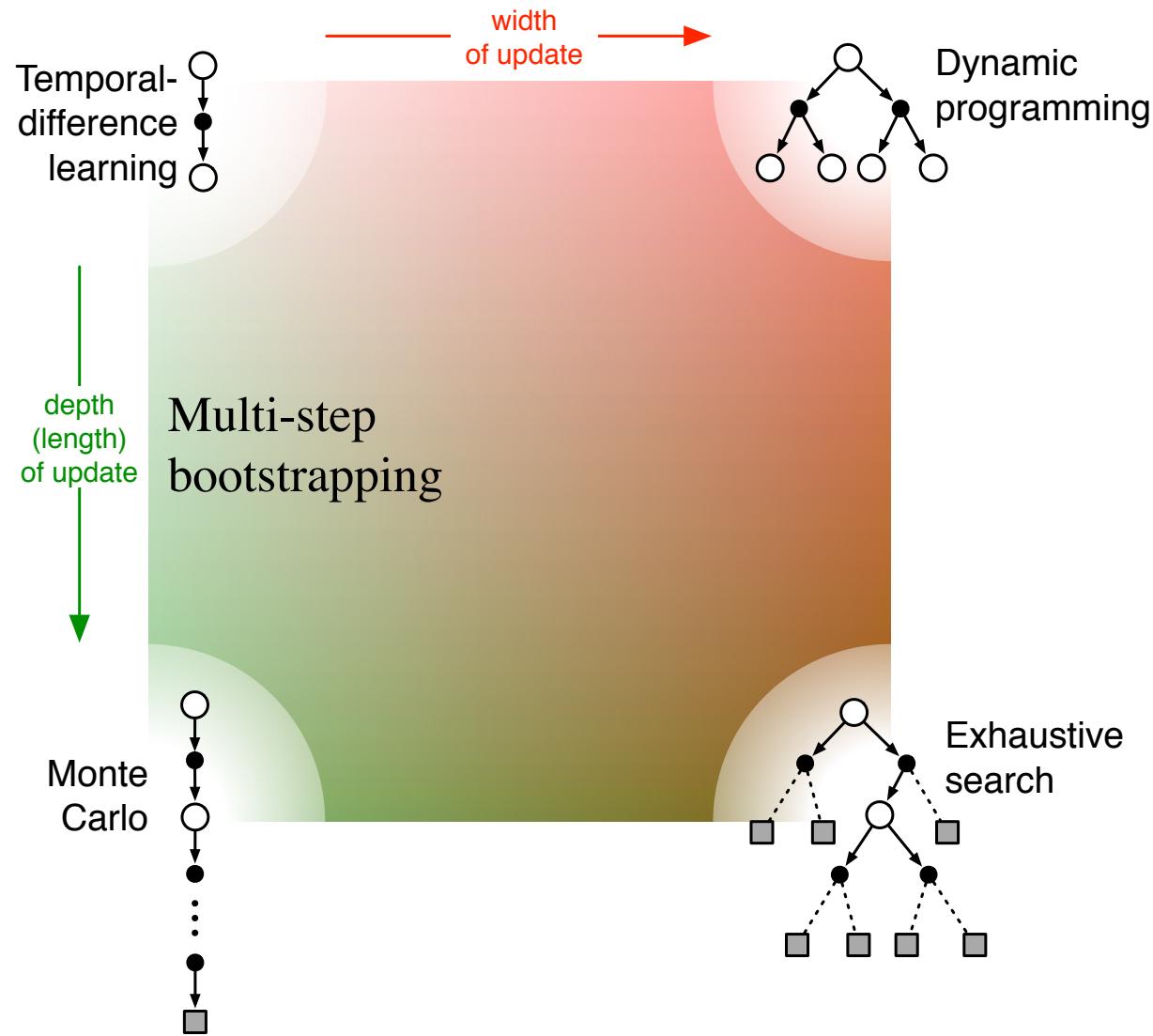


Unified View



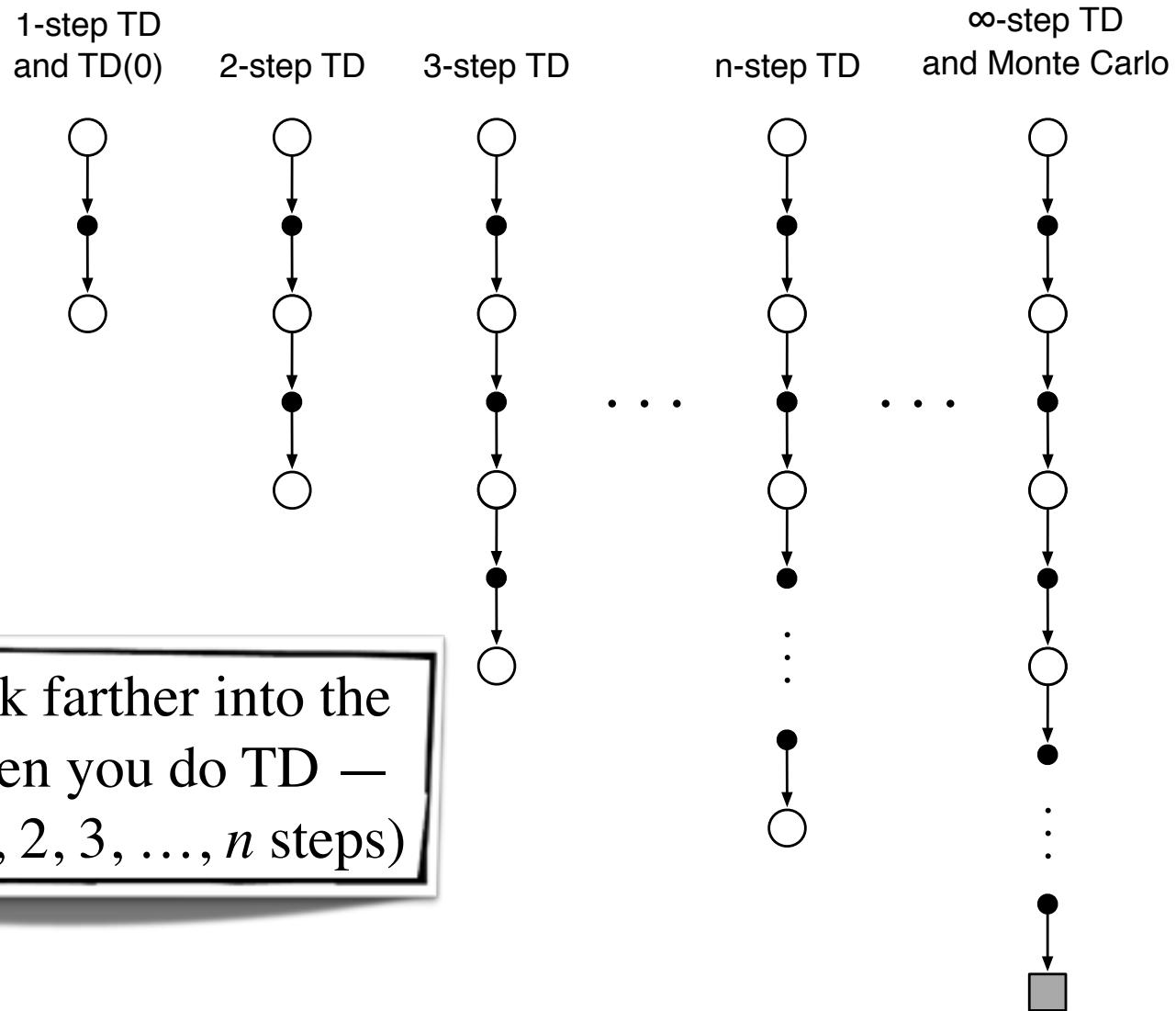
Chapter 7:

Multi-step Bootstrapping

Unifying Monte Carlo and TD

key algorithms: n-step TD, n-step Sarsa, Tree-backup, $Q(\sigma)$

***n*-step TD Prediction**

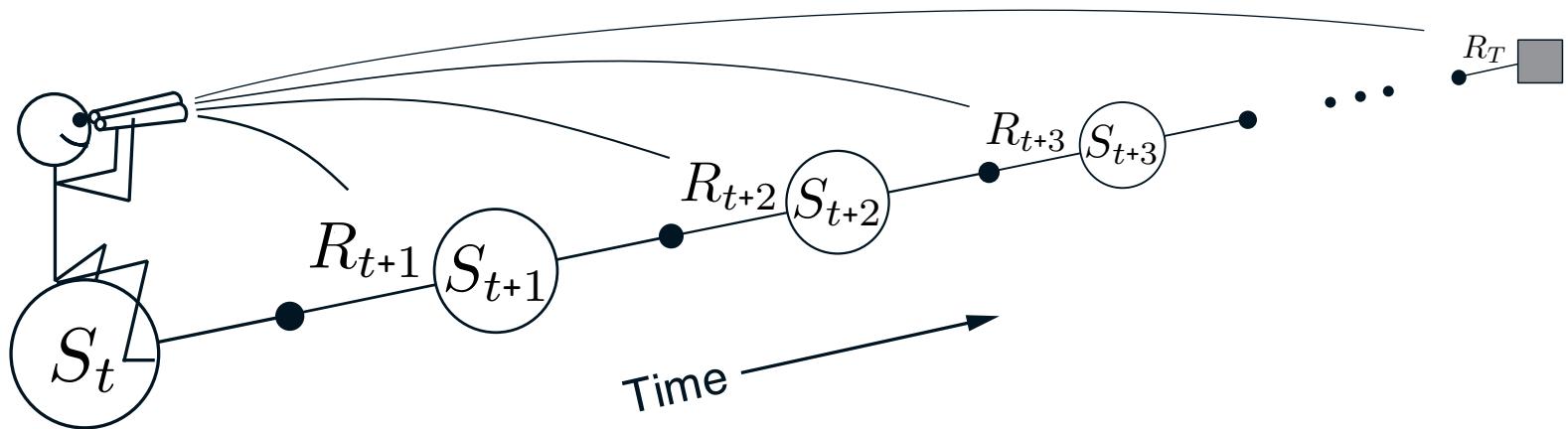


Mathematics of n -step TD Returns/Targets

- Monte Carlo: $G_t \doteq R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \cdots + \gamma^{T-t-1} R_T$
- TD: $G_{t:t+1} \doteq R_{t+1} + \gamma V_t(S_{t+1})$
 - Use V_t to estimate remaining return
- n -step TD:
 - 2 step return: $G_{t:t+2} \doteq R_{t+1} + \gamma R_{t+2} + \gamma^2 V_{t+1}(S_{t+2})$
 - n -step return: $G_{t:t+n} \doteq R_{t+1} + \gamma R_{t+2} + \cdots + \gamma^{n-1} R_{t+n} + \gamma^n V_{t+n-1}(S_{t+n})$
with $G_{t:t+n} \doteq G_t$ if $t+n \geq T$)

Forward View of TD(λ)

- Look forward from each state to determine update from future states and rewards:



***n*-step TD**

- Recall the *n*-step return:

$$G_{t:t+n} \doteq R_{t+1} + \gamma R_{t+2} + \cdots + \gamma^{n-1} R_{t+n} + \gamma^n V_{t+n-1}(S_{t+n})$$

- Of course, this is not available until time *t+n*
- The natural algorithm is thus to **wait** until then:

$$V_{t+n}(S_t) \doteq V_{t+n-1}(S_t) + \alpha [G_{t:t+n} - V_{t+n-1}(S_t)], \quad 0 \leq t < T$$

- This is called ***n*-step TD**

n-step TD for estimating $V \approx v_\pi$

Initialize $V(s)$ arbitrarily, $s \in \mathcal{S}$

Parameters: step size $\alpha \in (0, 1]$, a positive integer n

All store and access operations (for S_t and R_t) can take their index mod n

Repeat (for each episode):

Initialize and store $S_0 \neq \text{terminal}$

$$T \leftarrow \infty$$

For $t = 0, 1, 2, \dots$:

If $t < T$, then:

Take an action according to $\pi(\cdot | S_t)$

Observe and store the next reward as R_{t+1} and the next state as S_{t+1}

If S_{t+1} is terminal, then $T \leftarrow t + 1$

$\tau \leftarrow t - n + 1$ (τ is the time whose state's estimate is being updated)

If $\tau \geq 0$:

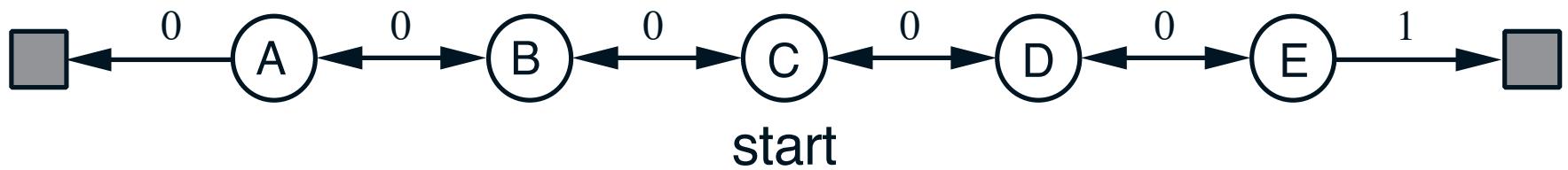
$$G \leftarrow \sum_{i=\tau+1}^{\min(\tau+n,T)} \gamma^{i-\tau-1} R_i$$

If $\tau + n < T$, then: $G \leftarrow G + \gamma^n V(S_{\tau+n})$

$$V(S_\tau) \leftarrow V(S_\tau) + \alpha [G - V(S_\tau)]$$

Until $\tau = T - 1$

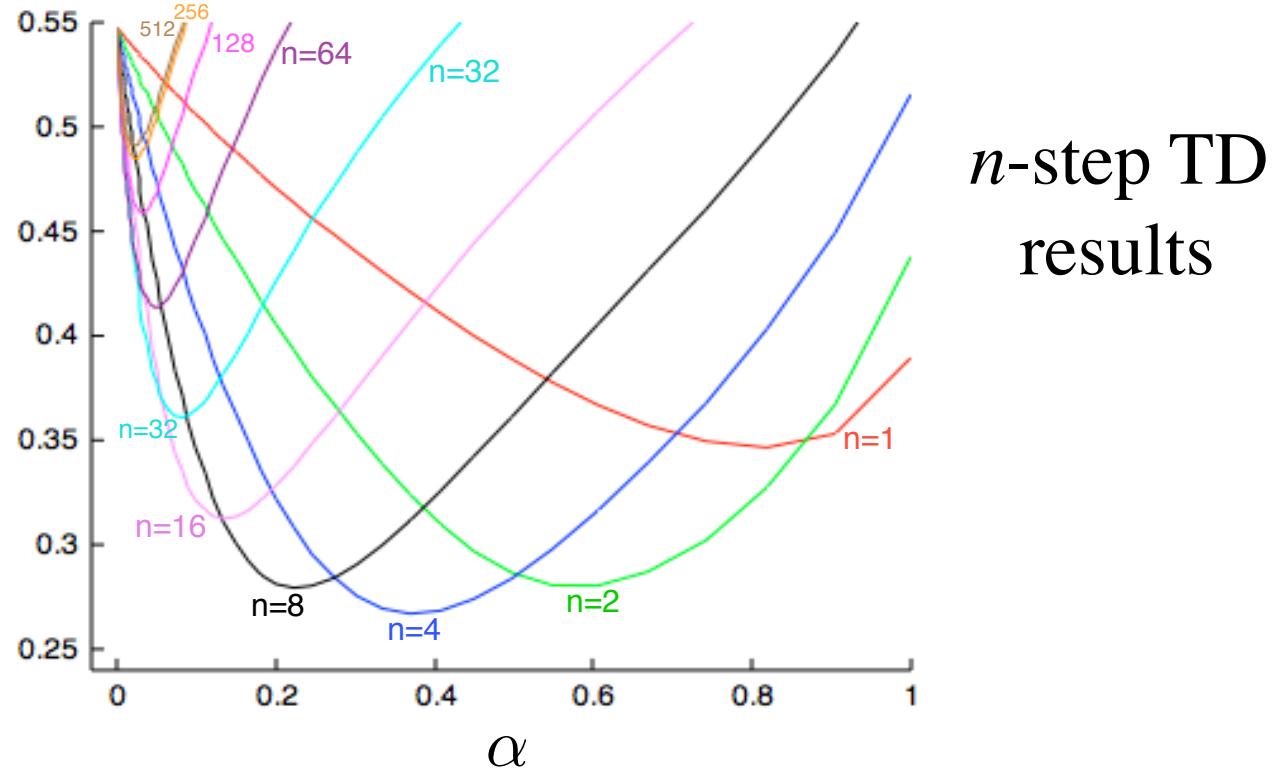
Random Walk Examples



- ➊ How does 2-step TD work here?
- ➋ How about 3-step TD?

A Larger Example – 19-state Random Walk

Average RMS error over 19 states and first 10 episodes



- An intermediate α is best
- An intermediate n is best
- Do you think there is an optimal n ? for every task?

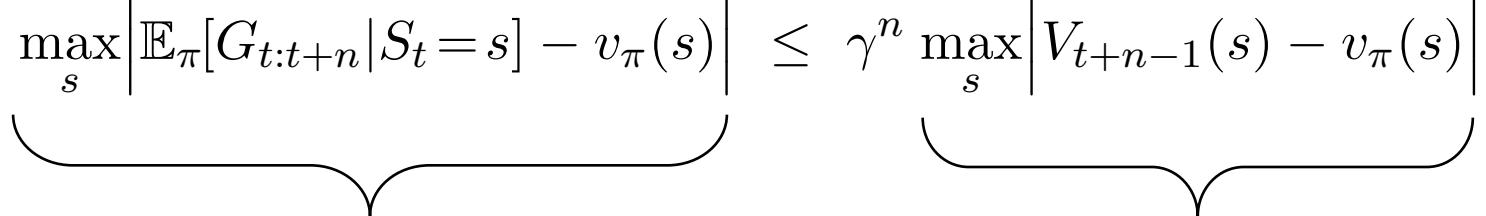
Conclusions Regarding n -step Methods (so far)

- Generalize Temporal-Difference and Monte Carlo learning methods, sliding from one to the other as n increases
 - $n = 1$ is TD as in Chapter 6
 - $n = \infty$ is MC as in Chapter 5
 - an intermediate n is often much better than either extreme
 - applicable to both continuing and episodic problems
- There is some cost in computation
 - need to remember the last n states
 - learning is delayed by n steps
 - per-step computation is small and uniform, like TD
- Everything generalizes nicely: error-reduction theory, Sarsa, off-policy by importance sampling, Expected Sarsa, Tree Backup
- The very general n -step $Q(\sigma)$ algorithm includes everything!

Error-reduction property

- Error reduction property of n -step returns

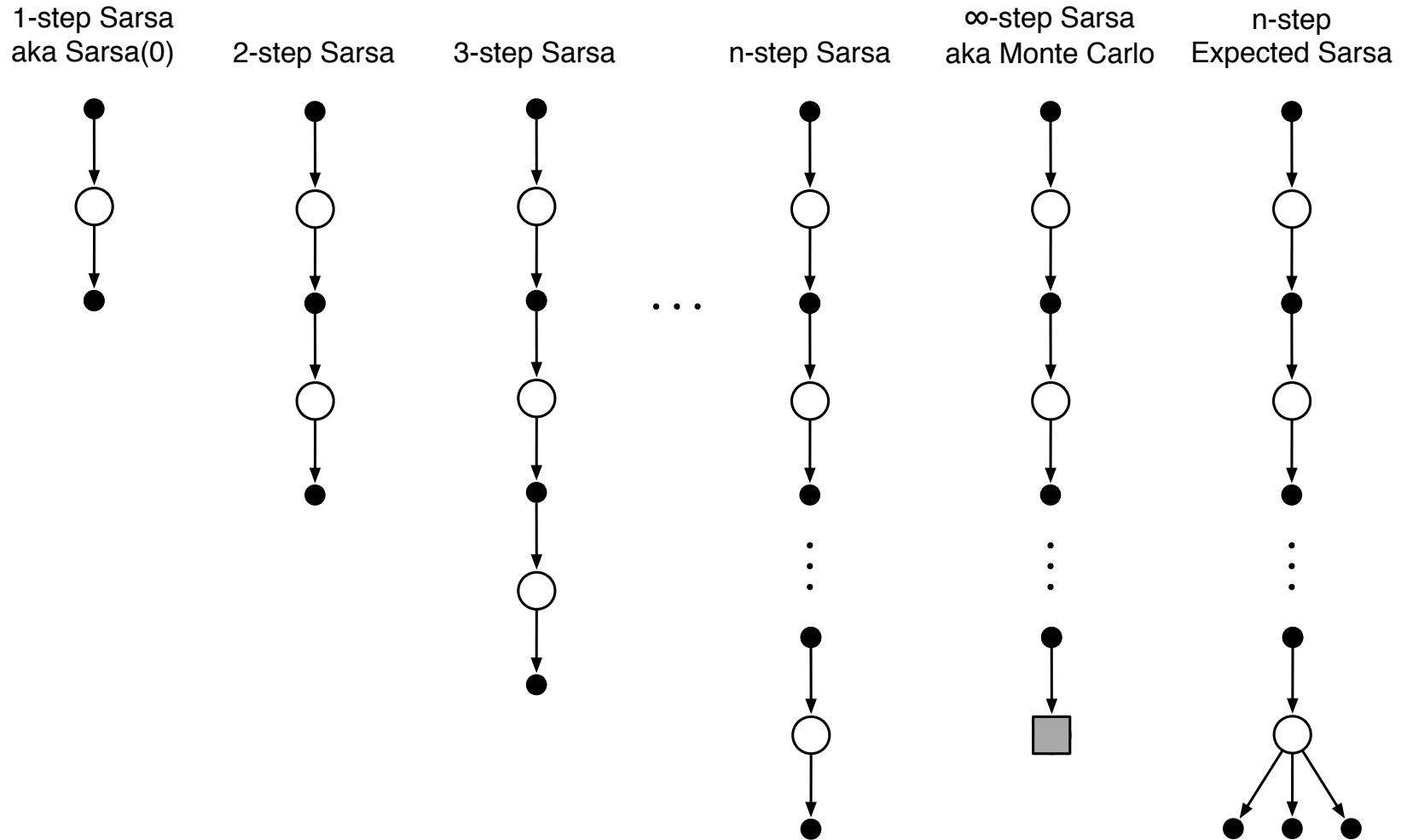
$$\max_s \left| \mathbb{E}_\pi[G_{t:t+n} | S_t = s] - v_\pi(s) \right| \leq \gamma^n \max_s \left| V_{t+n-1}(s) - v_\pi(s) \right|$$



Maximum error using n -step returnMaximum error using V

- Using this, you can show that n -step methods converge

It's much the same for action values



On-policy n -step Action-value Methods

- Action-value form of n -step return

$$G_{t:t+n} \doteq R_{t+1} + \gamma R_{t+2} + \cdots + \gamma^{n-1} R_{t+n} + \gamma^n Q_{t+n-1}(S_{t+n}, A_{t+n})$$

- n -step Sarsa:

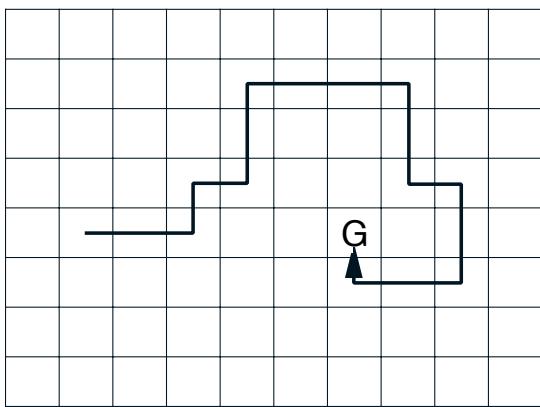
$$Q_{t+n}(S_t, A_t) \doteq Q_{t+n-1}(S_t, A_t) + \alpha [G_{t:t+n} - Q_{t+n-1}(S_t, A_t)]$$

- n -step Expected Sarsa is the same update with a slightly different n -step return:

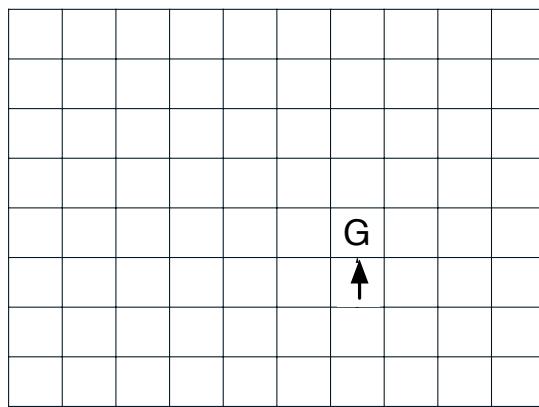
$$G_{t:t+n} \doteq R_{t+1} + \cdots + \gamma^{n-1} R_{t+n} + \gamma^n \sum_a \pi(a|S_{t+n}) Q_{t+n-1}(S_{t+n}, a)$$

n-step Methods Can Accelerate Learning

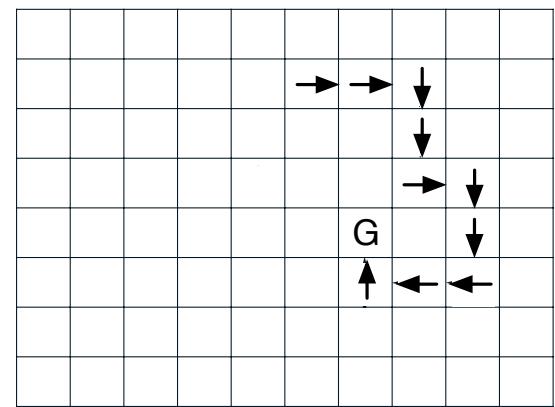
Path taken



Action values increased
by one-step Sarsa



Action values increased
by 10-step Sarsa



Off-policy n -step Methods by Importance Sampling

- Recall the *importance-sampling ratio*:

$$\rho_{t:h} \doteq \prod_{k=t}^{\min(h, T-1)} \frac{\pi(A_k | S_k)}{b(A_k | S_k)}$$

- We get off-policy methods by weighting updates by this ratio
- Off-policy n -step TD:

$$V_{t+n}(S_t) \doteq V_{t+n-1}(S_t) + \alpha \rho_{t:t+n-1} [G_{t:t+n} - V_{t+n-1}(S_t)]$$

- Off-policy n -step Sarsa:

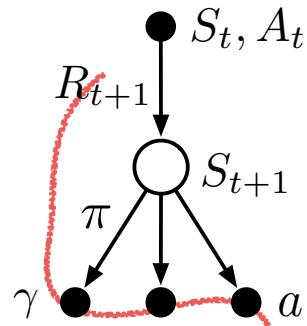
$$Q_{t+n}(S_t, A_t) \doteq Q_{t+n-1}(S_t, A_t) + \alpha \rho_{t+1:t+n-1} [G_{t:t+n} - Q_{t+n-1}(S_t, A_t)]$$

- Off-policy n -step Expected Sarsa:

Just like above except expected n -step return and ρ goes to $t+n-2$

Off-policy Learning w/o Importance Sampling: The n -step Tree Backup Algorithm

Expected Sarsa
and 1-step Tree Backup



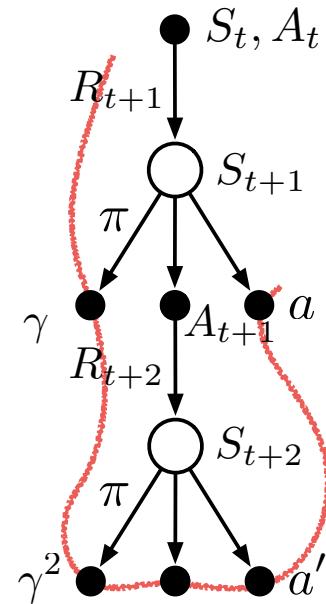
$$R_{t+1} + \gamma \sum_a \pi(a|S_{t+1}) Q(S_{t+1}, a)$$

Target →

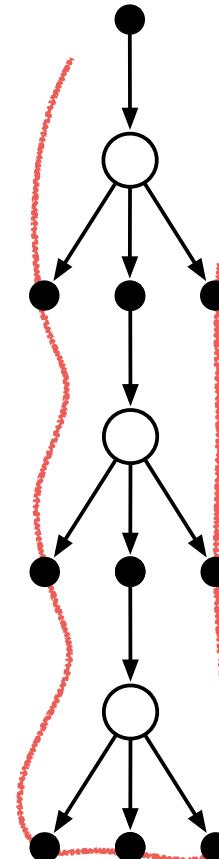
$$R_{t+1} + \gamma \sum_{a \neq A_{t+1}} \pi(a|S_{t+1}) Q(S_{t+1}, a)$$

$$+ \gamma \pi(A_{t+1}|S_{t+1}) \left(R_{t+2} + \gamma \sum_{a'} \pi(a'|S_{t+2}) Q(S_{t+2}, a') \right)$$

2-step Tree Backup



3-step TB

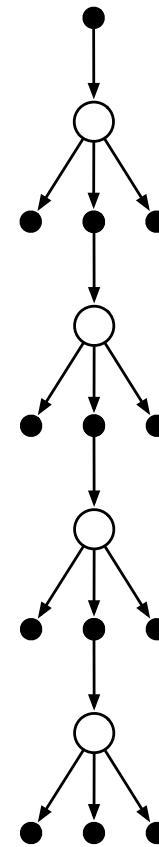


A Unifying Algorithm: n -step $Q(\sigma)$

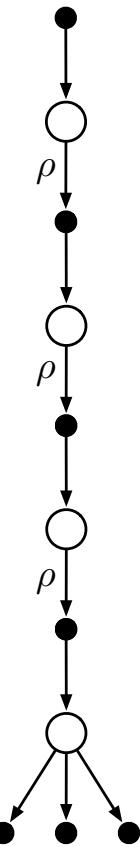
4-step
Sarsa



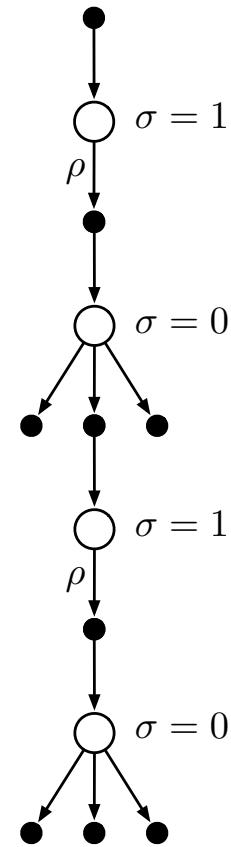
4-step
Tree backup



4-step
Expected Sarsa



4-step
 $Q(\sigma)$



Choose whether to sample or take the expectation *on each step* with $\sigma(s)$

Conclusions Regarding n -step Methods

- Generalize Temporal-Difference and Monte Carlo learning methods, sliding from one to the other as n increases
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