Analyse 1 - Anna Lachowska
Résumé

Benjamin Bovey

November 2018

1 Identités algébriques

1.1 Polynômes:

$x, y \in \mathbb{R}$:

$$(x+y)^2 = x^2 + 2xy + y^2$$
$$(x-y)^2 = x^2 - 2xy + y^2$$
$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$
$$(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$
$$x^2 - y^2 = (x-y)(x+y)$$
$$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$$
$$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$$

1.2 Logarithmes:

On assume que la notation log sans indice précisé dénote le logarithme naturel.
$a, b \in \mathbb{R}_{>0},\ c \in \mathbb{R}$

$$\log(ab) = \log(a) + \log(b)$$
$$\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$$
$$\log(a^c) = c \cdot \log(a)$$
$$\log(e) = 1$$
$$\log_a(1) = 0$$
$$\log_a(a) = 1,\ a \neq 1$$

1.3 Trigonométrie:

$$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$$
$$\begin{cases} \sin(2x) = 2\cos(x)\sin(x) \\ \\ \end{cases}$$
$$\cos^2(x) + \sin^2(x) = 1$$
$$\sin(-x) = -\sin(x) \quad \text{(impaire)}$$
$$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$$
$$\begin{cases} \cos(2x) = 1 - 2\sin^2(x) \\ \qquad\quad\ = -1 + 2\cos^2(x) \\ \qquad\quad\ = \cos^2(x) - \sin^2(x) \end{cases}$$
$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$
$$\cos(-x) = \cos(x) \quad \text{(paire)}$$

1.3.1 Quelques valeurs de $\cos(x)$ et $\sin(x)$

x	$\sin(x)$	$\cos(x)$	$\tan(x)$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	∞

2 Limites utiles

Soient P_n et Q_n deux suites polynomiales:

$$\lim_{n\to\infty} \frac{P_n}{Q_n} = \begin{cases} 0, & \text{si } \deg P_n < \deg Q_n \\ \frac{p_n}{q_n}, & \text{si } \deg P_n = \deg Q_n, \text{ avec } p_n \text{ et } q_n \text{ les coefficients du terme de plus haut degré} \\ \infty, & \text{si } \deg P_n > \deg Q_n \end{cases} \tag{2.1}$$

$$\lim_{n\to\infty} \frac{1}{n^p} = 0 \quad \forall p \in \mathbb{R}_+^* \tag{2.2}$$

$$\lim_{n\to\infty} \sqrt[n]{a} = 1 \quad \forall a \in \mathbb{R}_+ \tag{2.3}$$

$$\lim_{n\to\infty} \frac{p^n}{n!} = 0 \quad \forall p \in \mathbb{R}_+^* \tag{2.4}$$

$$\lim_{n\to\infty} \frac{\sin \frac{1}{n}}{\frac{1}{n}} = 1 \tag{2.5}$$

$$\lim_{n\to\infty} \sin \frac{1}{n} = 0 \tag{2.6}$$

$$\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e \tag{2.7}$$

$$\lim_{n\to\infty} \left(1 - \frac{1}{n}\right)^n = \frac{1}{e} = e^{-1} \tag{2.8}$$

$$\lim_{n\to\infty} \frac{n!}{n^n} = 0 \tag{2.9}$$

$$\sum_{k=0}^{\infty} r^k = \begin{cases} \frac{1}{1-r}, & |r| < 1 \\ \text{diverge}, & |r| \geq 1 \end{cases} , r \in \mathbb{R} \tag{2.10}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \text{ converge } \forall p \in \mathbb{R}_{>1} \tag{2.11}$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \text{ diverge.} \tag{2.12}$$

$$\sum_{n=0}^{\infty} |a_n| \text{ converge } \Rightarrow \sum_{k=0}^{\infty} a_n \text{ converge. } (\not\Leftarrow) \tag{2.13}$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 \tag{2.14}$$

$$\lim_{x\to 0} \sin \frac{1}{x} \quad \text{n'existe pas.} \tag{2.15}$$

$$\lim_{x\to 0} x \cdot \sin \frac{1}{x} = \lim_{x\to 0} x \cdot \cos \frac{1}{x} = 0 \tag{2.16}$$

$$e^x \stackrel{\text{déf.}}{=} \sum_{n=0}^{\infty} \frac{x^n}{n!} \tag{2.17}$$

$$\lim_{x\to 0} \frac{e^x - 1}{x} = 1 \qquad \lim_{x\to\infty} \frac{\log(x)}{x^\alpha} = 0, \ \alpha > 0 \tag{2.18}$$

3 Formes indéterminées

$$\infty - \infty, \quad \frac{\infty}{\infty}, \quad \frac{0}{0}, \quad 0 \cdot \infty, \quad 0^0, \quad \infty^0, \quad 1^\infty$$