A rational mechanics course where everything is made with Python code

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• Professor: prepares lessons $\xrightarrow{by\ heart}$ blackboard/slides

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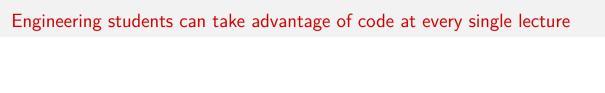
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- Student: course repository \xrightarrow{clones} its own modifiable one
- Recycles professor's code to solve new problems



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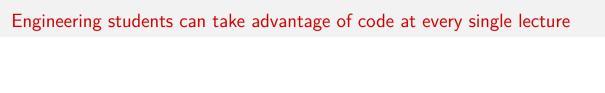


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- Now they'll employ computational algebra after they learnt algebra and calculus
 - ► Focus on new skills, not in automatable calculations
 - Employing numerical calculus they solve what is impossible in a blackboard/paper





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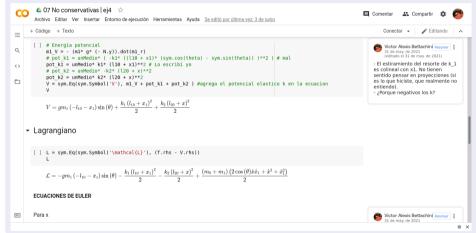
- An example problem is solved by the professor provided code
- The student recycles it to solve other related problems
- Gradually becoming autonomous by reusing not the provided but his own code

Tools — Jupyter notebook: text + equations + executable code



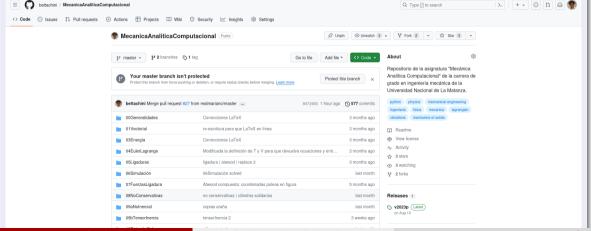
Tools — Google Colaboratory runs Jupyter notebooks on-line

- The platform allows professors to edit and comment on the work of students.
- Students can collaborate remotely, working on the same Jupyter notebook.



Tools — All course material in a GitHub repository

- Clearly organised, freely accessible, and easy to update.
- Google Colaboratory loads Jupyter notebooks directly from GitHub.



Tools — From the 1st class — LATEX: concise mathematical notation

• LATEX typesetting follows strictly the standards of the American Mathematical Society

Considero que el potencial V es nulo en el origen de coordenadas, es decir que donde se encuentra su mínimo $\omega=0, V(\omega=0)=-mg\ell'$ y por tanto

$$V(\varphi) = mg(-\ell\cos\varphi) = -mg\ell\cos\varphi,$$

Como vemos la aproximación funciona bastante bien. Conformes con ella calculamos la fuerza

$$\vec{F} = -\vec{\nabla}V = -\left(\frac{\partial}{\partial r}, \frac{1}{r}\frac{\partial}{\partial \varphi}, \frac{\partial}{\partial z}\right)V(\varphi)$$

Pero solo nos interesa expresar la 2.a ley de Newton para lo que pasa en $\hat{oldsymbol{arphi}}$

$$\vec{mr} \cdot \hat{\varphi} = -\frac{1}{r} \frac{\partial}{\partial \varphi} V(\varphi)$$

En el lado izquierdo de la expresión de la aceleración en cílindricas $\ddot{\vec{r}} = (\ddot{r} - r\dot{\phi}^2)\hat{r} + (\dot{r}\dot{\phi}^2 + r\ddot{\phi})\hat{\phi} + \ddot{z}\hat{z}$, nos quedamos solo con la componente en $\hat{\phi}$,

$$\ddot{\vec{r}} \cdot \hat{\varphi} = \dot{r} \dot{\varphi}^2 + r \ddot{\varphi}$$

y como el hilo del péndulo es rígido e inextensible $r \equiv \ell$ solo queda de esto

$$\ddot{\vec{r}} \cdot \hat{\varphi} = \ell \ddot{\varphi}$$

En el lado derecho la derivada del potencial respecto a φ es

$$\frac{\partial}{\partial \varphi} V(\varphi) = mg\ell \sin(\varphi)$$

Tools — 1st class — Matplotlib: precise and reproducible graphics

• Explicit Python code for producing graphics. Students are enticed to play with it.

```
# graficación
fig. ax = plt.subplots(figsize=(12, 4))
ax.plot(tiempos, phi(tiempos), 'o-')
ax.set xlabel('Tiempo [sl')
ax.set vlabel(r'$\varphi$ [rad]')
Text(0, 0.5, '$\\varphi$ [rad]')
     0.15
     0.10
     0.05
     0.00
    -0.05
    -0.10
    -0.15
              O
                                                                                                      10
                                                     Tiempo [s]
```

Tools — 3rd class — SymPy: symbolic calculations

• Student's effort centred on new subjects by freeing them of calculus and algebraic tasks.

```
[8]: m2_v_cuadrado = m2_v.dot(m2_v)
m2_v_cuadrado
```

[8]:
$$\ell^2 \sin^2(\varphi)\dot{\varphi}^2 + (\ell\cos(\varphi)\dot{\varphi} + \dot{x})^2$$

Con esto la energía cinética queda

$$T(\dot{x}_1, \varphi, \dot{\varphi}) = \frac{m_1}{2} \left(\dot{\vec{r}}_1\right)^2 + \frac{m_2}{2} \left(\dot{\vec{r}}_2\right)^2$$
$$= \frac{m_1}{2} \dot{x}^2 + \frac{m_2}{2} \left(\dot{x}^2 + 2\dot{x}\ell\cos\varphi\dot{\varphi} + l^2\dot{\varphi}^2\right)$$

- [9]: # Energía cinética
 unMedio = sym.Rational(1,2) # Rational: fracción de enteros, alternativamente podría haberse usado 0.5
 m1_T = unMedio* m1* m1_v_cuadrado
 m2_T = unMedio* m2* m2_v_cuadrado
 T = sym.Eq(sym.Symbol('T'), (m1_T + m2_T)) # simplify: simplifica usando factor común y otras operaciones
 T
- $T = \frac{m_1 \dot{x}^2}{2} + \frac{m_2 \left(\ell^2 \sin^2 (\varphi) \dot{\varphi}^2 + (\ell \cos (\varphi) \dot{\varphi} + \dot{x})^2 \right)}{2}$

Tools — 4th class — Equations for Lagrangian dynamics

Ecuaciones de Euler-Lagrange

Para x

[8]:
$$m_1\ddot{x} + m_2\left(-\ell\sin(\phi)\dot{\phi}^2 + \ell\cos(\phi)\ddot{\phi} + \ddot{x}\right) = 0$$

Esta es una ecuación diferencial lineal de segundo orden homogena. De aquí podría despejarse \ddot{x}

$$\ddot{x} = \frac{\ell m_2 \left(\sin \left(\phi \right) \dot{\phi}^2 - \cos \left(\phi \right) \ddot{\phi} \right)}{m_1 + m_2}$$

Pero queda en función de otra aceleración $\dot{\phi}$.

Para ϕ

[10]: phi_EL = sym.Eq(L.rhs.diff(phi) - L.rhs.diff(phi.diff(t)).diff(t), 0).simplify() # ecuación igualando a cero
phi FL

Tools — 4th class — Automatation of resolutions

Mathematical complexity doesn't limit the scope of tackled mechanical problems.

```
[14]: sistemaEcuaciones = [
              x EL,
               phi EL,
         variablesDespeje = [x.diff(t,2), phi.diff(t,2)] # despejar aceleraciones generalizadas
         variablesDespeje sol= sym.nonlinsolve(sistemaEcuaciones, variablesDespeje ).args[0]
[15]: x pp = sym.Eq(variablesDespeie[0], variablesDespeie sol.args[0] ) # [m s-2]
         phi pp = sym.Eg(variablesDespeie[1], variablesDespeie sol.args[1]) # [m s-2]
         x pp. phi pp
         \left| \ddot{x} = \frac{-\ell g m_2 \sin(\phi) + \frac{\ell m_2 \left(\ell m_2 \cos(\phi) \dot{\phi}^2 + g m_1 + g m_2\right) \sin(\phi)}{m_1 + m_2 \sin^2(\phi)}}{\ell m_2 \cos(\phi)}, \ \ddot{\phi} = -\frac{\left(\ell m_2 \cos(\phi) \dot{\phi}^2 + g m_1 + g m_2\right) \sin(\phi)}{\ell \left(m_1 + m_2 \sin^2(\phi)\right)} \right|
```

Tools — 5th class — SciPy: numerical computation of results

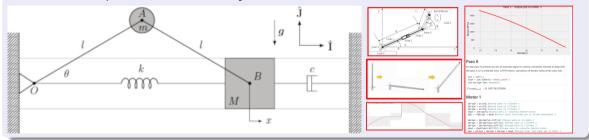
```
[22]: # defino una función con el sistema de derivadas
     # t : no se usa en este sistema pero lo dejamos para uso posterior
     # y : lista de estado con [y[0], y[1], y[2], y[3]]
     # v[0]: x
     # v[1]: x punto
     # v[2]: phi
     # y[3]: phi punto
     # dvdt : lista de derivadas
     def y punto(t, y):
         dvdt = [v[1],
                x pp numpy(y[0], y[1], y[2], y[3]),
                v[31.
                phi pp numpy(v[0], v[1], v[2], v[3]),
         return dvdt
[23]: # Integración de a pasos en el tiempo
     y ode2 = solve ivp(y punto, (t rango[0], t rango[-1]), y inicial, t eval = t rango)
[25]: y ode2.y[0]
0.87877042. 0.87745354. 0.87702754. 0.87352768. 0.86357726.
            0.84474673, 0.81565733, 0.77559949, 0.72423163, 0.66166451,
```

Tools — 5th class — Graphical analysis of numerical results

```
solucion = y ode2
[26]:
      nombreCoordenada = 'x'
      fig, ax = plt.subplots(nrows= 1, ncols= 2, squeeze=False, figsize=(12, 4)) # dos figuras en la misma fila
      fig.suptitle('Integración numérica para $'+ nombreCoordenada + '$', fontsize=16)
      ax[0,0].plot(solucion.t, solucion.y[0]) # posición x
      ax[0,0].set(xlabel='t [s]', ylabel= '$' + nombreCoordenada+ '$ [m]', title='Posición')
      ax[0,1].plot(solucion.t, solucion.y[1]) # velocidad x
      ax[0.1].set(xlabel='t [s]', vlabel='$\dot{' + nombreCoordenada+ '}\$ [m/s]', title='Velocidad')
[26]: [Text(0.5, 0, 't [s]'),
       Text(0, 0.5, '\$\dot{x}\ [m/s]'),
       Text(0.5, 1.0, 'Velocidad')]
                                              Integración numérica para x
                              Posición
                                                                                   Velocidad
                                                               0.0
                                                              -0.2
           0
                                                              -0.4
      Ε -1
× -2
        vbettachini@unlam.edu.ar
                                              A code centred mechanics subject
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                                                                                                                  14 / 21
```

Tools — 7th class — Adding complexity

• Code from previous classes is **recycled** to model more realistic devices.



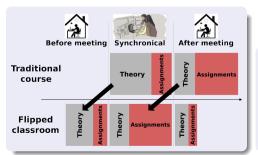
Tools — 8th class — GitHub Copilot: Al assistance for coding

- Now that the students grasp the basics they're invited to take advantage of AI
- After commenting in plain text what they need code is suggested

```
lagrangiano = (T.rhs - V.rhs).expand()
         t = sym.Symbol('t') # como se deriva respecto al tiempo con la función diff se declara t como símbolo
         return sym.Eq(
             lagrangiano.diff(coordenadaGeneralizada)
             - lagrangiano.diff(coordenadaGeneralizada.diff(t)).diff(t)
         ).simplify()
    x1 EL = eulerLagrange(T, V, x1)
    x1 EL
\frac{\pi^2 M \ddot{x}_1}{2} - g m_1 + g m_2 + m_1 \ddot{x}_1 + m_2 \ddot{x}_1 = 0
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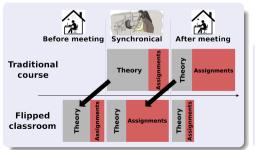
#Despeiar x1PuntoPunto

• New theory (Jupyter notebooks and videos) and assignments published on-line



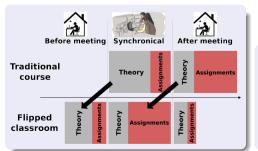
Synchronic	Theory	Assignments
Before	Read and apply	Start them
During	Consultations	Complete them
After	Additional consultations	TA's corrections

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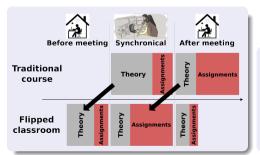
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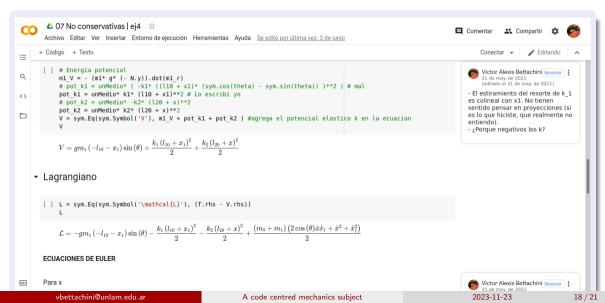
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- synchronic meetings with TA's to finish assignments

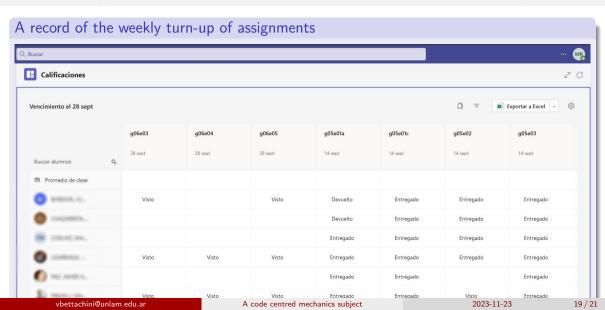


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Methodology — Google Colaboratory: asynchronic remote assistance



Methodology — Microsoft Teams: individualized student follow-up



Thank You!

Summary

A course centred on code

- Material: text + equations + executable code in digital notebooks in a repository
- Assignments: professor's code recycling
- On-line:
 - Remote collaboration and correction
 - Doesn't require powerful nor at campus computers
 - Al assistance for coding
 - Dated record of each student's work

Flipped classroom

- Theory: emphasis on student's autonomus reading
- Reinforced by: suggested bibliography and short professor's videos
- Consultations: mostly on-line asynchronic and publicly accessible
- Synchronic mettings: TA's personal assistance for completing assignments