}{L_s}(\phi_{\text{sq}} - Mi_{\text{rq}})$	$R3_{\text{rq}}$: $i_{\text{rq}} = \frac{1}{L_r}(\phi_{\text{rq}} - Mi_{\text{sq}})$
$R4_{\text{sd}}$: $v_{\text{Rsd}} = R_s i_{\text{sd}}$	$R4_{\text{rd}}$: $v_{\text{Rrd}} = R_r i_{\text{rd}}$
$R4_{\text{sq}}$: $v_{\text{Rsq}} = R_s i_{\text{sq}}$	$R4_{\text{rq}}$: $v_{\text{Rrq}} = R_r i_{\text{rq}}$
Electromagnetic coupling:	
$R5_{\text{sd}}$: $T_{\text{sd}} = -p\, i_{\text{sd}}\, \phi_{\text{sq}}$	$R5_{\text{rd}}$: $T_{\text{rd}} = -p\frac{M}{L_r}\, i_{\text{sd}}\, \phi_{\text{rq}}$
$R5_{\text{sq}}$: $T_{\text{sq}} = p\, i_{\text{sq}}\, \phi_{\text{sd}}$	$R5_{\text{rq}}$: $T_{\text{rq}} = p\frac{M}{L_r}\, i_{\text{sq}}\phi_{\text{rd}}$
$R6_{\text{sd}}$: $e_{\text{sd}} = -\phi_{\text{sq}}\, \omega s$	$R6_{\text{rd}}$: $e_{\text{rd}} = -\phi_{\text{rq}}\, \omega r$
$R6_{\text{sq}}$: $e_{\text{sq}} = -\phi_{\text{sd}}\omega s$	$R6_{\text{rq}}$: $e_{\text{rq}} = -\phi_{\text{rd}}\omega r$
$R7_{\text{s}}$: $T_s = T_{\text{sd}} + T_{\text{sq}}$	$R7_{\text{r}}$: $T_r = T_{\text{rd}} + T_{\text{rq}}$
Speed of the reference frame : $R8$: $\omega_s = \omega_r + p$	

after introducing (2.101), the results are:

$$T = p\left[I_{\text{s\_dq0}}\right]^{\text{T}}\frac{3}{2}M_{\text{sr}}\frac{d\left[R_p\right]}{d\theta}\left[I_{\text{r\_dq0}}\right] \tag{2.104}$$

and using Eq. 2.73, the result in scalar form is:

$$T = pM\left(i_{\text{sq}}i_{\text{rd}} - i_{\text{sd}}i_{\text{rq}}\right) \tag{2.105}$$

When using the equations that link flux to currents (Eq. 2.98), it is possible to determine other torque equivalent expressions more adapted to the conception of the squirrel-cage machine control. Among the ones that are used most, there is the equation in the rotor flux frame, which brings into play the (measurable) currents to the stator and the flows to the rotor. Based on the expressions of the rotor currents (relations $R3_{\text{rd}}$ and $R3_{\text{rq}}$, Table 2.2), it results in:

$$T = p\frac{M}{L_r}\left(i_{\text{sq}}\phi_{\text{rd}} - i_{\text{sd}}\,\phi_{\text{rq}}\right) \tag{2.106}$$

Similarly, by substituting the expressions of the rotor currents by relations R3sd, R3sq (Table 2.2) in Eq. 2.105, it is possible to determine an expression of the torque, which uses the stator flux components:

$$T = p\left(i_{\text{sq}}\phi_{\text{sd}} - i_{\text{sd}}\phi_{\text{sq}}\right) \tag{2.107}$$

2.4.3 General Model in the Park Reference Frame

2.4.3.1 Setting in Equation of the Electromagnetical Part

Using the mesh law, Eqs. 2.94 and 2.95 are rewritten to make virtual induction e.m.f. corresponding to axes d ($R1_{\mathrm{sd}}$) and ($R1_{\mathrm{sq}}$), and to axes q ($R1_{\mathrm{rd}}$) and ($R1_{\mathrm{rq}}$) appear (Table 2.2).

This formulation uses virtual voltages across the terminals of resistances corresponding to axes d [($R4_{\mathrm{sd}}$) and ($R4_{\mathrm{rd}}$)] and to axes q [($R4_{\mathrm{sq}}$) and ($R4_{\mathrm{rq}}$)] along with electromotive rotation forces relative to axes d [($R6_{\mathrm{sd}}$) and ($R6_{\mathrm{rd}}$)] and to axes q [($R6_{\mathrm{sq}}$) and ($R6_{\mathrm{rq}}$)]. Integrating these induction e.m.f. allows the calculation of the various components of the fluxes. It is then possible to determine the currents by using the inverse equations of coupling with the fluxes (relation $(2.98)^{-1}$), which can be expressed by:

$$\begin{bmatrix} i_{\mathrm{sq}} \\ i_{\mathrm{rq}} \end{bmatrix} = \frac{1}{L_r L_s - M^2} \begin{bmatrix} L_r & -M \\ -M & L_s \end{bmatrix} \begin{bmatrix} \phi_{\mathrm{sq}} \\ \phi_{\mathrm{rq}} \end{bmatrix} \tag{2.108}$$

$$\begin{bmatrix} i_{\mathrm{sd}} \\ i_{\mathrm{rd}} \end{bmatrix} = \frac{1}{L_r L_s - M^2} \begin{bmatrix} L_r & -M \\ -M & L_s \end{bmatrix} \begin{bmatrix} \phi_{\mathrm{sd}} \\ \phi_{\mathrm{rd}} \end{bmatrix} \tag{2.109}$$

2.4.3.2 Equivalent Park Machine

v_{sd} and v_{sq} are the voltages applied to the windings of a fictive stator, the magnetic axes of which would be axes d and q that produce the same magnetic effects as obtained with true windings. v_{rd} and v_{rq} are the voltages applied to the windings of a fictive rotor, the magnetic axes of which would be axes d and q producing the same magnetic effects as obtained with true windings.

Thus, the Park transform allows the substitution of the real three-phase system by another (see Fig. 2.9) composed of:

- two stator (of resistance R_s and inductance L_s) and rotor (of resistance R_r and inductance L_r) windings rotating at an angular speed ω_s and run through by direct and quadrature currents. The frame of reference $\left(\vec{O}_{\mathrm{sa}}, \vec{O}_{\mathrm{sb}}, \vec{O}_{\mathrm{sc}}\right)$ is stationary.
- two stationary R-L circuits, run through by homopolar currents (i_{r0} and i_{s0}).

A coupling remains between the rotor and stator windings which are located on the same axis. The coupling is expressed by the mutual inductance M.

It is demonstrated that if the machine is supplied by a balanced three-phase system, a field rotates in the air gap at an angular speed, which is equal to the speed of the pulsation of the voltages, and therefore equal to the currents, as well [Caron and Hautier 1995]. When choosing a frame (d, q) linked to the rotating

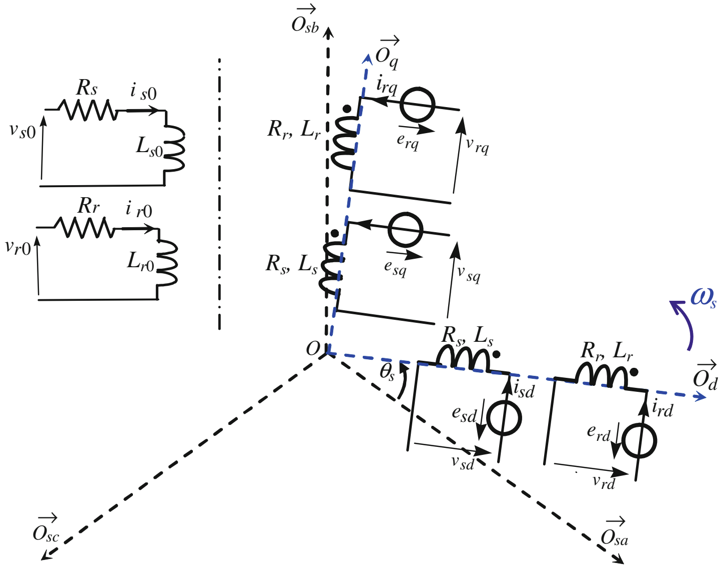

Fig. 2.9 Equivalent two-phase windings rotating in the park frame

field and then rotating at an angular speed, which is equal to the speed of the voltages pulsation supplying the true machine, the equivalent system in that frame creates a similar magnetic field. Quantities v_{sd} and $v_{\mathrm{sq}}$are then continuous voltages under steady conditions. The stationary three-phase windings—run through by sinusoidal currents—are then substituted by rotating two-phase windings—run through by direct currents. Frame (d, q)—linked to rotor quantities—then has to turn at the same speed for the mutual inductances to be constant. Condition (2.85) has to be checked to obtain this. The rotor and the stator of the machine—then called « Park machine » —are rotating at the same speed, so that flux and currents are linked by an independent time expression.

2.4.4 Model Using the Rotor Flux

2.4.4.1 Causal Ordering Graph Principle and Model

Several equivalent expressions can be used for the calculation of the torque. The expression using the flux to the rotor (Eq. 2.106) can be divided into two parts:

$$R5_{\mathrm{rq}} \rightarrow T_{\mathrm{rq}} = p\frac{M}{L_r}\, i_{\mathrm{sq}}\, \phi_{\mathrm{rd}} \tag{2.110}$$

and

$$R5_{\mathrm{rd}} \rightarrow T_{\mathrm{rd}} = -p\frac{M}{L_r} i_{\mathrm{sd}}\ \phi_{\mathrm{rq}} \tag{2.111}$$

The expression is then written as follows:

$$R7_s \rightarrow T = T_{\mathrm{rd}} + T_{\mathrm{rq}} \tag{2.112}$$

This expression is used for the implementation of the control known as "rotor flux-oriented".

The COG of this model (Fig. 2.10) emphasizes two gyrator couplings [relations ($R5_{\mathrm{rd}}$), ($R5_{\mathrm{rq}}$), ($R6_{\mathrm{rd}}$) and ($R6_{\mathrm{rq}}$)], since the expression of the electromagnetic torque shows that the torque is composed of two components called c_{rd} (for axis d) andc_{rq} (for axis q). The Park machine can also be considered as the association of two fictive direct current machines that are mechanically and electrically coupled. This mathematical model can be considered as a system of which the input quantities are a stator voltage vector and the mechanical speed imposed by the mechanical load (see paragraph 1.4.2.4.). The output quantities of this model are the generated electromechanical torque and the vector of the stator currents. A global representation of a squirrel-cage machine can easily be obtained by canceling the rotor voltages.

The Park transform allows the calculation of the direct and quadrature components of the voltages applied to the machine. The transform inverts the currents in the three-axe frame. The orientation angle of these two transforms is calculated by integrating the pulsation of the frame linked to the stator, with a reset when the stator voltage of the phase is zero, at:

$$\theta_s = \int_{t_0}^{t_0+\Delta t} \omega_s \ \mathrm{dt} \ + \ \theta_s(t_0) \tag{2.113}$$

Figure 2.11 shows the choice of linking the Park reference frame to the pulsation of the stator quantities. Consequently, the pulsation of these quantities is null in that frame. So, the Park transform is a mathematical operator which allows the conversion of sinusoidal quantities of pulsation $\omega_s$in a quantity of null value. This transform then allows the determination of a medium equivalent model for quantities with a given pulsation or with a pulsation more or less equal to it.

2.4.4.2 State Space Representation

In equations $(R2_{\mathrm{sd}}, R2_{\mathrm{sq}}, R2_{\mathrm{rd}}, R2_{\mathrm{rq}})$, the different components of the stator and rotor fluxes are obtained by integration—they are also called state variables.

As the fluxes and currents are linked by expressions $(R3_{\mathrm{sd}}, R3_{\mathrm{sq}}, R3_{\mathrm{rd}}, R3_{\mathrm{rq}})$, the different current components, or even a combination of fluxes and currents, can be chosen as state variables. For the conception of a motor control, it can be

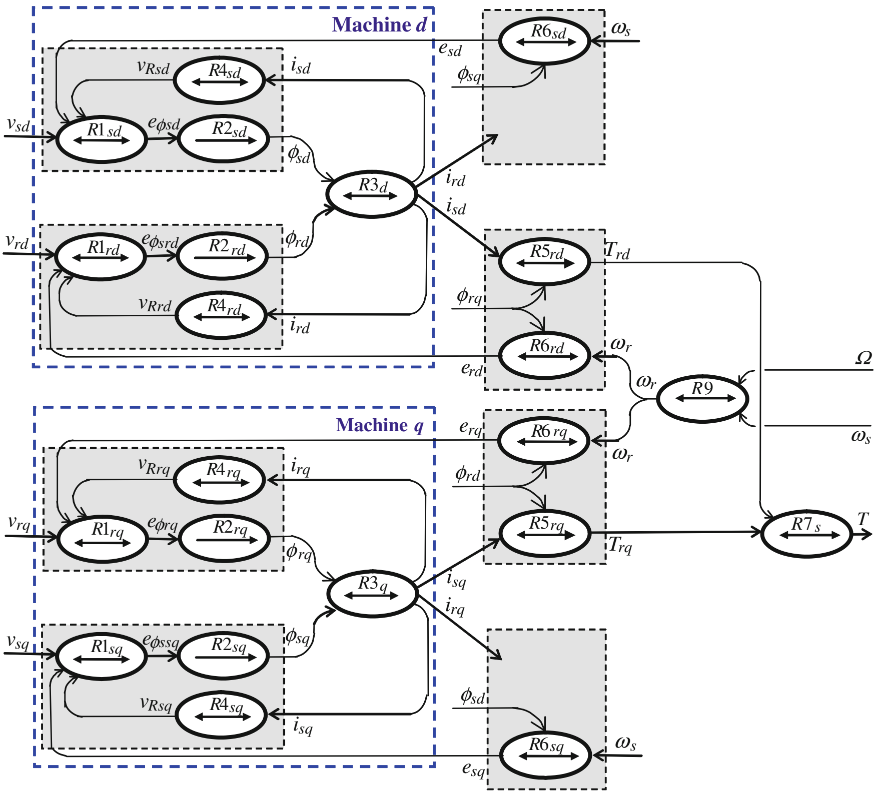

Fig. 2.10 C.O.G. of Park's model using fluxes to the rotor (the relations are carried over in Table 2.2)

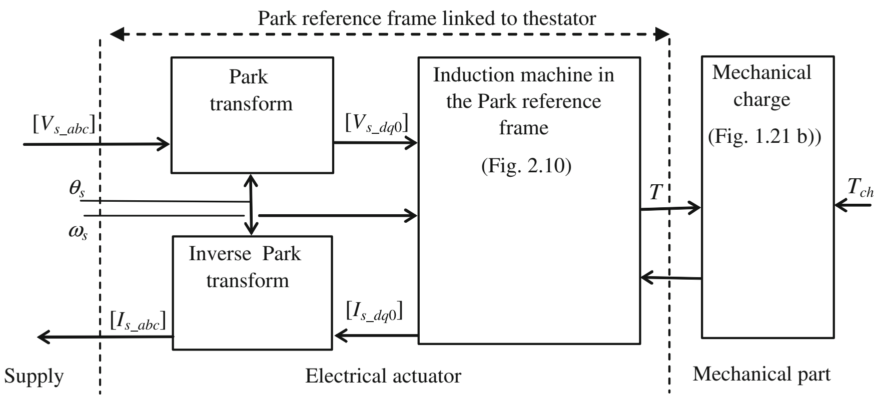

Fig. 2.11 Macro representation of the Park model of the induction machine in a frame of reference linked to the stator

interesting to choose the stator current components for state variables. In fact, this current as well as the rotor flux components—in as much as this flux will be controlled—can be easily measured. The state space representation then consists of determining only the differential equations that allow the definition of these variables by integration with intermediary mathematical relations. Thus, relations $R1_{\mathrm{rd}}, R4_{\mathrm{rd}}$ and $R6_{\mathrm{rd}}$ are injected in the derivative of this flux $(R2_{\mathrm{rd}})$ as to determine the differential equation of the rotor flux direct component. When applying the same procedure to other fluxes, this results in:

$$\frac{\mathrm{d}\phi_{\mathrm{sd}}}{\mathrm{d}t} = v_{\mathrm{sd}} - R_s i_{\mathrm{sd}} + \omega_s \phi_{\mathrm{sq}} \tag{2.114a}$$

$$\frac{\mathrm{d}\phi_{\mathrm{sq}}}{\mathrm{d}t} = v_{\mathrm{sq}} - R_s i_{\mathrm{sq}} - \omega_s \phi_{\mathrm{sd}} \tag{2.114b}$$

$$\frac{\mathrm{d}\phi_{\mathrm{rd}}}{\mathrm{d}t} = v_{\mathrm{rd}} - R_r i_{\mathrm{rd}} + \omega_r \phi_{\mathrm{rq}} \tag{2.114c}$$

$$\frac{\mathrm{d}\phi_{\mathrm{rq}}}{\mathrm{d}t} = v_{\mathrm{rq}} - R_r i_{\mathrm{rq}} - \omega_r \phi_{\mathrm{rd}} \tag{2.114d}$$

When introducing relations $R3_{\mathrm{rd}}$ and $R3_{\mathrm{rq}}$ in $R3_{\mathrm{sd}}$ and $R3_{\mathrm{sq}}$, and defining the dispersion coefficient:

$$\sigma = 1 - \frac{M^2}{L_r L_s}, \tag{2.115}$$

This results in:

$$\phi_{\mathrm{sd}} = \sigma L_s i_{\mathrm{sd}} + \frac{M}{L_r}\phi_{\mathrm{rd}} \tag{2.116a}$$

$$\phi_{\mathrm{sq}} = \sigma L_s\, i_{\mathrm{sq}} + \frac{M}{L_r}\phi_{\mathrm{rq}} \tag{2.116b}$$

Finally, the following equations are obtained when replacing the stator flux components by (2.116a) and the rotor flux components by $R3_{\mathrm{rd}}$ and $R3_{\mathrm{rq}}$ in the differential equations of the fluxes (2.114a):

$$\sigma L_s \frac{\mathrm{d}i_{\mathrm{sd}}}{\mathrm{d}t} = v_{\mathrm{sd}} - R_s i_{\mathrm{sd}} + \omega_s \sigma L_s i_{\mathrm{sq}} - \frac{M}{L_r}\frac{\mathrm{d}\phi_{\mathrm{rd}}}{\mathrm{d}t} + \omega_s \frac{M}{L_r}\phi_{\mathrm{rq}} \tag{2.117a}$$

$$\sigma L_s \frac{\mathrm{d}i_{\mathrm{sq}}}{\mathrm{d}t} = v_{\mathrm{sq}} - R_s i_{\mathrm{sq}} - \omega_s \sigma L_s i_{\mathrm{sd}} - \frac{M}{L_r}\frac{\mathrm{d}\phi_{\mathrm{rq}}}{\mathrm{d}t} - \omega_s \frac{M}{L_r}\phi_{\mathrm{rd}} \tag{2.117b}$$

$$\frac{\mathrm{d}\phi_{\mathrm{rd}}}{\mathrm{d}t} = v_{\mathrm{rd}} - \frac{R_r}{L_r}\phi_{\mathrm{rd}} + \frac{R_r}{L_r}M i_{\mathrm{sd}} + \omega_r \phi_{\mathrm{rq}} \tag{2.117c}$$

$$\frac{\mathrm{d}\phi_{\mathrm{rq}}}{\mathrm{d}t} = v_{\mathrm{rq}} - \frac{R_r}{L_r}\phi_{\mathrm{rq}} + \frac{R_r}{L_r}Mi_{\mathrm{sq}} - \omega_r\phi_{\mathrm{rd}} \tag{2.117d}$$

In the two last equations the derivatives of the rotor fluxes appear, which can be replaced by their expression (2.117c) and (2.117d):

$$\sigma L_s\frac{\mathrm{d}i_{\mathrm{sd}}}{\mathrm{d}t} = v_{\mathrm{sd}} - \frac{M}{L_r}v_{\mathrm{rd}} - R_{\mathrm{sr}}i_{\mathrm{sd}} + \omega_s\sigma L_s i_{\mathrm{sq}} + \frac{M}{L_r}(\omega_s - \omega_r)\,\phi_{\mathrm{rq}} + \frac{M}{L_r^2}R_r\phi_{\mathrm{rd}} \tag{2.118a}$$

$$\sigma L_s\frac{\mathrm{d}i_{\mathrm{sq}}}{\mathrm{d}t} = v_{\mathrm{sq}} - \frac{M}{L_r}v_{\mathrm{rq}} - R_{\mathrm{sr}}i_{\mathrm{sq}} - \omega_s\sigma L_s i_{\mathrm{sd}} - \frac{M}{L_r}(\omega_s - \omega_r)\,\phi_{\mathrm{rd}} + \frac{M}{L_r^2}R_r\phi_{\mathrm{rq}} \tag{2.118b}$$

$$\frac{\mathrm{d}\phi_{\mathrm{rd}}}{\mathrm{d}t} = v_{\mathrm{rd}} - \frac{R_r}{L_r}\phi_{\mathrm{rd}} + \frac{R_r}{L_r}M\,i_{\mathrm{sd}} + \omega_r\phi_{\mathrm{rq}} \tag{2.119a}$$

$$\frac{\mathrm{d}\phi_{\mathrm{rq}}}{\mathrm{d}t} = v_{\mathrm{rq}} - \frac{R_r}{L_r}\phi_{\mathrm{rq}} + \frac{R_r}{L_r}M\,i_{\mathrm{sq}} - \omega_r\phi_{\mathrm{rd}} \tag{2.119b}$$

with R_{sr}, the total resistance viewed from the stator is:

$$R_{\mathrm{sr}} = R_s + R_r\frac{M^2}{L_r^2} = R_s + \frac{(1-\sigma)L_s}{L_r}R_r \tag{2.120}$$

When using Eq. 2.86, these four relations are expressed in vector form as follows:

$$\begin{bmatrix} \dot{\phi}_{\mathrm{rd}} \\ \dot{\phi}_{\mathrm{rq}} \\ \dot{i}_{\mathrm{sd}} \\ \dot{i}_{\mathrm{sq}} \end{bmatrix} = \begin{bmatrix} -\frac{R_r}{L_r} & \omega_r & M\frac{R_r}{L_r} & 0 \\ -\omega_r & -\frac{R_r}{L_r} & 0 & M\frac{R_r}{L_r} \\ \frac{M R_r}{\sigma L_s L_r^2} & \frac{M}{\sigma L_s L_r}p\Omega & -\frac{R_{\mathrm{sr}}}{\sigma L_s} & \omega_s \\ \frac{M}{\sigma L_s L_r}p\Omega & \frac{M R_r}{\sigma L_s L_r^2} & -\omega_s & -\frac{R_{\mathrm{sr}}}{\sigma L_s} \end{bmatrix} \begin{bmatrix} \phi_{\mathrm{rd}} \\ \phi_{\mathrm{rq}} \\ i_{\mathrm{sd}} \\ i_{\mathrm{sq}} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{-M}{\sigma L_r L_s} & 0 & \frac{1}{\sigma L_s} & 0 \\ 0 & \frac{-M}{\sigma L_r L_s} & 0 & \frac{1}{\sigma L_s} \end{bmatrix} \begin{bmatrix} v_{\mathrm{rd}} \\ v_{\mathrm{rq}} \\ v_{\mathrm{sd}} \\ v_{\mathrm{sq}} \end{bmatrix} \tag{2.121}$$

This mathematical formulation of the model will be used to elaborate the rotor-flux-oriented control without a priori knowledge of the squirrel-cage induction machine in Chap. 3.

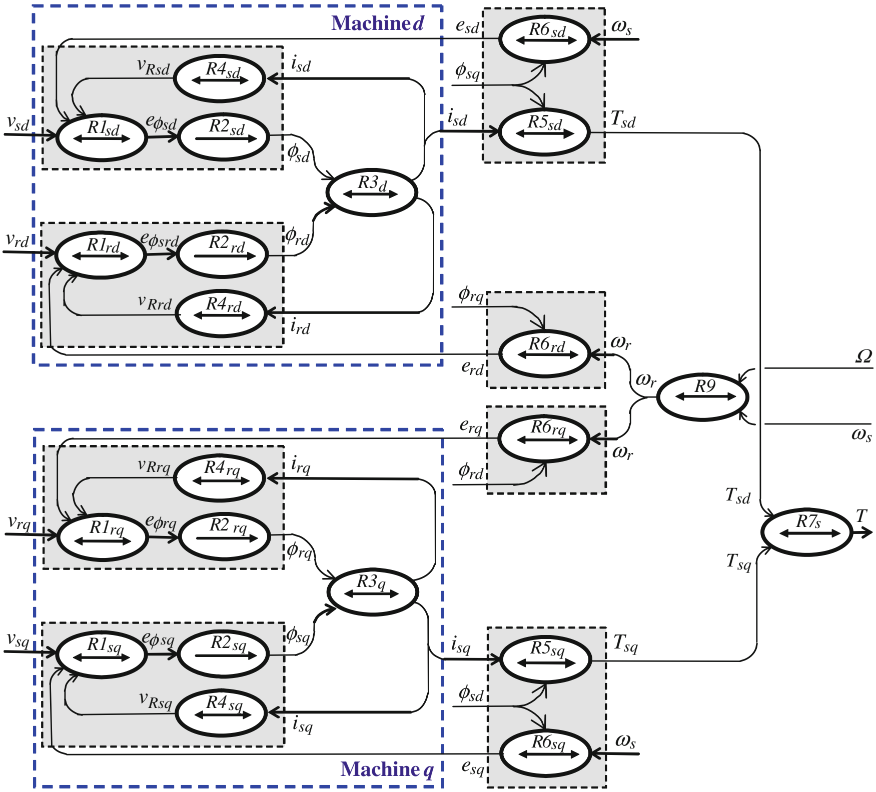

Fig. 2.12 C.O.G. of Park's model using fluxes to the stator (the relations are reported in Table 2.2)

2.4.5 *Model Using the Stator Flux*

2.4.5.1 Model Principle and Causal Ordering Graph

In the stator flux frame of reference, the torque expression that uses fluxes to the stator (Eq. 2.107) can be divided into two parts:

$$R5_{\mathrm{sd}} \rightarrow T_{\mathrm{sq}} = p i_{\mathrm{sq}} \phi_{\mathrm{sd}} \tag{2.122}$$

and

$$R5_{\mathrm{sq}} \rightarrow T_{\mathrm{sd}} = -p i_{\mathrm{sd}} \phi_{\mathrm{sq}} \tag{2.123}$$

It is described as follows:

$$R7_s \rightarrow T = T_{\mathrm{sd}} + T_{\mathrm{sq}} \tag{2.124}$$

This expression is used to elaborate on what is called the “stator-flux-oriented” control (which will be dealt with in next chapter). As proven earlier, the COG of this model (Fig. 2.12) emphasizes two gyrator couplings [relations ($R5_{sd}$), ($R5_{sq}$), ($R6_{sd}$) and ($R6_{sq}$)], since the expression of the electromagnetic torque shows that the torque expression is composed of two components called T_{sd} and T_{sq}, respectively axis d and axis q.

Park’s machine can again be considered again as the association of two fictitious DC machines that are mechanically and electrically linked. This mathematical model is equivalent to a system with input quantities—such as a stator voltage vector and the mechanical speed imposed by the mechanical charge, and output quantities—such as the generated electromechanical torque and the vector of the stator currents.

2.4.5.2 State Space Representation

A second control system of this motor can be elaborated by using the stator and rotor current components from this model as state variables. The state space representation then consists of only determining the differential equations that allow the definition of these variables by integration with intermediary mathematical relations.

When introducing the relations $R3_{sd}$ and $R3_{sq}$ in $R3_{rd}$ and $R3_{rq}$, the following results are shown:

$$\phi_{rd} = \sigma L_r i_{rd} + \frac{M}{L_s}\phi_{sd} \tag{2.125a}$$

$$\phi_{rq} = \sigma L_r i_{rq} + \frac{M}{L_s}\phi_{sq} \tag{2.125b}$$

When replacing the stator flux components by (2.125a) and the stator current components by $R3_{sd}$ and $R3_{sq_{in}}$ the flux differential equations are obtained: (2.114a):

$$\frac{d\phi_{sd}}{dt} = v_{sd} - \frac{R_s}{L_s}\phi_{sd} + \frac{R_s}{L_s}Mi_{rd} + \omega_s\phi_{sq} \tag{2.126a}$$

$$\frac{d\phi_{sq}}{dt} = v_{sq} - \frac{R_s}{L_s}\phi_{sq} + \frac{R_s}{L_s}Mi_{rq} - \omega_s\phi_{sd} \tag{2.126b}$$

$$\sigma L_r\frac{di_{rd}}{dt} = v_{rd} - R_r i_{rd} + \omega_r\sigma L_r i_{rq} - \frac{M}{L_s}\frac{d\phi_{sd}}{dt} + \omega_r\frac{M}{L_s}\phi_{sq} \tag{2.126c}$$

$$\sigma L_r\frac{di_{rq}}{dt} = v_{rq} - R_r i_{rq} - \omega_r\sigma L_r i_{rd} - \frac{M}{L_s}\frac{d\phi_{sq}}{dt} - \omega_r\frac{M}{L_s}\phi_{sd} \tag{2.126d}$$

The last two equations show the derivatives of the stator fluxes that can be replaced by their expressions (2.126a) and (2.126b)

$$\frac{d\phi_{sd}}{dt} = v_{sd} - \frac{R_s}{L_s}\phi_{sd} + \frac{R_s}{L_s}Mi_{rd} + \omega_s\phi_{sq} \tag{2.127a}$$

$$\frac{d\phi_{sq}}{dt} = v_{sq} - \frac{R_s}{L_s}\phi_{sq} + \frac{R_s}{L_s}Mi_{rq} - \omega_s\phi_{sd} \tag{2.127b}$$

$$\sigma L_r\frac{di_{rd}}{dt} = v_{rd} - \frac{M}{L_s}v_{sd} - R_{rs}i_{rd} + \omega_r\sigma L_r i_{rq} + \frac{M}{L_s}(\omega_r - \omega_s)\phi_{sq} + \frac{M}{L_s^2}R_s\phi_{sd} \tag{2.127c}$$

$$\sigma L_r\frac{di_{rq}}{dt} = v_{rq} - \frac{M}{L_s}v_{sq} + R_{rs}i_{rq} - \omega_r\sigma L_r i_{rd} - \frac{M}{L_s}(\omega_s - \omega_r)\phi_{sd} + \frac{M}{L_s^2}R_s\phi_{sq} \tag{2.127d}$$

with R_{rs}, the total resistance shown by the rotor is:

$$R_{rs} = R_r + R_s\frac{M^2}{L_s^2} = R_r + \frac{(1-\sigma)L_r}{L_s}R_s \tag{2.128}$$

These four relations are shown in matrix form as follows:

$$\begin{bmatrix} \dot{\phi}_{sd} \\ \dot{\phi}_{sq} \\ \dot{i}_{rd} \\ \dot{i}_{rq} \end{bmatrix} = \begin{bmatrix} -\frac{R_s}{L_s} & \omega_s & M\frac{R_s}{L_s} & 0 \\ -\omega_s & -\frac{R_s}{L_s} & 0 & M\frac{R_s}{L_s} \\ \frac{MR_s}{L_s^2\sigma L_r} & \frac{M}{L_r\sigma L_s}(\omega_r - \omega_s) & \frac{-R_{rs}}{\sigma L_r} & \omega_r \\ \frac{M}{L_r\sigma L_s}(\omega_s - \omega_r) & \frac{MR_s}{L_s^2\sigma L_r} & -\omega_r & \frac{-R_{rs}}{\sigma L_r} \end{bmatrix} \begin{bmatrix} \phi_{sd} \\ \phi_{sq} \\ i_{rd} \\ i_{rq} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{-M}{\sigma L_r L_s} & 0 & \frac{1}{\sigma L_r} & 0 \\ 0 & \frac{-M}{\sigma L_r L_s} & 0 & \frac{1}{\sigma L_r} \end{bmatrix} \begin{bmatrix} v_{sd} \\ v_{sq} \\ v_{rd} \\ v_{rq} \end{bmatrix} \tag{2.129}$$

This mathematical formulation of the model is used to elaborate on the stator-flux-oriented control of the squirrel-cage induction machine [Degobert 1997], and for the direct torque control [Canudas de Wit 2000].

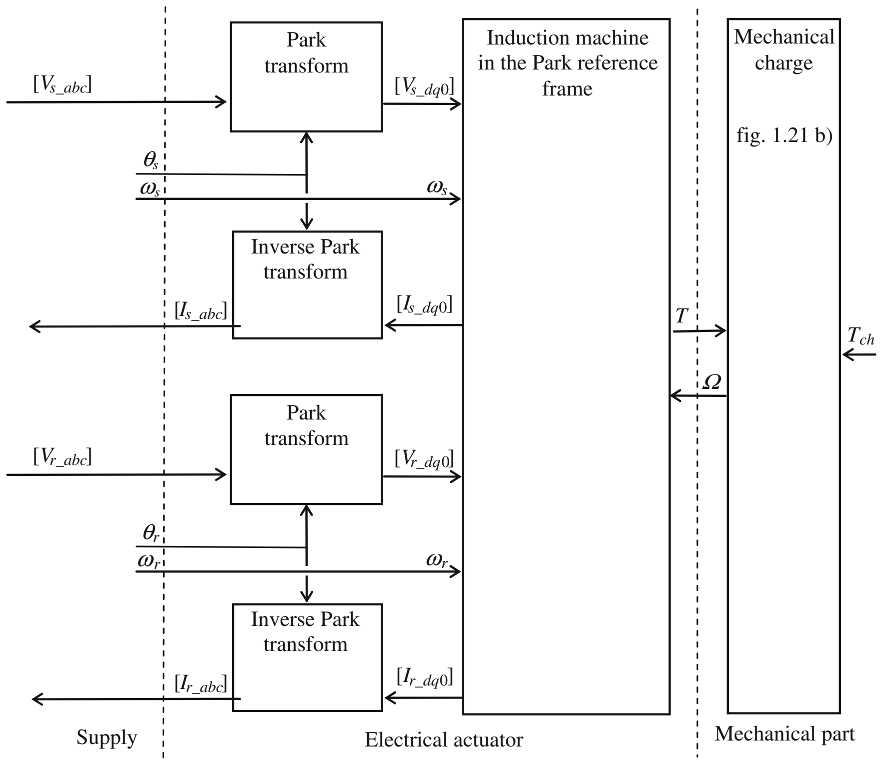

Fig. 2.13 Representation of the induction machine model into a system

2.4.6 Model for System Analysis

The previously developed models show that the induction machine is an electromechanical converter that:

- receives voltages to the stator and rotor as electrical quantities and relieves currents;
- receives speed as a mechanical quantity and that relieves an electromotive torque.

The second comment assumes that the mechanical part of the machine (usually negligible compared to the mechanical part of the load) is modeled externally with the mechanical part of the charge.

Figure 2.13 also illustrates the fact that the Park transform is a mathematical operator that allows converting sinusoidal quantities of pulsation ω_s into a null value quantity.

Thus, this transformation allows the determination of a similar average model for quantities that have a specific pulsation or that are more or less equal to this one.

It is useless to use a model that is neither based on the physical quantities internal to the machine (flux, e.m.f., etc.) nor on their coupling in order to carry out studies on industrial devices using this machine technology. It is, however, useful to have a model that explicits and especially considers the physical quantities exchanged between the machine and the external elements (power converters, mechanical charge, etc.) associated with it. As far as the electrical part of the machine is concerned, it is then necessary to determine the currents according to the applied voltages by replacing the fluxes in the Eq. 2.114a with their expressions ($R3_{\mathrm{sd}}$, $R3_{\mathrm{sq}}$, $R3_{\mathrm{rd}}$, $R3_{\mathrm{rq}}$) (Table 2.2). The direct axis components are expressed as follows:

$$L_s \frac{\mathrm{d}i_{\mathrm{sd}}}{\mathrm{d}t} + M \frac{\mathrm{d}i_{\mathrm{rd}}}{\mathrm{d}t} = v_{\mathrm{sd}} - R_s i_{\mathrm{sd}} + \omega_s \left(L_s i_{\mathrm{sq}} + M i_{\mathrm{rq}} \right) \tag{2.130}$$

$$L_r \frac{\mathrm{d}i_{\mathrm{rd}}}{\mathrm{d}t} + M \frac{\mathrm{d}i_{\mathrm{sd}}}{\mathrm{d}t} = v_{\mathrm{rd}} - R_r i_{\mathrm{rd}} + \omega_r \left(L_r i_{\mathrm{rq}} + M i_{\mathrm{sq}} \right) \tag{2.131}$$

When the expression of the derivative of the stator current direct component from the second equation is replaced in the first equation, the previous device is then expressed:

$$\frac{\mathrm{d}i_{\mathrm{sd}}}{\mathrm{d}t} = \frac{1}{\sigma L_s} \left[v_{\mathrm{sd}} - R_s i_{\mathrm{sd}} - \frac{M}{L_r} v_{\mathrm{rd}} - e'_{\mathrm{sd}} \right] \tag{2.132a}$$

$$\frac{\mathrm{d}i_{\mathrm{rd}}}{\mathrm{d}t} = \frac{1}{\sigma L_r} \left[v_{\mathrm{rd}} - R_r i_{\mathrm{rd}} - \frac{M}{L_s} v_{\mathrm{sd}} - e'_{\mathrm{rd}} \right] \tag{2.132b}$$

with the following coupling e.m.f., the following results are shown:

$$e'_{\mathrm{sd}} = -M \frac{R_r}{L_r} i_{\mathrm{rd}} - \omega_s \left(L_s i_{\mathrm{sq}} + M i_{\mathrm{rq}} \right) + \omega_r \frac{M}{L_r} \left(L_r i_{\mathrm{rq}} + M i_{\mathrm{sq}} \right) \tag{2.133a}$$

$$e'_{\mathrm{rd}} = -M \frac{R_s}{L_s} i_{\mathrm{sd}} - \omega_r \left(L_r i_{\mathrm{rq}} + M i_{\mathrm{sq}} \right) + \omega_s \frac{M}{L_r} \left(L_s i_{\mathrm{sq}} + M i_{\mathrm{rq}} \right) \tag{2.133b}$$

The same can be said of the quadrature components:

$$\frac{\mathrm{d}i_{\mathrm{sq}}}{\mathrm{d}t} = \frac{1}{\sigma L_s} \left[v_{\mathrm{sq}} - R_s i_{\mathrm{sq}} - \frac{M}{L_r} v_{\mathrm{rq}} - e'_{\mathrm{sq}} \right] \tag{2.134a}$$

$$\frac{\mathrm{d}i_{\mathrm{rq}}}{\mathrm{d}t} = \frac{1}{\sigma L_r} \left[v_{\mathrm{rq}} - R_r i_{\mathrm{rq}} - \frac{M}{L_s} v_{\mathrm{sq}} - e'_{\mathrm{rq}} \right] \tag{2.134b}$$

with the following coupling e.m.fs:

$$e'_{\mathrm{sq}} = -M \frac{R_r}{L_r} i_{\mathrm{rq}} + \omega_s (L_s i_{\mathrm{sd}} + M i_{\mathrm{rd}}) - \omega_r \frac{M}{L_r} (L_r i_{\mathrm{rd}} + M i_{\mathrm{sd}}) \tag{2.135a}$$

$$e'_{\mathrm{rq}} = -M\frac{R_s}{L_s}i_{\mathrm{sq}} + \omega_r(L_r i_{\mathrm{rd}} + M i_{\mathrm{sd}}) - \omega_s\frac{M}{L_r}(L_s i_{\mathrm{sd}} + M i_{\mathrm{rd}}) \tag{2.135b}$$

Therefore, the electromotive torque can be calculated with the Eq. 2.105.

2.5 The Effect of the Magnetic Saturation

Previously, the dynamic models have been developed in the assumption that the machine does not suffer from magnetic saturation (Sect. 2.2.1), which consists in considering a linear characteristic $B = f(H)$ (Fig. 1.5), or even a linear characteristic between flux and current. In Chap. 3, control systems are developed from these models. In order to validate these controls and to study their sensibility to the real presence of these saturations, it is necessary that a simple model of this magnetic saturation is introduced, which is valid under steady conditions. Thus, a model that makes a linear parameter β interfere is chosen to take into account the air gap, and another non linear parameter (elevation to the power s) to broach the specificity of the saturation point (so appears from experimental results of Fig. 2.15 for $i_{\mathrm{mn}} > 0,7$) :

$$i_{\mathrm{mn}} = \beta\phi_{\mathrm{sn}} + (1-\beta)\phi_{\mathrm{sn}}^{s} \tag{2.136}$$

$$M_n = \frac{\phi_{\mathrm{sn}}}{i_{\mathrm{mn}}} \tag{2.137}$$

i_{mn}, ϕ_{sn} and M_n are respectively the standardized values compared to their rated value (also called “per unit”):

– of the magnetizing current

$$i_m = \sqrt{(i_{\mathrm{sd}} + i_{\mathrm{rd}})^2 + (i_{\mathrm{sq}} + i_{\mathrm{rq}})^2} \tag{2.138}$$

– of the stator flux

$$\phi_s = \sqrt{\phi_{\mathrm{sd}}^2 + \phi_{\mathrm{sq}}^2} \tag{2.139}$$

– and of static inductance $\bar{M}$.

The organization of all equations leads to the calculation of the static inductance (Fig. 2.14).

The dynamic inductance does not necessarily have to be taken into account as the sensibility study will be carried out under steady conditions [Vas 1990].

The experimental and theoretical magnetic characteristics are compared with one another in order to determine the parameters [β and s of the expression (2.136)].

Therefore, a no-load test is carried out to determine the magnetic characteristic $U_s = f(I_m)$, with U_s representing the effective stator voltage and I_M representing

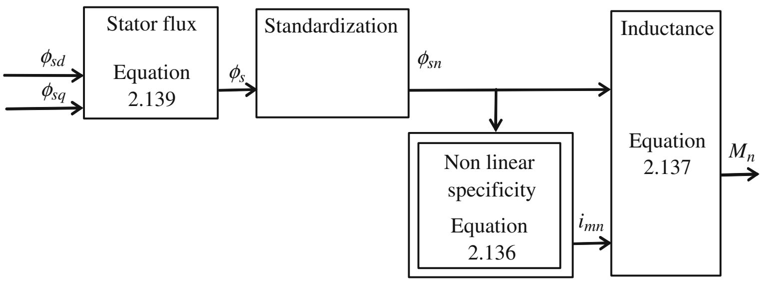

Fig. 2.14 Representation of the induction machine model into a device

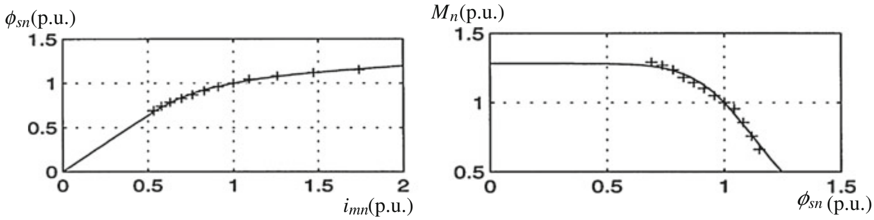

Fig. 2.15 Comparison between the experimental data (*dashed* line) and the saturation model (*solid* line)

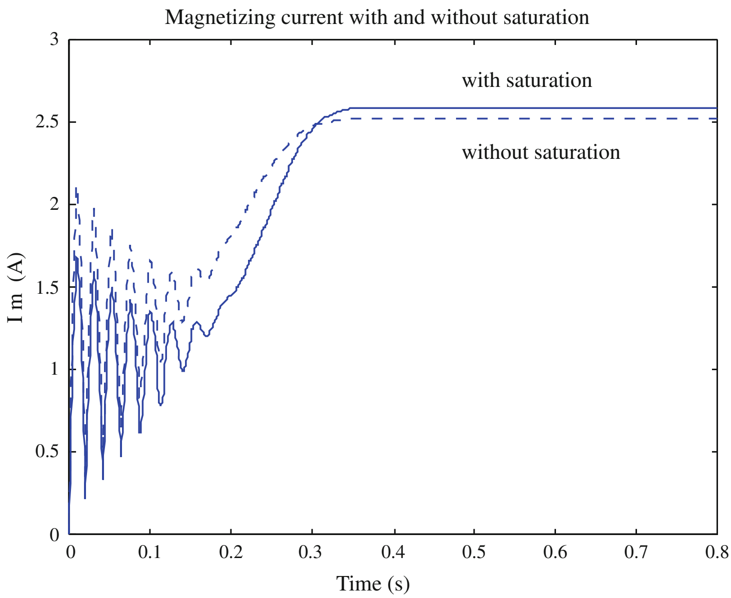

Fig. 2.16 Magnetizing current during the no-load starting of an induction machine, *solid* line: with magnetic saturation, *dashed* line: without magnetic saturation

the effective magnetizing current. β and s are chosen in order to make the experimental curve coincide with the theoretical one (Fig. 2.15). The equation of the magnetic characteristic is obtained by successive tests.

In order to demonstrate the importance of the magnetic saturation, Fig. 2.16 represents the magnetizing current obtained with and without the magnetic saturation (respectively in dashed and solid lines), and simulated when starting a no-load operation with an induction machine directly attached to the network. When the magnetic saturation is taken into consideration, the Fig. 2.16 shows that, in the beginning, the current peaks are less important, and that the magnetizing current is higher under steady conditions.

2.6 Conclusion

The physical and mathematical developments that are dealt with in this chapter have lead to determine a mathematical model reduced into a rotating frame of reference. The model parameters can be measured with different experimental procedures explained in [Caron and Hautier 1995]. The described modeling method can be used similarly for the modeling of three-phase induction motors [Sturtzer and Smigel 2000; Grenier et al. 2001]. The model.ing obtained in the Park reference frame will be used in the next chapter to elaborate on different speed control systems.

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