8.03 Notes

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1 Introduction

These notes are being taken after the 2024-2025 Academic Year. They are in preparation for the Fall '25 semester and are not intended to be comprehensive. I am using a mix of the MIT 8.03SC OpenCourseWare course taught by Yen-Jie Lee, (I took 8.20 with him, he's amazing!) as well as the recommended textbook, *The Physics of Waves* by Howard Georgi.

2 Harmonic Oscillation

2.1 The Humble Harmonic Oscillator

Let us construct a system in which a block is connected to a wall via a spring. The distance the block is from the wall where it is at rest, will be denoted x_0. Through Hooke's Law,

$$F_{spring} = -k(x - x_0)$$

we know that the force experienced on the block by the spring is proportional to its distance from its equilibrium point. We also know that force is proportional to acceleration, the second time derivative of position.

$$F = m\ddot{x}$$

Combining our two equations, we get:

$$m\ddot{x} = -k(x - x_0)$$

as our equation of motion (EoM). We can make our lives easier by redefining our coordinate system such that $x_0 = 0$ which makes our EoM much easier to work with.

$$\ddot{x} = \frac{-k}{m}x$$

This gives us a differential equation that has the general solution:

$$x(t) = \alpha\cos(\omega t) + \beta\sin(\omega t)$$

such that $\omega = \sqrt{\frac{k}{m}}$. This general problem has an infinitely many amount of solutions, but we can find something satisfactory for our system if we are given initial conditions. Because we're working with a second order differential, we'll probably be given two initial conditions for x and $\dot{x}$, usually at $t = 0$. Attempting to solve for $x(t)$ with our two initial values will give us the solution:

$$x(0)\cos(\omega t) + \frac{\dot{x}(0)}{\omega}\sin(\omega t)$$

2.1.1 Miscellaneous Things

ω is the angular frequency of our system. It's how our system oscillates. $\tau = \frac{2\pi}{\omega}$ is the period. every τ the system returns back to its equilibrium position. $\nu = \frac{1}{\tau}$ is the normal frequency.

2.2 Small Oscillations, Linearity, and Taylor Approximations!

A system is linear if it obeys two rules: Additivity and homogeneity. In other words (expressions), for functions $x(t)$ if we have two inputs,t_1 & t_2, then the function x must obey:

$$x(t_1) + x(t_2) = x(t_1 + t_2)$$

Furthermore, suppose that we have a scalar numbers c, then the function must also obey:

$$x(ct_1) = cx(t_1)$$

In other words, for a function to be linear, then for all inputs, it must satisfy

$$\alpha x(t_1) + \beta x(t_2) = x(\alpha t_1 + \beta t_2)$$

The implication of this is that for linear systems, we can essentially add them up or multiply them by some scalar. This is akin to the principle of superposition, and you'll find that this will make our lives so much easier!

If we go back to our beloved SB system, we can imagine that our equation of motion will likely have more factors involved than just acceleration and position. In fact the actual force equation looks something like this:

$$f(t) = \alpha\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + \beta\frac{\mathrm{d}x}{\mathrm{d}t} + \gamma x$$

since the LHS is non-zero, our equation is non-homogenous. Replacing the LHS with a 0 would make our equation homogenous.

Force is the negative differential of the Potential, $V(x)$. i.e,

$$F(x) = -\frac{\mathrm{d}}{\mathrm{d}x}V(x)$$

Once again, returning back to our SB system, at x_0, the spring force is equal to zero. This makes sense because x_0 is the equilibrium point. Since our force is zero, the potential function at x_0 is at a minimum (because force is the first derivative of potential!) Assuming our potential function is smooth, we can perform a Taylor approximation (TA) onto our force about the equilibrium to gain more insight into what's going on. Note that we're defining $x_0 = 0$

$$F(0) \to -V'(0) - xV''(0) - \frac{x^2}{2}V'''(0)...$$

Notice that the second order term looks really similar to Hooke's law, in that the spring force is proportional to distance from the equilibrium point! We can determine in harmonic oscillators if the system's equilibrium point is stable or unstable depending on if the second derivative of the potential is positive or not.