A rational mechanics course where everything is made with Python code

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New Media Pedagogy 23: research trends, methodological challenges and succesful implementations
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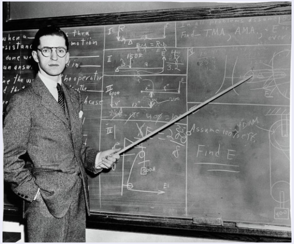

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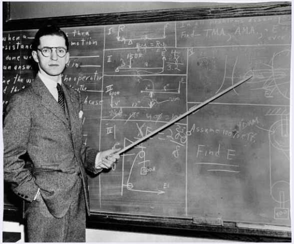

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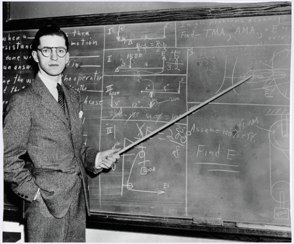

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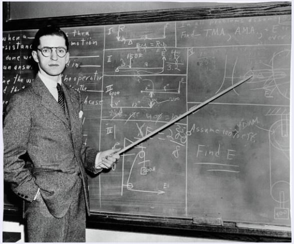

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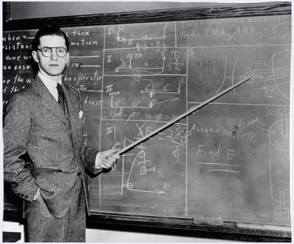

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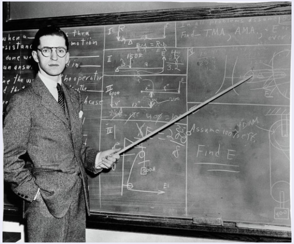

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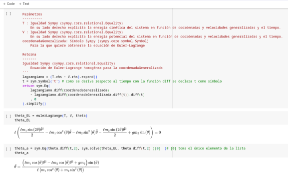

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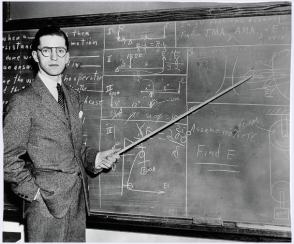

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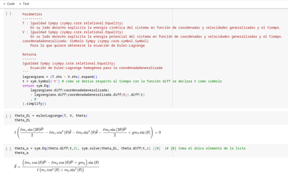

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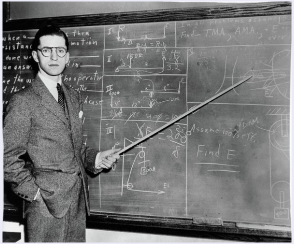

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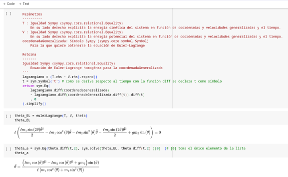

Digital course material with embedded code

- Professor: ideas $\xrightarrow{\textit{updates}}$ code/theory in repository
- Student: course repository $\xrightarrow{\textit{clones}}$ its own modifiable one
- **Recycles** professor's code to solve new problems

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```
[14]: sistemaEcuaciones = [
          x_EL,
          phi_EL,
      ]
      variablesDespeje = [x.diff(t,2), phi.diff(t,2)] # despejar aceleraciones generalizadas
      variablesDespeje_sol= sym.nonlinsolve(sistemaEcuaciones, variablesDespeje ).args[0]
```

```
[15]: x_pp = sym.Eq(variablesDespeje[0], variablesDespeje_sol.args[0] ) # [m s-2]
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$$\left(\ddot{x} = \frac{-\ell g m_2 \sin(\phi) + \frac{\ell m_2 \left(\ell m_2 \cos(\phi)\dot{\phi}^2 + g m_1 + g m_2\right) \sin(\phi)}{m_1 + m_2 \sin^2(\phi)}}{\ell m_2 \cos(\phi)}, \ \ddot{\phi} = -\frac{\left(\ell m_2 \cos(\phi)\dot{\phi}^2 + g m_1 + g m_2\right)\sin(\phi)}{\ell \left(m_1 + m_2 \sin^2(\phi)\right)} \right)$$

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  - Focus on new skills, not in automatable calculations
  - Employing numerical calculus they solve what is impossible in a blackboard/paper

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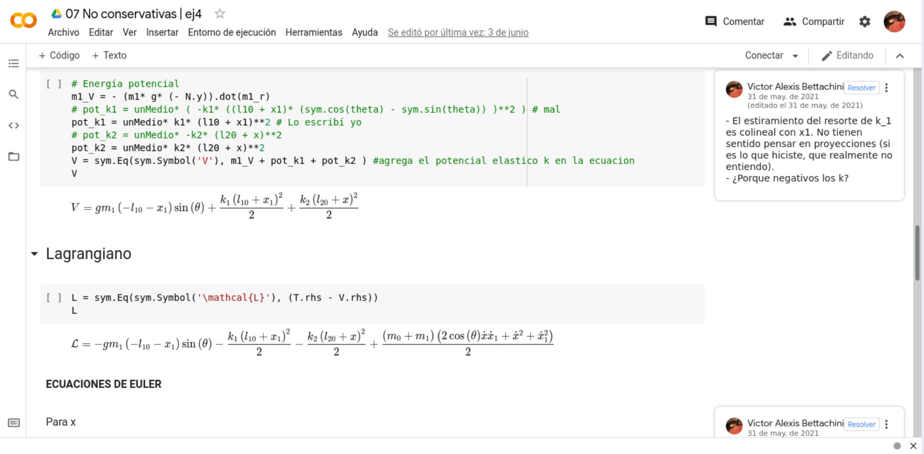
Papert (1980) "...the best learning takes place when the learner takes charge"

- An example problem is solved by the professor provided code
- The student **recycles** it to solve other related problems
- Gradually becoming autonomous by reusing not the provided but his own code

Tools — Jupyter notebook: text + equations + executable code

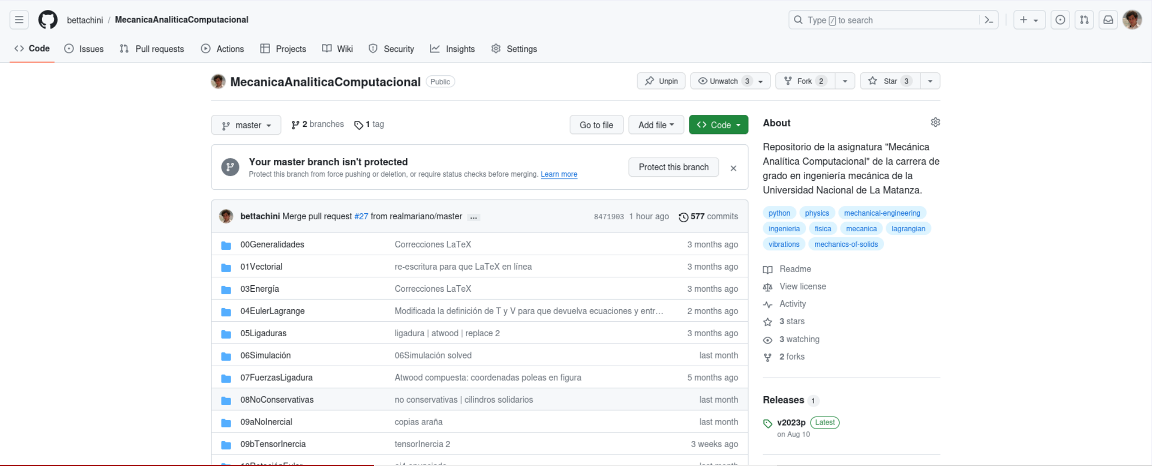
Tools — Google Colaboratory runs Jupyter notebooks on-line

- The platform allows professors to edit and comment on the work of students.
- Students can collaborate remotely, working on the same Jupyter notebook.



Tools — All course material in a GitHub repository

- Clearly organised, freely accessible, and easy to update.
- Google Colaboratory loads Jupyter notebooks directly from GitHub.



- LaTeX typesetting follows strictly the standards of the American Mathematical Society

Considero que el potencial V es nulo en el origen de coordenadas, es decir que donde se encuentra su mínimo $\varphi = 0, V(\varphi = 0) = -mg\ell$ y por tanto

$$V(\varphi) = mg\,(-\ell\cos\varphi) = -mg\ell\cos\varphi,$$

Como vemos la aproximación funciona bastante bien. Conformes con ella calculamos la fuerza

$$\vec{F} = -\vec{\nabla}V = -\left(\frac{\partial}{\partial r}, \frac{1}{r}\frac{\partial}{\partial\varphi}, \frac{\partial}{\partial z}\right)V(\varphi)$$

Pero solo nos interesa expresar la 2.a ley de Newton para lo que pasa en $\hat{\varphi}$

$$m\ddot{\vec{r}}\cdot\hat{\varphi} = -\frac{1}{r}\frac{\partial}{\partial\varphi}V(\varphi)$$

En el lado izquierdo de la expresión de la aceleración en cílindricas $\ddot{\vec{r}} = (\ddot{r} - r\dot{\varphi}^2)\hat{r} + (\dot{r}\dot{\varphi}^2 + r\ddot{\varphi})\hat{\varphi} + \ddot{z}\hat{z}$, nos quedamos solo con la componente en $\hat{\phi}$,

$$\ddot{\vec{r}}\cdot\hat{\varphi} = \dot{r}\dot{\varphi}^2 + r\ddot{\varphi}$$

y como el hilo del péndulo es rígido e inextensible $r \equiv \ell$ solo queda de esto

$$\ddot{\vec{r}}\cdot\hat{\varphi} = \ell\ddot{\varphi}$$

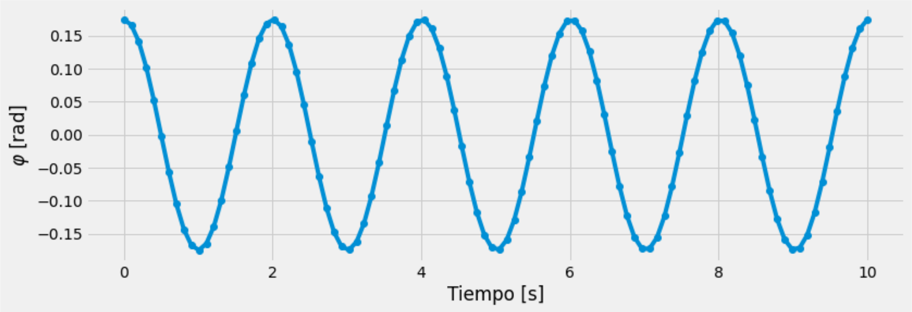
En el lado derecho la derivada del potencial respecto a φ es

$$\frac{\partial}{\partial\varphi}V(\varphi) = mg\ell\sin(\varphi)$$

- Explicit Python code for producing graphics. Students are enticed to play with it.

```
# graficación
fig, ax = plt.subplots(figsize=(12, 4))
ax.plot(tiempos, phi(tiempos), 'o-' )
ax.set_xlabel('Tiempo [s]')
ax.set_ylabel(r'$\varphi$ [rad]')
```

```
Text(0, 0.5, '$\\varphi$ [rad]')
```



- Student's effort centred on new subjects by freeing them of calculus and algebraic tasks.

[8]:
```
m2_v_cuadrado = m2_v.dot(m2_v)
m2_v_cuadrado
```

[8]: $\ell^2 \sin^2(\varphi)\dot{\varphi}^2 + (\ell \cos(\varphi)\dot{\varphi} + \dot{x})^2$

Con esto la energía cinética queda

$$
\begin{aligned}
T(\dot{x}_1, \varphi, \dot{\varphi}) &= \frac{m_1}{2}\left(\dot{\vec{r}}_1\right)^2 + \frac{m_2}{2}\left(\dot{\vec{r}}_2\right)^2 \\
&= \frac{m_1}{2}\dot{x}^2 + \frac{m_2}{2}\left(\dot{x}^2 + 2\dot{x}\ell\cos\varphi\dot{\varphi} + l^2\dot{\varphi}^2\right)
\end{aligned}
$$

[9]:
```
# Energía cinética
unMedio = sym.Rational(1,2) # Rational: fracción de enteros, alternativamente podría haberse usado 0.5
m1_T = unMedio* m1* m1_v_cuadrado
m2_T = unMedio* m2* m2_v_cuadrado
T = sym.Eq(sym.Symbol('T'), (m1_T + m2_T) ) # simplify: simplifica usando factor común y otras operaciones
T
```

[9]: $$T = \frac{m_1\dot{x}^2}{2} + \frac{m_2\left(\ell^2\sin^2(\varphi)\dot{\varphi}^2 + (\ell\cos(\varphi)\dot{\varphi} + \dot{x})^2\right)}{2}$$

Ecuaciones de Euler-Lagrange

Para x

[8]:
```
x_EL = sym.Eq(L.rhs.diff(x) - L.rhs.diff(x.diff(t)).diff(t), 0 ).simplify() # ecuación igualando a cero
x_EL
```

[8]:

$$m_1 \ddot{x} + m_2 \left(-\ell \sin(\phi)\dot{\phi}^2 + \ell \cos(\phi)\ddot{\phi} + \ddot{x}\right) = 0$$

Esta es una ecuación diferencial lineal de segundo orden homogena. De aquí podría despejarse $\ddot{x}$

[9]:
```
sym.Eq(x.diff(t,2),
       list( sym.solveset(x_EL, x.diff(t,2) ) )[0]  # solveset devuelve un set, que convertimos a lista
      ) # aceleración = x punto punto [m s-2]
```

[9]:

$$\ddot{x} = \frac{\ell m_2 \left(\sin(\phi)\dot{\phi}^2 - \cos(\phi)\ddot{\phi}\right)}{m_1 + m_2}$$

Pero queda en función de otra aceleración $\ddot{\phi}$.

Para ϕ

[10]:
```
phi_EL = sym.Eq(L.rhs.diff(phi) - L.rhs.diff(phi.diff(t)).diff(t), 0 ).simplify() # ecuación igualando a cero
phi_EL
```

- Mathematical complexity doesn't limit the scope of tackled mechanical problems.

[14]:
```
sistemaEcuaciones = [
    x_EL,
    phi_EL,
]
variablesDespeje = [x.diff(t,2), phi.diff(t,2)] # despejar aceleraciones generalizadas
variablesDespeje_sol= sym.nonlinsolve(sistemaEcuaciones, variablesDespeje ).args[0]
```

[15]:
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x_pp = sym.Eq(variablesDespeje[0], variablesDespeje_sol.args[0] ) # [m s-2]
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```
[22]: # defino una función con el sistema de derivadas
      # t : no se usa en este sistema pero lo dejamos para uso posterior
      # y : lista de estado con [y[0], y[1], y[2], y[3]]
      # y[0]: x
      # y[1]: x punto
      # y[2]: phi
      # y[3]: phi punto
      # dydt : lista de derivadas
      def y_punto(t, y):
          dydt = [y[1],
                  x_pp_numpy(y[0], y[1], y[2], y[3]),
                  y[3],
                  phi_pp_numpy(y[0], y[1], y[2], y[3]),
                 ]
          return dydt
```

```
[23]: # Integración de a pasos en el tiempo
      y_ode2 = solve_ivp(y_punto, (t_rango[0], t_rango[-1] ), y_inicial, t_eval = t_rango)
```

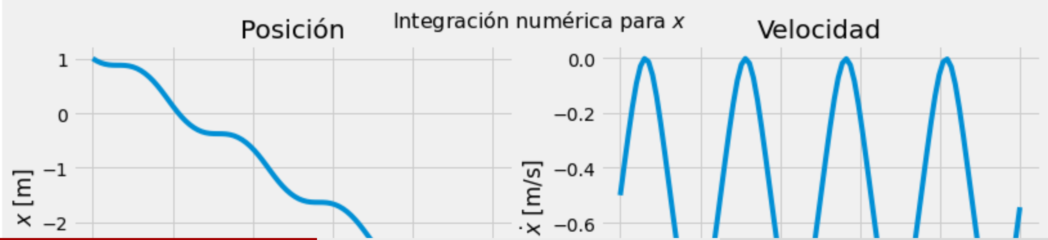
```
[25]: y_ode2.y[0]
```

```
[25]: array([ 1.        ,  0.95510744,  0.92131146,  0.89820932,  0.88468059,
              0.87877042,  0.87745354,  0.87702754,  0.87352768,  0.86357726,
              0.84474673,  0.81565733,  0.77559949,  0.72423163,  0.66166451,
```

```
[26]: solucion = y_ode2
      nombreCoordenada = 'x'

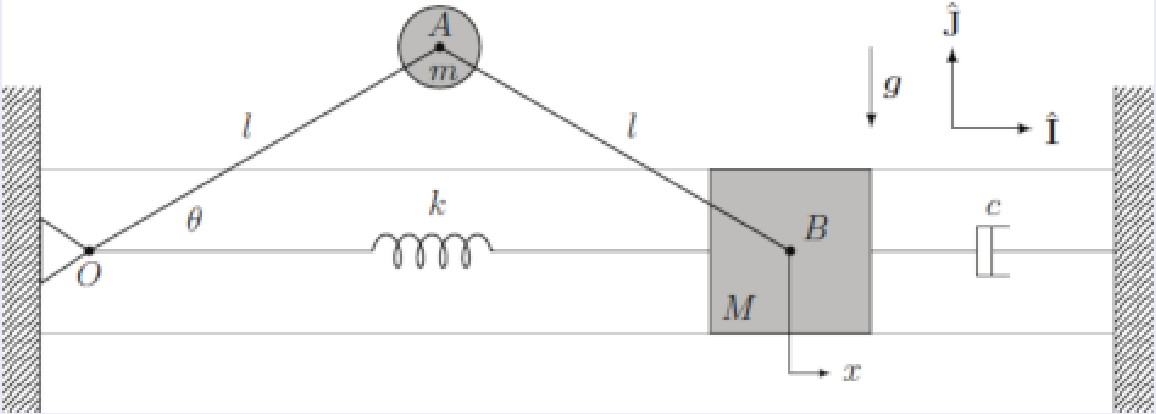
      fig, ax = plt.subplots(nrows= 1, ncols= 2, squeeze=False, figsize=(12, 4)) # dos figuras en la misma fila
      fig.suptitle('Integración numérica para $'+ nombreCoordenada + '$', fontsize=16)
      ax[0,0].plot(solucion.t, solucion.y[0]) # posición x
      ax[0,0].set(xlabel='t [s]', ylabel= '$' + nombreCoordenada+ '$ [m]', title='Posición')
      ax[0,1].plot(solucion.t, solucion.y[1]) # velocidad x
      ax[0,1].set(xlabel='t [s]', ylabel='$\dot{' + nombreCoordenada+ '}$ [m/s]', title='Velocidad')
```

```
[26]: [Text(0.5, 0, 't [s]'),
       Text(0, 0.5, '$\\dot{x}$ [m/s]'),
       Text(0.5, 1.0, 'Velocidad')]
```



Tools — 7th class — Adding complexity

- Code from previous classes is **recycled** to model more realistic devices.



- Now that the students grasp the basics they're invited to take advantage of AI
- After commenting in plain text what they need code is suggested

```
    lagrangiano = (T.rhs - V.rhs).expand()
    t = sym.Symbol('t') # como se deriva respecto al tiempo con la función diff se declara t como símbolo
    return sym.Eq(
        lagrangiano.diff(coordenadaGeneralizada)
        - lagrangiano.diff(coordenadaGeneralizada.diff(t)).diff(t)
        , 0
    ).simplify()
```

[21]

```
x1_EL = eulerLagrange(T, V, x1)
x1_EL
```

[22]

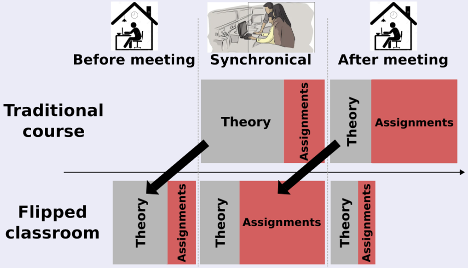
$$\frac{\pi^2 M \ddot{x}_1}{2} - g m_1 + g m_2 + m_1 \ddot{x}_1 + m_2 \ddot{x}_1 = 0$$

Esta es una ecuación diferencial lineal de segundo orden homogena. De aquí se puede despejar $\ddot{x}$

```
#Despejar x1PuntoPunto
x1PuntoPunto = sym.solveset(x1_EL, x1.diff(t, t)).args[0]
```

Methodology — Weekly cycle of our flipped classroom

- New theory (Jupyter notebooks and videos) and assignments published on-line



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Before	Read and apply	Start them
During	Consultations	Complete them
After	Additional consultations	TA's corrections

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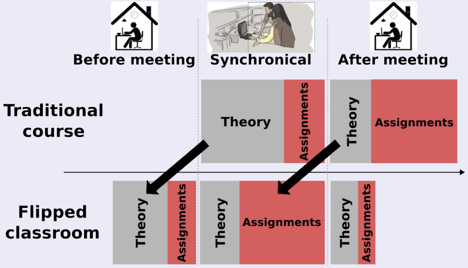

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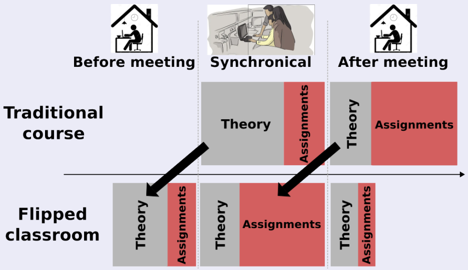

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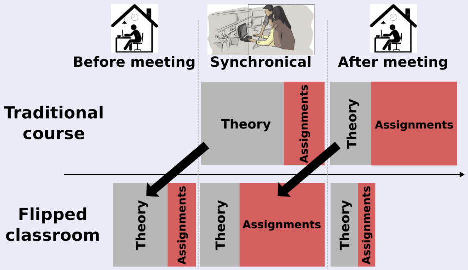
- On-line 24/7 **asynchronic** consultations that are **public** to other students
- **synchronic** meetings with TA's to finish assignments



Synchronic	Theory	Assignments
Before	Read and apply	Start them
During	Consultations	Complete them
After	Additional consultations	TA's corrections

Methodology — Google Colaboratory: asynchronic remote assistance

07 No conservativas | ej4

Archivo Editar Ver Insertar Entorno de ejecución Herramientas Ayuda Se editó por última vez: 3 de junio

Comentar Compartir

+ Código + Texto

Conectar Editando

```
[ ] # Energía potencial
    m1_V = - (m1* g* (- N.y)).dot(m1_r)
    # pot_k1 = unMedio* ( -k1* ((l10 + x1)* (sym.cos(theta) - sym.sin(theta)) )**2 ) # mal
    pot_k1 = unMedio* k1* (l10 + x1)**2 # Lo escribí yo
    # pot_k2 = unMedio* -k2* (l20 + x)**2
    pot_k2 = unMedio* k2* (l20 + x)**2
    V = sym.Eq(sym.Symbol('V'), m1_V + pot_k1 + pot_k2 ) #agrega el potencial elastico k en la ecuacion
    V
```

$$V = gm_1 \left(-l_{10} - x_1\right) \sin\left(\theta\right) + \frac{k_1 \left(l_{10} + x_1\right)^2}{2} + \frac{k_2 \left(l_{20} + x\right)^2}{2}$$

Victor Alexis Bettachini Resolver
31 de may. de 2021
(editado el 31 de may. de 2021)

- El estiramiento del resorte de k_1 es colineal con x1. No tienen sentido pensar en proyecciones (si es lo que hiciste, que realmente no entiendo).
- ¿Porque negativos los k?

Lagrangiano

```
[ ] L = sym.Eq(sym.Symbol('\mathcal{L}'), (T.rhs - V.rhs))
    L
```

$$\mathcal{L} = -gm_1 \left(-l_{10} - x_1\right) \sin\left(\theta\right) - \frac{k_1 \left(l_{10} + x_1\right)^2}{2} - \frac{k_2 \left(l_{20} + x\right)^2}{2} + \frac{\left(m_0 + m_1\right) \left(2 \cos\left(\theta\right) \dot{x} \dot{x}_1 + \dot{x}^2 + \dot{x}_1^2\right)}{2}$$

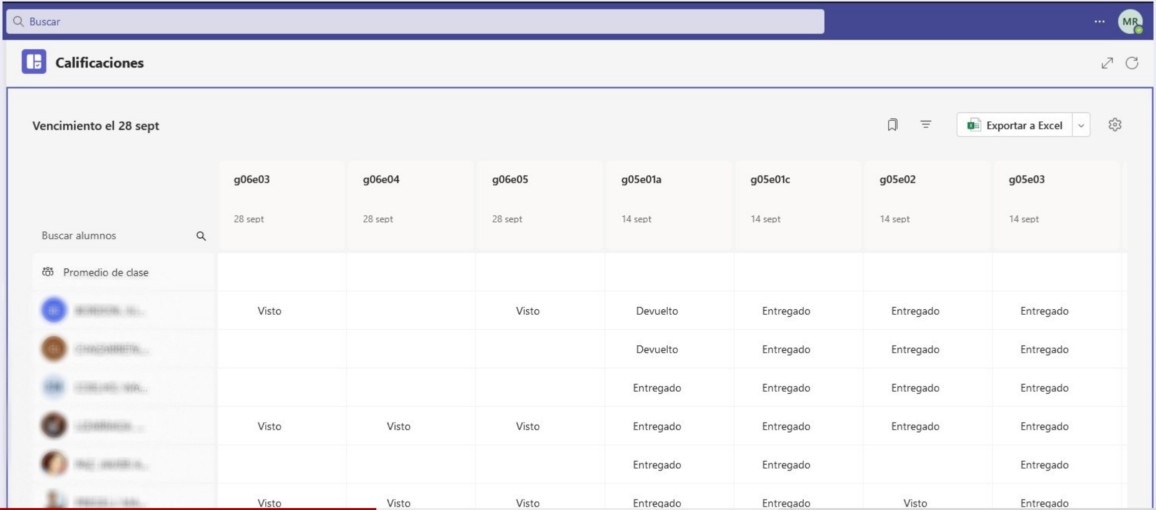
ECUACIONES DE EULER

Para x

Victor Alexis Bettachini Resolver
31 de may. de 2021

Methodology — Microsoft Teams: individualized student follow-up

A record of the weekly turn-up of assignments



Thank You!

Summary

A course centred on code

- Material: text + equations + executable code in digital notebooks in a repository
- Assignments: professor's code recycling
- On-line:
  - Remote collaboration and correction
  - Doesn't require powerful nor at campus computers
  - AI assistance for coding
  - Dated record of each student's work

Flipped classroom

- Theory: emphasis on student's autonomus reading
- Reinforced by: suggested bibliography and short professor's videos
- Consultations: mostly on-line asynchronic and publicly accessible
- Synchronic mettings: TA's personal assistance for completing assignments