1.

Nature number (n)	Integer
1	0
2	-1
3	1
4	-2
5	2

one-to-one

for even n $f(n) = -\frac{n}{2}$

odd n $f(n) = \frac{n-1}{2}$

for even n let $f(a) = f(b)$

$$-\frac{a}{2} = -\frac{b}{2}$$

$$a = b$$

for odd n let $f(a) = f(b)$

$$\frac{a-1}{2} = \frac{b-1}{2}$$

$$a = b$$

Onto: for $f(n) < 0$ $f(n) = -\frac{n}{2}$ $n = -2f(n)$

for $f(n) > 0$ $f(n) = \frac{n-1}{2}$ $n = 2f(n)+1$

2. $\mathbb{Q} = \{\frac{m}{n} \mid m, n \in \mathbb{Z}, n \neq 0\}$

	$n=1$ 1	$n=2$ 2	$n=3$ 3	$n=4$ 4	$n=5$ 5	$n=6$ 6	$n=7$ 7
1	$\pm 1/1$	$\pm 1/2$	$\pm 1/3$	$\pm 1/4$	$\pm 1/5$	$\pm 1/6$	$\pm 1/7$
2	$\pm 2/1$	$\cancel{\pm 2/2}$	$\pm 2/3$	$\cancel{\pm 2/4}$	$\pm 2/5$	$\pm 2/6$	$\pm 2/7$
3	$\pm 3/1$	$\pm 3/2$	$\cancel{\pm 3/3}$	$\pm 3/4$	$\pm 3/5$	$\cancel{\pm 3/6}$	$\pm 3/7$
4	$\pm 4/1$	$\pm 4/2$	$\pm 4/3$	$\cancel{\pm 4/4}$	$\pm 4/5$	$\pm 4/6$	$\pm 4/7$
5	$\pm 6/1$	$\pm 5/2$	$\pm 5/3$	$\pm 5/4$	$\cancel{\pm 5/5}$	$\pm 5/6$	$\pm 5/7$

for any $\frac{a}{b}$, we have a diagonal contains it

n	$f(n)$
1	-1
2	1
3	-2
4	2
5	$-\frac{1}{2}$
6	$\frac{1}{2}$
7	-3
8	3
9	$-\frac{1}{3}$
10	$\frac{1}{3}$

each diagnoal is a finite set of rationals and there are countable number of diagonals. It's not hard to see that for each element on the table, we can find a natural number correspond to it. Similarly, for any natural number, we can find a correspond fraction on the table. So we can have a bijection mapping here

3. We will use the fact the 2-tuples are countable

for 2 tuple (rewrite from fraction, but keep the repeated values e.g. $2/2$, $3/3$ ...)

n	f(n)
1	(1,1)
2	(2,1)
3	(1,2)
4	(3,1)
5	(2,2)
6	(1,3)

we can then creat another table

	1 n=1	2 n=2	3 n=3	4 n=4	5	6
1	(1,1,1)	(1,1,2)	(1,1,3)	(1,1,4)	(1,1,5)	(1,1,6)
2	(2,1,1)	(2,1,2)	(2,1,3)	(2,1,4)	(2,1,5)	(2,1,6)
3	(1,2,1)	(1,2,2)	(1,2,3)	(1,2,4)	(1,2,5)	(1,2,6)
4	(3,1,1)	(3,1,2)	(3,1,3)	(3,1,4)	(3,1,5)	(3,1,6)
5	(2,2,1)	(2,2,2)	(2,2,3)	(2,2,4)	(2,2,5)	(2,2,6)
6	(1,3,1)	(1,3,2)	(1,3,3)	(1,3,4)	(1,3,5)	(1,3,6)

Once again, draw diagnals, length of diagnal is finite and the number of diagnals is countable

n	f(n)
1	(1,1,1)
2	(2,1,1)

3	(1, 1, 2)
4	(1, 2, 1)
5	(2, 1, 2)
6	(1, 1, 3)

On top of that, the set of n-tuples $n \in \mathbb{N}$ is countably infinite

4.

a) This approach doesn't not work. let's say we start with set of size 1. the mapping we have is, we can see before we move on to set with size 2, we already run out of natural numbers. set of size 1 has bijection mapping to natural number

n	f(n)
1	1
2	2
3	3
4	4
⋮	⋮

b)
We can add one number each time, and find the premutation of it. (not include repeated ones)

① $\{1\}^{1}$

② $\{2\}^{2}, \{1\ 2\}^{3}, \{2\ 1\}^{4}$

③ $\{3\}^{5}, \{1\ 2\ 3\}^{6}, \{1\ 3\}^{7}, \{2\ 3\}^{8}, \{3\ 2\}^{9}, \{3\ 1\}^{10}$
$\{1\ 3\ 2\}^{11}, \{2\ 1\ 3\}^{12}, \{2\ 3\ 1\}^{13}, \{3\ 1\ 2\}^{14}, \{3\ 2\ 1\}^{15}$

④ . . . .

For each row, we have finite number of elements, and the number of such rows are countable. we can put index on each element, (marked in blue)

for example, if natural number 8 correspond to 23, and 321 correspond to 15 in the set of natural number.

5.

a) first we list all set with size 1

$\{a\}\ \{b\}\ \{c\} \cdots \{z\}$

then we list set with size 2

$\{aa\}\ \{ab\} \cdots \{az\}\ \{bz\} \cdots \{zz\}$

then we list set of size 3

$\{aaa\} \cdots \qquad \{zzz\}$

we can have countable number of such steps, and each time we can list finite number of sets. mapping will be like this →

so natural number has a bijection mapping to set of all finit strings

n	f(n)
1	$\{a\}$
2	$\{b\}$
3	$\{c\}$
4	$\{d\}$
5	$\{e\}$
⋮	⋮
27	$\{aa\}$
⋮	⋮
676	$\{zz\}$
⋮	⋮

b) Computers programs is made from numbers, alphabet, etc, they can also be regraded as finite strings, as we have proved earlier the subset of finite strings is countably, so the number of computer program is also countable.