Universidad Nacional de La Matanza

DIIT

Departamento de Ingeniería e Investigaciones Tecnológicas

A rational mechanics course where everything is made with Python code

Bettachini, Víctor A.; Real, Mariano A.; Palazzo, Edgardo

New Media Pedagogy 23

Value students and professors time

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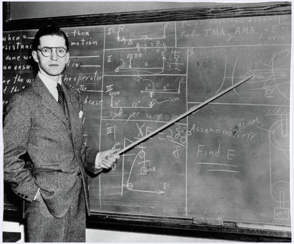

Classroom and practice: an excercise on transcription

- Professor: lessons $\xrightarrow{\textit{by heart}}$ blackboard/slides

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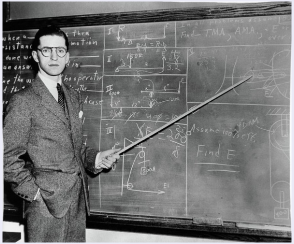

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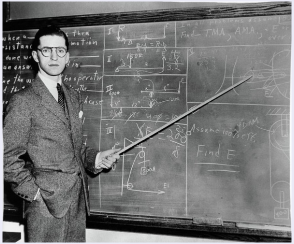

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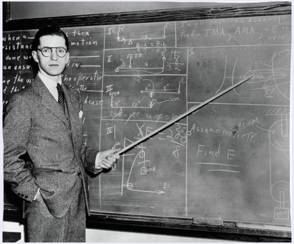

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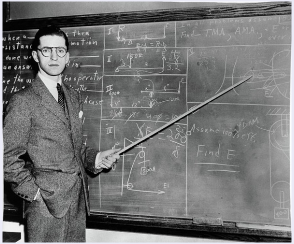

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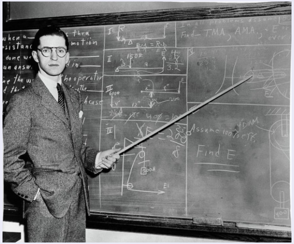

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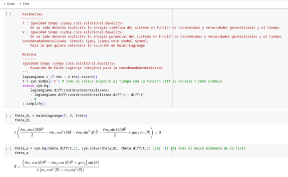

+ Code + Text

```
    Parámetros
    ----------
    T : Igualdad Sympy (sympy.core.relational.Equality)
        En su lado derecho explicita la energía cinética del sistema en función de coordenadas y velocidades generalizadas y el tiempo.
    V : Igualdad Sympy (sympy.core.relational.Equality)
        En su lado derecho explicita la energía potencial del sistema en función de coordenadas y velocidades generalizadas y el tiempo.
    coordenadaGeneralizada: Símbolo Sympy (sympy.core.symbol.Symbol)
        Para la que quiere obtenerse la ecuación de Euler-Lagrange

    Retorna
    -------
    Igualdad Sympy (sympy.core.relational.Equality)
        Ecuación de Euler-Lagrange homogénea para la coordenadaGeneralizada
    '''
    lagrangiano = (T.rhs - V.rhs).expand()
    t = sym.Symbol('t') # como se deriva respecto al tiempo con la función diff se declara t como símbolo
    return sym.Eq(
        lagrangiano.diff(coordenadaGeneralizada)
        - lagrangiano.diff(coordenadaGeneralizada.diff(t)).diff(t)
        , 0
    ).simplify()
```

```
theta_EL = eulerLagrange(T, V, theta)
theta_EL
```

$$\ell\left(\frac{\ell m_1 \sin(2\theta)\dot{\theta}^2}{2} - \ell m_1 \cos^2(\theta)\ddot{\theta} - \ell m_2 \sin^2(\theta)\ddot{\theta} - \frac{\ell m_2 \sin(2\theta)\dot{\theta}^2}{2} + g m_2 \sin(\theta)\right) = 0$$

```
theta_a = sym.Eq(theta.diff(t,2), sym.solve(theta_EL, theta.diff(t,2) )[0]  )# [0] toma el único elemento de la lista
theta_a
```

$$\ddot{\theta} = \frac{\left(\ell m_1 \cos(\theta)\dot{\theta}^2 - \ell m_2 \cos(\theta)\dot{\theta}^2 + g m_2\right)\sin(\theta)}{\ell\left(m_1 \cos^2(\theta) + m_2 \sin^2(\theta)\right)}$$

- Professor: ideas $\rightarrow$ new code/notes in repository

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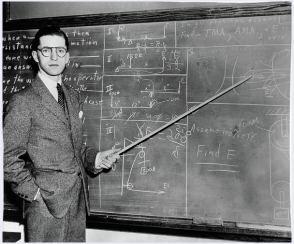

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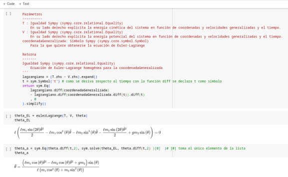

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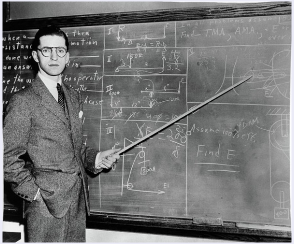

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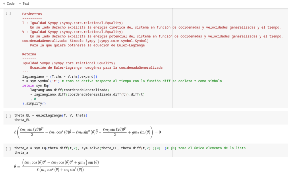

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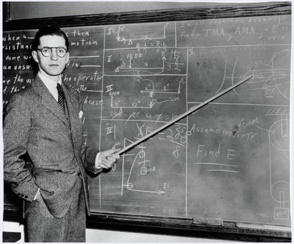

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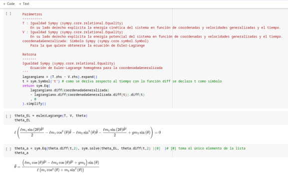
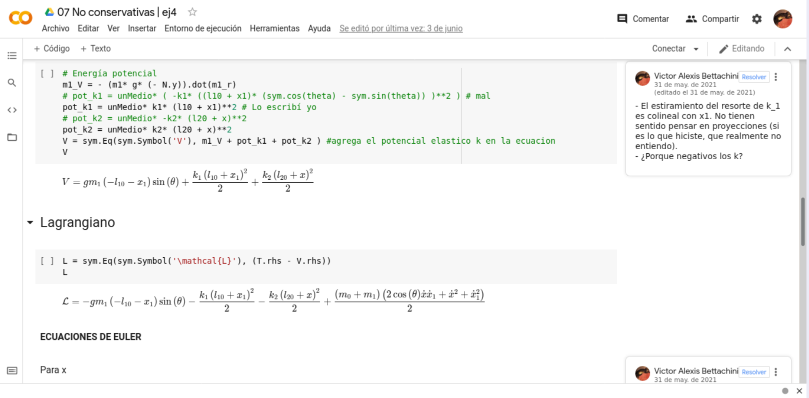


- Professor: ideas $\rightarrow$ new code/notes in repository
- Student: course repository $\rightarrow$ its own modifiable one
- Use code to solve problems = **recycle** professor's code
- Modifiying it solves different problems

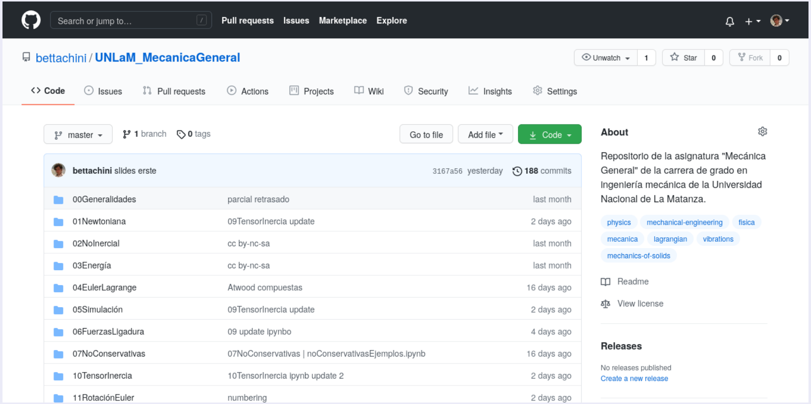
The course setup: Google Colaboratory

- It runs Jupyter notebooks online, and it's free.
- Students can collaborate remotely, working on the same notebook.
- Teachers can edit and comment the work of students.



The course setup: GitHub

- Course material, accesible in a clear way, and easy to keep updated.
- Google Colaboratory loads Jupyter Notebooks directly from GitHub.



Course Overview

Class 1: Concise Mathematical Notation

- Latex typesetting, standard for the American Mathematical Society

Considero que el potencial V es nulo en el origen de coordenadas, es decir que donde se encuentra su mínimo $\varphi = 0, V(\varphi = 0) = -mg\ell$ y por tanto

$$V(\varphi) = mg\left(-\ell\cos\varphi\right) = -mg\ell\cos\varphi,$$

Como vemos la aproximación funciona bastante bien. Conformes con ella calculamos la fuerza

$$\vec{F} = -\vec{\nabla}V = -\left(\frac{\partial}{\partial r}, \frac{1}{r}\frac{\partial}{\partial\varphi}, \frac{\partial}{\partial z}\right)V(\varphi)$$

Pero solo nos interesa expresar la 2.a ley de Newton para lo que pasa en $\hat{\varphi}$

$$m\ddot{\vec{r}} \cdot \hat{\varphi} = -\frac{1}{r}\frac{\partial}{\partial\varphi}V(\varphi)$$

En el lado izquierdo de la expresión de la aceleración en cilíndricas $\ddot{\vec{r}} = (\ddot{r} - r\dot{\varphi}^2)\hat{r} + (\dot{r}\dot{\varphi}^2 + r\ddot{\varphi})\hat{\varphi} + \ddot{z}\hat{z}$, nos quedamos solo con la componente en $\hat{\phi}$,

$$\ddot{\vec{r}} \cdot \hat{\varphi} = \dot{r}\dot{\varphi}^2 + r\ddot{\varphi}$$

y como el hilo del péndulo es rígido e inextensible $r \equiv \ell$ solo queda de esto

$$\ddot{\vec{r}} \cdot \hat{\varphi} = \ell\ddot{\varphi}$$

En el lado derecho la derivada del potencial respecto a φ es

$$\frac{\partial}{\partial\varphi}V(\varphi) = mg\ell\sin(\varphi)$$

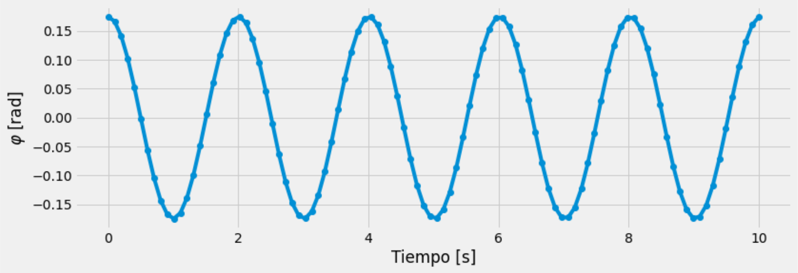
Course Overview

Class 1: precise and reproducible graphics

- The use of Python code for producing graphics is explicit, making the students play with it.

```
# graficación
fig, ax = plt.subplots(figsize=(12, 4))
ax.plot(tiempos, phi(tiempos), 'o-' )
ax.set_xlabel('Tiempo [s]')
ax.set_ylabel(r'$\varphi$ [rad]')
```

```
Text(0, 0.5, '$\\varphi$ [rad]')
```



Class 3: symbolic calculations

- These students have completed courses in Calculus and Algebra, now it's time to focus on applying the tools acquired in those courses.

[8]:
```
m2_v_cuadrado = m2_v.dot(m2_v)
m2_v_cuadrado
```

[8]: $\ell^2 \sin^2(\varphi)\dot{\varphi}^2 + (\ell\cos(\varphi)\dot{\varphi} + \dot{x})^2$

Con esto la energía cinética queda

$$\begin{aligned} T(\dot{x}_1, \varphi, \dot{\varphi}) &= \frac{m_1}{2}\left(\dot{\vec{r}}_1\right)^2 + \frac{m_2}{2}\left(\dot{\vec{r}}_2\right)^2 \\ &= \frac{m_1}{2}\dot{x}^2 + \frac{m_2}{2}\left(\dot{x}^2 + 2\dot{x}\ell\cos\varphi\dot{\varphi} + l^2\dot{\varphi}^2\right) \end{aligned}$$

[9]:
```
# Energía cinética
unMedio = sym.Rational(1,2) # Rational: fracción de enteros, alternativamente podría haberse usado 0.5
m1_T = unMedio* m1* m1_v_cuadrado
m2_T = unMedio* m2* m2_v_cuadrado
T = sym.Eq(sym.Symbol('T'), (m1_T + m2_T) ) # simplify: simplifica usando factor común y otras operaciones
T
```

[9]: $T = \frac{m_1\dot{x}^2}{2} + \frac{m_2\left(\ell^2\sin^2(\varphi)\dot{\varphi}^2 + (\ell\cos(\varphi)\dot{\varphi} + \dot{x})^2\right)}{2}$

Course Overview

Class 4: Dynamics

Ecuaciones de Euler-Lagrange

Para x

[8]:
```
x_EL = sym.Eq(L.rhs.diff(x) - L.rhs.diff(x.diff(t)).diff(t), 0 ).simplify() # ecuación igualando a cero
x_EL
```

[8]:

$$m_1 \ddot{x} + m_2 \left(-\ell \sin(\phi)\dot{\phi}^2 + \ell \cos(\phi)\ddot{\phi} + \ddot{x}\right) = 0$$

Esta es una ecuación diferencial lineal de segundo orden homogena. De aquí podría despejarse $\ddot{x}$

[9]:
```
sym.Eq(x.diff(t,2),
       list( sym.solveset(x_EL, x.diff(t,2) ) )[0]  # solveset devuelve un set, que convertimos a lista
      ) # aceleración = x punto punto [m s-2]
```

[9]:

$$\ddot{x} = \frac{\ell m_2 \left(\sin(\phi)\dot{\phi}^2 - \cos(\phi)\ddot{\phi}\right)}{m_1 + m_2}$$

Pero queda en función de otra aceleración $\ddot{\phi}$.

Para ϕ

[10]:
```
phi_EL = sym.Eq(L.rhs.diff(phi) - L.rhs.diff(phi.diff(t)).diff(t), 0 ).simplify() # ecuación igualando a cero
phi_EL
```

Course Overview

Class 4: Automation of resolutions

- No heavy calculations consuming the energy of the students.
- Their complexity does not limit the mechanical problems that can be tackled.

```
[14]: sistemaEcuaciones = [
          x_EL,
          phi_EL,
      ]
      variablesDespeje = [x.diff(t,2), phi.diff(t,2)] # despejar aceleraciones generalizadas
      variablesDespeje_sol= sym.nonlinsolve(sistemaEcuaciones, variablesDespeje ).args[0]
```

```
[15]: x_pp = sym.Eq(variablesDespeje[0], variablesDespeje_sol.args[0] ) # [m s-2]
      phi_pp = sym.Eq(variablesDespeje[1], variablesDespeje_sol.args[1] ) # [m s-2]
      x_pp, phi_pp
```

[15]:

$$\left(\ddot{x} = \frac{-\ell g m_2 \sin(\phi) + \frac{\ell m_2 \left(\ell m_2 \cos(\phi)\dot{\phi}^2 + g m_1 + g m_2\right) \sin(\phi)}{m_1 + m_2 \sin^2(\phi)}}{\ell m_2 \cos(\phi)}, \ \ddot{\phi} = -\frac{\left(\ell m_2 \cos(\phi)\dot{\phi}^2 + g m_1 + g m_2\right) \sin(\phi)}{\ell \left(m_1 + m_2 \sin^2(\phi)\right)} \right)$$

Course Overview

Class 5: Numerical analysis

- Explicit solutions.

```
[22]: # defino una función con el sistema de derivadas
      # t : no se usa en este sistema pero lo dejamos para uso posterior
      # y : lista de estado con [y[0], y[1], y[2], y[3]]
      # y[0]: x
      # y[1]: x punto
      # y[2]: phi
      # y[3]: phi punto
      # dydt : lista de derivadas
      def y_punto(t, y):
          dydt = [y[1],
                  x_pp_numpy(y[0], y[1], y[2], y[3]),
                  y[3],
                  phi_pp_numpy(y[0], y[1], y[2], y[3]),
                 ]
          return dydt

[23]: # Integración de a pasos en el tiempo
      y_ode2 = solve_ivp(y_punto, (t_rango[0], t_rango[-1] ), y_inicial, t_eval = t_rango)

[25]: y_ode2.y[0]

[25]: array([ 1.        ,  0.95510744,  0.92131146,  0.89820932,  0.88468059,
              0.87877042,  0.87745354,  0.87702754,  0.87352768,  0.86357726,
              0.84474673,  0.81565733,  0.77559949,  0.72423163,  0.66166451,
              0.588266  ,  0.50468237,  0.41250381,  0.31433661,  0.21366454,
              0.11444308,  0.02023394, -0.06599563, -0.14244216, -0.20809592,
             -0.26250272, -0.30576388, -0.33796804, -0.35953138, -0.37175469,
```

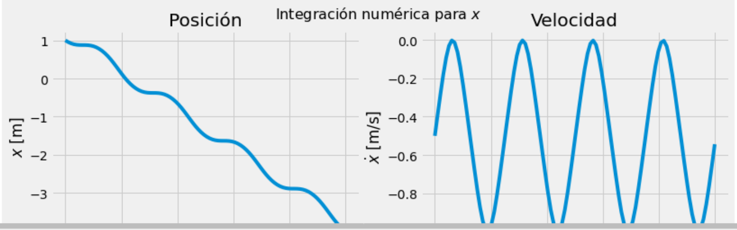
Course Overview

Class 5: analysis of results

```
[26]: solucion = y_ode2
      nombreCoordenada = 'x'

      fig, ax = plt.subplots(nrows= 1, ncols= 2, squeeze=False, figsize=(12, 4)) # dos figuras en la misma fila
      fig.suptitle('Integración numérica para $'+ nombreCoordenada + '$', fontsize=16)
      ax[0,0].plot(solucion.t, solucion.y[0]) # posición x
      ax[0,0].set(xlabel='t [s]', ylabel= '$' + nombreCoordenada+ '$ [m]', title='Posición')
      ax[0,1].plot(solucion.t, solucion.y[1]) # velocidad x
      ax[0,1].set(xlabel='t [s]', ylabel='$\dot{' + nombreCoordenada+ '}$ [m/s]', title='Velocidad')
```

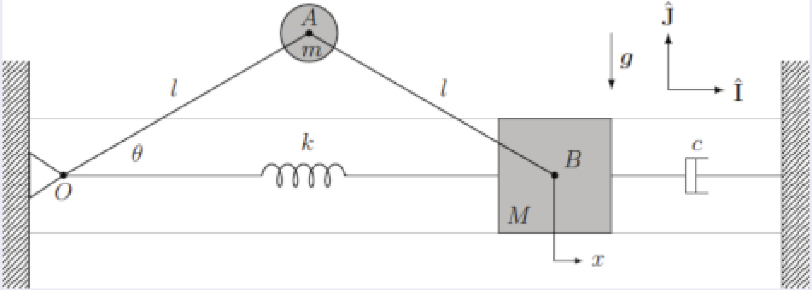
```
[26]: [Text(0.5, 0, 't [s]'),
       Text(0, 0.5, '$\\dot{x}$ [m/s]'),
       Text(0.5, 1.0, 'Velocidad')]
```



0 $_ 1 Python 3 (ipykernel) | Idle Saving completed Mode:

Class 7: Adding complexity

- Reuse code from previous classes to study complex situations.
- Similar to real world problems.



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```

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  - Employing numerical calculus they solve what is impossible in a blackboard/paper

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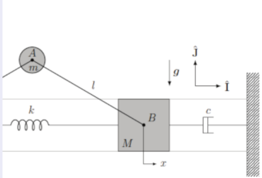

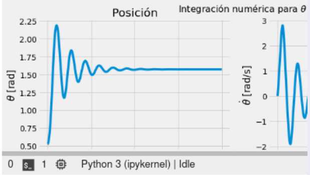

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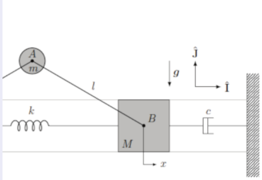

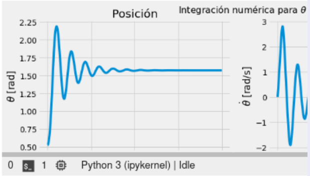

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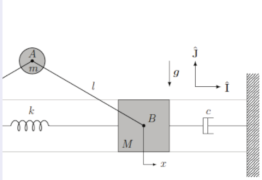

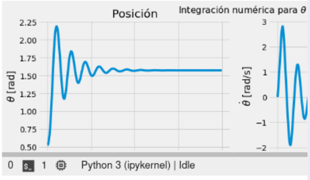

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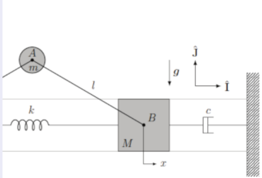

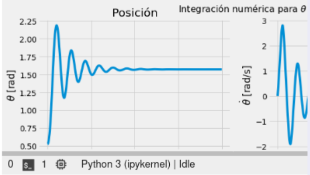

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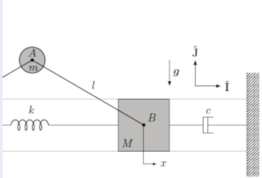

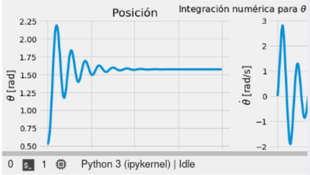

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- Gradually he becomes autonomous by reusing not the provided but his own code

All course material can be edited on-line

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On-line programmable notebook: text + equations + code

File Edit View Run Kernel Tabs Settings Help

PÉNDULOENHEBRADOSOLVED.IPYNB

Launcher | cursoJupyter.ipynb | pénduloEnhebradoSolved.ip

Markdown | Python 3

3. Obtenga una expresión para la tensión que ejerce la barra

$$Q_d = \lambda_1 \frac{\partial f_1}{\partial d} = \lambda_1$$

Por tanto hay que resolver el sistema con las 3 ecuaciones de Euler-Lagrange y la única de ligadura para determinar λ_1. Esta última hay que resolverla para su caso homogéneo y expersar su derivada segunda para que esté en el mismo orden que las de Euler-Lagrange, a fin de cuentas estamos resolviendo sistemas diferenciales de 2.o orden.

[14]:
```
f_1
```

[14]: $f_1 = -l + d$

Determinamos también $\ddot{\theta}_1$ y $\ddot{\theta}_2$ pues serán necesarias para los cálculos numéricos posteriores.

[15]:
```
sistema = [theta1_EL.expand(),
           theta2_EL.expand(),
           d_EL.expand(),
           sym.Eq(f_1.rhs.diff(t,2), 0), # esto es igual a d punto punto = 0
          ]
variables = [theta1.diff(t,2), theta2.diff(t,2), lambda_1]
variables_sol = sym.nonlinsolve(sistema, variables).args[0]
```

[16]:
```
lambda_1_sol = sym.Eq(lambda_1, variables_sol.args[2])
lambda_1_sol.simplify()
```

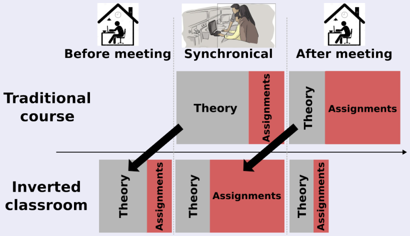
[16]: $\lambda_1 = \quad m \left(2a \cos(\theta_1 - \theta_2)\dot{\theta}_1^2 + g \cos(2\theta_1 - \theta_2) + g \cos(\theta_2) + 2d\dot{\theta}_2^2 - 2\ddot{d}\right)$

Synchronical and asynchronical work on the code

New theory alongisde its worked examples in programmable notebooks

- On-line 24/7 **asynchronical** consultations that are **public** for others to see

Flipped classroom



Synchronic	Theory	Assignments
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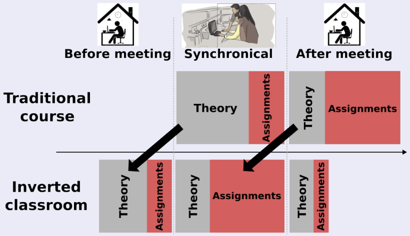

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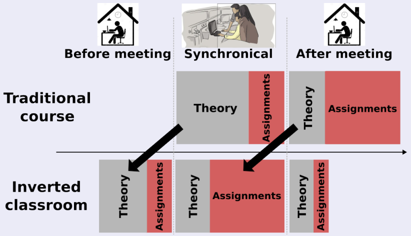

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- On a weekly basis these **must** be turned-in for scoring

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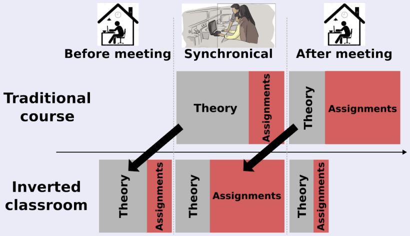

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Asynchronic corrections and remote assistance

Student's work can be commented and edited in Google Colaboratory

07 No conservativas | ej4

Archivo Editar Ver Insertar Entorno de ejecución Herramientas Ayuda Se editó por última vez: 3 de junio

Comentar Compartir

+ Código + Texto

Conectar Editando

```
[ ] # Energía potencial
    m1_V = - (m1* g* (- N.y)).dot(m1_r)
    # pot_k1 = unMedio* ( -k1* ((l10 + x1)* (sym.cos(theta) - sym.sin(theta)) )**2 ) # mal
    pot_k1 = unMedio* k1* (l10 + x1)**2 # Lo escribí yo
    # pot_k2 = unMedio* -k2* (l20 + x)**2
    pot_k2 = unMedio* k2* (l20 + x)**2
    V = sym.Eq(sym.Symbol('V'), m1_V + pot_k1 + pot_k2 ) #agrega el potencial elastico k en la ecuacion
    V
```

$$V = g m_1 \left(-l_{10} - x_1\right) \sin\left(\theta\right) + \frac{k_1 \left(l_{10} + x_1\right)^2}{2} + \frac{k_2 \left(l_{20} + x\right)^2}{2}$$

Victor Alexis Bettachini Resolver
31 de may. de 2021
(editado el 31 de may. de 2021)

- El estiramiento del resorte de k_1 es colineal con x1. No tienen sentido pensar en proyecciones (si es lo que hiciste, que realmente no entiendo).
- ¿Porque negativos los k?

Lagrangiano

```
[ ] L = sym.Eq(sym.Symbol('\mathcal{L}'), (T.rhs - V.rhs))
    L
```

$$\mathcal{L} = - g m_1 \left(-l_{10} - x_1\right) \sin\left(\theta\right) - \frac{k_1 \left(l_{10} + x_1\right)^2}{2} - \frac{k_2 \left(l_{20} + x\right)^2}{2} + \frac{\left(m_0 + m_1\right)\left(2 \cos\left(\theta\right) \dot{x} \dot{x}_1 + \dot{x}^2 + \dot{x}_1^2\right)}{2}$$

ECUACIONES DE EULER

Exams

The exam is just another excercise.

- The students send their notebook.
- Feedback is inserted in between the student's work.

```
[ ]: sym.Eq(flig_rho_rep, flig_rho)
```

[119]: $\mathrm{Q}_{\rho}(\lambda) = \lambda$

Esta fuerza efectivamente es igual a λ, pero ¿por qué? No das una justificación.

Entonces la fuerza de ligadura Q_{ρ}, o lo que es lo mismo, la fuerza que hace la barra rigida esta dada por la ecuacion:

```
[ ]: f_lig_rho = sym.Eq(sym.Function('Q_rho')(theta1, theta1.diff(t), theta2, theta2.diff(t)) , lambda_final)
     f_lig_rho
```

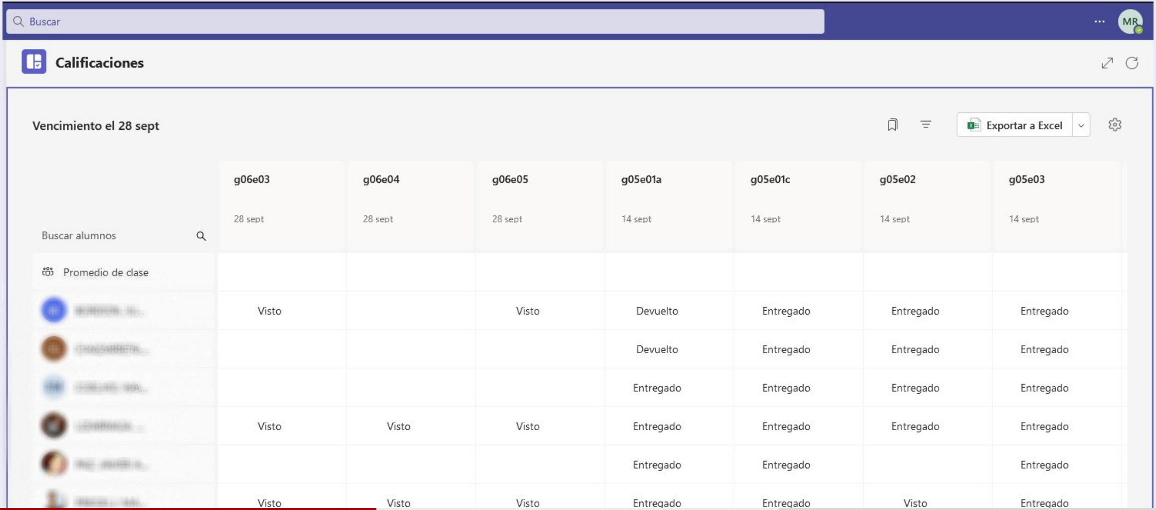
[120]: $\mathrm{Q}_{\rho}\left(\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2\right) = -m\left(\ell\dot{\theta}_2^2 + a\sin\left(\theta_1 - \theta_2\right)\ddot{\theta}_1 + a\cos\left(\theta_1 - \theta_2\right)\dot{\theta}_1^2 + g\cos\left(\theta_2\right)\right)$

No es la expresión buscada.
Entre las variables hay una que no está marcada en la izquierda de la igualdad, donde decís que la expresas en función de los θ_i y $\dot{\theta}_i$.

Podés ver que te quedó expresado en función de $\ddot{\theta}_1$. Esta puede obtenerse de resolver el sistema con las otras dos ecuaciones de Euler-Lagrange.

Individualized student follow-up at Microsoft Teams

A record of the weekly turn-up of assignments



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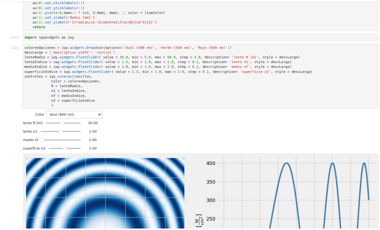

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