ME40343: Helicopter Dynamics

ADVANCED HELICOPTER DYNAMICS

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S1 2018

PART 1: LECTURE PLAN

1: Introduction
2: Actuator Disc Theory (ADT) Hover
3: ADT Vertical Flight
4: ADT Forward Flight
5: Blade Element Theory (BET)
6: Blade Element Momentum Theory (BEMT)
7: Coursework – The Wind Turbine

Lectures 1–4: **ACTUATOR DISC THEORY***

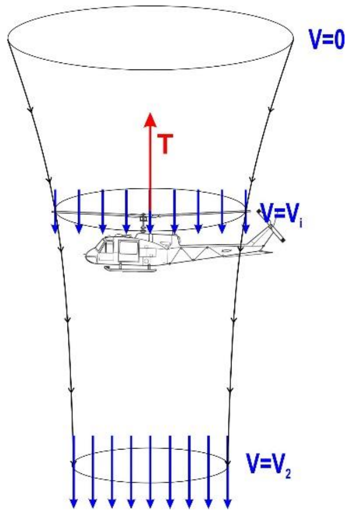
Lectures 4–6: **BET / BEMT***

* 1 Question Each in Exam Paper

ROTOR AERODYNAMICS

1. Actuator Disc Theory

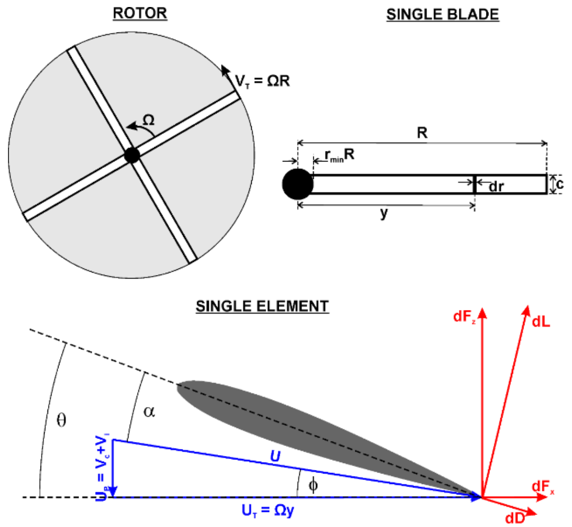
Newton's second law: **F = ma**



1D 'Conceptual' Approach

2. Blade Element Theory

Integration of individual airfoil elements



Assumes Uniform Induced Velocity

BET

1. Derivation of thrust and power for hover / axial flight

$$\boldsymbol{C_T = \frac{1}{2}\int_{r_{min}}^{1} \sigma C_l\, r^2 dr}$$

$$\boldsymbol{\sigma = \frac{N_b c}{\pi R}}$$

$$\boldsymbol{C_P = C_Q = \frac{1}{2}\int_{r_{min}}^{1} \sigma(\, C_l\, \lambda r^2 + \, C_d r^3\,) dr}$$

2. Assumes uniform inflow (constant λ like ADT)

3. Cl and Cd will be function of radius and α

4. Can be approximated through linearized aerodynamics

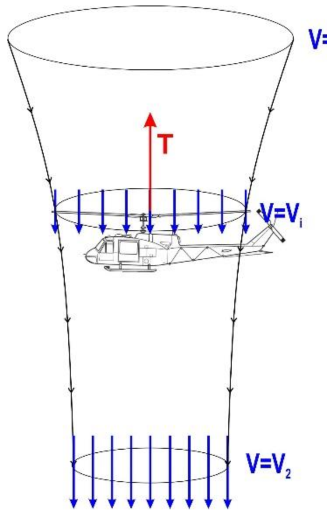
$$\boldsymbol{C_T = \frac{1}{2} C_{l\alpha} \int_{r_{min}}^{1} \sigma\left(\theta r^2 - \lambda r\right) dr}$$

5. Lets you design the blade, not just estimate performance

ROTOR AERODYNAMICS

1. Actuator Disc Theory

Newton's second law: **F = ma**



1D 'Conceptual' Approach

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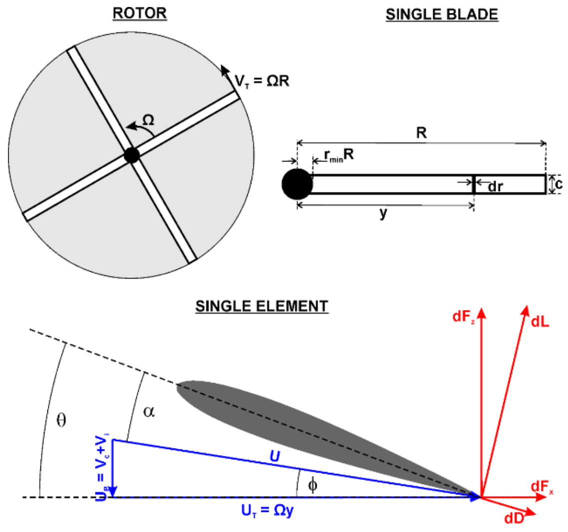
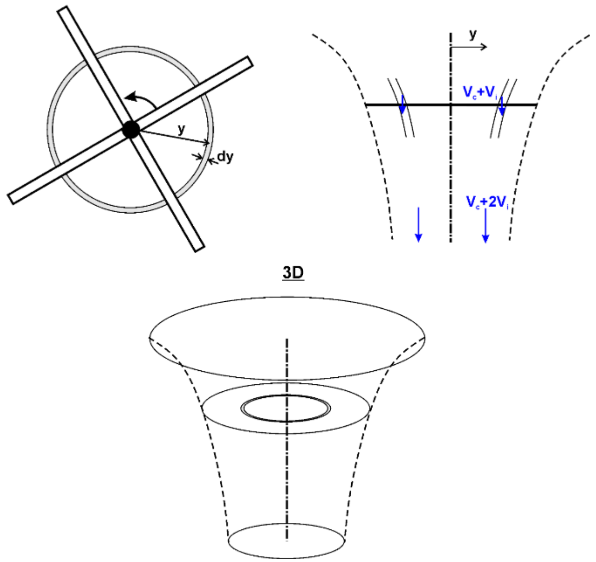


Assumes Uniform Induced Velocity

3. Blade Element Momentum Theory

Hybrid, balance BET and ADT for each radii



Numerical Solution

LECTURE 6

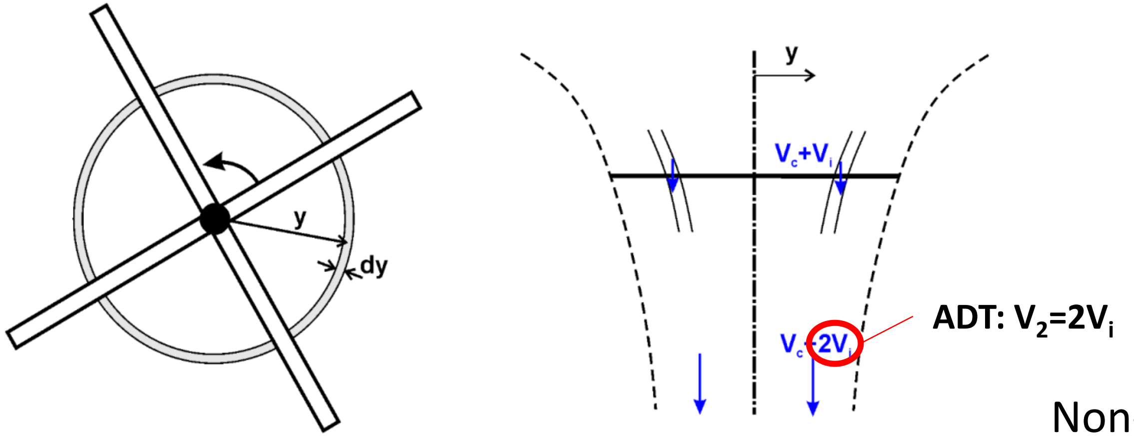
6: Blade Element Momentum Theory

1. Derive BEMT for hover & axial flight
2. Investigate effect of inflow distribution
3. Demonstrate effect of twist
4. Demonstrate case for minimum power

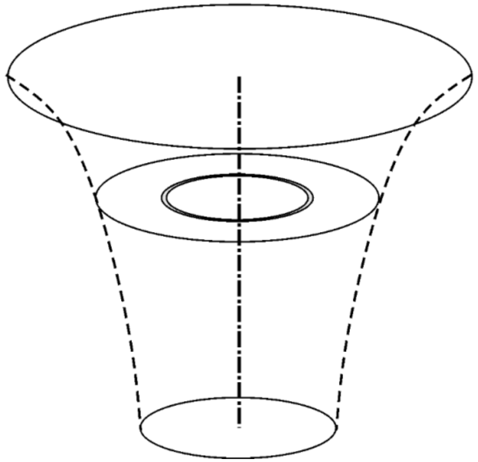
DERIVATION

Similar to BET except the inflow is non-uniform. Each annuli treated as independent and a balance found between momentum theory (ADT) and force (BET)

2D



3D



ANNULI MOMENTUM THEORY

$$dT = \dot{m} * (\mathrm{V_{out}} - \mathrm{V}_{in}) \qquad \text{(scalar)}$$

$$= \rho(V_c + V_i)dA * (2V_i)$$

$$= \rho(V_c + V_i) * 2V_i * 2\pi y dy$$

$$\frac{dy}{R} = d\left(\frac{y}{R}\right) = dr$$

Non-Dimensional: $dC_T = \dfrac{dT}{\rho A(\Omega R)^2}$

$$= 4\frac{(V_c + V_i)}{\Omega R}\frac{V_i}{\Omega R}\frac{y}{R}\frac{dy}{R}$$

$$\boxed{\boldsymbol{dC_T = 4\lambda\lambda_i r dr}}$$

Induced Power (Force * Velocity):

$$\boxed{\boldsymbol{dC_P = \lambda dC_T = 4\lambda^2\lambda_i r dr}}$$

DERIVATION

So far only momentum theory, no BET.

$$\boxed{dC_T = 4\lambda\lambda_i r dr}$$

$$\boxed{dC_P = 4\lambda^2\lambda_i r dr}$$

$$\frac{(V_c + V_i)}{\Omega R} \qquad \frac{V_i}{\Omega R}$$

V_c (normally) known, but what about V_i?

Assume a Value?

LECTURE 6

6: Blade Element Momentum Theory

1. Derive BEMT for hover & axial flight
2. Investigate effect of inflow distribution
3. Demonstrate effect of twist
4. Demonstrate case for minimum power

ASSUMED INFLOW

$$\boldsymbol{dC_T = 4\lambda\lambda_i r dr}$$

$$\boldsymbol{dC_P = 4\lambda^2\lambda_i r dr}$$

Consider hover so that $\lambda = \lambda_i = \lambda_h$:

$$dC_T = 4{\lambda_h}^2 r dr$$

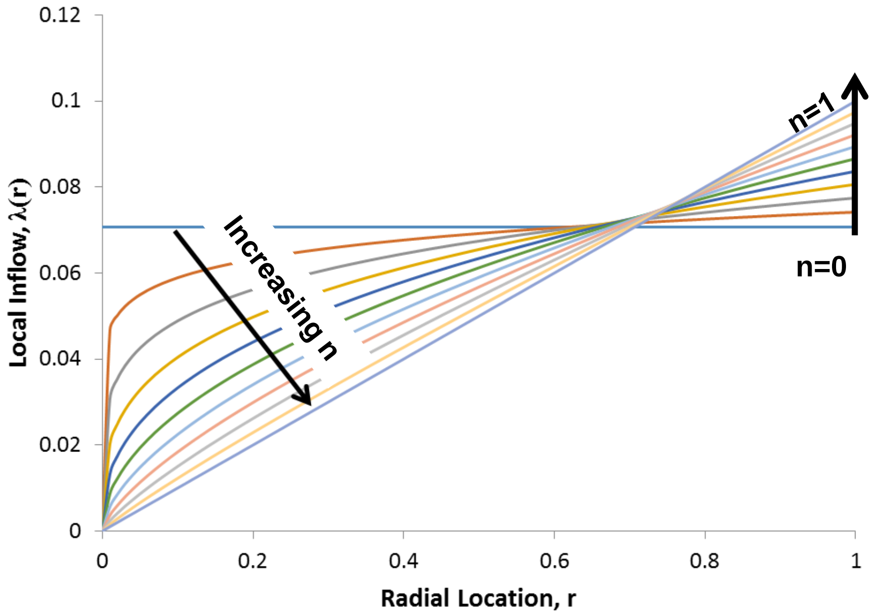
$$dC_P = 4{\lambda_h}^3 r dr$$

Integrate:

$$C_T = \int_0^1 4{\lambda_h}^2 r\, dr$$

$$C_P = \int_0^1 4{\lambda_h}^3 r\, dr$$

Make up inflow distribution: $\lambda(r) = \lambda_{tip} r^n \quad for \quad n \geq 0$



ASSUMED INFLOW

THRUST COEFFICIENT

$$C_T = \int_0^1 4{\lambda_h}^2 r\,dr$$

$$\lambda(r) = \lambda_{tip} r^n \qquad for \qquad n \geq 0$$

$$C_T = \int_0^1 4\left(\lambda_{tip} r^n\right)^2 r\,dr$$

Rearrange: $= 4{\lambda_{tip}}^2 \int_0^1 r^{2n} r\,dr$

Adjust Indices: $= 4{\lambda_{tip}}^2 \int_0^1 r^{2n+1}\,dr$

Integrate: $= \dfrac{2{\lambda_{tip}}^2}{n+1}$

Rearrange: $\boldsymbol{\lambda_{tip} = \sqrt{n+1}\sqrt{\frac{C_T}{2}}}$

Expression for inflow in terms of desired C_T.

ASSUMED INFLOW

POWER COEFFICIENT

$$C_P = \int_0^1 4{\lambda_h}^3 r\,dr$$

$$\lambda(r) = \lambda_{tip} r^n \qquad for \qquad n \geq 0$$

$$C_P = \int_0^1 4\left(\lambda_{tip} r^n\right)^3 r\,dr$$

$$\lambda_{tip} = \sqrt{n+1}\sqrt{\frac{C_T}{2}}$$

Integrate:

$$= \frac{4{\lambda_{tip}}^3}{3n+2}$$

Substitute:

$$= \frac{4\left(\sqrt{n+1}\sqrt{\frac{C_T}{2}}\right)^3}{3n+2}$$

Rearrange:

$$C_P = \frac{4(n+1)^{3/2}}{3n+2}\frac{{C_T}^{3/2}}{\sqrt{2}}$$

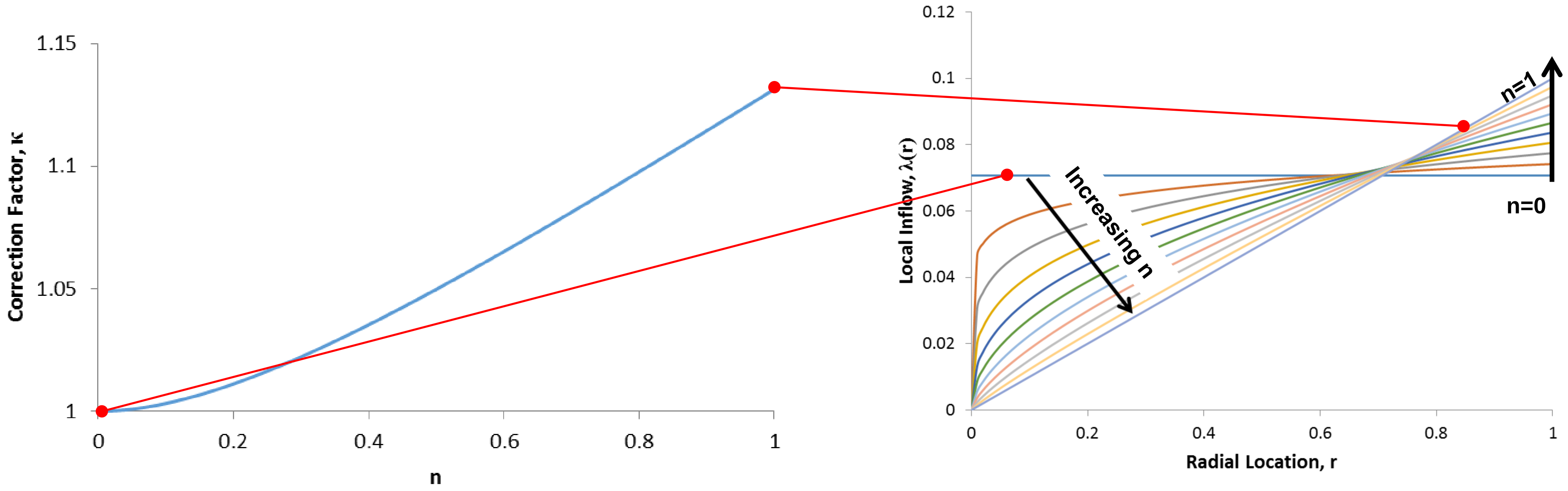
$$\boxed{\boldsymbol{C_{P_c} = \kappa \frac{{C_T}^{\frac{3}{2}}}{\sqrt{2}}}}$$

$$\boldsymbol{\kappa = \frac{4(n+1)^{3/2}}{3n+2}}$$

ASSUMED INFLOW

POWER COEFFICIENT

$$\kappa = \frac{4(n+1)^{3/2}}{3n+2}$$



More Non-Uniformity = More Power

DERIVATION

Rather than assume an inflow, equate BET to momentum theory to **calculate an inflow.**

$$\boldsymbol{dC_T = \frac{1}{2}\sigma C_{l\alpha}\left(\theta r^2 - \lambda r\right) dr}$$

(BET: Linearized Aerodynamics)

$$\boldsymbol{dC_T = 4\lambda\lambda_i r dr}$$

(Momentum)

$$\frac{1}{2}\sigma C_{l\alpha}\left(\theta r^2 - \lambda r\right)dr = 4\lambda(\lambda - \lambda_c)rdr$$

Rearrange: $$\lambda^2 + \left(\frac{\sigma C_{l\alpha}}{8} - \lambda_c\right)\lambda - \frac{\sigma C_{l\alpha}}{8}\theta r = 0$$

QUADRATIC: $ax^2 + bx + c = 0$

Where x = λ, a=1, b=$\left(\frac{\sigma C_{l\alpha}}{8} - \lambda_c\right)$, c=$-\frac{\sigma C_{l\alpha}}{8}\theta r$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\boxed{\boldsymbol{\lambda(r,\lambda_c) = \sqrt{\left(\frac{\sigma C_{l\alpha}}{16} - \frac{\lambda_c}{2}\right)^2 + \frac{\sigma C_{l\alpha}}{8}\theta r} - \left(\frac{\sigma C_{l\alpha}}{16} - \frac{\lambda_c}{2}\right)}}$$

DERIVATION

$$\lambda(r,\lambda_c) = \sqrt{\left(\frac{\sigma C_{l\alpha}}{16} - \frac{\lambda_c}{2}\right)^2 + \frac{\sigma C_{l\alpha}}{8}\theta r} - \left(\frac{\sigma C_{l\alpha}}{16} - \frac{\lambda_c}{2}\right)$$

How to solve?

Numerically.
Split rotor into n = 1…*N* annuli, every annulus is solved independently and then added together.

Force from one annuli:

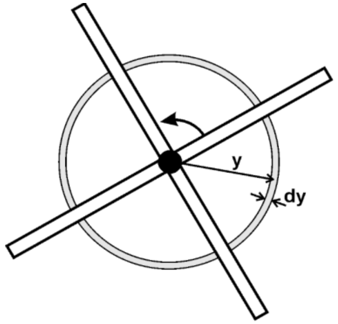
$$\Delta C_{T,n} = \frac{\sigma C_{l\alpha}}{2}\left(\theta(r_n) r_n^2 - \lambda(r_n) r_n\right)\Delta r$$

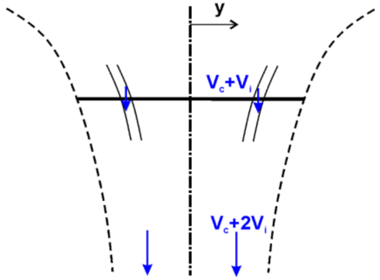
Force from rotor:

$$C_T = \sum_{n=1}^{N} \Delta C_{T,n}$$

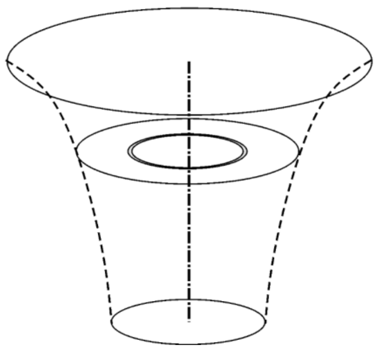
Power from rotor:

$$C_P = C_Q = \sum_{n=1}^{N} \lambda_n \Delta C_{T,n}$$





3D



LECTURE 6

6: Blade Element Momentum Theory

1. Derive BEMT for hover & axial flight
2. Investigate effect of inflow distribution
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ZERO TWIST

$$\boldsymbol{\lambda(r,\lambda_c) = \sqrt{\left(\frac{\sigma C_{l\alpha}}{16} - \frac{\lambda_c}{2}\right)^2 + \frac{\sigma C_{l\alpha}}{8}\theta r} - \left(\frac{\sigma C_{l\alpha}}{16} - \frac{\lambda_c}{2}\right)}$$

Consider hover (λ_c=0):

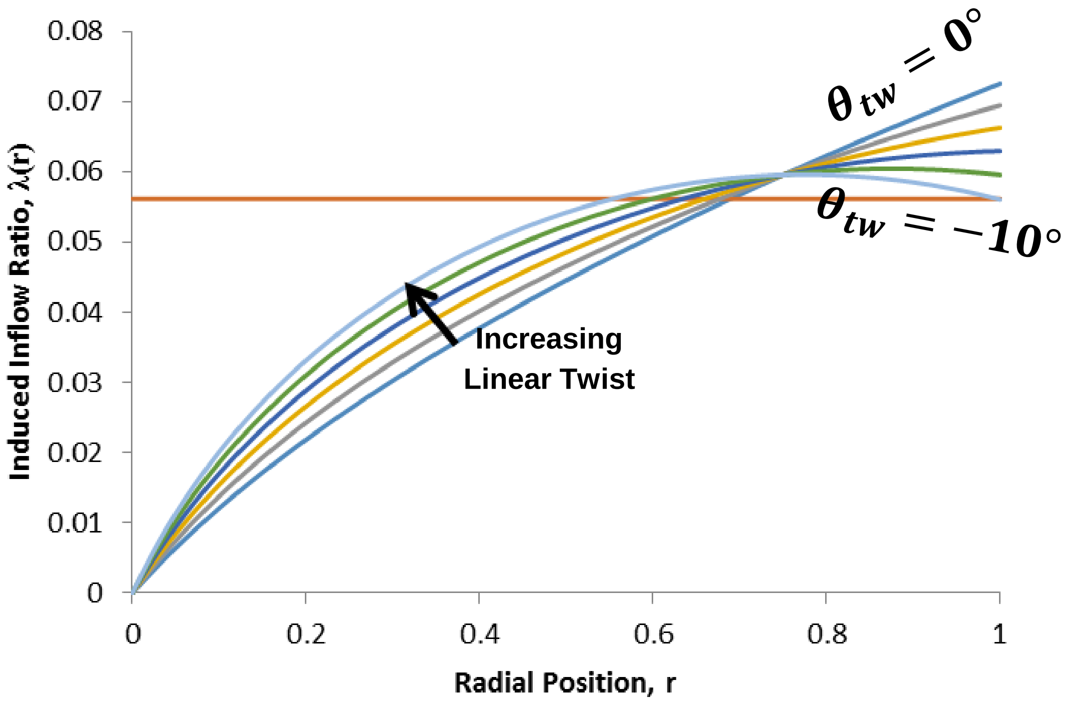
$$\lambda(r) = \sqrt{\left(\frac{\sigma C_{l\alpha}}{16}\right)^2 + \frac{\sigma C_{l\alpha}}{8}\theta r} - \left(\frac{\sigma C_{l\alpha}}{16}\right)$$

$$\boldsymbol{\lambda(r) = \frac{\sigma C_{l\alpha}}{16}\left(\sqrt{1 + \frac{32}{\sigma C_{l\alpha}}\theta r} - 1\right)}$$

LINEAR TWIST

Linear Twist: $\boldsymbol{\theta(r) = \theta_o + r\theta_{tw}}$

$$\boldsymbol{\lambda(r) = \frac{\sigma C_{l\alpha}}{16}\left(\sqrt{1 + \frac{32}{\sigma C_{l\alpha}}\theta r} - 1\right)}$$



Inflow can become more uniform = lower power!

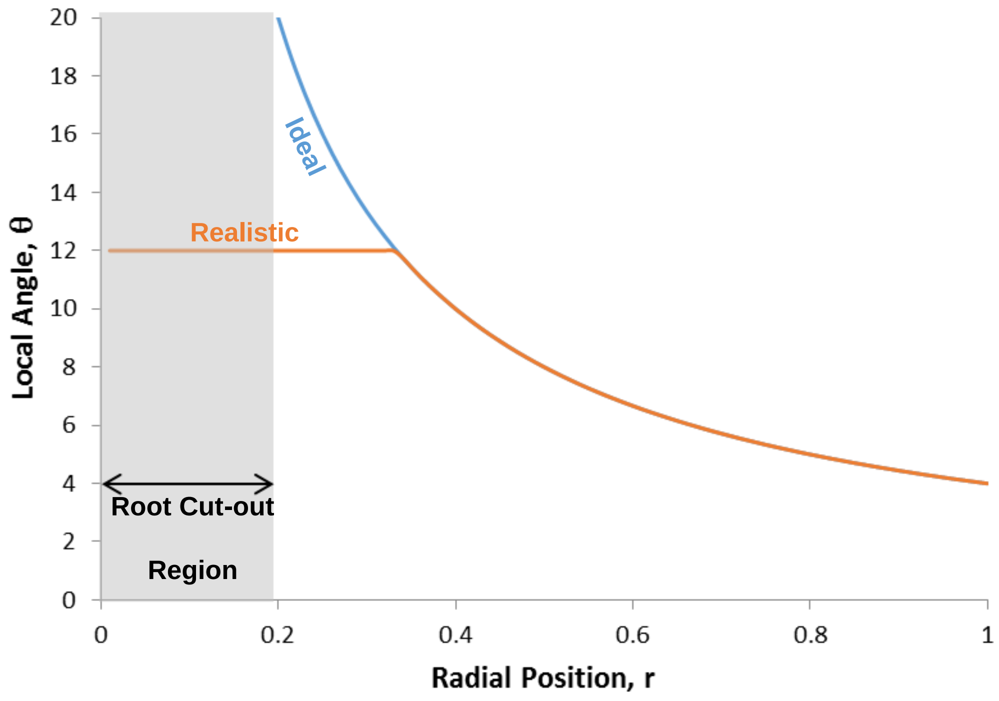
IDEAL TWIST

$$\boldsymbol{\lambda(r) = \frac{\sigma C_{l\alpha}}{16}\left(\sqrt{1 + \frac{32}{\sigma C_{l\alpha}}\theta r} - 1\right)}$$

Will achieve uniform inflow in hover if:

$$\boldsymbol{\theta(r) = \frac{\theta_{tip}}{r}}$$

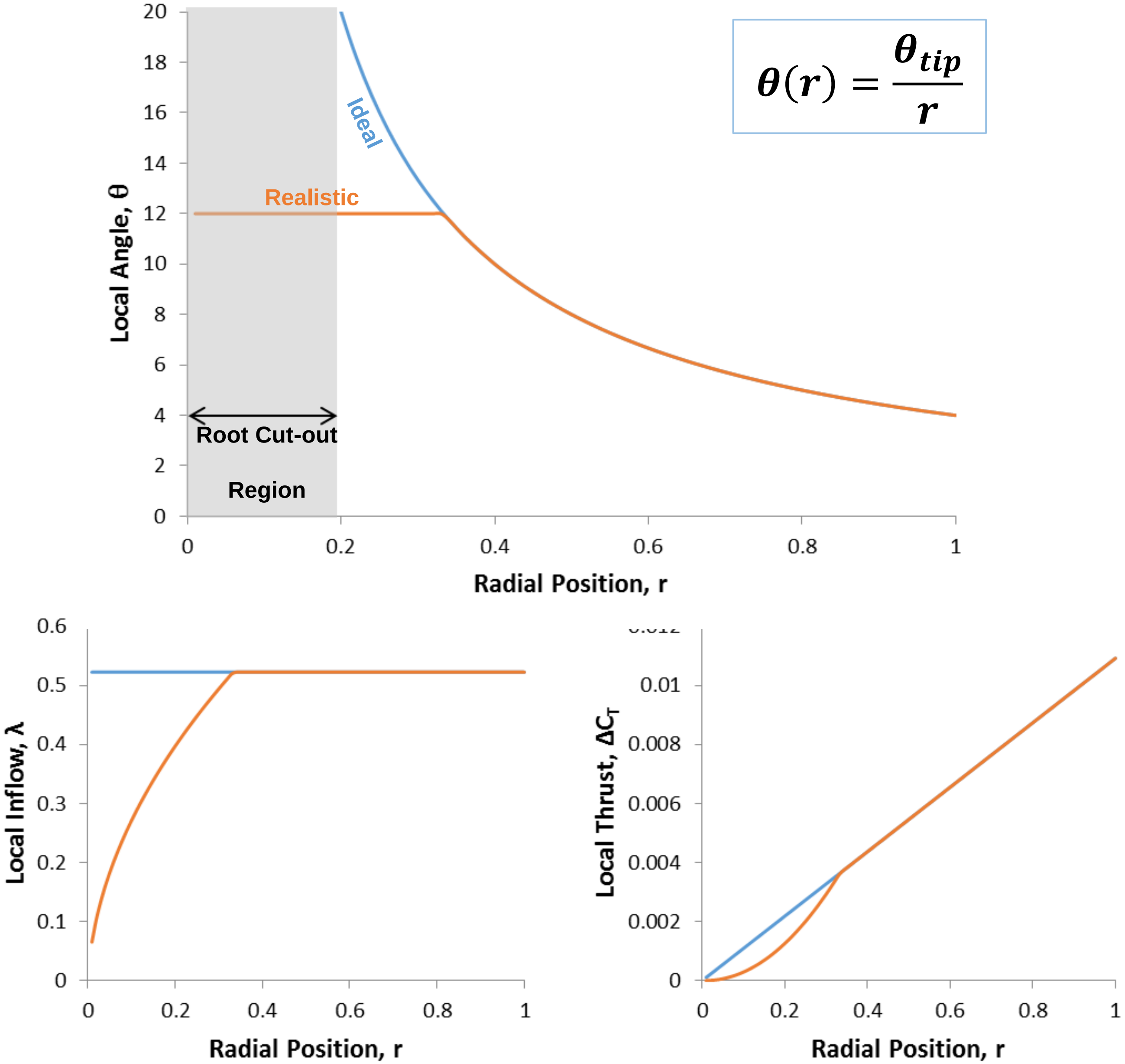
$$\lambda = \frac{\sigma C_{l\alpha}}{16}\left(\sqrt{1 + \frac{32}{\sigma C_l}\theta_{tip}} - 1\right)$$



However not realistic, θ goes to infinity at r = 0 as based on linearized aerodynamics (ignores stall)

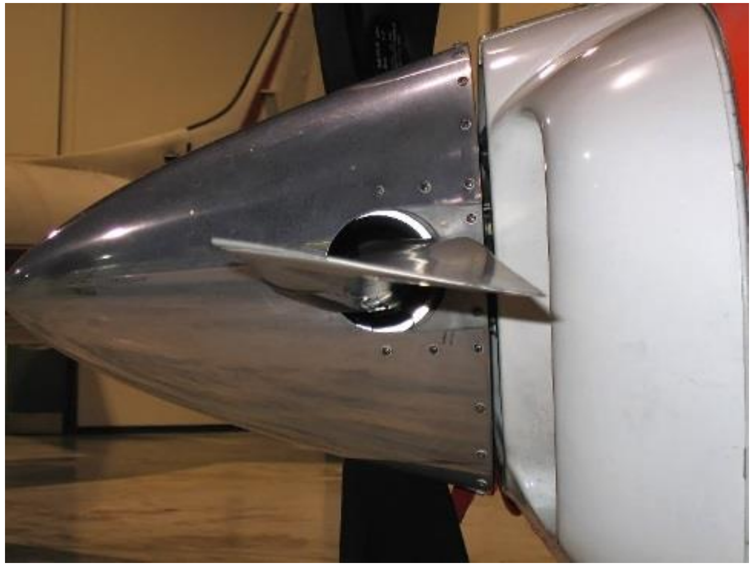
IDEAL TWIST

$$\theta(r) = \frac{\theta_{tip}}{r}$$



IDEAL TWIST

Forward Flight Compromise: Ideal twist is for hover, forward flight will have different requirements.



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1. Derive BEMT for hover & axial flight
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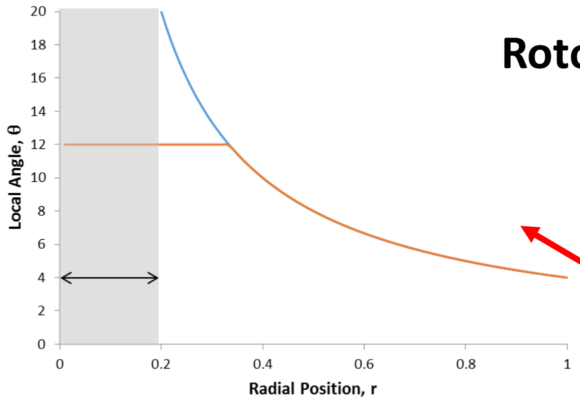
MINIMUM POWER

Ideal Twist = Uniform inflow = Minimum Induced Power

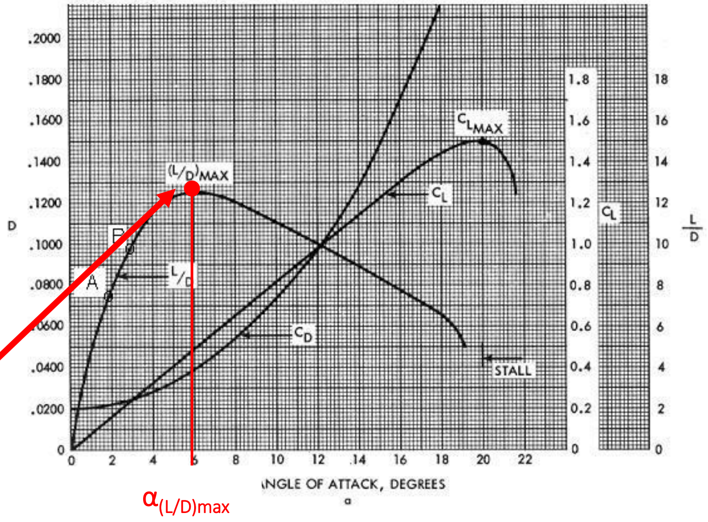
Rotor Hover Power = Induced Power + Profile Power



$$C_P = C_Q = \kappa \frac{{C_T}^{3/2}}{\sqrt{2}} + \frac{1}{8}\sigma C_d$$

$$\boldsymbol{\theta(r) = \frac{\theta_{tip}}{r}}$$

$\alpha = \alpha_{(L/D)max}$



Minimum Induced Power prefers
high α near root

Minimum Profile Power prefers
low α near root.

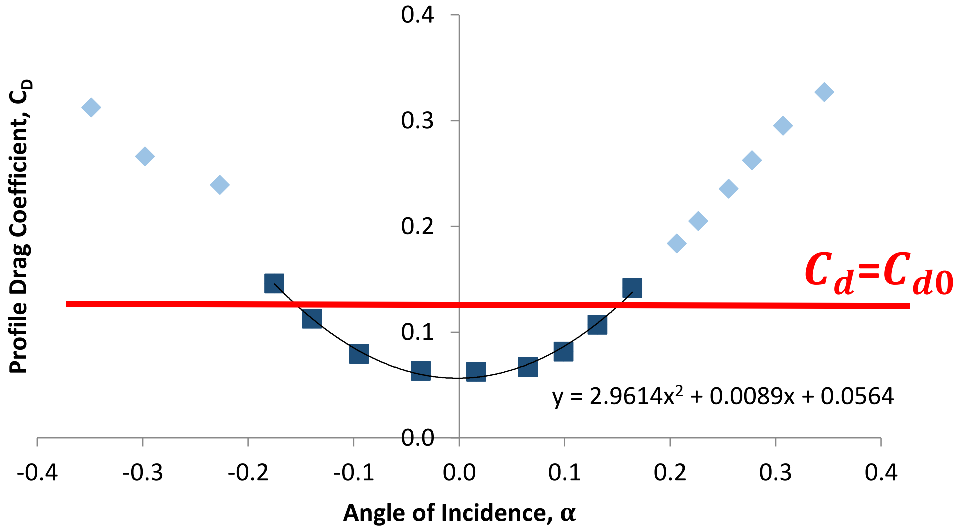
Compromise = Optimum Hovering Rotor

MINIMUM POWER

$$C_P = C_Q = \kappa \frac{{C_T}^{3/2}}{\sqrt{2}} + \frac{1}{8}\sigma C_d$$

To find optimum need drag to not be a constant.

$$C_d(\alpha) = C_{d0} + d_1\alpha + d_2\alpha^2$$



OPTIMUM HOVERING ROTOR

How to meet both requirements?

$$\boldsymbol{\theta(r) = \frac{\theta_{tip}}{r}}$$

Linearized Aero (BET):

$$\boxed{dC_T = \frac{1}{2}\sigma C_{l\alpha}\left(\theta r^2 - \lambda r\right)dr}$$

$$= \frac{1}{2}\sigma C_{l\alpha}\left(\frac{\theta_{tip}}{r} - \frac{\lambda}{r}\right)r^2 dr$$

$=\alpha=\alpha_{(L/D)max}$

$$= \frac{1}{2}\sigma C_{l\alpha}\alpha_1 r^2 dr$$

Equate to Momentum Equation (5.15) : $dC_T = 4\lambda^2 r dr = \dfrac{1}{2}\sigma C_{l\alpha}\,\alpha_1 r^2 dr$

Rearrange: $\lambda = \sqrt{\dfrac{\sigma C_{l\alpha}\,\alpha_1 r}{8}}$

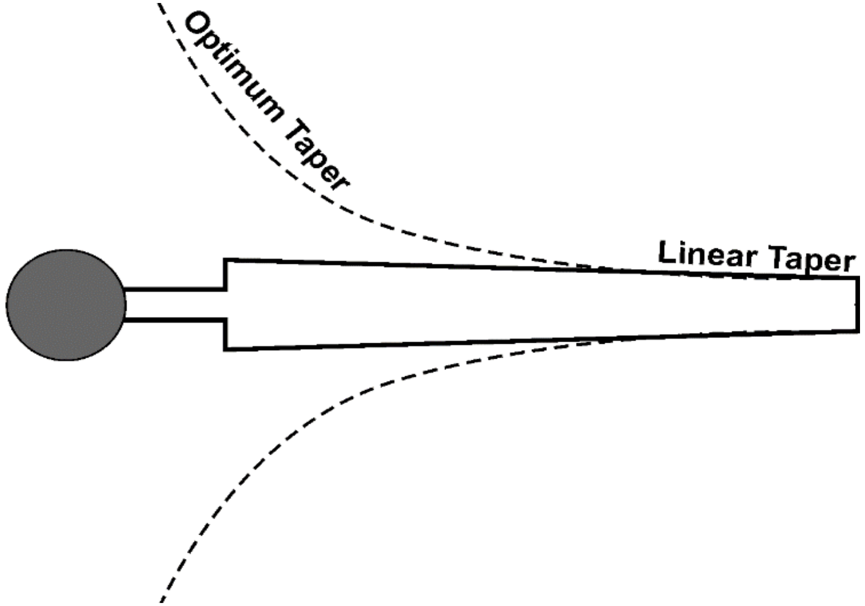
OPTIMUM HOVERING ROTOR

$$\lambda = \sqrt{\frac{\sigma C_{l\alpha}\,\alpha_1 r}{8}}$$

Constants

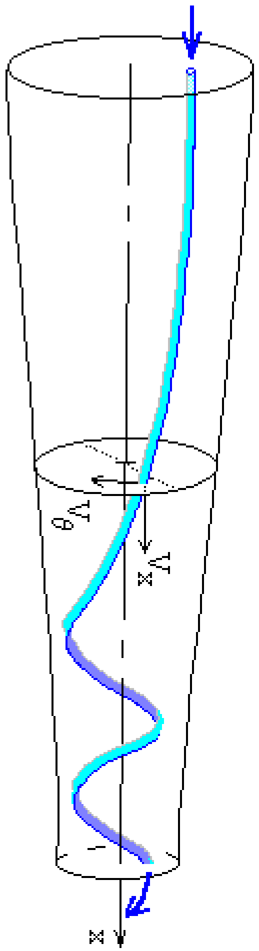
Uniform inflow (minimum power) achieved when: $\sigma r = \left(\frac{N_b c}{\pi R}\right) r = Constant$

$$\boldsymbol{c = \frac{c_{tip}}{r}}$$



Goes to infinity at r = 0, not practical. Use linear taper instead.

SWIRL



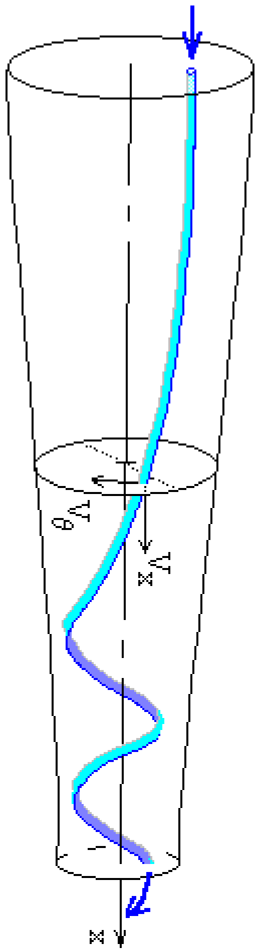
Flow will be accelerated normal to rotor plane (mainly by lift) but also around rotor plane (mainly by drag).

To include add wake rotational interference factor a':

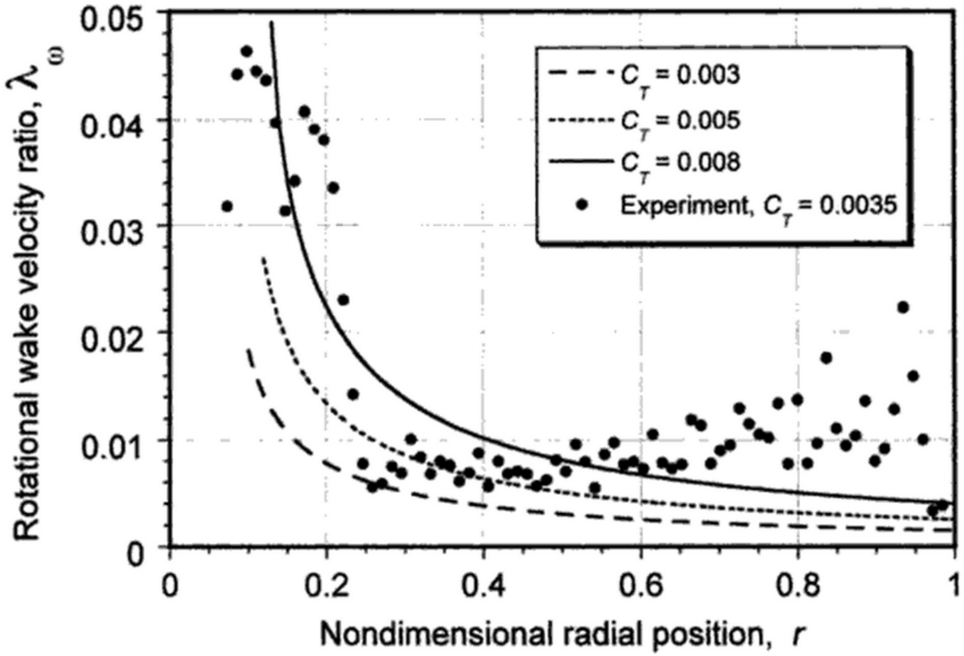
$$(1 - a')\Omega dQ_i = v_i dT$$

Hard to solve.

SWIRL



For helicopters the effect is small (<1%)



However for propellers / wind turbines it becomes significant.

SUMMARY

$$\lambda(r,\lambda_c) = \sqrt{\left(\frac{\sigma C_{l\alpha}}{16} - \frac{\lambda_c}{2}\right)^2 + \frac{\sigma C_{l\alpha}}{8}\theta r} - \left(\frac{\sigma C_{l\alpha}}{16} - \frac{\lambda_c}{2}\right)$$

$$\Delta C_{T,n} = \frac{\sigma C_{l\alpha}}{2}\left(\theta(r_n) r_n^2 - \lambda(r_n) r_n\right)\Delta r$$

$$C_T = \sum_{n=1}^{N} \Delta C_{T,n}$$

$$C_P = C_Q = \sum_{n=1}^{N} \lambda_n \Delta C_{T,n}$$

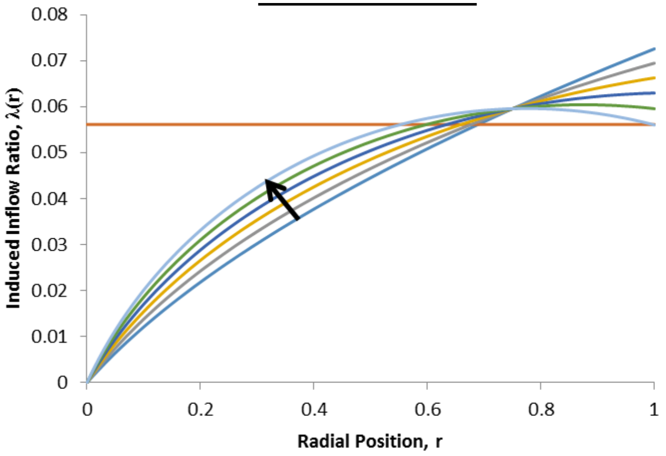
1. BEMT Balances momentum and blade forces to predict non-uniform inflow.
2. Non-uniform inflow always increases induced power
3. Non-uniformity minimised through ideal twist but neglects stall and profile power.

$$\theta(r) = \frac{\theta_{tip}}{r}$$

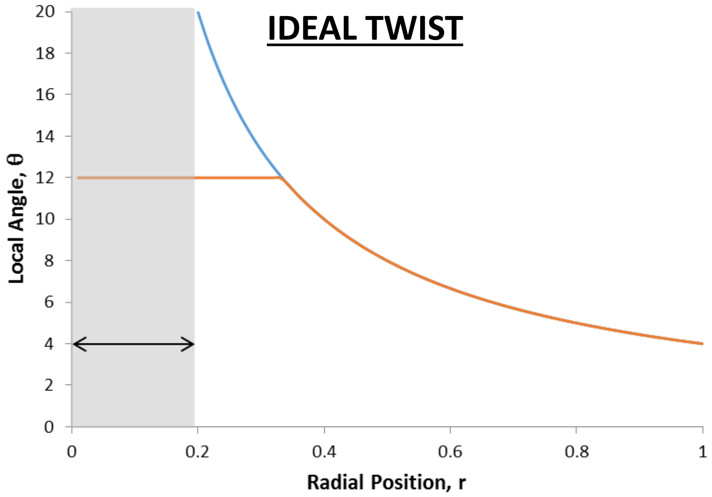
4. Total power minimised through optimum hovering rotor (maintain $(L/D)_{min}$ and uniform inflow through chordwise variation). Not realistic use taper instead.

$$c = \frac{c_{tip}}{r}$$

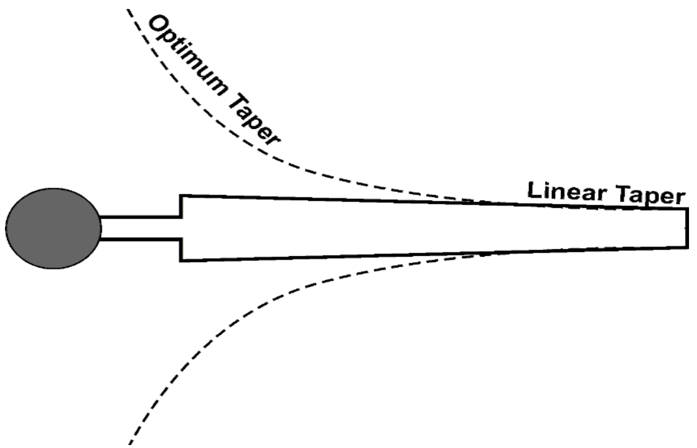
LINEAR TWIST



IDEAL TWIST



TAPER



Next?

Tutorial Questions: ADT Q1 – 13, BET Q1-4 & BEMT Q5-7.

Further Reading: Part 1 Notes.

Next Lecture: Coursework.