ME40343: Helicopter Dynamics

ADVANCED HELICOPTER DYNAMICS

Dr David Cleaver

S1 2018

PART 1: LECTURE PLAN

1: Introduction
2: Actuator Disc Theory (ADT) Hover
3: ADT Vertical Flight
4: ADT Forward Flight
5: Blade Element Theory (BET)
6: Blade Element Momentum Theory (BEMT)
7: Coursework – The Wind Turbine

ACTUATOR DISC THEORY* (1–4)

BET / BEMT* (5–6)

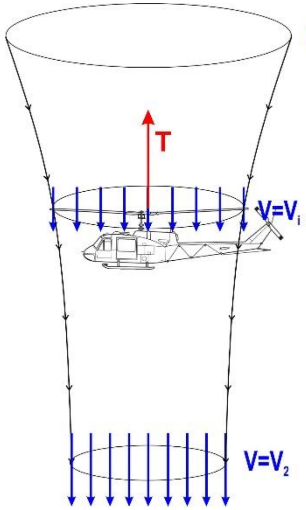
* 1 Question Each in Exam Paper

ROTOR AERODYNAMICS

APPLY TO WIND TURBINE

1. Actuator Disc Theory

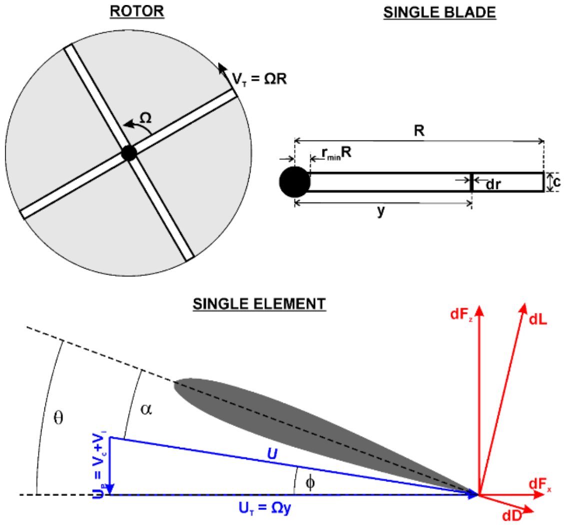
Newton's second law: **F = ma**



1D 'Conceptual' Approach

2. Blade Element Theory

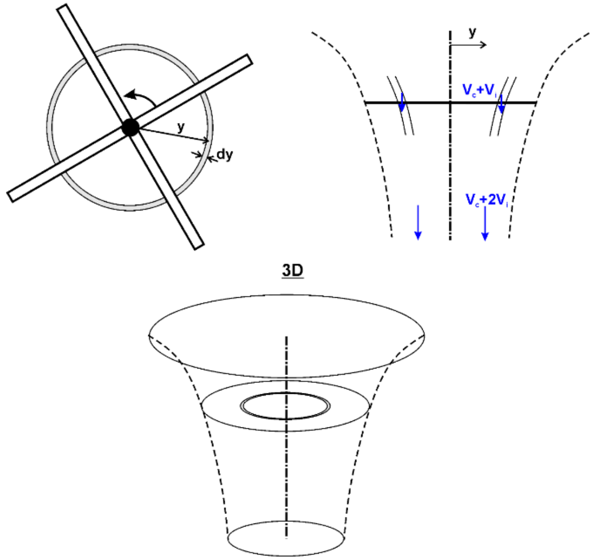
Integration of individual airfoil elements



Assumes Uniform Induced Velocity

3. Blade Element Momentum Theory

Hybrid, balance BET and ADT for each radii



Numerical Solution

BEMT

$$\lambda(r,\lambda_c) = \sqrt{\left(\frac{\sigma C_{l\alpha}}{16} - \frac{\lambda_c}{2}\right)^2 + \frac{\sigma C_{l\alpha}}{8}\theta r} - \left(\frac{\sigma C_{l\alpha}}{16} - \frac{\lambda_c}{2}\right)$$

$$\Delta C_{T,n} = \frac{\sigma C_{l\alpha}}{2}\left(\theta(r_n) r_n^2 - \lambda(r_n) r_n\right)\Delta r$$

$$C_T = \sum_{n=1}^{N} \Delta C_{T,n} \qquad C_P = C_Q = \sum_{n=1}^{N} \lambda_n \Delta C_{T,n}$$

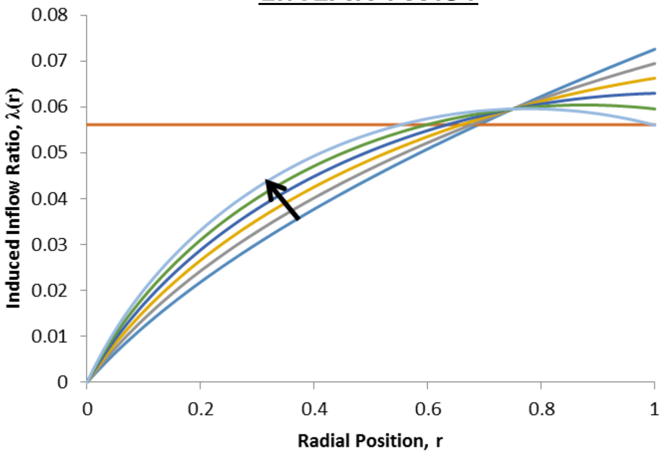
1. BEMT Balances momentum and blade forces to predict non-uniform inflow.
2. Non-uniform inflow always increase power
3. Non-uniformity minimised through ideal twist but neglects stall and profile power.

$$\theta(r) = \frac{\theta_{tip}}{r}$$

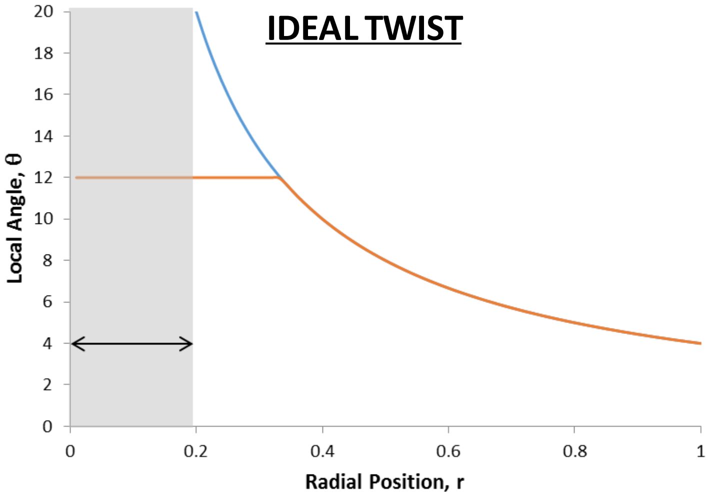
4. Total power minimised through optimum hovering rotor (maintain $(L/D)_{max}$ and uniform inflow through chordwise variation). Not realistic use taper instead.

$$c = \frac{c_{tip}}{r}$$

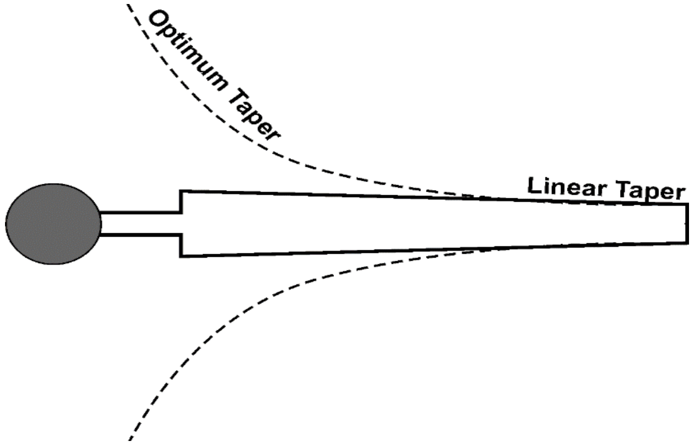
LINEAR TWIST



IDEAL TWIST



TAPER



LECTURE 7

7: The Inverse Problem: The Wind Turbine

1. The Wind Turbine
2. Derive BEMT for wind turbine
3. Coursework
4. MATLAB

UNIVERSITY OF BATH

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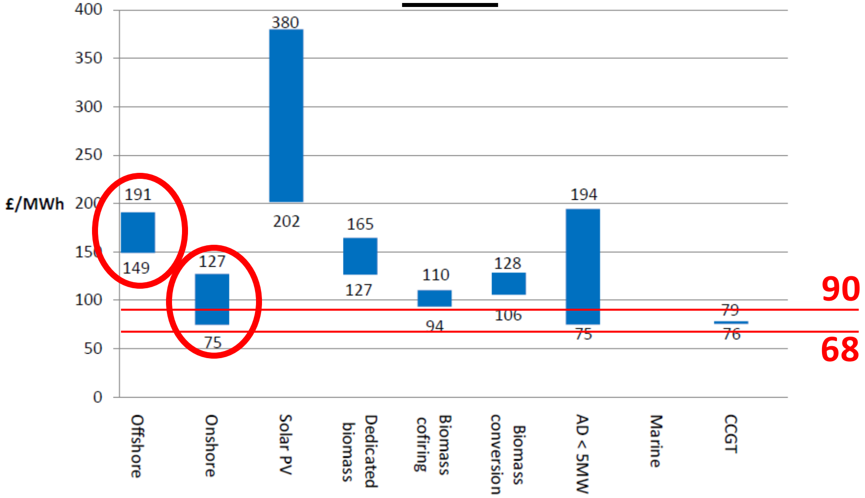
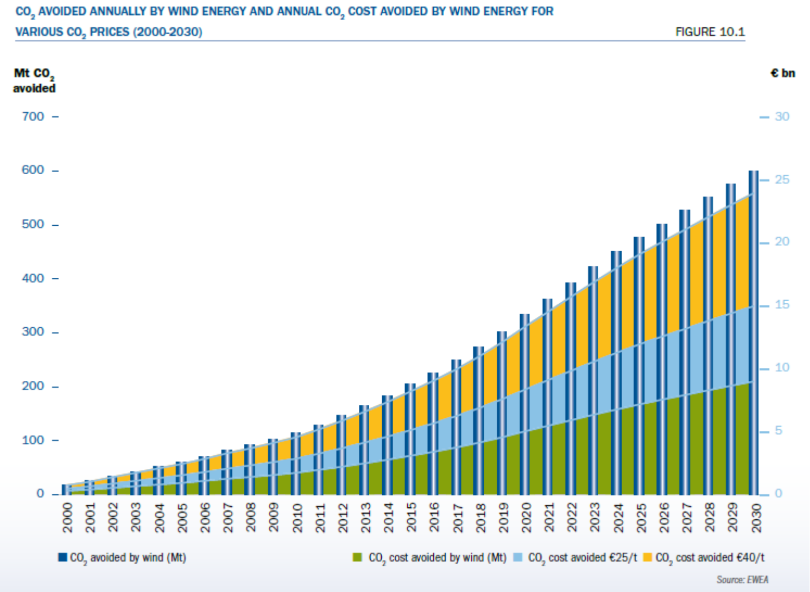
7: COURSEWORK

THE WIND TURBINE

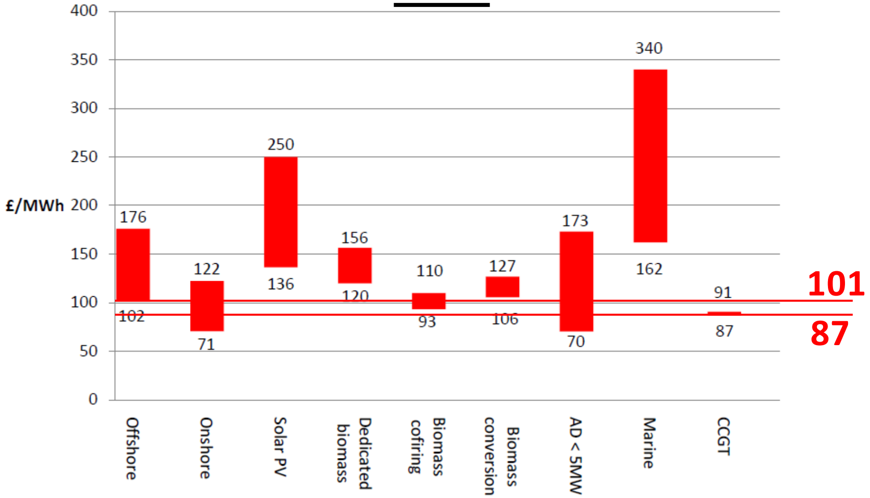
2010



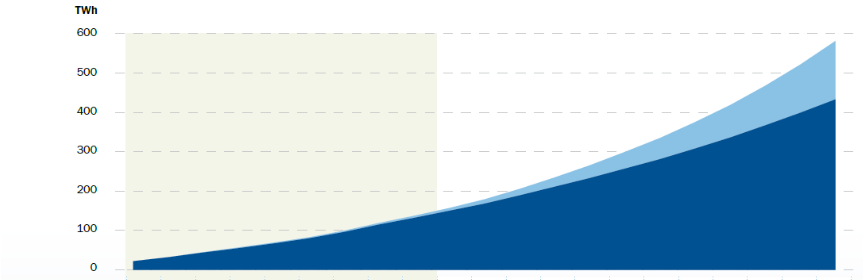
CO2 Cost



2020



THE WIND TURBINE

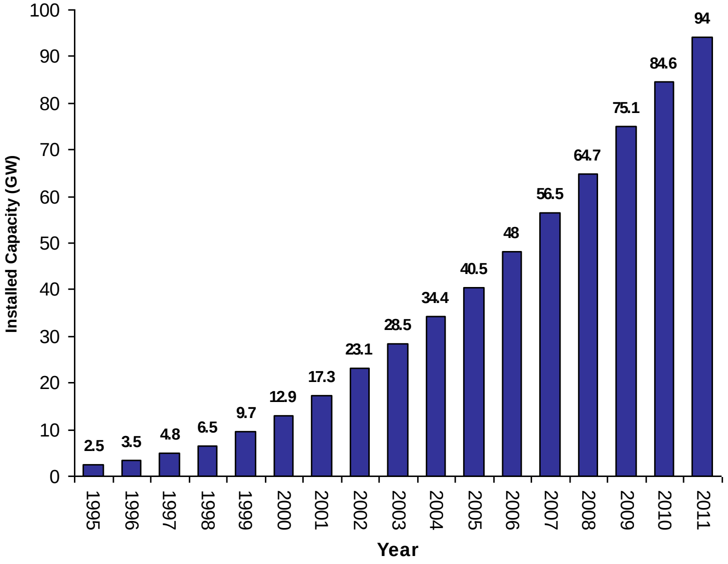


	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Offshore	0.1	0.3	1.0	1.9	2.2	2.6	3.3	4.1	5.3	6.9	10.9	16.4	23.5	32.3	42.3	53.7	67.1	82.4	100.5	122.4	148.3
Onshore	22	32	44	55	67	80	96	115	132	150	168	189	211	233	257	281	308	336	367	399	433
TOTAL	23	32	44	57	70	83	100	119	137	157	179	205	234	265	299	335	375	419	467	522	582

Source: EWEA

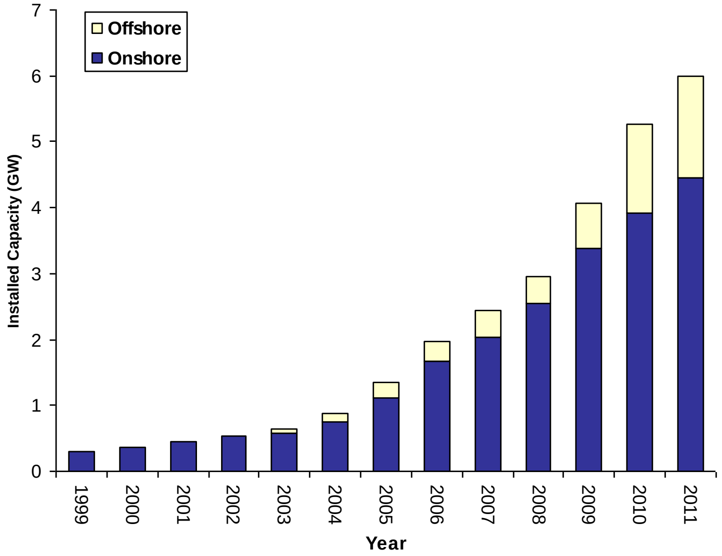
- Fastest growing energy sector ~16% p.a
- ~100GW Installed capacity
- €32.5bn GDP (2010)

Europe



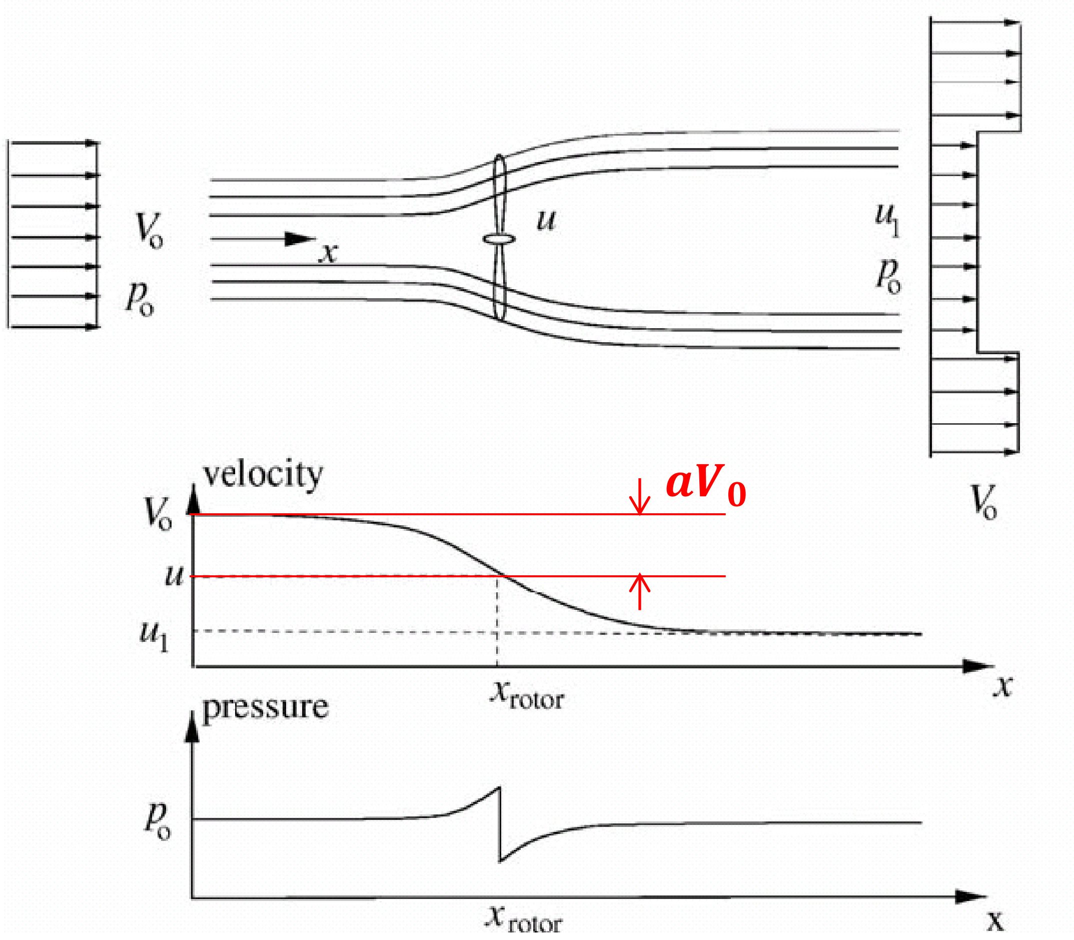
- Fastest growing energy sector ~ 30% p.a.
- 2020 forecast of 26GW
- At cost of ~£60bn

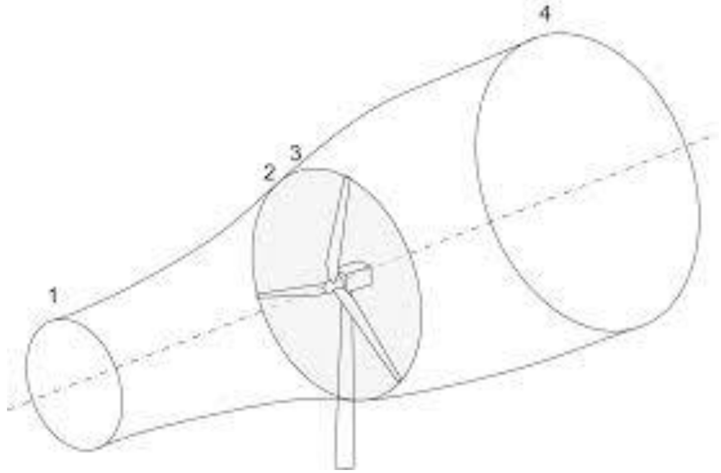
UK



THE WIND TURBINE

ADT





Axial Induction Factor a:

$$u = (1 - a)V_0$$

a = proportion of velocity absorbed

W.T. equivalent of induced velocity

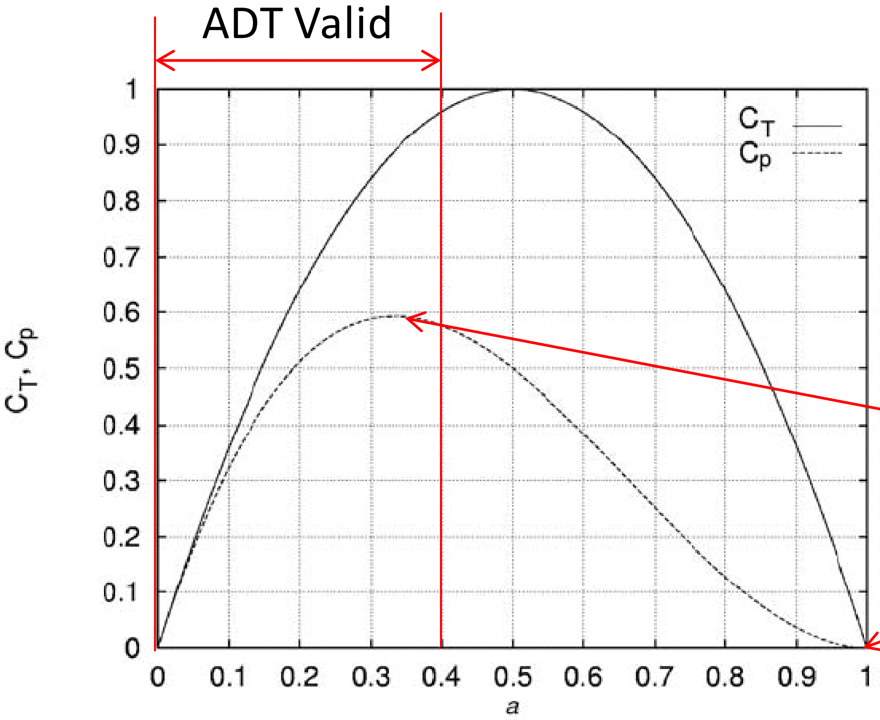
THE WIND TURBINE

ADT

See Hansen chapter 4 for derivation:

$$C_T = 4a(1-a)$$

$$C_P = 4a(1-a)^2$$



Maxima / minima when $\frac{dC_P}{da} = 0$:

$$\frac{dC_P}{da} = 4(1-a)(1-3a) = 0$$

a=1: minima

a=1/3: maxima

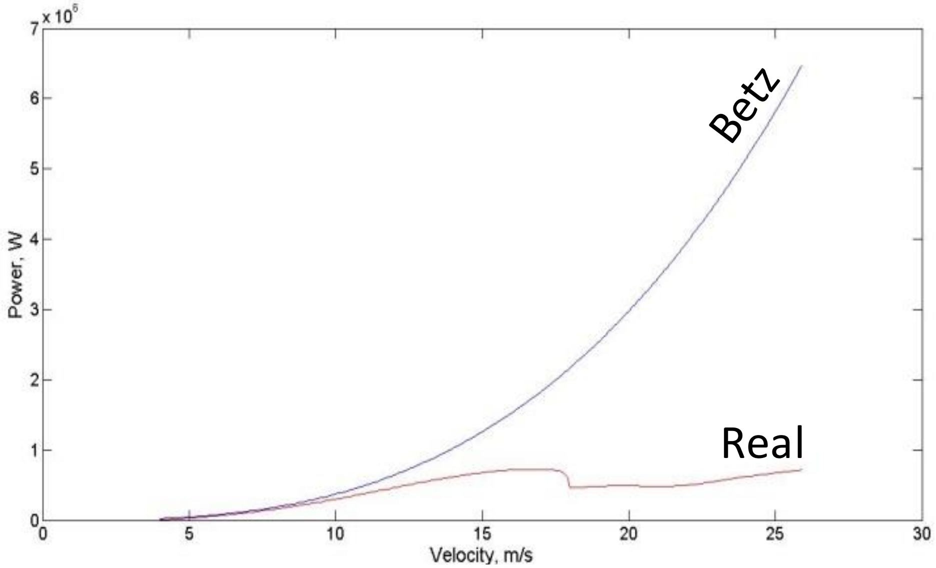
$$C_{P,max} = 4 * \frac{1}{3}\left(1 - \frac{1}{3}\right)^2 = \frac{16}{27}$$

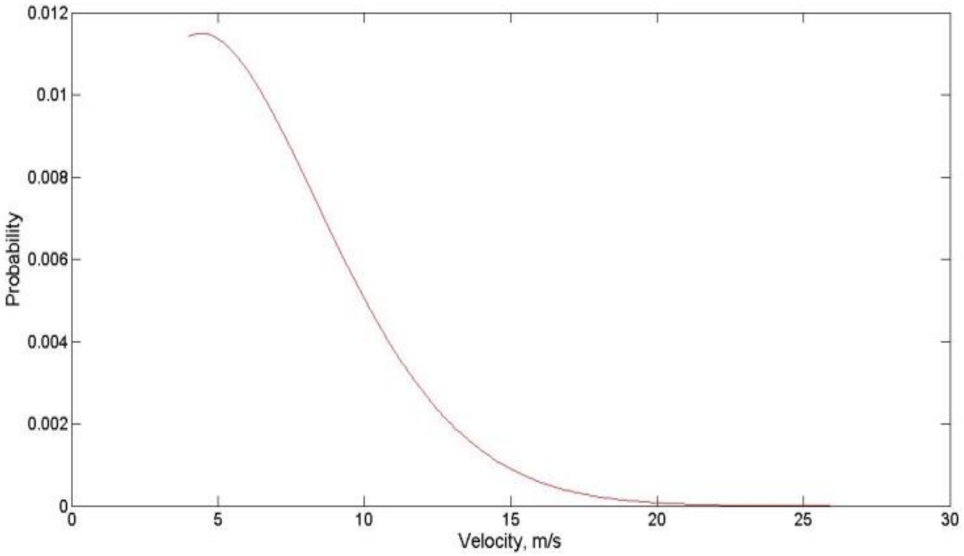
BETZ LIMIT

THE WIND TURBINE

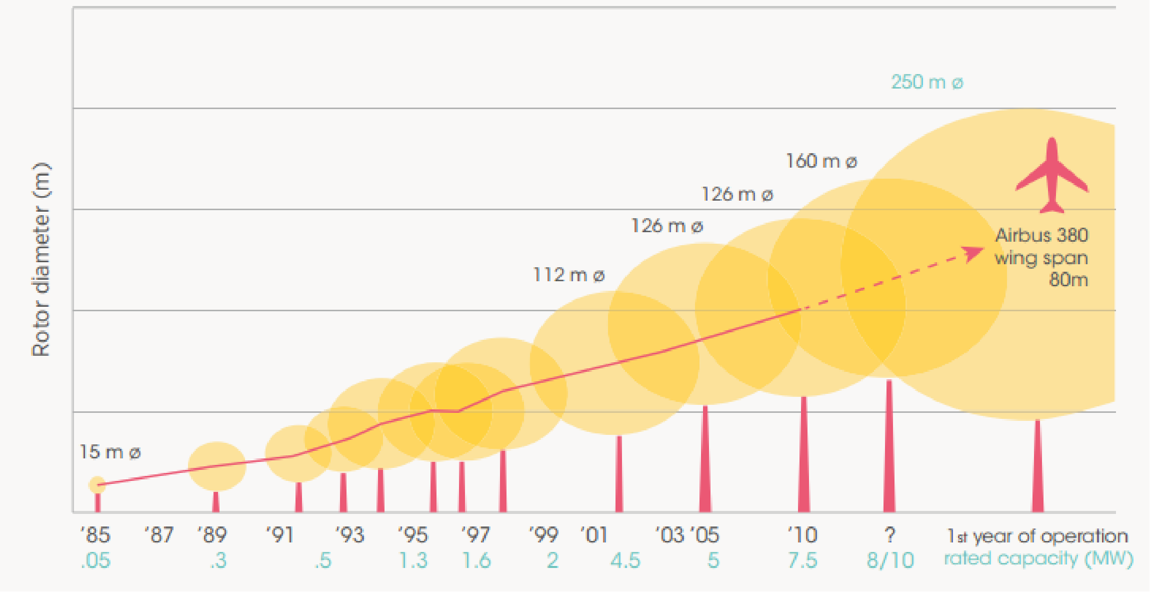
$$C_{P,max} = \frac{16}{27} = \frac{P}{\frac{1}{2}\rho {V_0}^3 A}$$

VELOCITY

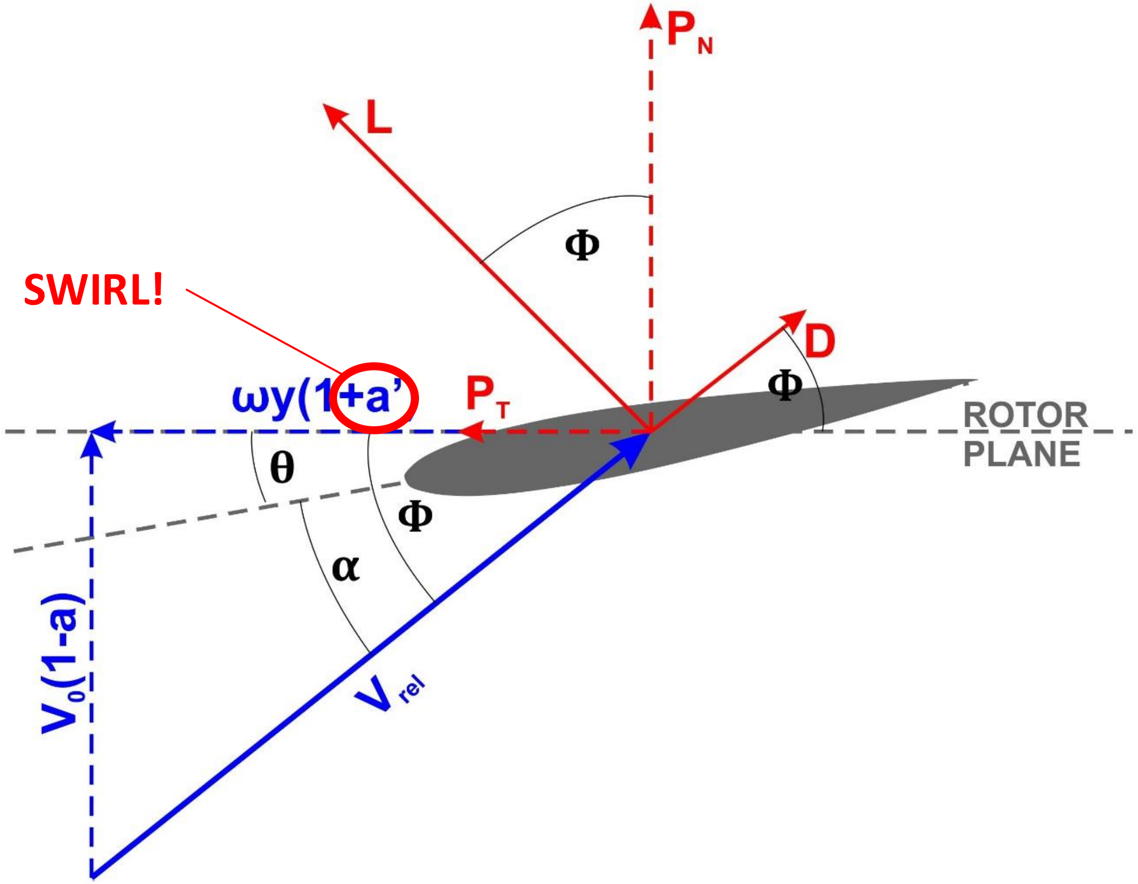




AREA



BEMT



See Hansen chapter 6 for more detail.

Momentum Equations:

For Annulus

$$dT = 4\pi y\rho V_0{}^2 a(1-a)dy$$

$$dM = 4\pi y^3\rho V_0\,\omega(1-a)a'dy$$

BET Equations:

$$dT = \frac{1}{2}\rho B\left(\frac{V_0\,(1-a)}{sin\Phi}\right)^2 cC_n dy$$

$$dM = \frac{1}{2}\rho y B\frac{V_0\,(1-a)\varpi y(1+a')}{sin\Phi cos\Phi}cC_t dy$$

BEMT

Momentum Equations

$$dT = 4\pi r\rho V_0^{\,2} a(1-a)dr$$

$$dM = 4\pi r^3\rho V_0\,\omega(1-a)a'dr$$

BET Equations

$$dT = \frac{1}{2}\rho B\left(\frac{V_0\,(1-a)}{\sin\Phi}\right)^2 cC_n dr$$

$$dM = \frac{1}{2}\rho r B\frac{V_0\,(1-a)\varpi r(1+a')}{\sin\Phi\cos\Phi}cC_t dr$$

Equate (dT=dT and dM=dM) gives:

BEMT

$$a = \frac{1}{\frac{4\sin^2\phi}{\sigma C_n}+1} \qquad a' = \frac{1}{\frac{4\sin\phi\cos\phi}{\sigma C_t}-1}$$

Iterative Solution

BEMT

1. Initial guess for a and a’
2. Calculate flow angle ϕ:

$$tan\phi = \frac{(1-a)V_0}{(1+a')\omega y}$$

3. Calculate angle $\alpha = \phi - \theta$
4. Use MATLAB function (in Moodle) to get Cl(α, Re) and Cd(α, Re)

   [Cl, Cd] = ForceCoefficient(Alpha, Re)

5. Convert to normal and tangential forces

$$C_n = C_l cos\phi + C_d sin\phi \qquad C_t = C_l sin\phi - C_d cos\phi$$

6. Calculate a and a’

$$a = \frac{1}{\frac{4sin^2\phi}{\sigma C_n}+1} \qquad a' = \frac{1}{\frac{4sin\phi cos\phi}{\sigma C_t}-1}$$

7. If a and a’ are outside a specified tolerance, loop back to step 2 with these new values.

   Error = abs(a_{OUT}-a_{IN})+abs(a'_{OUT}-a'_{IN})

Error > tolerance

Error < tolerance

Calculate Forces

COURSEWORK

Design a technically optimal wind turbine.

1. Understand Wind Turbines
Growing sector closely related to aerospace.

2. Apply BEMT
Very important, widely used for a range of problems (rotorcraft, propellers, wind turbines…)

3. Develop Programming Skills
Extremely useful skill

4. Apply Optimisation
Applicable to a very wide range of problems, becoming standard.

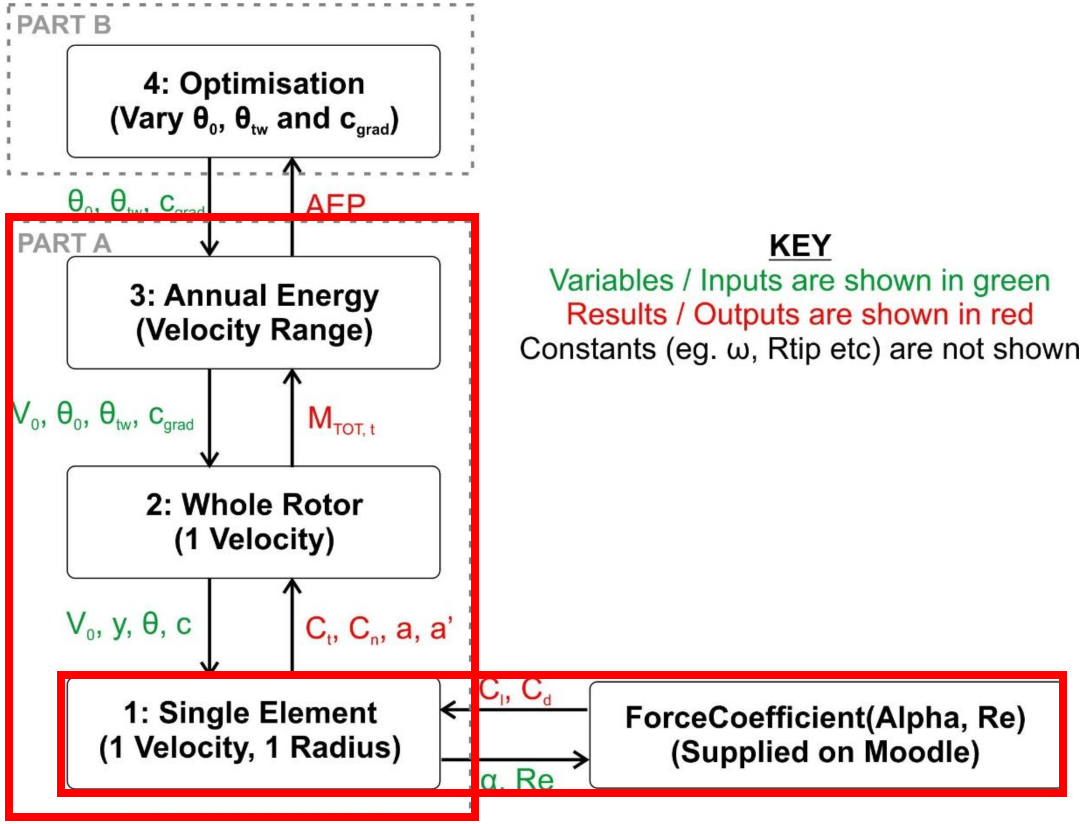
COURSEWORK

Part A (50%): BEMT Model
Write a model to calculate the Annual Energy Production (AEP) of a given design.

Part B (50%): Optimisation
Use MATLAB optimisation routines to optimise: θ_0, θ_{tw}, and $c_{grad.}$
Critically assess and discuss model and design limitations.

COURSEWORK

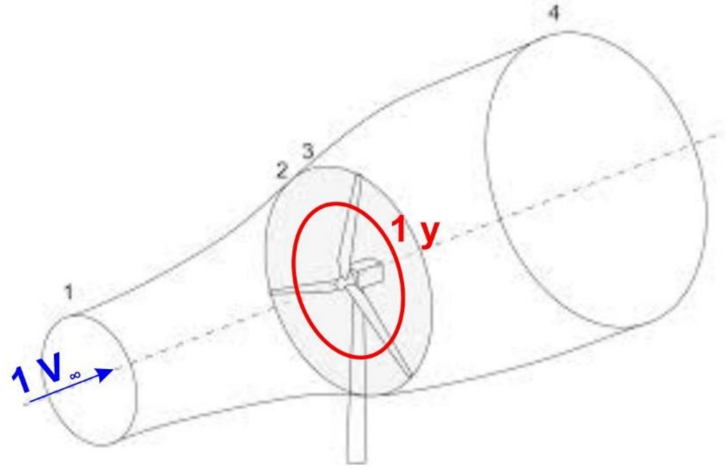
Part A: BEMT Model



3: Annual Energy Production
Repeat for velocity range and multiply by probability to get AEP.

2: Whole Rotor
Integration across rotor to get total force for 1 velocity.

1: Single Element
BEMT Loop. Calculate force at 1 radius for 1 velocity.



COURSEWORK

1: Single Element

At a given radius and velocity, calculate the forces.

UNSTABLE ITERATION?

1. Initial guess for a and a’
2. Calculate flow angle ϕ:

$$tan\phi = \frac{(1-a)V_0}{(1+a')\omega y}$$

3. Calculate angle $\alpha = \phi - \theta$
4. Use MATLAB function (in Moodle) to get Cl(α, Re) and Cd(α, Re)

[Cl, Cd] = ForceCoefficient(Alpha, Re)

5. Convert to normal and tangential forces

$$C_n = C_l cos\phi + C_d sin\phi \qquad C_t = C_l sin\phi - C_d cos\phi$$

6. Calculate a and a’

$$a = \frac{1}{\frac{4sin^2\phi}{\sigma C_n} + 1} \qquad a' = \frac{1}{\frac{4sin\phi cos\phi}{\sigma C_t} - 1}$$

7. If a and a’ are outside a specified tolerance, loop back to step 2 with these new values.

Error = abs(a_{OUT}-a_{IN})+abs(a'_{OUT}-a'_{IN})

Error>tolerance

A) ADD RELAXATION FACTOR

$a=k*(a_{OUT}-a_{IN})+a_{IN}$

$a'=k*(a'_{OUT}-a'_{IN})+a'_{IN}$

B) ADD COUNTER

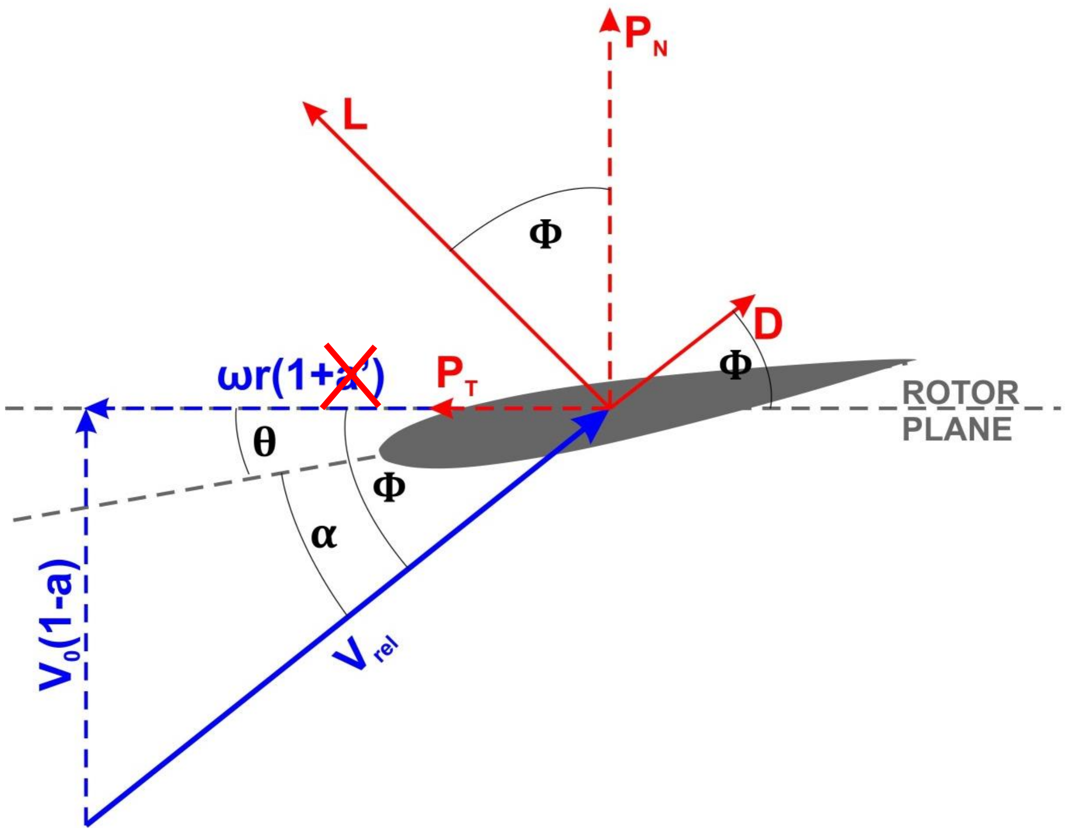
If Count > 100
  loop without a’
end

Error<tolerance

Calculate Forces

COURSEWORK

UNSTABLE ITERATION?



COURSEWORK

ALTERNATE a LOOP

1. Initial guess for a
2. Calculate flow angle ϕ:

$$\tan\phi = \frac{(1-a)V_0}{\omega y}$$

3. Calculate angle $\alpha = \phi - \theta$
4. Use MATLAB function (in Moodle) to get Cl(α, Re) and Cd(α, Re)

[Cl, Cd] = ForceCoefficient(Alpha, Re)

5. Convert to normal and tangential forces

$$C_n = C_l\cos\phi + C_d\sin\phi \qquad C_t = C_l\sin\phi - C_d\cos\phi$$

6. Calculate a

$$a = \frac{1}{\frac{4\sin^2\phi}{\sigma C_n} + 1}$$

7. If a is outside a specified tolerance, loop back to step 2 with these new values.

Error = abs(a_{OUT}-a_{IN})

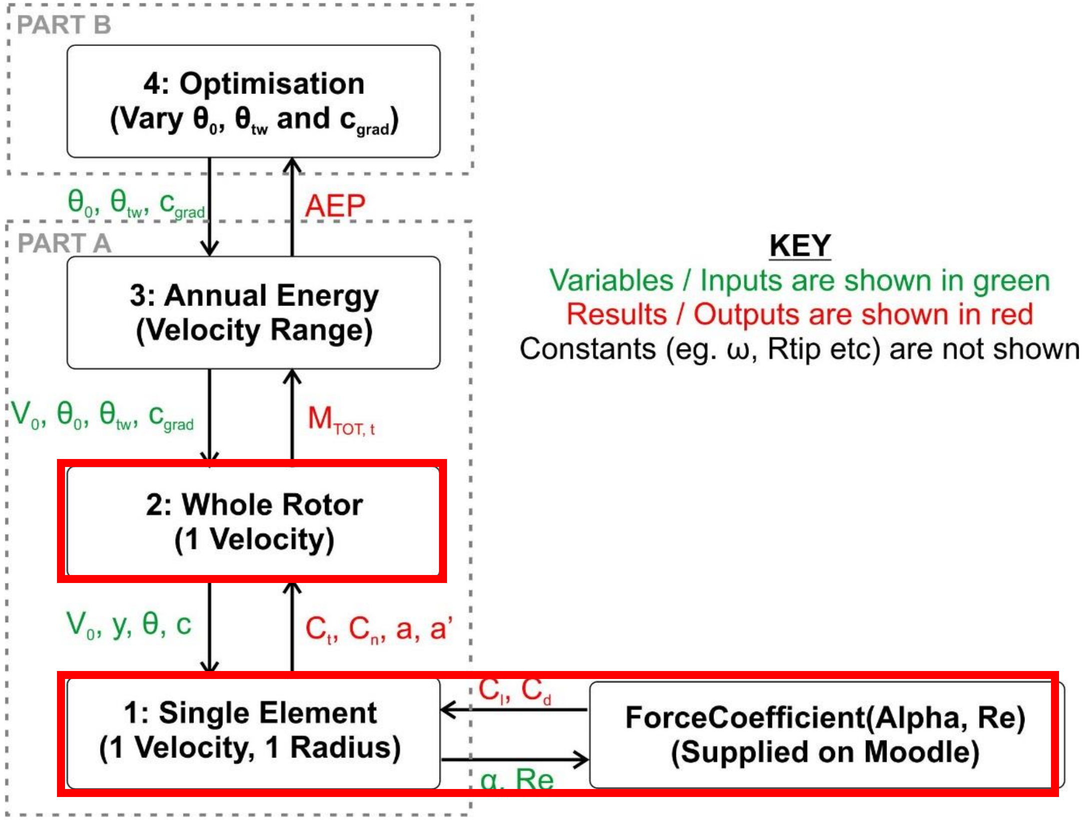
Error>tolerance

Error<tolerance

Calculate Forces

COURSEWORK

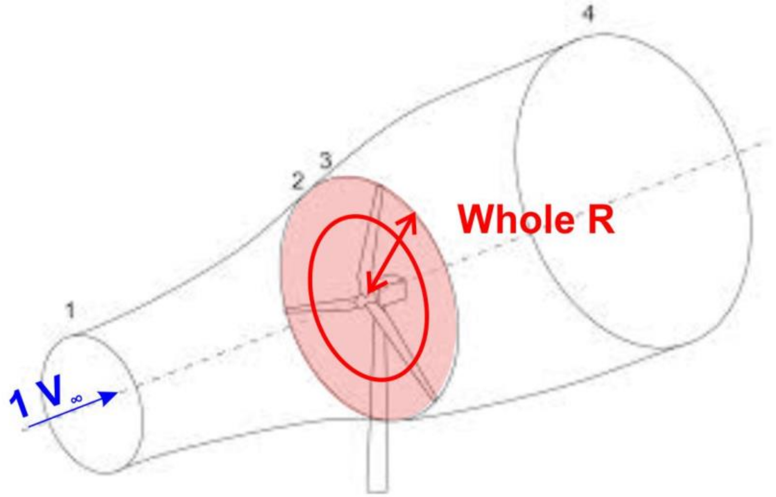
Part A



3: Annual Energy Production
Repeat for velocity range and multiply by probability to get AEP.

2: Whole Rotor
Integration across rotor to get total force for 1 velocity.

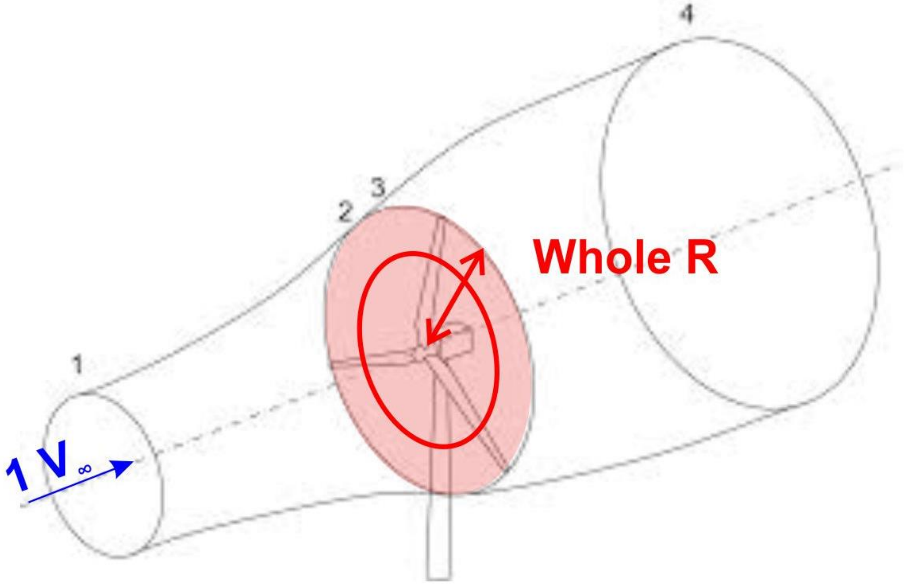
1: Single Element
BEMT Loop. Calculate force at 1 radius for 1 velocity.

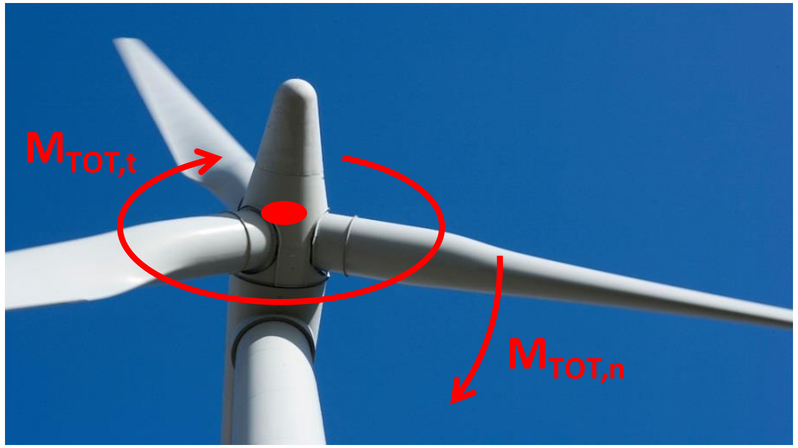


COURSEWORK

2: Whole Rotor

For one wind speed, calculate total moments (torque and bending)





1. Loop single element calculation to cover 1..N radii

2. Calculate two moments

$$M_i = \left(\frac{1}{2}\rho V_{REL}{}^2 c C_?\right)\Delta y * y_i$$

3. Numerically Integrate moments

$$M_{TOT,?} = \sum_{i=1}^{N} M_i$$

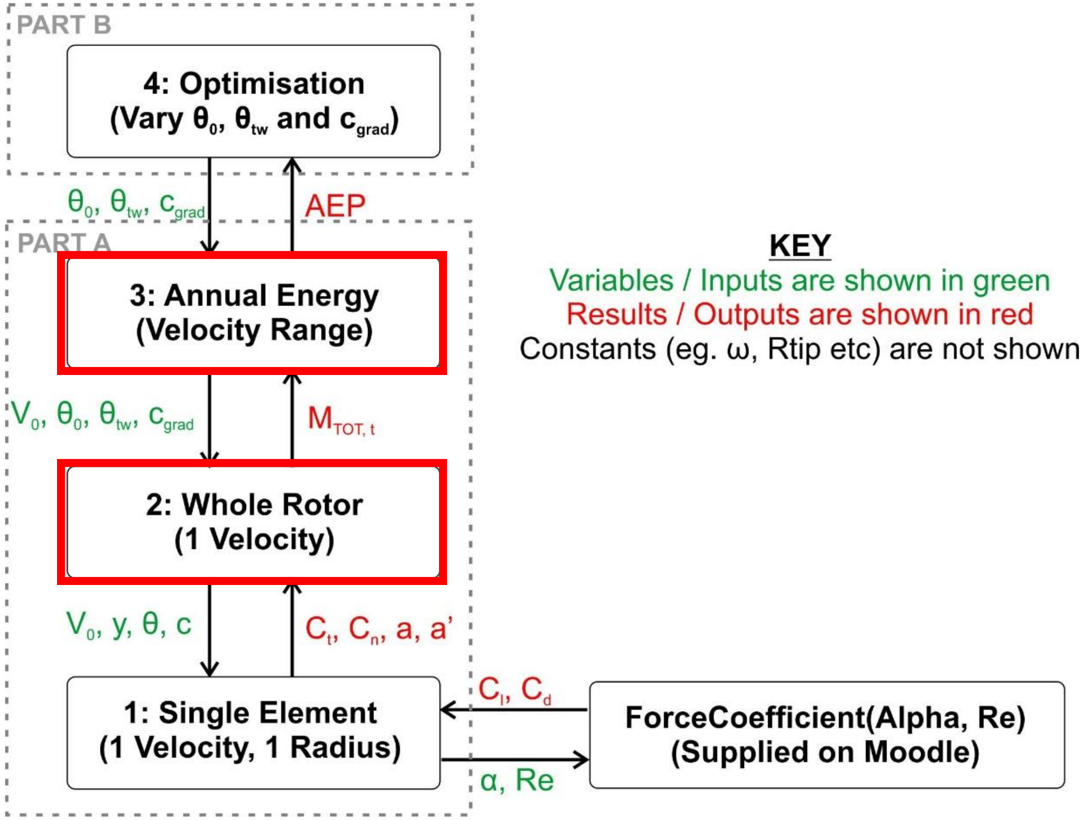
4. Calculate power:

$$P = M_{TOT,t} * B * \omega$$

$M_{TOT,t}$ = *in-plane moment for power extraction*
$M_{TOT,n}$ = *out-of-plane blade bending moment*

COURSEWORK

Part A



KEY

Variables / Inputs are shown in green

Results / Outputs are shown in red

Constants (eg. ω, Rtip etc) are not shown

3: Annual Energy Production

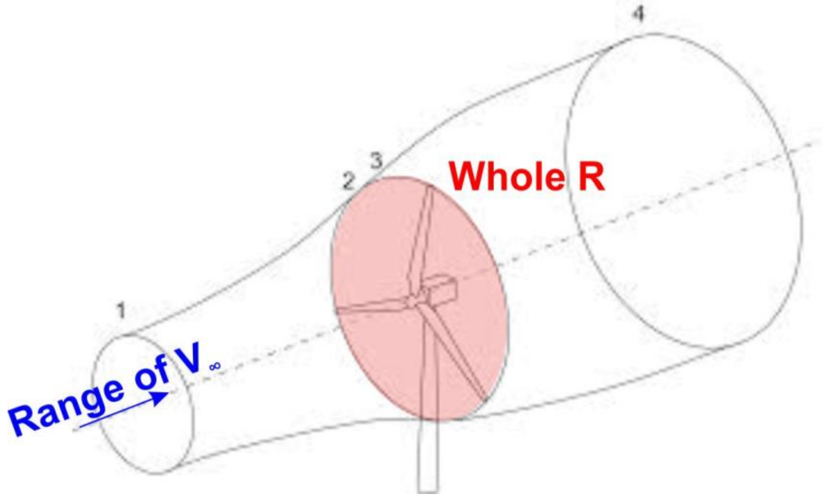
Repeat for velocity range and multiply by probability to get AEP.

2: Whole Rotor

Integration across rotor to get total force for 1 velocity.

1: Single Element

BEMT Loop. Calculate force at 1 radius for 1 velocity.

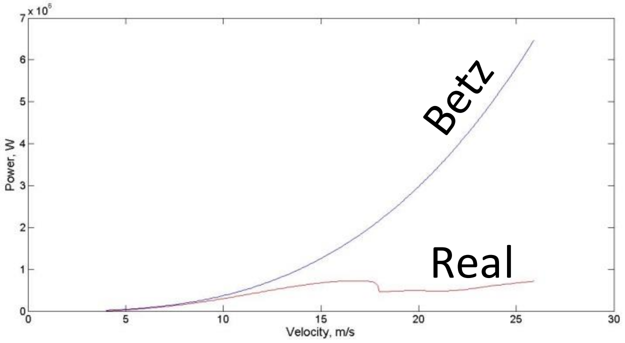


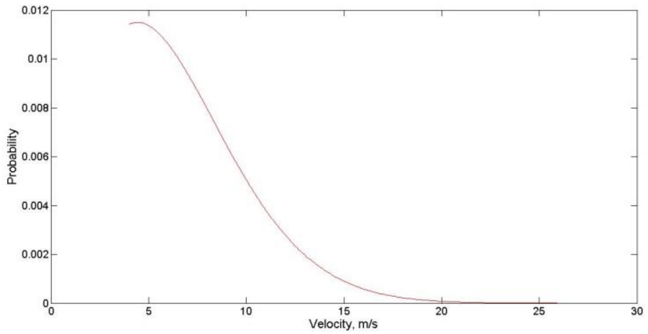
COURSEWORK

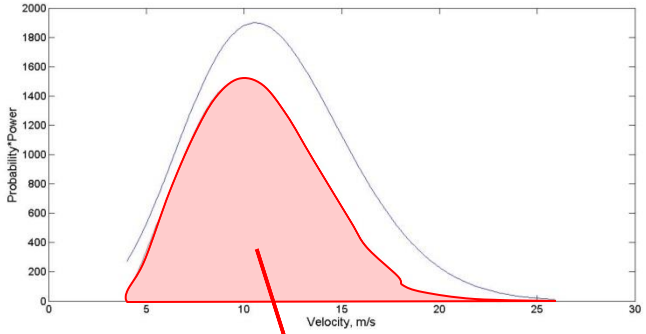
3: Annual Energy

Loop for range of wind speeds then multiply by probability to get AEP

Power X **Probability** = **Power per ΔV**







Integration = AEP

Loop Whole Rotor for velocity range.

$$f(V_i < V_0 < V_{i+1}) = exp\left\{-\left(\frac{V_i}{A}\right)^k\right\} - exp\left\{-\left(\frac{V_{i+1}}{A}\right)^k\right\}$$

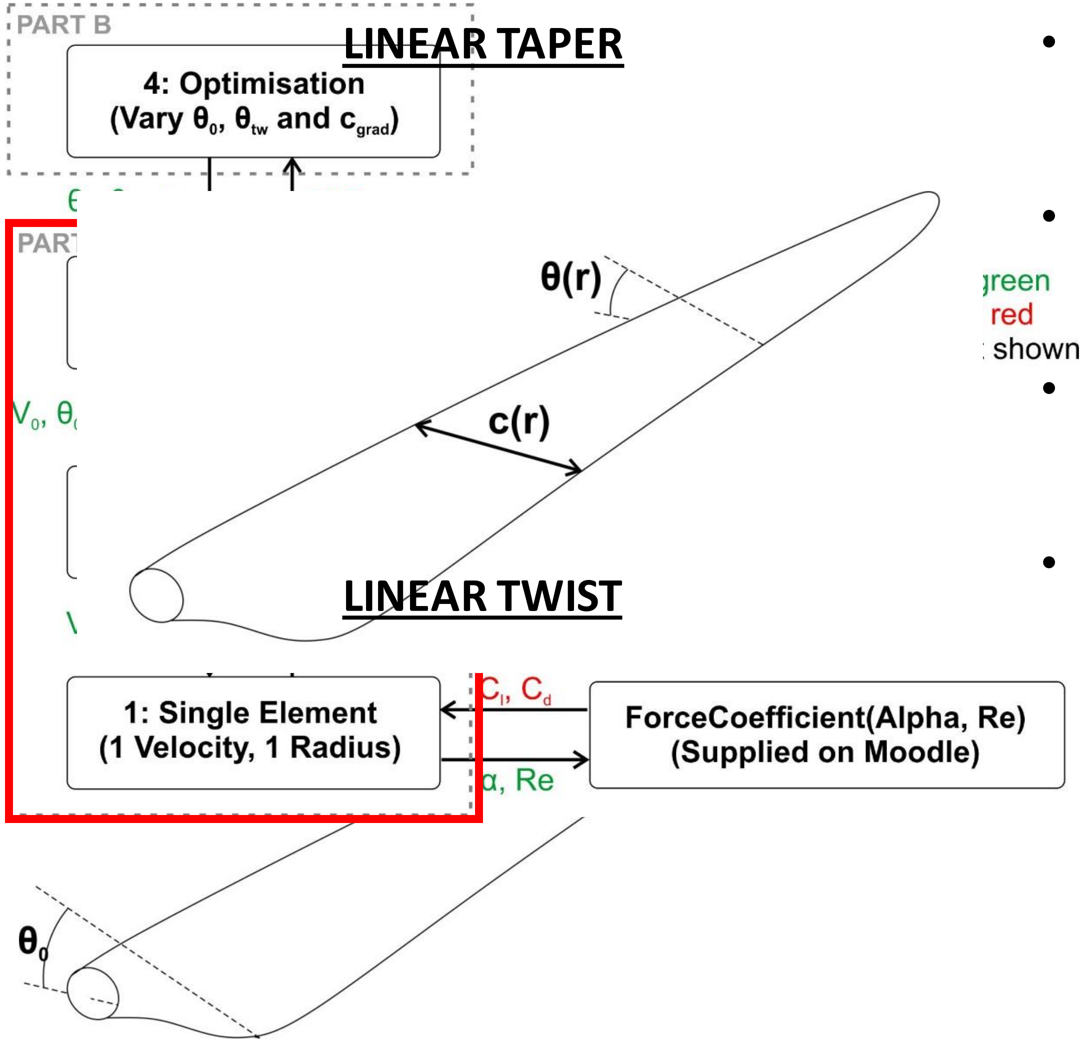
Constants for a site
A = 7
k = 1.8

Convert to W*hr/year

$$\boldsymbol{AEP = \sum_{i=1}^{N} \frac{1}{2}[P(V_i) + P(V_{i+1})] * f(V_i < V_0 < V_{i+1}) * 8760}$$

COURSEWORK

Part A



- Can now calculate AEP for a particular W.T. design. But which design?
- For a given airfoil and radius blade can be defined through: θ(r) and c(r).
- Part A use linear taper and linear twist with constant blade area. 3 variables: θ_0, θ_{tw} and c_{grad}.
- Starting values for part A:

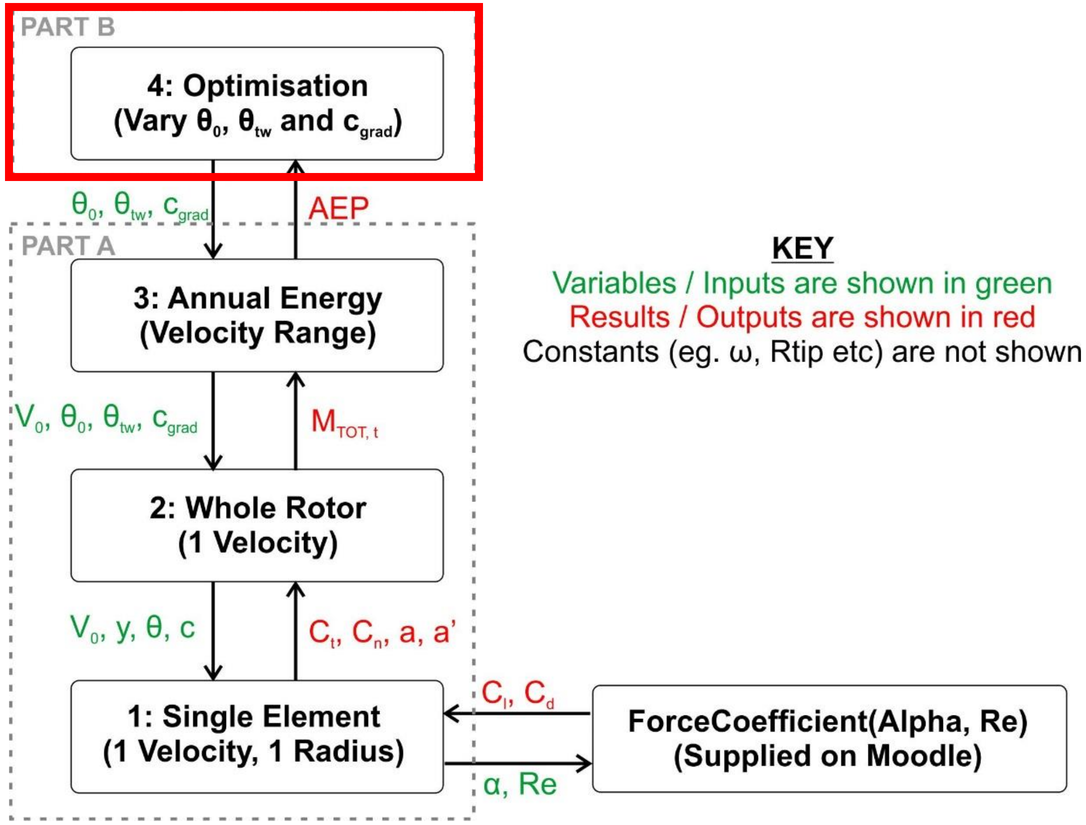
$$\theta_0 = 12^o$$
$$\theta_{tw} = -0.4^o$$
$$c_{grad} = 0$$

PART B: Optimise θ_0, θ_{tw} and c_{grad} for maximum AEP

COURSEWORK

Part B

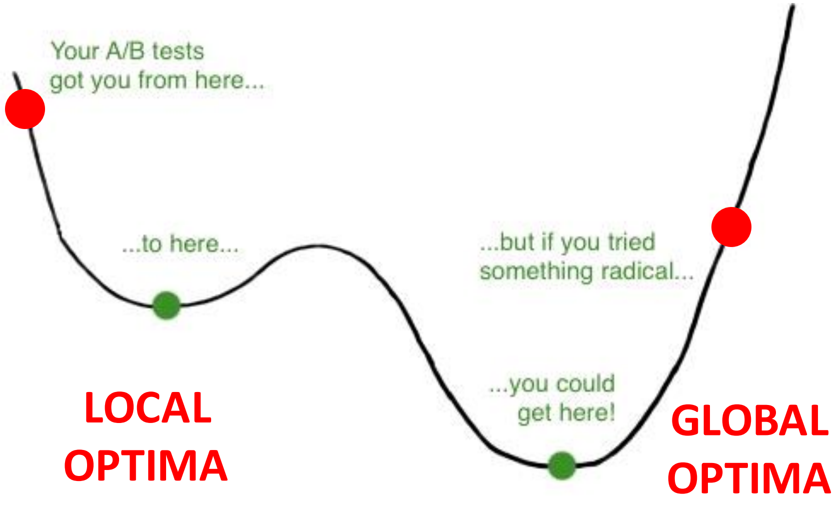
Optimise θ_0, θ_{tw} and c_{grad} for maximum AEP



- Use MATLAB optimisation function.
- Loops through part A code varying θ_0, θ_{tw} and c_{grad} until AEP is maximum.

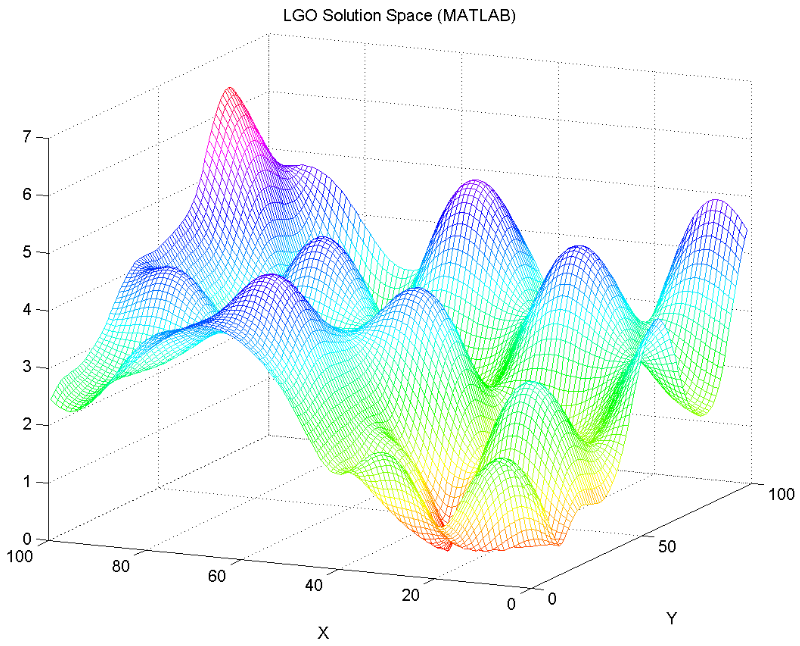
COURSEWORK

Optimisation



- Start from initial guess.
- Uses gradient to go 'downhill' and find minimum.

- New field with potential to revolutionise design in all fields of engineering.
- Particular interest from aerospace industry.
- Built on definition of a cost function. Goal is to minimise cost.



COURSEWORK

Optimisation

fun – is the name of the cost function

x0 – initial guess

LB – lower bound / limits of variables

UB – upper bound / limits of variables

x – optimised variables

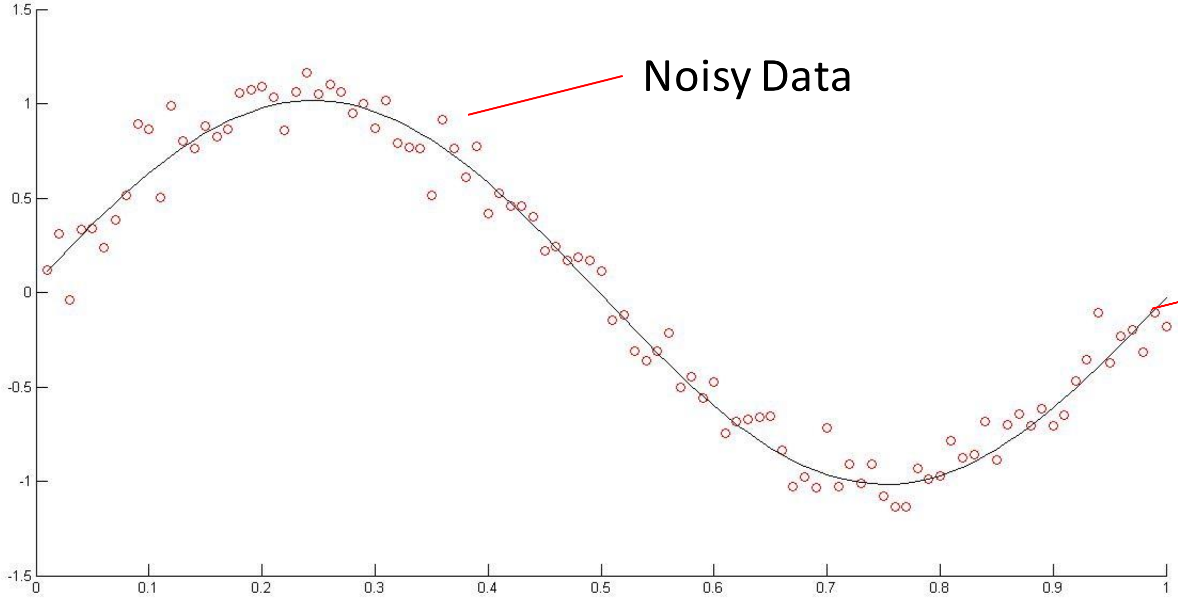
x=FMINSEARCHBND(fun,x0,LB,UB)

Outputs

Inputs

COURSEWORK

Sine Curve Example



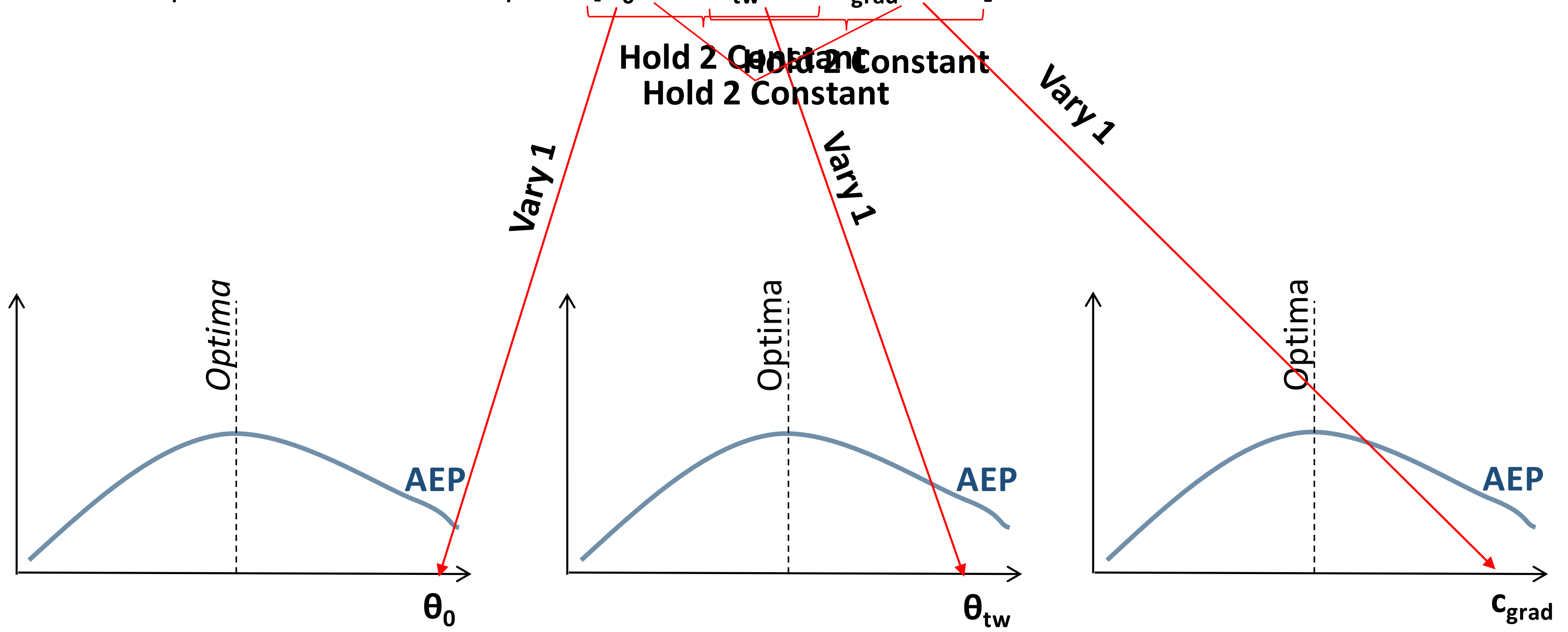
$$y = A sin(2\pi f t + \phi)$$

Optimise A, f and ϕ to get best fit.

COURSEWORK

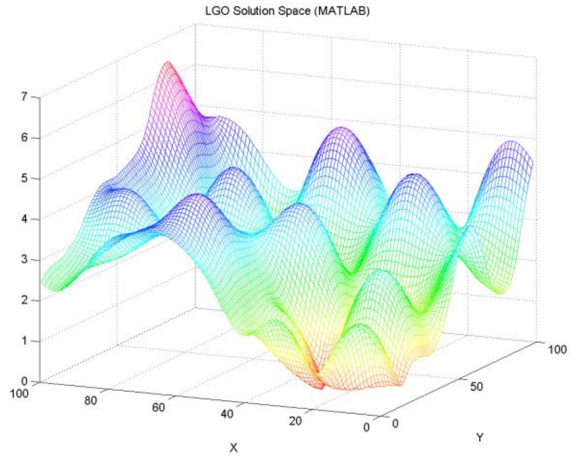
1. Check Optima:

Example Fminseachbnd output = [θ_0=15˚ θ_{tw}=20˚ c_{grad}=0.09]

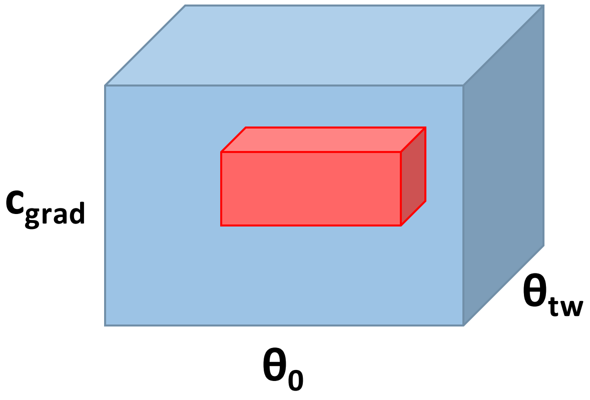


COURSEWORK

2. **Good Starting Point:**



3. **Minimise Parameter Space**



COURSEWORK

Part B: Discussion

(i) What are the limitations of the model? Describe the principle limitations of the model including possible improvements.

(ii) What are the limitations of the design? This could include aspects outside the scope of the optimisation / model like current, emerging and future technologies or changes to the design constraints.

Extra credit for demonstrating the above effects / improvements.

COURSEWORK

Deadline: 4pm 16th November

Report < 2,000 words.

	Mark (%)	Description
Summary	5	
Introduction	5	Brief.
Methodology	5	Very brief.
Results & Discussion	45	Majority of report, two sub-sections: A. AEP – explain the behaviour B. Optimal Design - use figures to demonstrate why it is optimal and what would happen if you were to vary the 3 input parameters. Discuss and further considerations
Conclusions	5	Be concise.
References & Appendices	25	Include your final MATLAB code here.
Report Quality	10	Clarity. The goal is to convey your meaning as clearly and succinctly as possible. Use figures, tables and bullet points / lists to help you.

Next?

Tutorial Questions: ADT Q1 – 13 (solutions on Moodle), BET Q1-4 & BEMT Q5-7.

Tutorials in 4E 2.39

Further Reading: Hansen (excellent text). On Moodle / through library.

Next Lecture: Forward Flight.