Introduction

Summary of the Analysis II course given by Sir Thomas Mountford.

1 Notes

- A *metric space* is a vector space equipped with a metric, that is a function that measures distance between every two elements of the space, whereas a norm space is simply a vector space equipped with a norm: all metric spaces are norm spaces, but the converse is not true.
- BASIC CONCEPT OF TOPOLOGY: a *neighborhood*, or *open ball* of x $B(\vec{x}, r) = \{\vec{y} \ : \ d(\vec{x}, \vec{y}) < r\}$ is a "zone around a point" (a set of points) such that these points are less than a certain distance around $\vec{x}$ (actually it's not, it's "any set containing $B(\vec{x}, \epsilon)$ for some $\epsilon > 0$
- OPEN SET: An open set $O \subseteq \mathbb{R}^n$ is a set such that $\forall \vec{x} \in O \exists \epsilon_x > 0$ such that $B(\vec{x}, \epsilon_x) \subset O$ (So O is open $\iff$ O is an ... for each of its points)
- PROPERTIES: union over any collection of open sets is open; the intersection of two open sets is open
- CLOSED SET: a set $F \subset \mathbb{R}^n$ is closed if $F^C = \{\vec{x} \ : \ \vec{x} \notin F\}$ is open. Intuitively, think of a disc from which you remove all points on the circle. Ex: $\overline{B}(\vec{x}, r) = \{\vec{y} \ : \ d(\vec{x}, \vec{y}) \leq r\}$ is closed. Ex: in one dimension, take $F = [0, 1]$. Then compute $F^C =]-\infty, 0[\cup]1, \infty[$ is a union of open sets, and therefore is an open set. Therefore, F is a closed set by definition. (F^C means "F complement")
- BOUNDARY: Given $S \subset \mathbb{R}^n$, the boundary of S, that is, δs, is the collection $\{\vec{x} \ : \ \forall \epsilon > 0 \ B(\vec{x}, \epsilon) \cap S \neq \emptyset \wedge B(\vec{x}, \epsilon) \cap S^C \neq \emptyset\}$
- given S, the closure (adherence) of S, $\overline{S}$, is $S \cup \delta S$