ME40343: Helicopter Dynamics

ADVANCED HELICOPTER DYNAMICS

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S1 2018

PART 1: LECTURE PLAN

1: Introduction
2: Actuator Disc Theory (ADT) Hover
3: ADT Vertical Flight
4: ADT Forward Flight
5: Blade Element Theory (BET)
6: Blade Element Momentum Theory (BEMT)
7: Coursework – The Wind Turbine

ACTUATOR DISC THEORY*

BET / BEMT*

* 1 Question Each in Exam Paper

UNIVERSITY OF BATH

Dr D Cleaver

ME40343: Helicopter Dynamics

ADT

1. Applies momentum theory to a rotor.
2. Accuracy improved through 3 non-ideal corrections.
3. Vertical Flight: 3 equations, 4 Working States (VRS dangerous; IA = safety case)
4. Forward Flight: 1 equation, be wary of limits of validity.

HOVER

$$\lambda_i = \frac{V_i}{V_T} \qquad \sigma = \left(\frac{N_b c}{\pi R}\right) \qquad C_T = \frac{T}{\rho A (\Omega R)^2} \qquad C_P = \frac{P}{\rho A (\Omega R)^3}$$

$$V_h = \sqrt{\left(\frac{T}{A}\right)\frac{1}{2\rho}} \qquad C_P = \frac{C_T^{\frac{3}{2}}}{\sqrt{2}} \qquad \lambda_h = \sqrt{\frac{C_T}{2}}$$

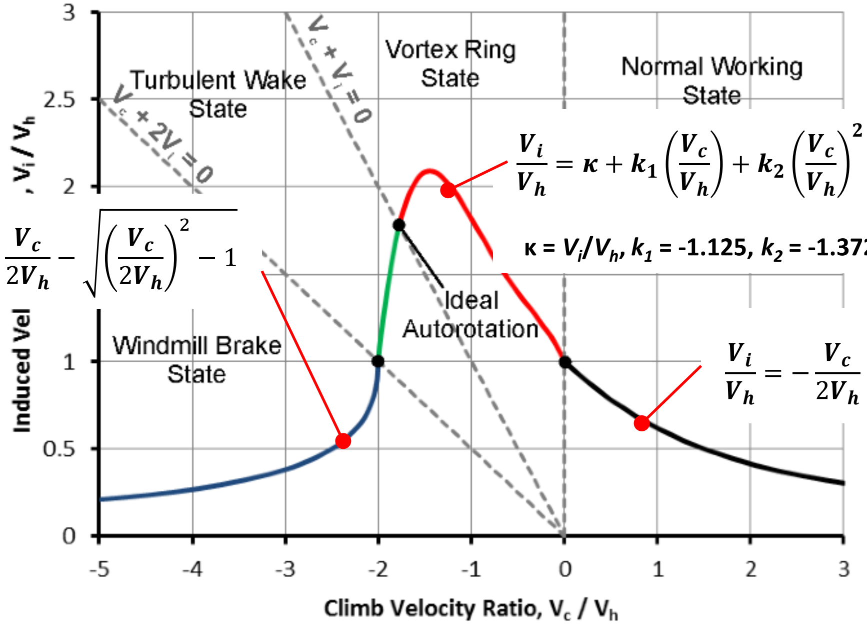
$$A_e = B^2 A$$

$$FM = \frac{ideal\ power}{actual\ power} = \frac{\frac{C_T^{\frac{3}{2}}}{\sqrt{2}}}{\frac{\kappa C_T^{\frac{3}{2}}}{\sqrt{2}} + \frac{1}{8}\sigma C_{d0}}$$

$$B \approx 1 - \frac{c_0(1 + 0.7\tau)}{1.5R}$$

$$B \approx 1 - \frac{1.386\lambda}{N_b}$$

VERTICAL



$$\frac{V_i}{V_h} = \kappa + k_1\left(\frac{V_c}{V_h}\right) + k_2\left(\frac{V_c}{V_h}\right)^2 + k_3\left(\frac{V_c}{V_h}\right)^3 + k_4\left(\frac{V_c}{V_h}\right)^4$$

$\kappa = V_i/V_h$, k_1 = -1.125, k_2 = -1.372, k_3 = -1.718, k_4 = -0.655

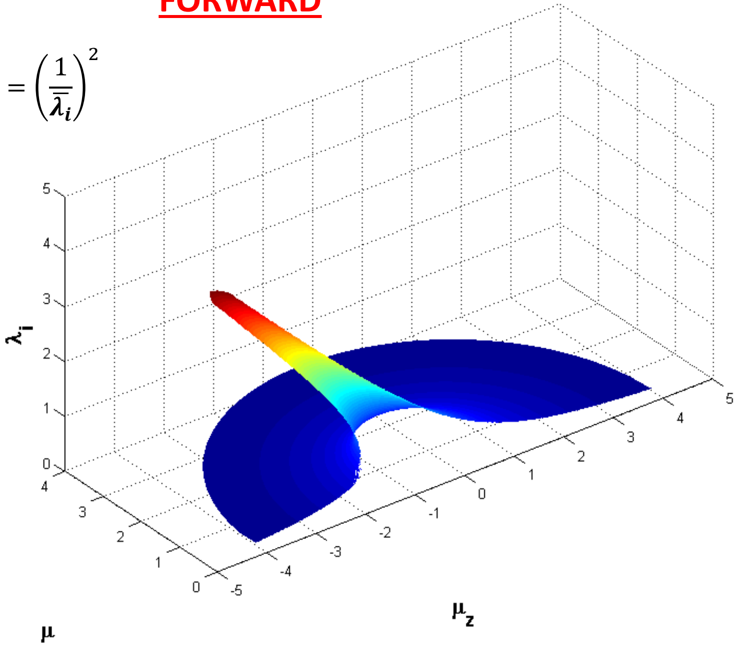
$$\frac{V_i}{V_h} = -\frac{V_c}{2V_h} - \sqrt{\left(\frac{V_c}{2V_h}\right)^2 - 1}$$

$$\frac{V_i}{V_h} = -\frac{V_c}{2V_h} + \sqrt{\left(\frac{V_c}{2V_h}\right)^2 + 1}$$

$$\frac{P}{P_h} = \frac{V_c}{V_h} + \frac{V_i}{V_h}$$

FORWARD

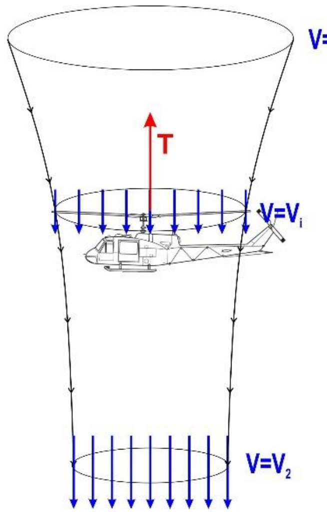
$$(\overline{\mu_x})^2 + (\overline{\mu_z} + \overline{\lambda_i})^2 = \left(\frac{1}{\overline{\lambda_i}}\right)^2$$



ROTOR AERODYNAMICS

1. Actuator Disc Theory

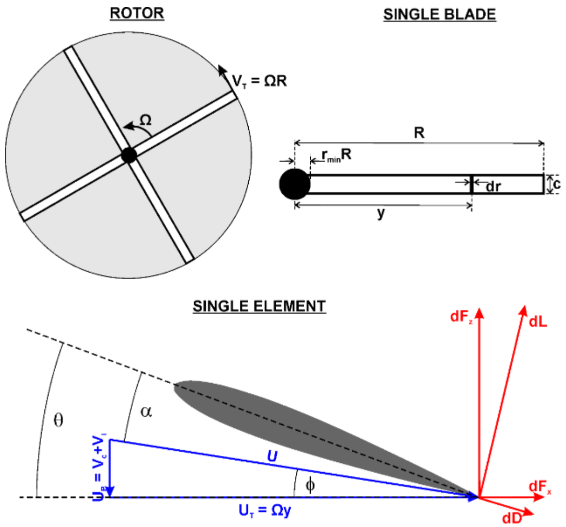
Newton's second law: **F = ma**



1D 'Conceptual' Approach

2. Blade Element Theory

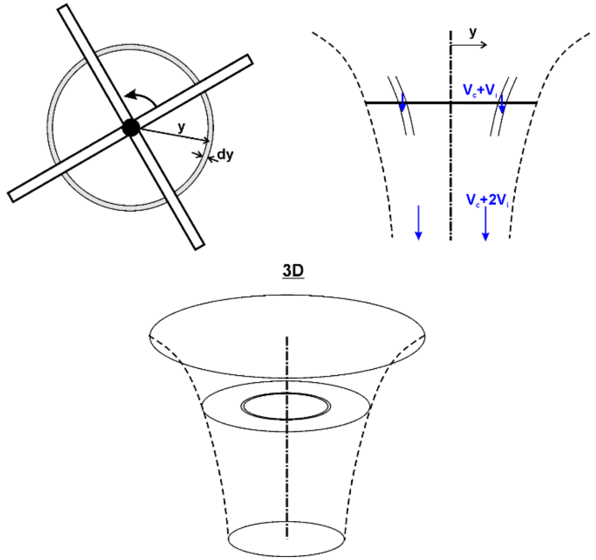
Integration of individual airfoil elements



Assumes Uniform Induced Velocity

3. Blade Element Momentum Theory

Hybrid, balance BET and ADT for each radii



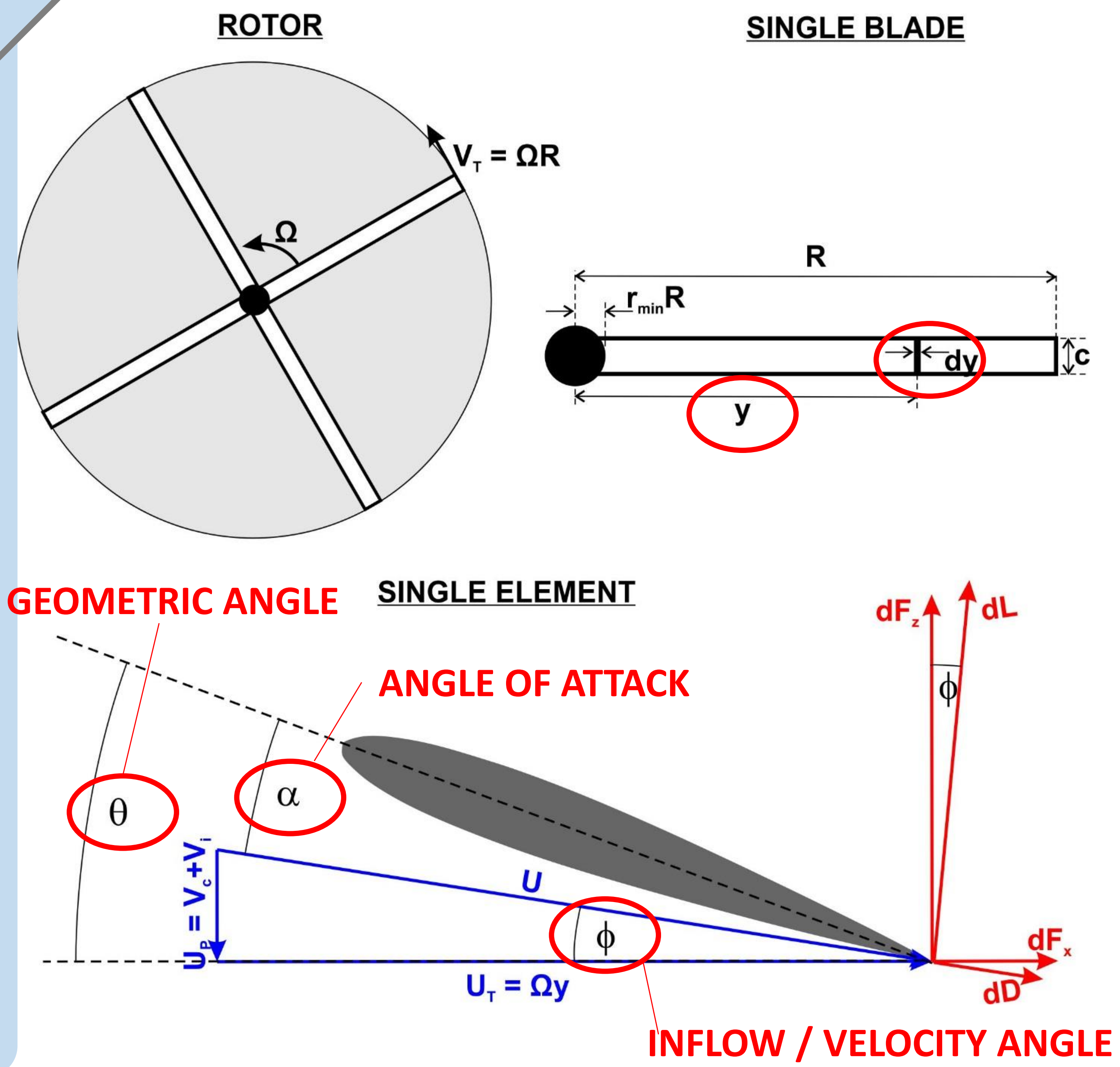
Numerical Solution

LECTURE 5

5: Blade Element Theory

1. Derive BET for hover & axial flight
2. Apply to untwisted blade
3. Apply to twisted blade

DERIVATION



DEFINITIONS

Important Angles:

$$\phi = tan^{-1}\left(\frac{V_c + V_i}{\Omega y}\right)$$

$$\alpha = \theta - \phi$$

Element Forces:

$$dL = \frac{1}{2}\rho U^2 c\, C_l\, dy$$

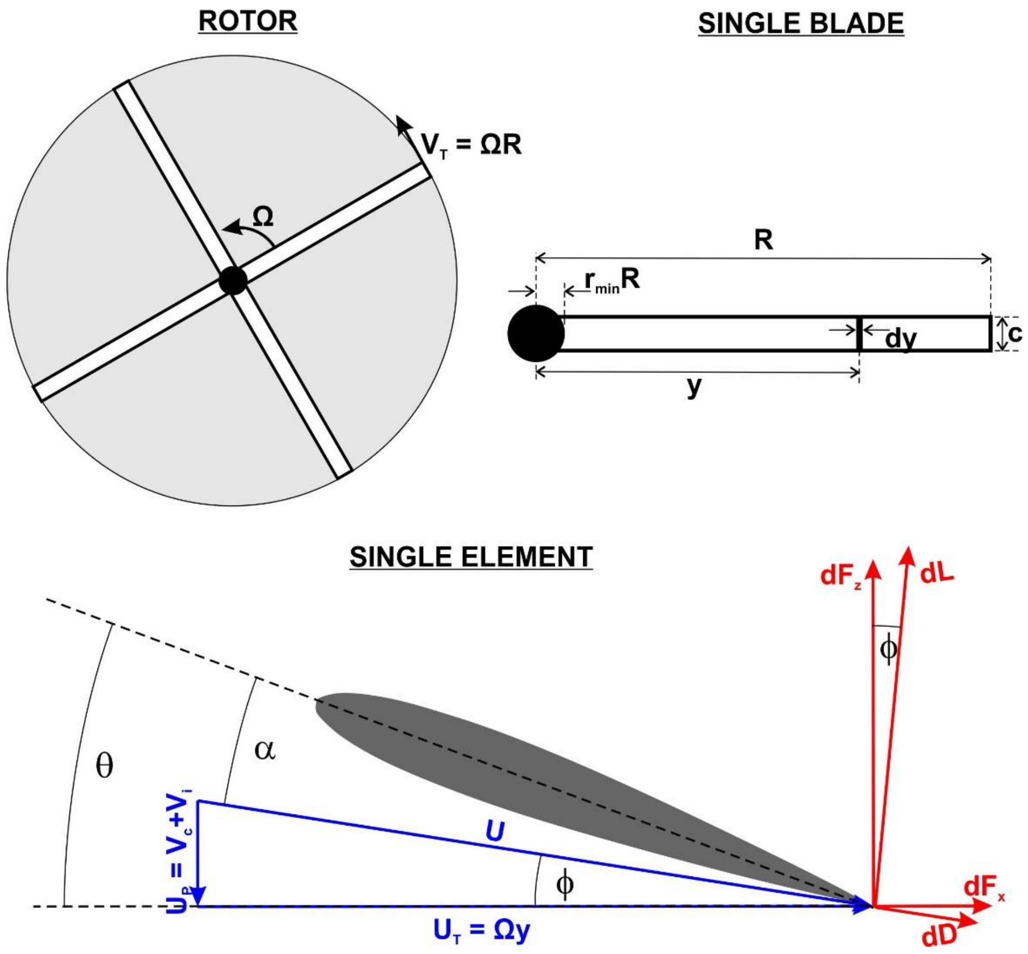
$$dD = \frac{1}{2}\rho U^2 c\, C_d\, dy$$

In Rotor Plane:

$$dF_z = dLcos\phi - dDsin\phi$$

$$dF_x = dLsin\phi + dDcos\phi$$

DERIVATION



SIMPLIFYING ASSUMPTIONS

1. ϕ is small

$$U = \sqrt{{U_T}^2 + {U_P}^2} \approx U_T$$

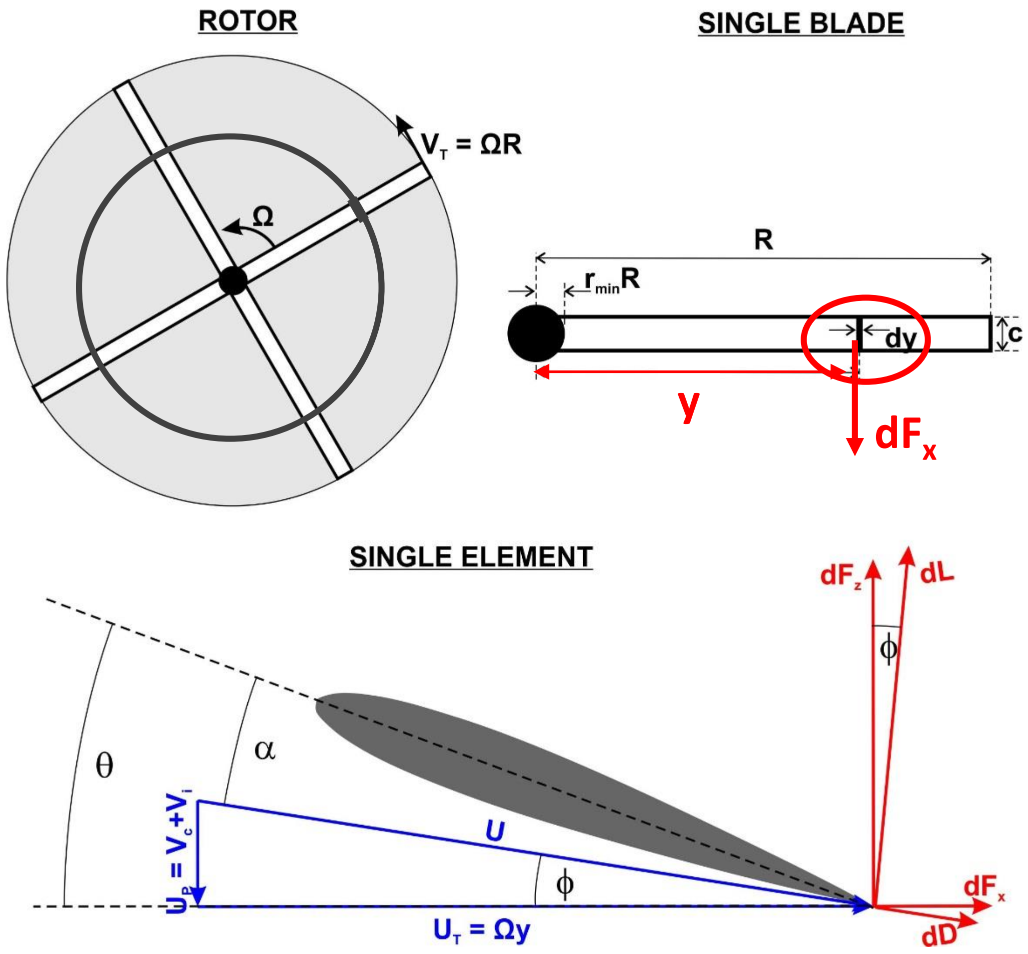
$$\phi \approx \frac{V_c + V_i}{\Omega y} \qquad sin\phi \approx \phi \qquad cos\phi \approx 1$$

2. dD is small compared to dL

$$dF_z = dLcos\phi - dDsin\phi \longrightarrow dF_z = dL$$

$$dF_x = dLsin\phi + dDcos\phi \longrightarrow dF_x = dL\phi + dD$$

DERIVATION



WHOLE ROTOR

Apply to all blades (annulus)

1. Thrust (vertical force)

$$dF_z = dL$$

$$dT = N_b dF_z = N_b dL$$

2. Torque (dFx * distance)

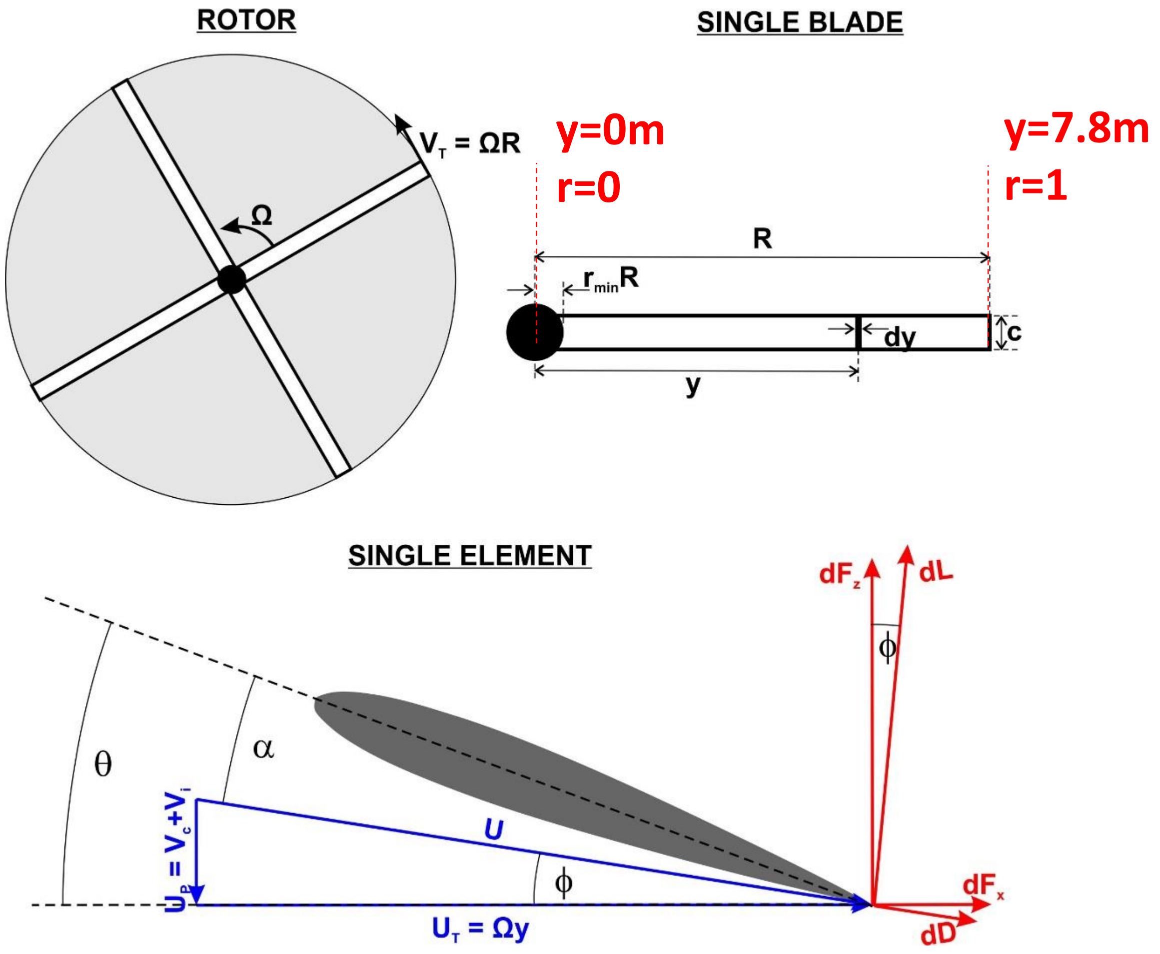
$$dF_x = dL\phi + dD$$

$$dQ = N_b dF_x y = N_b (dL\phi + dD) y$$

3. Power (Torque * rotational velocity)

$$dP = N_b (dL\phi + dD)\Omega y$$

DERIVATION



NON-DIMENSIONALIZATION

$$r = \frac{y}{R}$$

$$\frac{U}{\Omega R} = \frac{\Omega y}{\Omega R} = r$$

$$\lambda = \frac{V_C + V_i}{\Omega R} = r\phi$$

$$\phi \approx \frac{V_c + V_i}{\Omega y}$$

$$dC_T = \frac{dT}{\rho(\Omega R)^2 A}$$

$$dC_Q = \frac{dQ}{\rho(\Omega R)^2 AR}$$

$$dC_P = \frac{dP}{\rho(\Omega R)^3 A}$$

DERIVATION

THRUST

$$\boldsymbol{dC_T = \frac{dT}{\rho(\Omega R)^2 A}}$$

$$\boldsymbol{dT = N_b dL}$$

$$= \frac{N_b dL}{\rho(\Omega R)^2 A}$$

$$\boldsymbol{dL = \frac{1}{2}\rho U^2 c\, C_l\, dy}$$

$$= \frac{N_b \frac{1}{2}\rho U^2 c\, C_l\, dy}{\rho(\Omega R)^2 \pi R^2}$$

$$\boldsymbol{\frac{U}{\Omega R} = r}$$

$r = \frac{y}{R}$: **dy = Rdr**

$$= \frac{1}{2}\frac{N_b c}{\pi R} C_l\, r^2 dr$$

$$\boldsymbol{\sigma = \frac{N_b c}{\pi R}}$$

$$\boxed{dC_T = \frac{1}{2}\sigma C_l\, r^2 dr}$$

Thrust coefficient for a rotor annulus.

DERIVATION

POWER / TORQUE

$$dC_P = dC_Q = \frac{dQ}{\rho(\Omega R)^2 AR}$$

$$dQ = N_b(dL\phi + dD)y$$

$$dL = \frac{1}{2}\rho U^2 c\, C_l\, dy$$

$$= \frac{N_b(dL\phi + dD)y}{\rho(\Omega R)^2 \pi R^3}$$

$$dD = \frac{1}{2}\rho U^2 c\, C_d\, dy$$

$$= \frac{\frac{1}{2}\rho U^2 c N_b(\, C_l\, \phi + \, C_d\,)y dy}{\rho(\Omega R)^2 \pi R^3}$$

$$\frac{U}{\Omega R} = r$$

$r = \frac{y}{R}$: dy = Rdr

$$= \frac{1}{2}\frac{N_b c}{\pi R}\left(\, C_l\, \frac{\lambda}{r} + \, C_d\,\right) r^3 dr$$

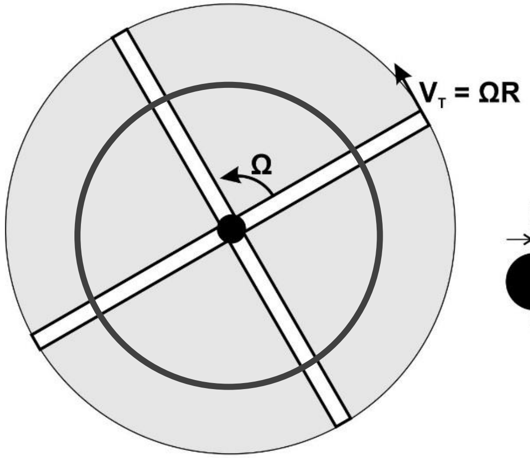
$$\sigma = \frac{N_b c}{\pi R}$$

$$\boxed{dC_P = dC_Q = \frac{1}{2}\sigma(\, C_l\, \lambda r^2 + \, C_d r^3\,)dr}$$

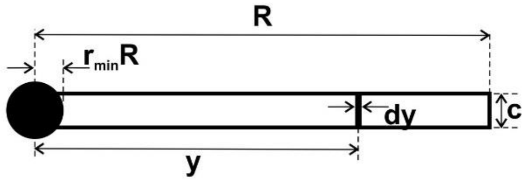
Power/Torque coefficient for a rotor annulus.

DERIVATION

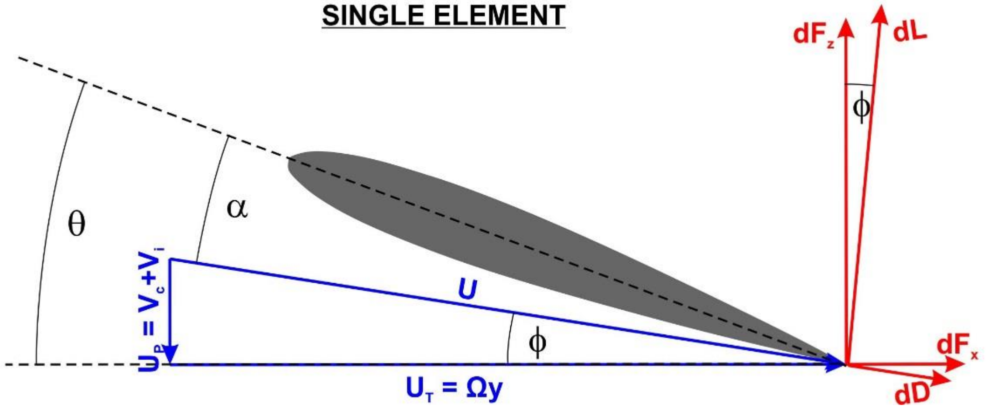
ROTOR



SINGLE BLADE



SINGLE ELEMENT



INTEGRATION

For single annulus:

$$\boldsymbol{dC_T = \frac{1}{2}\sigma C_l\, r^2 dr}$$

$$\boldsymbol{dC_P = dC_Q = \frac{1}{2}\sigma(\, C_l\, \lambda r^2 + \, C_d r^3\,)dr}$$

For whole rotor:

$$\boldsymbol{C_T = \frac{1}{2}\int_{r_{min}}^{r_{max}} \sigma C_l\, r^2 dr}$$

$$\boldsymbol{C_P = C_Q = \frac{1}{2}\int_{r_{min}}^{r_{max}} \sigma(\, C_l\, \lambda r^2 + \, C_d r^3\,)dr}$$

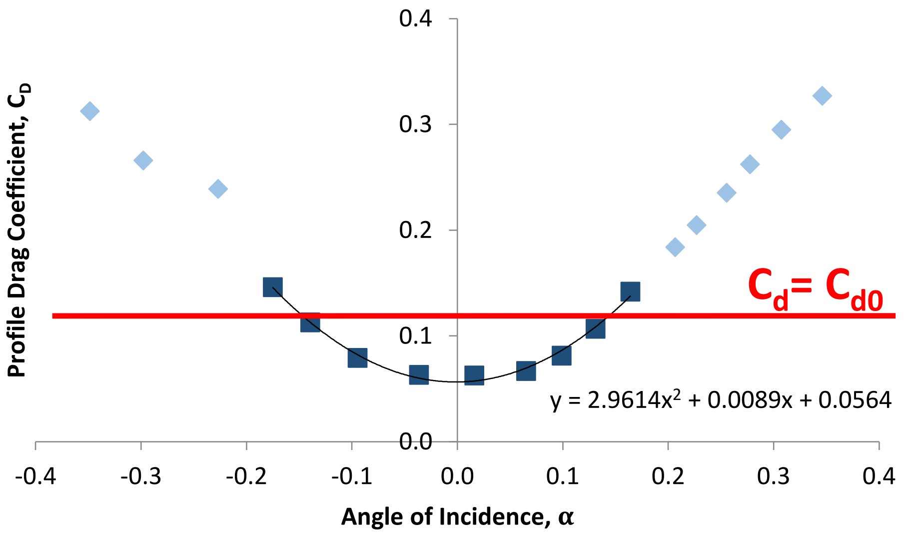
INTERPRETATION

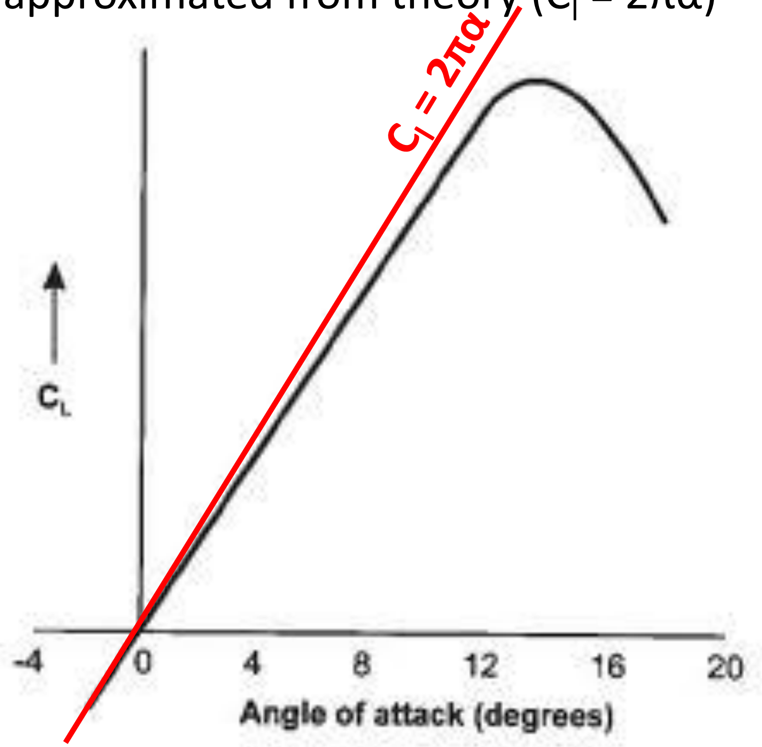
$$C_T = \frac{1}{2}\int_{r_{min}}^{r_{max}} \sigma C_l\, r^2\, dr$$

$$C_P = C_Q = \frac{1}{2}\int_{r_{min}}^{r_{max}} \sigma(C_l \lambda r^2 + C_d r^3)\, dr$$

$$\sigma = \frac{N_b c}{\pi R}$$

1. **Solidity** – outside if assumed constant over blade (no taper)
2. **Tip Loss** – can be corrected for through Prandtl tip loss $R_e = BR$.
3. **Drag Coefficient** – can be estimated from experimental data / approximated with constant ($C_d = C_{d0}$)
4. **Lift Coefficient** – can be estimated from experimental data / approximated from theory ($C_l = 2\pi\alpha$)





APPLICATION

Linearized Aerodynamics

$$C_l = C_{l\alpha}(\alpha - \alpha_0)$$

$$= C_{l\alpha}(\theta - \alpha_0 - \phi)$$

$$= C_{l\alpha}(\theta - \phi)$$

α_0 = 0 as symmetrical / already adjusted for

$$\boxed{C_T = \frac{1}{2}\int_{r_{min}}^{r_{max}} \sigma C_l\, r^2 dr}$$

$$\boxed{\lambda = r\phi}$$

$$= \frac{1}{2}\sigma C_{l\alpha}\int_{r_{min}}^{r_{max}} (\theta - \phi)\, r^2 dr$$

$$\boxed{C_T = \frac{1}{2}C_{l\alpha}\int_{r_{min}}^{r_{max}} \sigma\left(\theta r^2 - \lambda r\right) dr}$$

(Only applies to linearized problems!)

APPLICATION

BET EXAMPLE 1: Untwisted Blade, Uniform Inflow

Apply to a rigid 4-bladed, **untwisted constant-chord** *rotor subject to a* **uniform inflow** *in* **hover***.*

a) How does thrust coefficient vary with pitch angle? Show graphically.

b) What is the collective angle required to achieve a thrust coefficient of 0.01?

c) How does this change if you correct for tip losses?

$r_{min} = 0$

R=3 m

c = 0.2 m

$$C_{l\alpha} = 2\pi$$

$$\sigma = 0.085$$

APPLICATION

BET EXAMPLE 1: Untwisted Blade, Uniform Inflow

Linearized Aerodynamics:

$$\boldsymbol{C_T = \frac{1}{2} C_{l\alpha} \int_{r_{min}}^{r_{max}} \sigma\left(\theta r^2 - \lambda r\right) dr}$$

Untwisted ($\theta = \theta_0$), no taper ($\sigma = constant$):

$$C_T = \frac{1}{2}\sigma C_{l\alpha} \int_0^1 (\theta_0 r^2 - \lambda r)\, dr$$

Integrate

$$= \frac{1}{2}\sigma C_{l\alpha}\left[\frac{\theta_0 r^3}{3} - \frac{\lambda r^2}{2} + c\right]_0^1$$

$$\boldsymbol{C_T = \frac{1}{2}\sigma C_{l\alpha}\left(\frac{\theta_0}{3} - \frac{\lambda}{2}\right)}$$

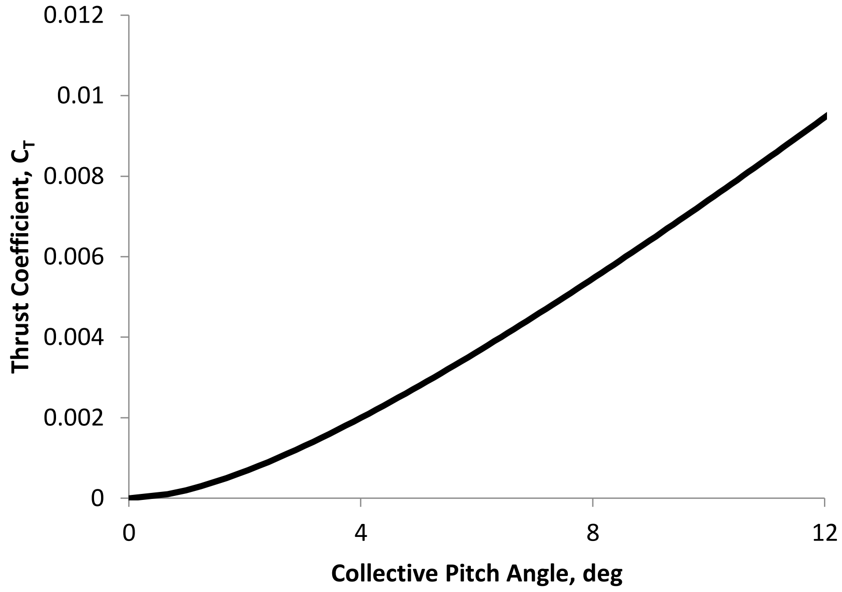
APPLICATION

BET EXAMPLE 1: Untwisted Blade, Uniform Inflow

$$\boldsymbol{C_T = \frac{1}{2}\sigma C_{l\alpha}\left(\frac{\theta_0}{3} - \frac{\lambda}{2}\right)}$$

Need value for inflow, use ADT: $\lambda_h = \sqrt{C_T/2}$

$$C_T = \frac{1}{2}\sigma C_{l\alpha}\left(\frac{\theta_0}{3} - \frac{1}{2}\sqrt{\frac{C_T}{2}}\right)$$



Rearrange

$$\boldsymbol{\theta_0 = \frac{6C_T}{\sigma C_{l\alpha}} + \frac{3}{2}\sqrt{\frac{C_T}{2}}}$$

APPLICATION

BET EXAMPLE 1: Untwisted Blade, Uniform Inflow

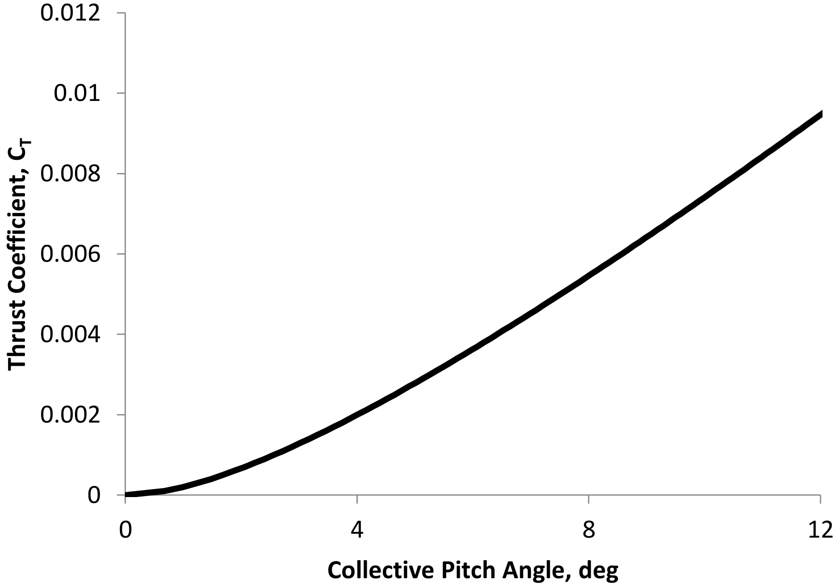
b) What is the collective angle required to achieve a thrust coefficient of 0.01?

$$\boxed{\boldsymbol{\theta_0 = \frac{6C_T}{\sigma C_{l\alpha}} + \frac{3}{2}\sqrt{\frac{C_T}{2}}}}$$

Apply Values:

$$= \frac{6 * 0.01}{0.085 * 2 * \pi} + \frac{3}{2}\sqrt{\frac{0.01}{2}}$$

$$\boldsymbol{\theta_0 = 12.5°}$$



APPLICATION

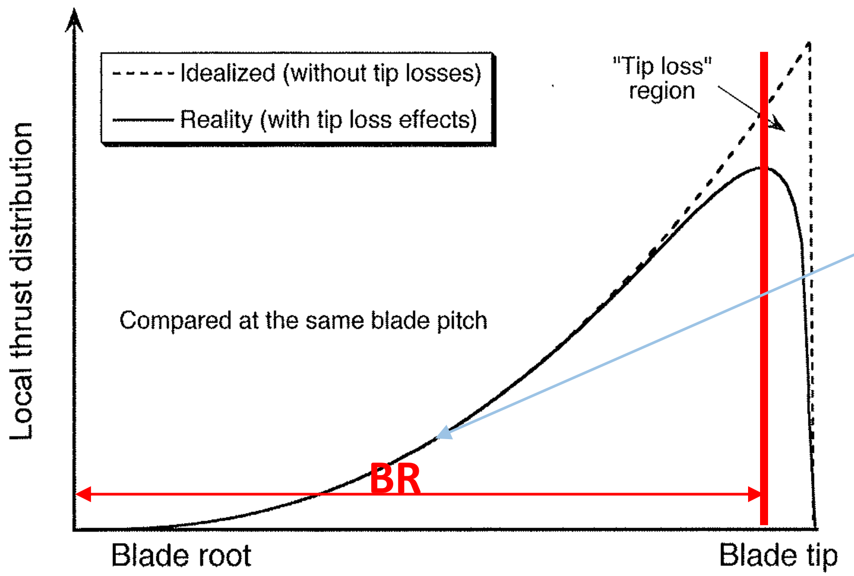
BET EXAMPLE 1: Untwisted Blade, Uniform Inflow

c) *How does this change if you correct for tip losses?*

Untapered (constant-chord)

$$\boxed{\boldsymbol{B \approx 1 - \frac{1.386\lambda}{N_b}}}$$

$$= 1 - \frac{1.386\sqrt{\frac{C_T}{2}}}{N_b} = 0.975$$



Increased V_i in this region to compensate

APPLICATION

BET EXAMPLE 1: Untwisted Blade, Uniform Inflow

Change in λ_h due to reduced area:

$$\lambda_h = \frac{1}{V_T}\sqrt{\left(\frac{T}{AB^2}\right)\frac{1}{2\rho}}$$

$$= \frac{1}{B}\frac{1}{V_T}\sqrt{\left(\frac{T}{A}\right)\frac{1}{2\rho}} = \frac{\lambda_{old}}{B}$$

APPLICATION

BET EXAMPLE 1: Untwisted Blade, Uniform Inflow

Linearized Aerodynamics:

$$\boxed{\boldsymbol{C_T = \frac{1}{2} C_{l\alpha} \int_{r_{min}}^{r_{max}} \sigma\left(\theta r^2 - \lambda r\right) dr}}$$

Untwisted $(\theta = \theta_0)$, no taper $(\sigma = constant)$:

$$C_T = \frac{1}{2}\sigma C_{l\alpha} \int_0^1 \left(\theta_0 r^2 - \frac{\lambda_{old}}{B} r\right) dr$$

Integrate

$$= \frac{1}{2}\sigma C_{l\alpha} \left[\frac{\theta_0 r^3}{3} - \frac{\lambda r^2}{2B} + c\right]_0^1$$

$$\boldsymbol{C_T = \frac{1}{2}\sigma C_{l\alpha}\left(\frac{\theta_0}{3} - \frac{\lambda}{2B}\right)}$$

APPLICATION

BET EXAMPLE 1: Untwisted Blade, Uniform Inflow

$$\boldsymbol{C_T = \frac{1}{2}\sigma C_{l\alpha}\left(\frac{\theta_0}{3} - \frac{\lambda}{2B}\right)}$$

Rearrange

$$\theta_0 = \frac{6C_T}{\sigma C_{l\alpha}} + \frac{3}{2B}\sqrt{\frac{C_T}{2}}$$

Apply Values

$$= \frac{6 * 0.01}{0.085 * 2 * \pi} + \frac{3}{2 * 0.975}\sqrt{\frac{0.01}{2}}$$

$$\boldsymbol{\theta_0 = 12.7°}$$

Makes sense. Higher V_i which needs higher angles

APPLICATION

BET EXAMPLE 2: Linearly Twisted Blade, Uniform Inflow

$$\boldsymbol{\theta(r) = \theta_o + r\theta_{tw}}$$

At Root

Twist across Blade

a) Derive an expression for the thrust coefficient for this case with θ_{tw} = 2°.

b) Using the same variables as for BET Example 1, graphically show how this changes thrust coefficient?

c) What is the new value of $\boldsymbol{\theta_o}$ required for C_T = 0.01?

APPLICATION

BET EXAMPLE 2: Linearly Twisted Blade, Uniform Inflow

$$\boldsymbol{\theta(r) = \theta_o + r\theta_{tw}}$$

Linearized Aerodynamics:

$$\boldsymbol{C_T = \frac{1}{2} C_{l\alpha} \int_{r_{min}}^{r_{max}} \sigma\left(\theta r^2 - \lambda r\right) dr}$$

Twisted with no taper:

$$= \frac{1}{2}\sigma C_{l\alpha} \int_0^1 \left((\theta_o + r\theta_{tw})r^2 - \lambda r\right) dr$$

Expand:

$$= \frac{1}{2}\sigma C_{l\alpha} \int_0^1 (\theta_o r^2 + \theta_{tw} r^3 - \lambda r)\, dr$$

Integrate:

$$= \frac{1}{2}\sigma C_{l\alpha} \left[\theta_o \frac{r^3}{3} + \theta_{tw}\frac{r^4}{4} - \lambda\frac{r^2}{2} + c\right]_0^1$$

$$\boldsymbol{C_T = \frac{1}{2}\sigma C_{l\alpha}\left(\frac{\theta_o}{3} + \frac{\theta_{tw}}{4} - \frac{\lambda}{2}\right)}$$

APPLICATION

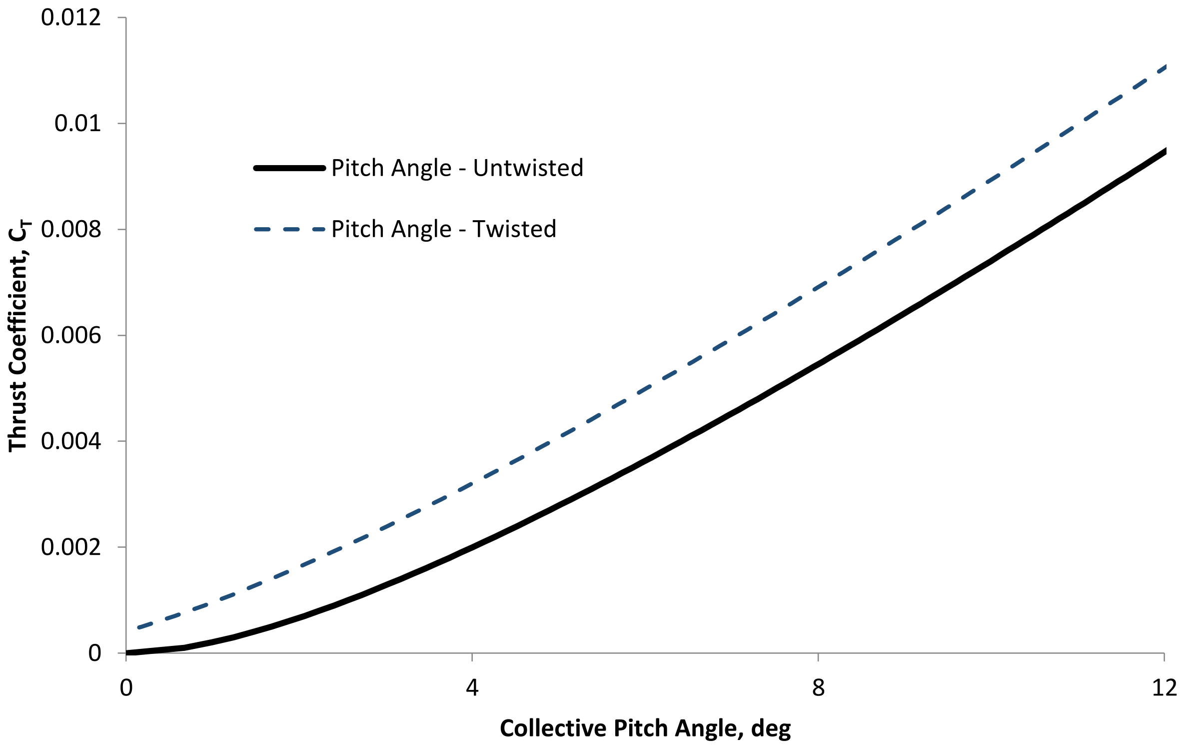
BET EXAMPLE 2: Linearly Twisted Blade, Uniform Inflow

b) Using the same variables as for BET Example 1, graphically show how this changes thrust coefficient?

$$\boldsymbol{C_T = \frac{1}{2}\sigma C_{l\alpha}\left(\frac{\theta_o}{3} + \frac{\theta_{tw}}{4} - \frac{\lambda}{2}\right)}$$

Rearrange:

$$\theta_0 = \frac{6C_T}{\sigma C_{l\alpha}} - \frac{3\theta_{tw}}{4} + \frac{3}{2}\sqrt{\frac{C_T}{2}}$$



APPLICATION

BET EXAMPLE 2: Linearly Twisted Blade, Uniform Inflow

c) *What is the new value of θ_o required for C_T = 0.01?*

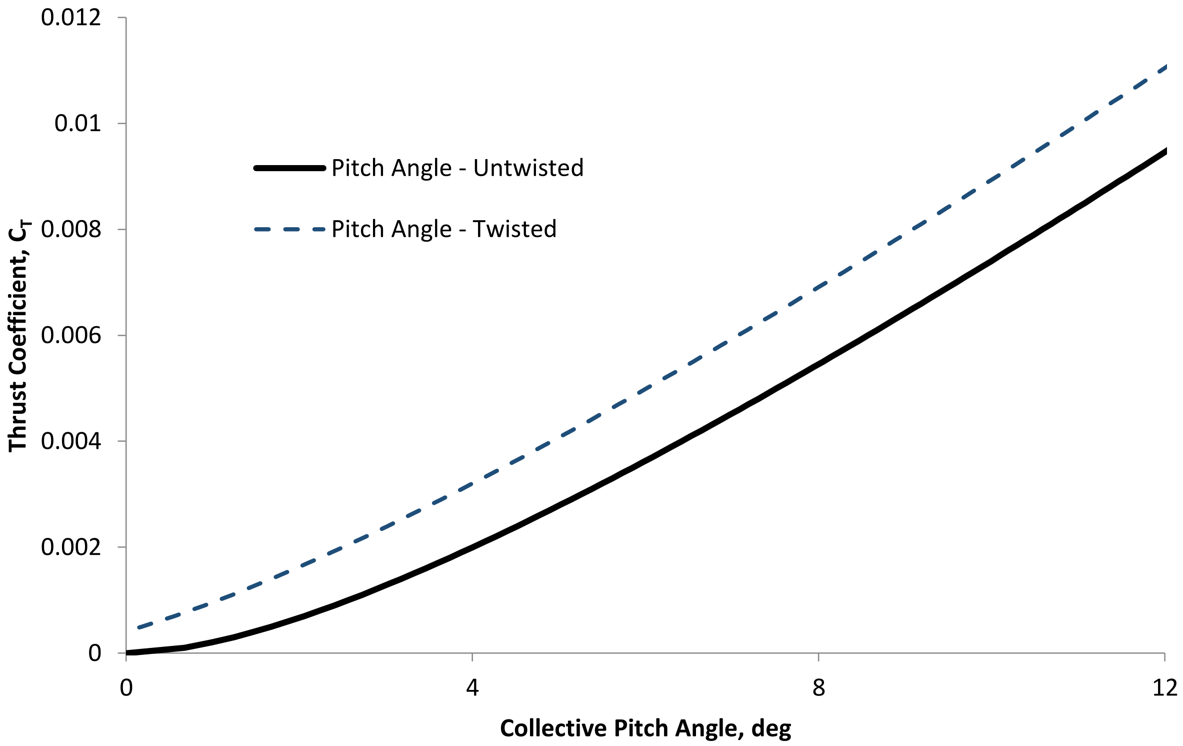
$$\boldsymbol{\theta_0 = \frac{6C_T}{\sigma C_{l\alpha}} - \frac{3\theta_{tw}}{4} + \frac{3}{2}\sqrt{\frac{C_T}{2}}}$$

Substitute Values:

$$= \frac{6 * 0.01}{0.085 * 2\pi} - \frac{3 * (2 * \pi/180)}{4} + \frac{3}{2}\sqrt{\frac{0.01}{2}}$$

$$\boldsymbol{\theta_0 = 0.192\ rad\ OR\ 11.0^\circ}$$

Exactly 1.5° less than untwisted blade.



For linear variation ¾ span representative of whole blade.

SUMMARY

1. Derivation of thrust and power for hover / axial flight

$$\boldsymbol{C_T = \frac{1}{2}\int_{r_{min}}^{r_{max}} \sigma C_l\, r^2 dr}$$

$$\boldsymbol{\sigma = \frac{N_b c}{\pi R}}$$

$$\boldsymbol{C_P = C_Q = \frac{1}{2}\int_{r_{min}}^{r_{max}} \sigma(\, C_l\, \lambda r^2 + \, C_d r^3\,)dr}$$

2. Assumes uniform inflow (constant λ from ADT)

3. C_l and C_d will be function of radius and α

4. Can be approximated through linearized aerodynamics

$$\boldsymbol{C_T = \frac{1}{2} C_{l\alpha}\int_{r_{min}}^{r_{max}} \sigma\left(\theta r^2 - \lambda r\right) dr}$$

5. Lets you design the blade, as well as estimate performance

Next?

Tutorial Questions: ADT Q1 – 13

BET Q1-4

Further Reading: Blade Element Momentum Theory Notes.

Next Lecture: Blade Element Momentum Theory.