Análisis de redes de acceso ópticas

(PONs + EDFAs + seguridad + *y otras yerbas*)

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1 Introducción (lo que hay)

Se buscan nuevos esquemas de acceso para satisfacer distintos requerimientos: comunicaciones punto a punto, acceso a otra red (como ser Internet), suporte de *multicast*, seguridad, soporte a ataques (*jamming*).

El esquema propuesto en el paper [OBAHG09] presenta una arquitectura que es segura (desde el punto de vista de cada ONU) y que soportaría *multicast* y *unicast* en varias comunicaciones (pero esto no está especificado cómo hacerlo: falta contestar si el hardware lo puede hacer). Sin embargo, uno de los puntos fuertes es la posibilidad de utilizarse como un *switch* óptico público, donde la seguridad y el soporte a ataques es importante. Un segmento interesado en este tipo de redes podría ser el de los bancos. Un análisis pormenorizado del esquema propuesto en [OBAHG09] revela que el mismo se puede modelizar como circuitos virtuales conmutados: se realiza de una comunicación por vez. La cuestión es si podrían considerarse como circuitos virtuales dinámicos: varias comunicaciones pueden tener lugar a la vez. En principio esto depende de la capacidad de la electrónica. Es importante recalcar que se busca que la electrónica esté dimensionada para el canal inicial (digamos 10 *Gbps*), y que luego podemos usar la óptica para complementar otras funcionalidades. En particular la óptica puede brindar varias longitudes de onda λ para proponer otros esquemas.

Luego tenemos el esquema presentado en [F*et al.*09], cuyas características más sobresalientes son la posibilidad de brindar comunicaciones entre los ONUs, con el OLT y además soporte de *multicast*. (Se incluirán más detalles una vez leído el paper.) Una posible extensión para esta arquitectura es incorporar un 'canal seguro' siguiendo los lineamientos del otro paper.

1.1 Propuestas

Una mejora que se puede hacer a [OBAHG09] es la utilización de múltiples λ para incrementar el espacio disponible en el dominio de colisión, disminuyendo así la cantidad de '1' que son enviados por lo que el aprovechamiento aumentará. Es importante notar que el aprovechamiento está pensado en función de las capacidades de la electrónica[1], aquí la misma soporta hasta 10 *Gbps*.

Dado este esquema podríamos idealmente hacer un sistema en donde cada ONU pueda enviar (y recibir) hasta 10 *Gbps*, repartidos contra varias contrapartes, ya sea en comunicaciones *unicast* ó *multicast*. El mayor problema es ver hasta donde la FPGA que tenemos (Virtex 5) puede soportar varios transceptores y a qué velocidades.

Tomando los valores utilizados, podemos llegar hasta 128 ONUs, y para que sea comparable al esquema de las PONs, consideremos que la capacidad se divide hasta en 32 clientes:

$$\frac{10\ Gbps}{32} = 312.5\ Mps\ \ . \tag{1}$$

[1] Jorge había recalcado que se quería mantener la electrónica (un sólo transceptor en principio) e incrementar la óptica; sin embargo es importante notar el límite no es *un solo* transceptor sino la capacidad a la que puede transmitir y recibir.

Luego, considerando que nuestros 128 clientes pueden comunicarse a la vez, necesitaríamos una capacidad total de:

$$312.5\ Mps \times 128 = 40\ Gbps\ . \tag{2}$$

Luego considerando un aprovechamiento del 17% del 'canal', necesitamos un ancho de banda de:

$$\frac{40\ Gbps}{0.17} > 235\ Gbps\ , \tag{3}$$

que se logran con 24 λ. Ahora, utilizando el esquema de tribits ó cuadribits hemos visto que se puede llegar hasta un aprovechamiento del 30% (a confirmar en las simulaciones que está realizando Alfredo de 1 *Gb*), por lo que el número de λ pasaría ser de 14 para mantener la capacidad de transmisión de cada canal según la ecuación 1. Luego, esta capacidad de cada nodo será dividida entre los canales en los que transmita, sean *unicast* ó *multicast*.

Cabe aclarar que un canal *multicast* es aquel donde hay una sola fuente y un grupo de receptores. Si se desea que todos los receptores actúen como fuentes, entones habrá que utilizar múltiples canales, uno por cada fuente[2].

Uno de los puntos a considerar es el tiempo de conmutación de longitud de onda para el láser y para los filtros sintonizarles: ¿nos permite una conmutación en tiempo de trama o aún mayor?

Otro problema es ver cómo se mantiene la sincronización:

- todo el tiempo sincronizado: obliga un esquema de *round-robin* para atender las diferentes comunicaciones abiertas.
- sincronización en cada ráfaga: al estilo *ethernet* donde hay un tiempo de la trama (aquí sería una ráfaga de tramas) que se pierde en la sincronización; nuevamente hay que ver cómo se hace[3].

Luego, una forma de analizar el uso de las λ es desde la óptica de las VLANs, donde cada λ puede representar una VLAN. Esta posibilidad hay que analizarla en más profundidad.

1.2 Problemas encontrados en [OBAHG09]

Si bien nuestro sistema es seguro desde el punto de vista de cada cliente (ONU), no necesariamente lo es cuando accedemos a las fibras del *up-stream*: hay sólo un cliente transmitiendo por esa fibra. Las soluciones propuestas son:

1. Enviar unos falsos; esto tiene el problema que reduciría la utilización del canal.
2. Hacer tramas más grandes de manera tal que se superpongan entre ellas; hay que verificar que este esquema sigue siendo seguro.
3. Es posible modificar el protocolo saliendo del esquema *time-hopping*, donde se utilizará TDM pero cada canal va codificado haciendo una función XOR bit a bit con un generador aleatorio (similar a RSA4). Esto mantiene las comunicaciones punto a punto seguras, no desperdicia ancho de banda pero no es resistente a los ataques tipo *jamming*.

2 Plan de trabajo

Tareas en curso:

[2]Este es uno de los problemas que intenta resolver CBT, donde hay un nodo que actúa de raíz, y cada emisor envía su mensaje éste mediante una comunicación *unicast*, luego es el nodo raíz quien distribuye por el árbol *multicast* el mensaje.

[3]Una posibilidad de que se envíen en forma iterativa una secuencia de sincronismo (por ejemplo unos 100 bits, reiniciando el contador en cada vez al último valor utilizado).

- Alfredo está realizando la simulación con 1 *Gbps* y los cuadribits para obtener la curva del aprovechamiento. [**Resultados esperados para la semana del 16/11/2009**]

- Alfredo va analizar la opción 2 de la sección 1.2 para ver si es segura.[**Resultados esperados para la semana del 16/11/2009**]

- Alfredo va a calcular de manera muy conservadora, el número de canales que un ONU puede manejar, basándose en una Virtex 5. [**Resultados esperados para la semana del 16/11/2009**]

- Victor va terminar los cálculos del error que introduce el canal óptico, para verificar que sean correctos nuestros resultados. [**Resultados esperados para la semana del 16/11/2009**]

- Nacho coordina y escribe éste documento, ayudando con los problemas que se presenten tanto a Alfredo como a Victor.

- Diego discute con Victor los concerniente al canal óptico.

Tareas para realizar:

- 7 u 8 de diciembre del 2009 haríamos una conferencia por la mañana (10:30hs a 13hs).

- ¿Cómo se presenta la arquitectura? (circuitos virtuales dinámicos, varios canales, *unicast*, *multicast*, etc.) Tal vez se podría presentar las dos arquitecturas de ambos papers. A ver/discutir.

3 Network physical layer

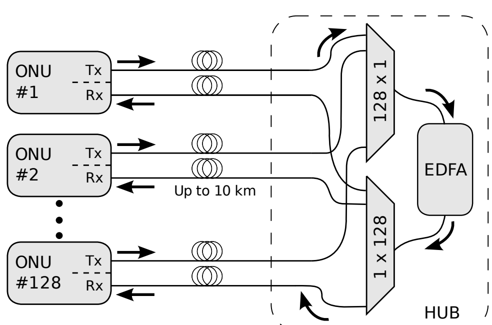

Figure 1: One hub links up to 128 ONUs.

Network presents a star network topology connecting up to 128 ONUs distanced up to 10 km from an optical hub. A single-laser wavelength of 1550 nm is used by all network ONUs to allow amplification by an Erbium-doped waveguide amplifier (EDFA).

Traffic generated at each node's Tx as an amplitude shift keying (ASK) return to zero (RZ) optical signal is routed through upstream fibre links, up to 10 km long. Traffic from all ONUs is merged by a 128×1 splitter before reaching hub's EDFA. As many bit '1' optical intensities are added at each bit slot as active Tx there are.

This superposition of traffics is amplified at EDFA and routed back through downstream fibre links, alike to upstream ones, to all ONUs by a 1×128 splitter. Each ONU APD type receiver provides the required sensitivity ($\leq -28\,dBm$) to operate the network with an acceptable BER.

3.1 Power budget

ONU Tx (see 3.3.2): $P_{\text{out}} = 2\,\text{dBm}$

Upstream (see 3.4.1): 3 dB attenuation $\Rightarrow P_{\text{out}} = -1\,\text{dBm}$

128×1 splitter (see 3.4.2): 25 dB attenuation (see 3.4.1) $\Rightarrow P_{\text{out}} = -26\,\text{dBm}$

EDFA (see **??**): 27 dB gain $\Rightarrow P_{\text{out}} = 1\,\text{dBm}$

1×128 splitter (see 3.6.1): 25 dB attenuation (see 3.4.1) $\Rightarrow P_{\text{out}} = -24\,\text{dBm}$

Downstream (see 3.6.2): 3 dB attenuation $\Rightarrow P_{\text{out}} = -27\,\text{dBm}$

ONU Rx: $-28\,\text{dBm}$ sensitivity $\Rightarrow P_{\text{margin}} = 1\,\text{dBm}$

3.2 Simulation programme

A programme simulating physical optical channel behaviour was made in order to estimate bit error rate occurring in the optical channel. Following the traffic sense shown by arrows in figure 1 a description on the simulation of actions of each device is given next.

Simulation describes optical traffic in base band, that is a not modulated signal.

In order to make easier to recognize simulation programme variables and function names they are written in this document with the same name and noted as follows: `VARIABLE`.

Programme structures, functions or values assumed for the simulation involved at each point appear inside boxes as follows:

`Simulation Programme function`

3.3 ONU's transmitters (Txs)

3.3.1 Optical power/ electrical field amplitude

Simulation run starts by mimicking the generation of opticaly encoded data traffic at transmitter. As optical bits trains traveling through fibres are described in the simulation as time dependent values of electric field E, related to optical intensity by $I = |E|^2$. If an homogenous optical intensity I is considered inside the spot (radius w_0), optical power P is [Agr01, section 1.3.1],

$$P = I\pi w_0^2 \tag{4}$$

Discarding optical beam geometrical factors it is assumed that $P = I$. Only electric field amplitude is accounted for (no phase information), that is, E is described by a real number, optical power is assumed to be $P = I = |E|^2$. Only z dependent component of electric field E, $A(z)$ is taken into account [Agr02, p.21 Agrawal].

3.3.2 Tx: '0' '1' bits power and extinction ration

Data traffic generated at each ONU is segmented into time bit slots (length `1/B`, bit rate `B`= $10\,GHz$) sampled a number of times (`NPtos_Bit`= 64, sampling frequency $640\,GHz$).

`InputParameters` function
`Carrier` structure

`NPtos_Bit` (No. samples per bit)	$2^6 = 64$
`Duty_Cycle`	$1.0/3.0 = 0.333$
`B` (Bit rate)	$10E9 = 10\ Gb/s$
`Bit_Slot`	$1/B = 100\ ps$
`dt` (time step)	`Bit_Slot`/(`NPtos_Bit` -0.5)

Used codification is:

Line coding	RZ-L ('1' peaks, '0' base)

Tx's semiconductor lasers being conmuted to bit '0' state are not compleatly off, they are still emiting a certain power P_0. This later is related to power for bit '1' P_1, the on-state optical power, by the *extinction ratio* defined as $r_{ex} = \frac{P_1}{P_0}$ [RS02, section 5.3]. So assuming a constant electric field amplitude for bit slot '0', its amplitude is given by $\sqrt{r_{ex} \times P_1}$. Manufacturers do not inform P_0 and P_1 but instead a single figure, Tx mean power $P\_mean = 0.5\,(P_1 + P_0)$, considering a balanced number of emited bit '0' and '1' and that both bits are represented by a constant power during all bit slot duration (NRZ coding). So P_0 is obtained as follows,

$$P_0 = \frac{2 \times\ P\_mean}{1 + r_{ex}} \tag{5}$$

High power Tx are pricey (see appendix A.1) meanwhile our proposal aims to use low cost devices.
A $P\_mean = 2\,dBm$ is considered in the simulation as a that of a standard Tx [CES08].
A minimal extinction ratio of $r_{ex} = 3.5\,dB$ is standard for transceivers[CES08][Mer07, p. 5].
In the simulation a less ideal value of $10\,dB$ is used.

At `InputParameters` function in it's `Gauss` structure, pulse amplitude parameters are established:

`P` (Tx mean output power)	$10^{(2\,dBm/10)/1000}\,W$
`r_ex` (extinction ratio)	$10^{(10.0\,dB/10)}$
`P_0` (bit '0' power)	$2 \times\ P/(1 + r_{ex})$
`A_0` (bit '0' amplitude)	$\sqrt{P_0}$
`P_peak` (bit '1' power)	$P_0 \times\ r\_ex$
`A_peak` (bit '1' amplitude)	$\sqrt{P\_peak}$

In the simulation programme bit '1' slots are represented by the electrical field amplitude of a super-Gaussian overpulse superposed to bit '0' amplitude $A\_0 = \sqrt{P\_0}$. A super-gaussian amplitude overpulse is created from an evenly sampled time vector $t[i]$ of a bit slot. Overpulse maximum amplitude is the difference between bit '1' peak amplitude $A\_peak = \sqrt{P\_peak} = \sqrt{P_0 * r\_ex}$ and bit '0' amplitude A_0. This amplitude is modulated by the real part of the expression in [Agr01, section 3.2.4], that is:

$$\exp\left[-\frac{1}{2}\left(\frac{T}{T_0}\right)^{2m}\right] \tag{6}$$

$$RZBit1Gauss[i] = (A\_peak - A\_0) * \exp\left[-\tfrac{1}{2}\left(\tfrac{t[i]-(CentringFactor \times Bit\_Slot)}{T\_0\_amp}\right)^{2m}\right]$$

Being $CentringFactor$ the fraction of bit slot at which pulse is centred.

At `InputParameters` function, at `Gauss` structure parameters for the pulse are established:

`m` (supergaussian shape factor)	4
`CentringFactor` (pulse centre into bit slot)	0.25 (at edge beginning) (0.5 at centre)
`FWHM_pow` (bit '1' temporal FWHM power)	`Duty_Cycle`/ `Bit_Slot`
`T_0_amp` (bit '1' temporal width)	`FWHM_pow`/ $(2 \times\ \sqrt{\ln 2})$

At the `RZPulsesTrainGenerator` function `FAtt1` affects the merged traffic (see section 3.3.3)
$RZPulsesTrain[i] *= (*Network).FAtt1$

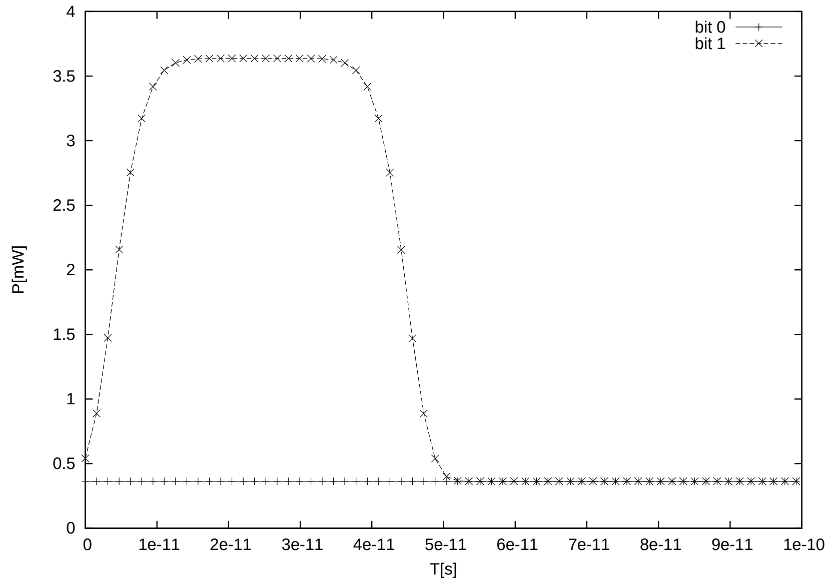

Figure 2: Bit '0' and '1' pulses. Whole bit slot is shown.

3.3.3 Tx: bit traffic stream

In the physical network each Tx generates an optical bit traffic as a concatenation of bit slots representing either a '0' bit or '1' bit. That traffic stream is then fed into a single upstream fibre for each Tx that carries it up to a splitter where it's mixed with traffic comming from other Txs.

In the way fibre links are treated in the simulation (see 3.4.1) the later is equivalent to generate a single stream as addition of that from all Txs and then feed this merged traffic into simulated links. Main assumption for that is that optical intensities add without any interference effect between beams (considered not coherent among themselves).

Bits to be transmitted are fed into the simulation programme through terminal's `stdin` stream and copied into a `InBitString` vector. Each byte into the file thus fed into the programme informs how many of all present ONUs (`transmitters`) are effectively transmitting '1' bits (`senders`) at each bit slot (one slot per byte). In order to obtain the optical intensity of merged traffic from all ONUs at each bit slot $transmiters - senders$ bit '0' optical intensities P_0 should be added to $senders$ bit '1' optical intensities,

$$\begin{aligned} RZPulsesTrain[i] &= \\ (transmiters - senders) \times P\_0 &+ senders \times (RZBit1Gauss[i] + A\_0)^2 \\ (transmiters - senders) \times A\_0^2 &+ senders \times (RZBit1Gauss[i]^2 + 2 \times RZBitGauss[i] \times A\_0 + A\_0^2) \\ transmiters \times A\_0^2 &+ senders \times (RZBit1Gauss[i]^2 + 2 \times RZBitGauss[i] \times A\_0) \\ transmiters \times P\_0 &+ senders \times RZBit1Gauss[i] \times (RZBit1Gauss[i] + 2 \times A\_0) \end{aligned} \tag{7}$$

being `RZBit1Gauss` bit '1' over-amplitude on top of bit '0' flat amplitude `A_0`. Calculation of the square root of this quantity allows to recover the electric field amplitude of merged traffic.

At the `RZPulsesTrainGenerator` function, the merged traffic `RZPulsesTrain` is generated (under **RZ PULSES TRAIN (Amplitude)**)

```
RZPulsesTrain[k*(*Carrier).NPtos_Bit+i]= RZBit1Gauss[i]+ 2* (*gauss).A_0
RZPulsesTrain[k*(*Carrier).NPtos_Bit+i]*= senders* RZBit1Gauss[i]
RZPulsesTrain[k*(*Carrier).NPtos_Bit+i]+= transmitters* (*gauss).P_0
RZPulsesTrain[k*(*Carrier).NPtos_Bit+i]= sqrt( RZPulsesTrain[k*(*Carrier).NPtos_Bit+ i])
```

Being k a descriptor for bit slot number.

3.4 Tx → EDFA input

3.4.1 Upstream fibre link: ONU's Tx → 128×1 coupler

Electric field amplitude of merged traffic is transported by the upstream fibre link up to the 128x1 coupler. Fibre links are composed by optic fibre stretchs connected inbetween by joints as usually optic fibre cables up to $4\,km$ in length are available commercialy. To reach greater distances cable sections must be joined either by connectors of by splicing the sections togheter. In our network proposal each ONU is up to $10\,km$ apart from the hub requiring at least two splices joints. Also there are one connector at ONU and another at splitter ends.

In the simulation electric field amplitude of merged traffic traversing fibres links are modified only by attenuation (no dispersion or other non linear effects are considered). Joints add to the attenuation as well as the fibre itself.

Power entering at a fibre P_0 is attenuated through fibre length L[Agr02, section 2.5.1]

$$P_{out} = P_0\, e^{-\alpha L} \tag{8}$$

being α fibre's attenuation constant.

Manufacturer's data on standard commercial fibres for $1550\,nm$ indicate $\alpha \simeq 0.2\,\frac{\text{dB}}{\text{km}}$ (see appendix B) that would amount in a $10\,km$ stretch to $\simeq 2\,dB$, being that the value used in the simulation.

Connectors add more attenuation than splices joints of fibres. For $1550\,nm$ connector attenuation $\simeq 0.35\,dB$, splices joints attenuation $\sim 0.05\,dB$[Cis, p.2].

To summarise fibre link estimated total attenuation (TA) is:

$$TA = n \times C + c \times J + L \times a = 2.8\,dB \simeq 3\,dB \tag{9}$$

n	connectors number	2
C	attenuation at single optical connector	$0.35\,dB$
c	splices joints number	2
J	single splice joint attenuation (dB)	$0.05\,dB$
a	optical fibre attenuation	$0.2dB/Km$
L	optical fibre length	$10\,km$

That amount is composed by $2\,dB$ at fibre and $0.8\,dB$ by insertion losses (connectors/splices). In the simulation a $TA = 3\,dB$ is assumed for the fibre link.

3.4.2 128×1 coupler

Beam splitters with single mode fiber for PON networks use the single mode behavior to split the beam. A splitter is done by physically splicing two fibers "together" as an X [Wik10]. That effectively divides the incoming traffic beam at one of its extremes and makes that two “copies” of that traffic emerges at the the two opposite extremes. Beam division causes that the power of output traffic is just half of the incoming one. That is acconted for in power budget as attenuation stating that each time the number of outputs is doubled output traffic had been attenuated by $\simeq 3\,dB$.

In the proposed network architecture traffic coming from all ONUs upon arrive at a 128×1 coupler and merge. A coupler is just the mentioned splitter working backwards, that is, it combines the optical powers on many optical fibers into a single one as long the beam is of the same wavelength.

In the simulation it is assumed that optical intensities add linearly. Each bit slot optical intensity (i.e. power) is added as if all traffic arrives with perfect synchronisation (no jitter) and identical attenuation. The physical network in order to behave that way would require delay adjustments and attenuators for all 128 inputs. So splitters are accounted for in the simulation only as attenuators of optical power.

No commercial 1×128 splitter models were found, so an estimation on the attenuation of an hypothetical one was made based in manufacturers data on models with a lesser number of terminals(see appendix C). In the simulation a conservative attenuation of $25\,dB$ was assumed for a 1×128 splitter.

Adding the attenuation at fibre link and at 128×1 splitter, beam from each Tx is attenuated $28\,dB$ before reaching EDFA input.

At the programme’s `Network` structure this attenuation `FAtt1` value is established:

`FAtt1` (Tx ->EDFA amplitude attenuation)	$10^{(-28\,dB/20)}$

At the `RZPulsesTrainGenerator` function `FAtt1` affects the merged traffic (see section 3.3.3)
$RZPulsesTrain[i]* = (*Network).FAtt1$

3.5 EDFA

3.5.1 EDFA amplification

EDFA purpose is to increase optical power to assure power reaching ONU’s receiver is above photodector sensitivity. EDFAs gain decreases with higher input power, altough as a simplification, in the simulation it provides a fixed gain of $27\,dB$ for any number of active Txs. This gain fixation in an actual physical network can be accomplished by saturating EDFA input with a wavelength generated locally at hub allowing a fixed EDFA output power.

At the programme’s `Network` structure `EDFAgain` value is established:

`EDFAgain` (amplitude)	$10^{27.0\,dB/20}$

At the `Link` function `EDFAgain` merged traffic `RZPulsesTrain` gets amplified
$RZPulsesTrain[i]* = (*Network).EDFAgain$

3.5.2 Amplified Spontaneous Emission noise

In addition to providing amplification EDFAs are also source for the Amplified Spontaneous Emission (ASE) noise. This has a spectrum approximately the same as the gain spectrum of the amplifier. Noise figure in an ideal EDFA is $\simeq 3\,dB$, while practical amplifiers have a noise figure as large as $6 \sim 8\,dB$ [Wik09a].

In order to be able to compare reported values for noise’s σ a standard definition of OSNR sampled into a given reference spectral bandwidth r_{BW}, centred in a certain *lambda*, is usually calculated following the TIA/EIA-526-19 standard. In this definition OSNR does not dependend on data format or pulse shape, but only on the average optical signal power and the average optical noise power[Kei03, sect. 14.2].

$$OSNR[dB] = 10\log\frac{P_{signal}}{P_{noise}} + 10\log\frac{B_m}{r\_BW} \quad (10)$$

where B_m is the noise-equivalent measurement bandwidth of the instrument being used, for example, each channel bandwidth in an optical spectrum analyzer.

ASE spectrum and statistics are simplified in the simulation assuming it to be a gaussian noise that's added to electric field at EDFA output with white, not the amplifier's, spectrum. This approximations allows to derive a simple relationship between noise amplitude standard deviation σ and its mean power. White noise autocorrelation is $\sigma^2\delta(t)$ [Wik09c]. Wiener-Khinchin theorem states that power spectral density of a random process is the Fourier transform of its autocorrelation function [Wik09d], then noise power spectral density $\phi(\omega) = \sigma^2\mathcal{F}(\delta(t))$. Being the signal power $P = \int d\omega\phi(\omega)$ [Wik09b], then $P = \sigma^2\int d\omega\mathcal{F}(\delta(t))$. As the integration is performed only into the ω range of the simulation P is finite (can be defined). So as $\mathcal{F}(\delta(t)) = \frac{1}{\sqrt{2\pi}}$ then mean noise power in this interval $P_{noise} = \sigma^2$. In that case, σ can be obtained from the expression 10 as

$$\sigma = \sqrt{P_{noise}} = \sqrt{\frac{P_{signal}}{OSNR} \times \frac{B_m}{r_{BW}}} \quad (11)$$

In the simulation the equivalent to B_m is the bandwidth into which optical power is integrated, that is the whole simulation bandwidth (see 3.5.3 to check how mean signal power is obtained). That bandwidth is the frequency sampling rate obtained as the reciprical of simulation time step `Carrier.dt` (see 3.3.2).

At the programme's function `BitSlots`, `OSigmaNoise` (σ) is calculated as:
$OSigmaNoise = \sqrt{\frac{Pmean\_EDFAout}{OSNR} \times \frac{1}{(*Network).r\_BW \times (*Carrier).dt}}$
`Pmean_EDFAout` stands for the mean power at EDFA output (see 3.5.3).

`r_BW` is the reference bandwidth centred around λ_0 (`Lambda_0`) converted to GHz at `InputParameters` function as $(*Network).r\_BW = c \times Br/(Lambda\_0 \times Lambda\_0)$

In this function are stated:

`c`	$299792458\, m/s$
`Lambda_0` (λ_0)	$1550\, nm$
`Br`	$0,1\, nm$ (It's costumary to express OSNR in an equivalent bandwidth of $0.1\, nm$)

`OSNR` is an input to the simulation converted into linear scale at `main` as $OSNR = 10^{(OSNR[dB]/10)}$

`Ofilt_BW` is the photodetector's input optical filter cut-off frequency, stated at `InputParameters` function, `Network` structure

`Ofilt_BW` (cut-off freq. optical filter)	$12,5\, GHz$

At the programme's `GenRuidoRapido` function an optical noise vector `ONoiseVector` is generated.
Its content are gaussian samples with `OSigmaNoise` dispersion.
The number of samples is the same as the traffic representation `RZPulsesTrain`.

At the programme's `Link` function optical noise vector `ONoiseVector` is added to traffic `RZPulsesTrain`
`RZPulsesTrain[i]+= ONoiseVector[i];`

3.5.3 Mean optical power at EDFA output

In order to estimate a figure for mean power emitted by ONUs it is considered that the collision of more than two '1' bits is a rare event, so the usual approximation of averaging the power for bit '0' and bit '1' is followed. The addition of power emitted from all ONUs at two cases should be averaged. One case with all ONUs transmitting '0' bits and other with one ONU transmitting a bit '1' and all the rest '0' bits. Marking the number of ONUs transmitting a bit '1' as *senders* and *tr* their total number, the former case power isn

$$P_{senders=0} = tr \times P\_0 \tag{12}$$

and the later

$$P_{senders=1} = (tr-1)P\_0 + (RZBit1Gauss + A\_0)^2 = tr \times P\_0 + RZBitGauss1^2 + 2 \times A\_0 \times RZBitGauss1 \tag{13}$$

So average power emitted by ONUs is

$$Pmean = \frac{1}{2}\left(P_{senders=0} + P_{senders=1}\right) \tag{14}$$

At the programme's `BitSlots` mean power emitted by ONUs `Pmean` is calculated as: $Pmean = transmitters \times P\_0 + \frac{1}{2 \times NPtos\_Bit} \sum_i^{NPtos\_Bit} RZBit1Gauss[i] \times (RZBitGauss1[i] + 2 \times A\_0)$ being `transmitters` ONUs number.

This power is first attenuated at fibre link and $1x128$ spitter, and then amplified by EDFA to obtain an estimation of mean power at EDFA output.

At the programme's `BitSlots` function mean power at EDFA output `Pmean_EDFAout` is calculated as: $Pmean\_EDFAout = \times Pmean$ $Pmean\_EDFAout* = Network.FAtt1 \times Network.FAtt1$ $Pmean\_EDFAout* = Network.EDFAgain \times Network.EDFAgain$

3.6 EDFA output → Rx

3.6.1 1×128 Splitter

At the 1×128 splitter traffic coming out of the EDFA is "copied" into each of the downstream fibre links. Power is equally shared among links that amounts to an "attenuation" of one output in comparison to the single input. The attenuation figure assumed is the same stated at section 3.6.1, $25\,dB$.

3.6.2 Fibre link: 1x128 splitter ->ONU's Rx

The downstream fibre link (1x128 splitter ->ONU's Rx) is alike to the upstream one (ONU's Tx ->128x1 splitter) its total attenuation is the same (see 3.4.1) $3\,dB$.

Adding the attenuation at the 1×128 splitter to that of this later fibre link, each beam reaching ONU's Rx is attenuated $28\,dB$ in comparison to EDFA's output.

At the programme's `Network` structure this attenuation `FAtt2` value is established:

`FAtt2` (EDFA ->Rx amplitude attenuation)	$10^{(-28\,dB/20)}$

At the `Link` function `FAtt2` affects the traffic at EDFA output)
$RZPulsesTrain[i]* = (*Network).FAtt2$

3.7 ONU's receivers (Rxs)

3.7.1 Optical filter

"In practice, an optical filter is used between the amplifier and the receiver to limit the optical bandwidth and thus reduce the spontaneous-spontaneous and shot noise components in the receiver." [RS02, extact from section 4.4.6]. In order to do that some commercial photodiodes aimed to the PON market integrate an optical filter [Ena08].

The effect of optical filters on traffic is usually described as a transfer function $H(\omega)$ acting on the traffic's amplitude spectrum $\tilde{A}(\omega)$ [Agr02, section 7.5]. An ideal band-pass filter would have a square $H(\omega) = 1$ in the selected unfiltered frequencies range. Band-pass bandwidth is chosen in accordance to link bit rate and coding.

In the simulation an approximation to the ideal filter is made using an order low-pass 6 Butterworth filter on the traffic previously to its photodetection. As in the simulation traffic is in base band (as already demodulated) filter is not a band-pass but a low-pass with cutoff (or corner) frequency half of band-pass bandwidth of a filter used in modulated traffic.

At the `Link` function traffic at the output of downstream link is filtered by the function:

```
OpticalFilter(RZPulsesTrain, (*Carrier).NPtos_Tot);
```

In the simulation programme an infinite impulse response (IIR) digital filter implemention provides a speedy computing of the filter effect. The code for this implementation was produced by a website [Fis99]. Accordingly to [PSPW02] optimal optical filter bandwidth for RZ coding with one third duty cycle (see 3.4.1) is 2.5× link bit rate, and for NRZ coding 2.5× link bit rate ($10\,GHz$). By convention optical bandwidths are measured in passband units [RS02, section 4.4.2], so half this frequency is the low-pass filter corner frequency used in the simulation.

`OpticalFilter` code represents a filter with the following characteristics:

Type	Butterworth
Order	6 (approximates an "ideal" square filter, $-36\,dB/dec$, $-3\,dB$ at $16\,GHz$)
Corner frequency (cutoff)	$12.5\,GHz$ ($0.5 \times 2.5 \times 10\,GHz$)

Being the corner frequency the one at which the magnitude of the response is $-3\,dB$.

3.7.2 Optical → electrical signal

ONU semiconductor photodetector converts incident optical power P_{in} into a photocurrent I_p [Agr02].

$$I_p = RP_{in} \tag{15}$$

where R is the photodetector responsivity.

The optical power reaching the ONU is obtained in the simulation by squaring the electrical field amplitude $P = |E|^2$ that passed through the optical filter (see 3.3.1).

At the `Link` function under `Optical -> Electrical signal (current)`
optical power reaching the photodector is obtained
$RZPulsesTrain[i]* = RZPulsesTrain[i]$

At the programme's `Network` structure photodetector responsivity `R` value is established:

R	$12\,A/W$ based on assumptions discussed at 3.9.2

At the `Link` function under `Optical -> Electrical signal (current)`
photodetector responsivity `R` affects the optical power)
$RZPulsesTrain[i]* = (*Network).FAtt2$

3.7.3 Receiver Noise

"Two fundamental noise mechanisms, shot noise and thermal noise, lead to fluctuations in the current even when the incident optical signal has a constant power.". Although shot noise is a stationary random process with Poisson statistics it is often approximated by Gaussian statistics. Thermal noise, also called Johnson or Nyquist noise, in modelled as a stationary Gaussian random process.[Agr02, section 4.4]

As the sum of Gaussian fluctuations has also Gaussian distribution in the simulation all receiver noise mechanisms are modelled as a single Gaussian fluctuation. In the same way as done for optical noise (see 3.5.2) the σ for the Gaussian fluctuations on the electrical photocurrent signal `DSigmaNoise` is obtained from an input value for SNR named `DSNR`. The rate of average electrical power to that of the noise defines `DSNR`. As electrical power varies as the square of the current, and both signal and noise currents are applied to same loads, this becomes [Agr02, section 4.4.2]

$$DSNR = \frac{I_p^2}{\sigma^2} \tag{16}$$

With the relationship between optical power and photocurrent (see 3.7.2) it can be stated then that

$$sigma = P_{in} \times R \times \frac{1}{\sqrt{DSNR}} \tag{17}$$

being P_{in} the mean incident optical power to the photodector and R its responsivity. In the simulation it is considered that mean signal optical power is not affected by detector's input optical filter.

At the programme's function `BitSlots` `DSigmaNoise` is calculated as:
`*DSigmaNoise= Pmean_Rxin* (*Network).R* sqrt(1/ DSNR)`

`Pmean\_Rxin` is the mean optical power reaching the photodector (see 3.7.4
`DSNR` is an input to the simulation. It's converted into linear scale at `main` as $DSNR = 10^{(DSNR[dB]/10)}$

At the programme's `GenRuidoRapido` function an electrical current noise vector `DNoiseVector` containing gaussian samples with `DSigmaNoise` dispersion.
The number of samples is the same as the traffic representation `RZPulsesTrain`.

At the programme's `Link` function optical noise vector `DNoiseVector`
is added to `RZPulsesTrain` that represents the photocurrent
`RZPulsesTrain[i]+= DNoiseVector[i]`

3.7.4 Mean optical power incident to photodector

Optical mean power at EDFA output (see 3.5.3) after traversing the downstream fibre link is attenuated before reaching any ONU photodector.

At the programme's `BitSlots` function mean power reaching a photodector `Pmean_Rxin` is calculated as:

```
Pmean_Rxin= Pmean_EDFAout;
Pmean_Rxin*= (*Network).FAtt2;
Pmean_Rxin*= (*Network).FAtt2;
```

3.7.5 Electrical filter

"...the receiver noise is proportional to the receiver bandwidth and can be reduced by using a low-pass filter whose bandwidth Δf is smaller than the bit rate." [Agr02, section 4.3.2].

As previously used to simulate the optical filter effect (see 3.7.1) an infinite impulse response (IIR) digital filter implemention provides a speedy computing for the electrical filter effect. Again the same recommendation for filter bandwidth is followed (see [PSPW02]). So it is assumed that optimal electrical filter bandwidth for RZ coding with one third duty cycle (see 3.4.1) is 1.4× link bit rate. By convention electrical bandwidths are measured in band units [RS02, section 4.4.2], so this frequency is the low-pass filter corner frequency used in the simulation.

At the `Link` function photocurrent `RZPulsesTrain` is filtered by the function:

```
ElectricalFilter(RZPulsesTrain, (*Carrier).NPtos_Tot, SamplingPoint, Network);
```

The code for the electrical filter was produced by a website [Fis99] with the following parameters.

`ElectricalFilter` code represents a filter with the following characteristics:

Type	Butterworth
Order	2
Corner frequency	$7\,GHz$ ($1,4 \times 10\,GHz$ in baseband units)

Being the corner frequency the one at which the magnitude of the response is $-3\,dB$.

Changing filter cut-off frequency causes a modification of the electrical pulses shape. Reducing cut-off frequency produces a delay in time of the peak for bit '1' pulse. This changes the eye diagram maximum openess, so the bit slot discretisation point to be sampled by the decision circuit changes (`SamplingPoint`). This sampling point is one from the first to the last (`NPtos_Bit`) in which each bit is discretised (3.3.2).

`Efilt_CutOff` is the photodetector's electrical filter cut-off frequency, stated at `InputParameters` function, `Network` structure

`Efilt_CutOff` (cut-off freq. electrical filter)	$7\,GHz$ (current)

`ElectricalFilter` establishes `SamplingPoint` accordingly to `(*Network).Efilt_BW`

`Network.Efilt_BW`	`SamplingPoint`
7	48
14	50

3.7.6 Decision circuit

The decision circuit compares a sample of the photocurrent taken at the same time in each bit slot to a threshold level, and decides whether the signal corresponds to bit 1 or bit 0. Optimal threshold for a standard receiver sorting pulses for '0' and '1' bits is experimentaly obtained from an eye diagram. At a given time in the bit slot the eye has its maximum apperture, that is photocurrents for '0' and '1' differ the most, so for detection proposes photocurrent is sampled at that time.

In the simulation programme instead of generating a whole eye diagram in each run, a sampling point is previously determined by inspection of eye diagrams generated in test runs. This point is represented in the simulation as an integer `SamplingPoint` $\leq$ number of time samples of a bit slot `NPtos_Bit`.

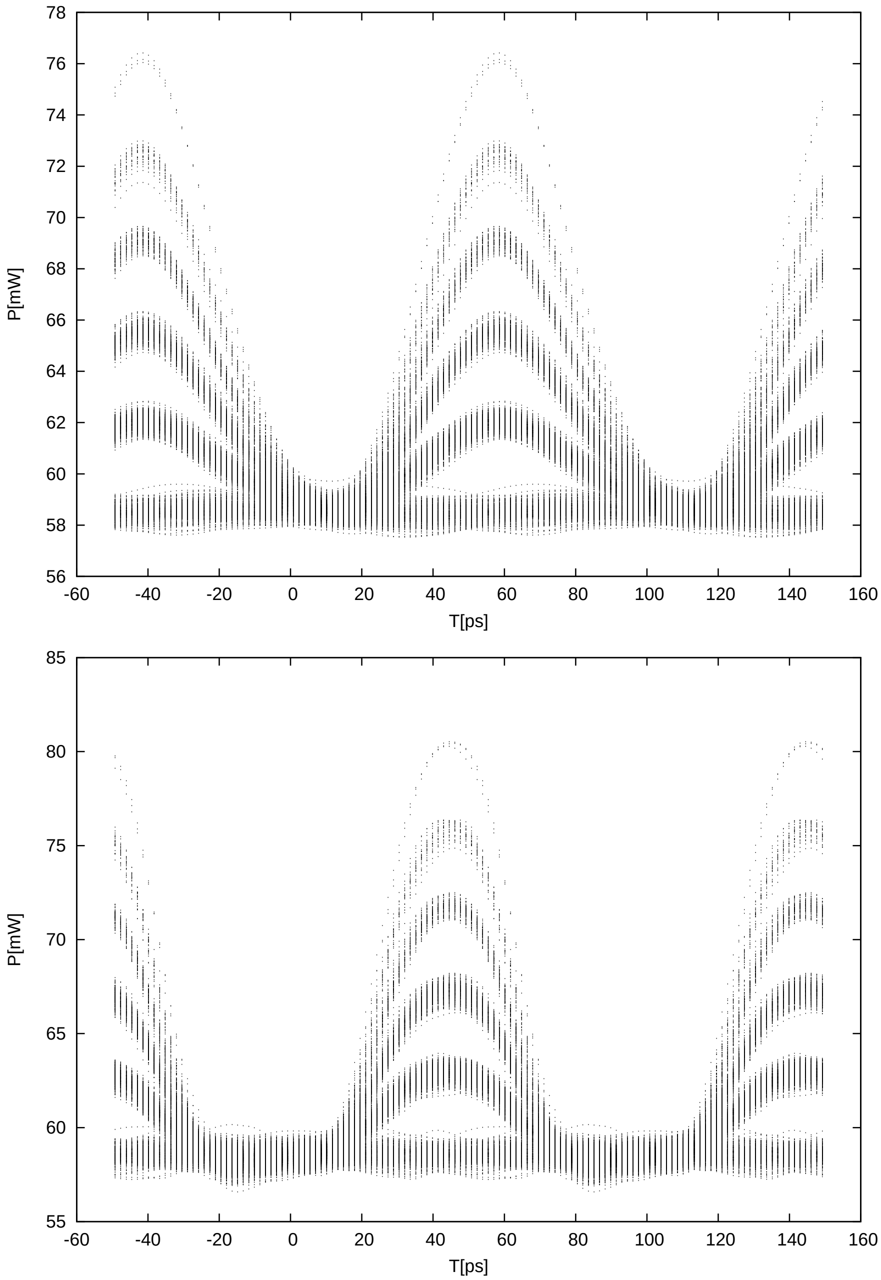

Figure 3: Efect of electrical filter cut-off frequency.Eye diagrams for $10\,GHz$ (top), $25\,GHz$ (bottom).

At the `Main` function each bit in the output of the electrical filter `RZPulsesTrain` (see 3.7.5) is sampled at `SamplingPoint`. Sampled current is compared to threshold current `Id`:

```
if(RZPulsesTrain[i* Carrier.NPtos_Bit+ SamplingPoint]> Id) BitsDisc[i]=1;
else BitsDisc[i]=0;
```

`Id` threshold current is calculated in the `Threshold1` function (see 3.7.7)

The detected bit string `BitsDisc` is then directed by the simulation programme to terminal's `stdout` stream. At the terminal this stream can be redirected to a file for further analysis.

3.7.7 Threshold1 routine

Performing several samplings in all the bit slots represented in an eye diagram (like the ones in Fig. 3) a mean and dispersion for bit '0' (I_0, σ_0) and bit '1' can be obtained. Accordingly to [Agr02, expression 4.5.9] optimal threshold photocurrent is

$$I_d = \frac{\sigma_0 I_1 + \sigma_1 I_0}{\sigma_0 + \sigma_1} \tag{18}$$

At the `BitSlots` function under `DETECTOR THRESHOLD Id` is requested:

```
Id= Threshold1(Carrier, RZBit1Gauss, OSigmaNoise, DSigmaNoise, Network, transmitters, gauss);
```

`RZBit1Gauss`, `OSigmaNoise`, `DSigmaNoise` were previously established at the `BitSlots` function `transmitters` being the number of ONUs is an input to the programme via terminal

`Threshold1` function sends bit state pairs (bits '0' and bits '1') through the whole loop. Current is sampled in a small band around `SamplingPoint` (± 5 samples) to obtain I_0 and I_1 as means of that samples and σ_0, σ_1 their dispersion. This operation is repeated a number (`NPulThr` currently set to 100) of

3.8 Simulation execution

3.9 Input data and parameters entry

Simulation was written in C language, so it's compiled before it can be runned. Simulation is started with the following command:
`noisesim #Tx DSNR[dB] OSNR[dB] <InputBitStream >OutputBitStream`

Being it's input/output parameters the following ones:

#Tx: clients number

DSNR[dB :] detector photocurrent SNR

OSNR[dB :] SNR at EDFA output

InputBitStream: each byte represents # ONUs on bit '1' state

OutputBitStream: each byte '1' or '0' received bit

In both input and output files (senders file and output bits files) ASCII number for each byte (000 to 255) denotes either number of active transmitter or bit status (ASCII number 000 or 001).

3.9.1 OSNR[dB] Input value

Spontaneous-emission-induced noise spectral density S_{sp} [Agr02, expression 6.1.15],

$$\begin{aligned} S_{sp} &= (G-1) n_{sp} h\nu \\ S_{sp} &= (500) \times 3,5 \times 6.626 \cdot 10^{-34}\, J \cdot s \times 1,934 \cdot 10^{12}\, Hz \\ S_{sp} &\simeq 2.244 \cdot 10^{-16}\, J \end{aligned} \tag{19}$$

being: G the amplifier gain, $G = 10^{27/10} \simeq 501$ ($G = 27\, dB$ see 3.5.1).

n_{sp} the spontaneous emission factor, $n_{sp} = 3.5$. "The value of n_{sp} depends on the level of population inversion within the amplifier. With complete inversion $n_{sp} = 1$, but it is typically higher, around $n_{sp} = 2 \sim 5$ for most amplifiers" [RS02, section 4.4.5]. An EDFA noise figure of $F_n \sim 7$ is assumed as $F_n \simeq 2 \times n_{sp}$ [RS02, expression 4.11].

h the Planck constant, $h = 6.626 \cdot 10^{-34}\, J \cdot s$

ν the optical frequency.$\nu = \frac{c}{\lambda} \simeq \frac{2{,}998 \cdot 10^8\, \frac{m}{s}}{1.55 \cdot 10^{-6}\, m} \simeq 193.4\, THz$ ($\lambda = 1550\, nm$, see 3)

So the amplified spontaneous (ASE) noise power at the output of the amplifier for each polarization mode is given by

$$\begin{aligned} P_N &= S_{sp} sim_{BW} \\ P_N &= 2.243 \cdot 10^{-16}\, J \times 6.4 \cdot 10^{11}\, Hz \\ P_N &\simeq 1.435 \cdot 10^{-4}\, W \end{aligned} \tag{20}$$

being sim_{BW} is the optical bandwidth into which the spectral density is considerated. In the simulation this optical bandwidth is the frequency sampling rate obtained as the reciprical of simulation time step `Carrier.dt`, that is $sim_{BW} = 6.4 \cdot 10^{11}\, Hz = 640\, GHz$ (see 3.3.2)

As two fundamental polarization modes are present in a single-mode fiber, the total noise power at the output of the amplifier is $2P_N$,

$$\begin{aligned} P_{ASE} &= 2 \times P_N \\ P_{ASE} &\simeq 2{,}872 \cdot 10^{-4}\, W \end{aligned} \tag{21}$$

Mean signal power at EDFA output is [1 client: $Pmean\_EDFAout =?$] [128 clients: $Pmean\_EDFAout = 2.970 \cdot 10^{-2}\, W$] (see 3.5.3).

With this latter figure the optical signal to noise ratio in the simulation bandwidth $OSNR_{sim\_BW}$ is

$$\begin{aligned} OSNR_{sim\_BW} &= \frac{Pmean\_EDFAout}{P_{ASE}} \simeq \frac{2.970 \cdot 10^{-2}\, W}{2{,}872 \cdot 10^{-4}\, W} \\ OSNR_{sim\_BW} &\simeq 103{,}43 = 20{,}147\, dB \end{aligned} \tag{22}$$

As the simulation input optical signal to noise ratio is expressed in the standard reference bandwidth $r_{BW} = 0{,}1\, nm$ (see 3.5.2), this latter figure is obtained from the expression 11 as,

$$\begin{aligned} OSNR_{r_{BW}}[dB] &= OSNR_{sim\_BW}[dB] + 10 \times \log \frac{sim\_BW}{r_{BW}} \\ OSNR_{r_{BW}}[dB] &\simeq 20{,}147\, dB + 10 \times \log \left(\frac{640\, GHz}{1.248 \cdot 10^{10}\, Hz} \right) \\ OSNR_{r_{BW}}[dB] &\simeq 37{,}247\, dB \end{aligned} \tag{23}$$

Theoretical check

An analytical expression for SNR at EDFA output [Agr02, expression 6.1.18] considering a detector with only shot noise, gives,

$$\begin{aligned} (SNR)_{out} &= \frac{GP_{in}}{4S_{sp}\Delta_f} \\ (SNR)_{out} &= \frac{GPtotal\_EDFAout}{4S_{sp} sim\_BW} \\ (SNR)_{out} &\simeq \frac{501 \times 2.970 \cdot 10^{-2}\, W}{4 \times 2.243 \cdot 10^{-16}\, J \times 6{,}4 \cdot 10^9\, Hz} \\ (SNR)_{out} &\simeq 2{,}591 \cdot 10^6 \end{aligned} \tag{24}$$

HELP!!!

3.9.2 DSNR[dB] Input value

Thermal noise

"The thermal noise current in a resistor R_L at temperature T can be mod eled as a Gaussian random process with zero mean and autocorrelation function $(4k_BT/R)\delta(\tau)$. Here k_B is Boltzmann's constant, $k_B = 1.3810^{-23}\,\frac{J}{K}$ and $\delta(\tau)$ is the Dirac delta function. Thus the noise is white, and in a bandwidth or frequency range B_e, the thermal noise current has the variance"[RS02, section 4.4.2]

$$\begin{aligned} \sigma^2_{thermal} &= \frac{4k_BT}{R_L}B_e \\ \sigma^2_{thermal} &\simeq \frac{4\times 1,381\cdot 10^{-23}\,\frac{J}{K}\times 298,17\,K}{50\,\frac{J}{s\cdot A^2}}\times 7\cdot 10^9\,Hz = 2,254\cdot 10^{-12}\,A^2 \end{aligned} \tag{25}$$

An arbitrary value of $R_L = 50\Omega$ is chosen to obtain an order-of-magnitude estimate at a $25\,C = 298,17\,K$ temperature [Mar91].

B_e is the effective noise bandwidth of the receiver (in base band units), $B_e = \int_0^\infty |H_T(f)|^2 df$ being $H_T(f)$ detectors' total transfer function (amplifier and low-pass filter) [Agr02, expression 4.4.4]. If the amplifier has a much larger bandwidth than the low-pass filter total transfer function is more or less that of the later, $H_T(f) \simeq H_F(f)$. Assuming this later to extend only up to electrical low-pass band output filter cutoff frequency, $B_e = 7\cdot 10^9\,Hz$ (see 3.7.5).

Shot noise

Shot noise for an APD receiver its can be estimated as [Agr02, expression 4.4.17]

$$\begin{aligned} \sigma^2_{shot} &= 2qM^2F_A(RP_{in}+I_d)B_e \\ \sigma^2_{shot} &\simeq 2\times 1,602\cdot 10^{-19}\,A\cdot s\times(10)^2\times 5.5\times\left(1.2\,\frac{A}{W}\times 4.708\cdot 10^{-5}\,W + 1\cdot 10^{-8}\,A\right)\times 7\cdot 10^9\,Hz \\ \sigma^2_{shot} &\simeq 6,970\cdot 10^{-11}\,A^2 \end{aligned} \tag{26}$$

Being $R_{APD} = MR_{pin}$ responsivity for an APD receiver (see 3.7.2) and R that of a standard p-i-n receiver, M is the avalanche multiplicative factor.

$R_{pin} = \frac{\eta q}{h\nu} \simeq 1,2\,A/W$, "...the responsivities achieved are on the order of... 1.2 A/W in the 1.55 μm band." [RS02, section 3.6.1].

$M \simeq 10$ for a standard InGaAs APD [Per06].

$q = 1,602\cdot 10^{-19}\,C$ $(C = As)$ the electron electric charge.

P_{in} is the mean optical power reaching the receiver.

I_d the receiver dark current. A typical $I_d = 10\,nA$ is a standard in APDs [Per06]
F_A the excess noise factor of the APD. A standard $F_A \simeq 5.5$ is assumed [Per06].

A further refinment for an APD preceded by a an EDFA is to consider to add to I_d the DC current due to ASE, that is the detector responsivity by the DC detected ASE power [JW01] [LCC97]

$$\sigma^2_{shot\_wASE} = 2qM^2F_AR(P_{in} + P^{DC}_{ASE} + I_d)B_e \tag{27}$$

ASE power at EDFA output P^{DC}_{ASE} in the simulation window is the one estimated in the expression 21 in $2,872 \cdot 10^{-4}\,W$. This power is then attenuated at the downstream link to a mere $4,551 \cdot 10^{-7}\,W$. This is further reduced again by detector's optical input filter. Assuming the later to be a perfect filter that slices a $25\,GHz$ bandwidth from the simulation bandwidth $640\,GHz$, power is proportionaly reduced to $P^{DC}_{ASE} = 2,489 \cdot 10^{-8}\,W$. Detector dark current is also considered.

$$\begin{aligned}
\sigma^2_{shot_wASE} &= 2qM^2F_A(R(P_{in} + P^{DC}_{ASE}) + I_d)B_e \\
\sigma^2_{shot_wASE} &\simeq 2 \times 1,602 \cdot 10^{-19}\,A \cdot s \times (10)^2 \times 5.5\times \\
&\left(1.2\,\frac{A}{W} \times \left(4,708 \cdot 10^{-5}\,W + 2,489 \cdot 10^{-8}\,W\right) + 1 \cdot 10^{-8}\,A\right) \times 7 \cdot 10^9\,Hz \\
\sigma^2_{shot_wASE} &\simeq 6,974 \cdot 10^{-11}\,A^2
\end{aligned} \tag{28}$$

Signal-ASE beat noise[LCC97]

$$\begin{aligned}
\sigma^2_{S-ASE} &= 4\frac{B_e}{B_o}M^2\left(I_pI_{ASE}\right) = 4\frac{B_e}{B_o}M^2\left(R^2P_{in}P_{ASE}\right) \\
\sigma^2_{S-ASE} &= 4 \times \frac{7 \cdot 10^9\,Hz}{2.5 \cdot 10^{10}\,Hz} \times (10)^2 \times \left(1,2\,\frac{A}{W}\right)^2 \times \left(4,708 \cdot 10^{-5}\,W \times 2,489 \cdot 10^{-8}\,W\right) \\
\sigma^2_{S-ASE} &\simeq 1,350 \cdot 10^{-10}\,A^2
\end{aligned} \tag{29}$$

Being B_o detectors input optical filter bandwidth.

ASE-ASE beat noise[LCC97]

$$\begin{aligned}
\sigma^2_{ASE-ASE} &= (2B_o - B_e)\frac{B_e}{B_o^2}\left(MI_{ASE}\right)^2 = (2B_o - B_e)\frac{B_e}{B_o^2}\left(MRP_{ASE}\right)^2 \\
\sigma^2_{ASE-ASE} &= (2 \times 2.5 \cdot 10^{10}\,Hz - 7 \cdot 10^9\,Hz) \times \frac{7 \cdot 10^9\,Hz}{(2.5 \cdot 10^{10}\,Hz)^2} \times \left(10 \times 1,2\,\frac{A}{W} \times 2,489 \cdot 10^{-8}\,W\right)^2 \\
\sigma^2_{ASE-ASE} &\simeq 3,211 \cdot 10^{-14}\,A^2
\end{aligned} \tag{30}$$

Detector electrical SNR

Electrical noise added to photocurrent are shot noise and thermal noise currents. These are considered to be independent, so total noise current can be modeled as a Gaussian random process with the following variance

$$\begin{aligned}
\sigma^2_{noise} &= \sigma^2_{shot} + \sigma^2_{thermal} \\
\sigma^2_{noise} &\simeq 6,974 \cdot 10^{-11}\,A^2 + 2,254 \cdot 10^{-12}\,A^2 \\
\sigma^2_{noise} &\simeq 7,199 \cdot 10^{-11}\,A^2
\end{aligned} \tag{31}$$

Electrical current at receiver output signal is $R_{APD} \times P_{in}$ (see 3.7.3), being P_{in} optical power after optical filtering (`Pmean_Rxin` in the simulatios, see 3.7.4) and R_{APD} APD detector responsivity. Electrical photocurrent signal to noise ratio is (see 3.7.3)

$$\begin{aligned} DSNR &= \frac{(R_{APD} \times P_{in})^2}{\sigma^2_{noise}} \\ DSNR &\simeq \frac{\left(12\,\frac{A}{W} \times 4,708 \cdot 10^{-5}\,W\right)^2}{7,199 \cdot 10^{-11}\,A^2} = 4,433 \cdot 10^3 \simeq 36,467\,dB \end{aligned} \tag{32}$$

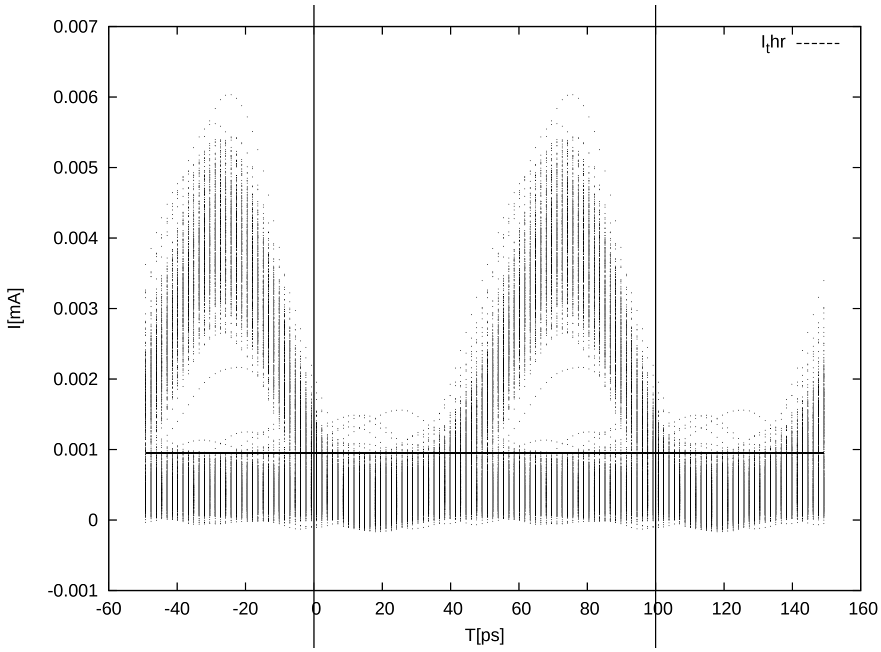

Figure 4: Eye diagrame for output current with $OSNR = 38,6\,dB$ and $DSNR = 15,8\,dB$. Horizontal line is the decision threshold current. Vertical lines mark bit slot limit.

Simulation with detector SNR only

As an alternative no optical noise could be added in the simulation and an DSNR including ASE beat noise could be used.

$$\begin{aligned} \sigma^2_{noise\_alt} &= \sigma^2_{thermal} + \sigma^2_{shot} + \sigma^2_{S-ASE} + \sigma^2_{ASE-ASE} \\ \sigma^2_{noise\_alt} &\simeq 2,254 \cdot 10^{-12}\,A^2 + 1,536 \cdot 10^{-12}\,A^2 + 2,811 \cdot 10^{-12}\,A^2 + 3,211 \cdot 10^{-14}\,A^2 \\ \sigma^2_{noise\_alt} &\simeq 6,704 \cdot 10^{-12}\,A^2 \end{aligned} \tag{33}$$

$$\begin{aligned} DSNR\_alt &= \frac{(R_{APD} \times P_{in})^2}{\sigma^2_{noise\_alt}} \\ DSNR\_alt &\simeq \frac{\left(12\,\frac{A}{W} \times 1.005 \cdot 10^{-6}\,W\right)^2}{6,704 \cdot 10^{-12}\,A} = 2,169 \cdot 10^1 \simeq 13.4\,dB \end{aligned} \tag{34}$$

An $OSNR[dB] = 999$ is used in this case.

3.10 Simulation validation

If the optimal threshold expression is used (see 3.7.7) expected BER is [RS02, expression 4.14]

$$BER = Q(\frac{I_1 - I_0}{\sigma_1 + \sigma_0}) \tag{35}$$

being

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{\frac{-y^2}{2}} dy = \frac{1}{2\sqrt{2}} erfc(x) \tag{36}$$

Neglecting the receiver thermal and shot noises a theoretical expression for the argument of Q is [RS02, expression 4.19]

$$\frac{2 \times \sqrt{\frac{Bo}{Be}} \times OSNR}{1 + \sqrt{1 + 4 \times OSNR}} \tag{37}$$

being Bo and Be optical and electrical filter bandwidths and $OSNR$ in linear scale.

A comparison between this theoretical prediction and a simulation of a RZ link with a $10\,GHz$ bit rate (con duty cycle 1/3), optical filter bandwidth $25\,GHz$, electrical filter bandwidth $14\,GHz$ is shown next.

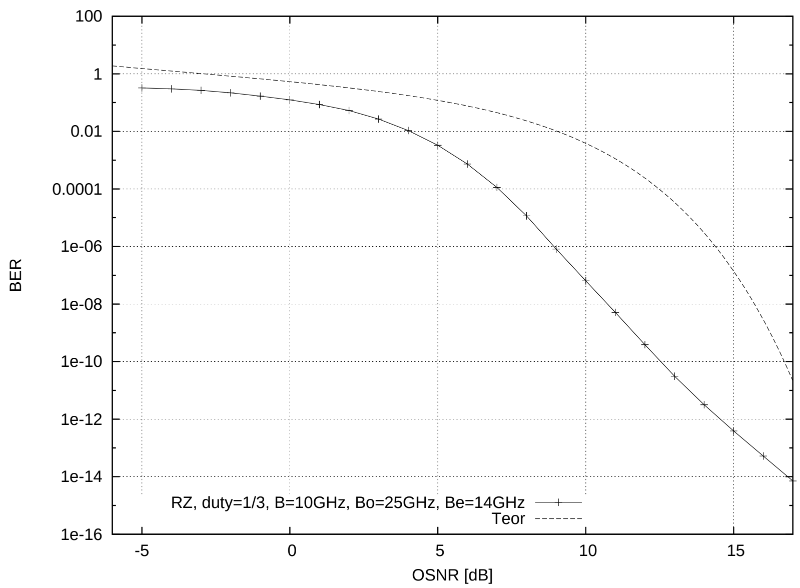

Figure 5: BER vs OSNR: comparison of simulation results to theoretical predictions.

4 Alternative architecture: no EDFA, 128×128 star coupler

4.1 128×128 star coupler

As currently there is no commercialy available 128×128 star coupler. From a linear fit an attenuation of 23.6 dB was estimated for an ideal 128×128 device.

Commercial max insertion loss *AC Photonics* [ACp]
32×32 17.5 dB
16×16 13.6 dB
8×8 10.2 dB

4×4 6.8 dB

BKTel [BKT]
16×16 14.0 dB
8×8 10.6 dB
4×4 7.0 dB

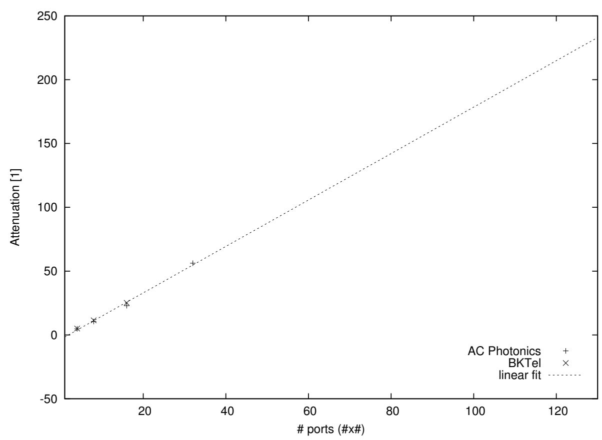

Figure 6: Linear fit to commercial star couplers attenuation data.

As an alternative for simulation, a set of couplers 32×32 could be interconnected to provide 128×128 connections.

4.2 Alteranative architecture: power budget

ONU Tx (see 3.3.2): $P_{\text{out}} = 2\,\text{dBm}$

Upstream (see 3.4.1): 3 dB attenuation $\Rightarrow P_{\text{out}} = -1\,\text{dBm}$

128×128 star coupler: 24 dB conservative attenuation $\Rightarrow P_{\text{out}} = -25\,\text{dBm}$

Downstream (see 3.6.2): 3 dB attenuation $\Rightarrow P_{\text{out}} = -28\,\text{dBm}$

ONU Rx: $-28\,\text{dBm}$ sensitivity $\Rightarrow P_{\text{margin}} = 0\,\text{dBm}$

5 Ideas 120718

- Noise at Tx
- PONs different fibre length

A Components cost

A.1 ONU: Tx power and transceiver cost

Rx, conditions to comply

1. sensitivty $\leq -28dBm$

Following table illustrate on the subject.
[http://www.ilxlightwave.com/laser-diode/laser-diode-new-used.html]

Model	λ [nm]	P [mW]	Cost [$]
Fermionics Lasertech with ILX LIV LD-1550	1548.811	1	300
Fitel FOL15DCWD-A81-19510-B [DFB]	1535.036	40	700

B Fibre attenuation

Accordingly to following figures we assume an attenuation $0.2\frac{\mathrm{dB}}{\mathrm{km}}$ in current commercial fibres.
Corning® ClearCurve® single-mode optical fiber (ITU-T G.652 compliant)
[http://www.corning.com:80/WorkArea/showcontent.aspx?id=25577]

λ [nm]	Attenuation [dB/km]	Dispersion [ps/(nm*km)]
1310	0.35	-
1550	0.21	≤ 18

Corning SMF 28 ULL Single Mode Fiber
[http://www.corning.com:80/WorkArea/showcontent.aspx?id=14357]

λ [nm]	Attenuation [dB/km]	Dispersion [ps/(nm*km)]
1310	0.4	-
1550	0.18	≤ 18

C Splitter attenuation

Wave-2-wave 1x16 (actual): [http://www.wave-2-wave.com/documents/DS-Splitters-v1-Web.pdf]: max insertion loss 17.3 dB.

Coretech 1x64 (actual): [http://www.coretechone.com/en/p/Product-200771314229_9.html]: max insertion loss 22.4 dB.

Zhone 1x64 (actual): [http://www.zhone.com/products/ZPON/ZPON.pdf]: max insertion loss 21.5 dB.

1x128 (hypothetical): 7 2x2 splitter stages (each one 3 dB) with 6 interstages joints high loss (0.5 dB) $\rightarrow 7 \times 3\ dB + 6 \times 0.5\ dB = 24\ dB$.

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