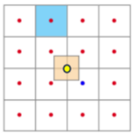
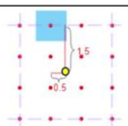
CH2 Digital Image Fundamentals

1. Nearest Neighbor Interpolation:

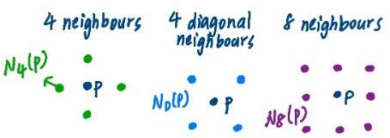
新點的值由最近的舊點決定

2. Bilinear Interpolation

3. Bicubic Interpolation

(a) 令紅點是舊圖的 pixel，灰線 grid 是新圖的 pixel。現在要用這 16 個紅點去計算出黃點 (new pixel 的值)。	
(b) 取個紅點與黃點的距離，舉例來說：黃點與藍底的那個紅點，x 方向的距離為 0.5，y 方向的距離為 1.5 (這裡都以 pixel 為單位)。	
(c) Apply 到每個紅點後，會得到一組 16 個數字的 x 軸距離，與一組 16 個數字的 y 軸距離，分別把這兩組數據帶入 function W，並各自存成一個 array $weight_x$和$weight_y$。	$W(x)=\begin{cases}(a+2)\lvert x\rvert^3-(a+3)\lvert x\rvert^2+1 & for\ \lvert x\rvert\le 1\\ a\lvert x\rvert^3-5a\lvert x\rvert^2+8a\lvert x\rvert-4a & for\ 1\le\lvert x\rvert\le 2\\ 0 & others\end{cases}$ x: 上一步算的距離
(d) 得到兩個權重矩陣後，做 Hadamard product：$紅點值 * weight\_x * weight\_y$，最後把乘完後的矩陣每個值加起來，就會得到黃點應該填入的值。	$p(x,y)=\sum_{i=0}^{3}\sum_{j=0}^{3}a_{ij}x^iy^j.$

4. Neighborhood $N_4(p), N_D(p), N_8(p)$



5. Adjacency

(1) 4-adjacency: Two pixels p q with values from V are 4-adjacent if q is in the set $N_4(p)$

(2) m(mixed)-adjacency:

(q is in the set $N_4(p)$) **OR**

(q is in the set $N_D(p)$ **AND** $N_4(p) \cap N_4(q)$沒有 V 裡的值)

6. Connectivity

(1) Connected set: 有 path 可以從 p 到 q $\forall p, q \in S$

(2) R 如果是 connected set，那它就是 a region of a image

(3) Adjacent: $R_i \cup R_j$是一個 connected set，否則稱為 disjoint

7. Boundary: R 裡的 pixel 有至少一個 background neighbor

8. Distance

(1) Euclidean: $D_e(p,q) = [(x-s)^2 + (y-t)^2]^{1/2}$

(2) City Block: $D_e(p,q) = |x-s| + |y-t|$

(3) Chess Board: $D_8(p,q) = \max(|x-s|, |y-t|)$

(4) m-adjacency path: 畫 m-ad 然後算要走幾步

9. 加減乘除運算的效果

(1) 相加 reduce noise (e.g. 10 張 noisy img 相加再除 10)

(2) 相減突出 details (e.g. 注射顯影劑後減掉未注射的)

(3) 相除後相消的感覺

(4) 相乘可以做 mask

CH3 Spatial Domain Filtering

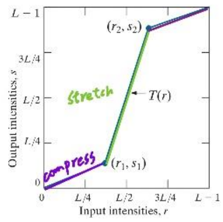
1. Log transformation: $s = clog(1+r),\ r \ge 0$

2. Power-Law (Gamma) trans: $s = cr^{\gamma},\ \gamma < 1$會提亮, 太亮/暗 + washed-out

3. Contrast Stretching:

$(r_1,s_1)=(r_{min},0)$
$(r_2,s_2)=(r_{max},L-1)$

$(r_1,s_1)=(m,0)$
$(r_2,s_2)=(m,L-1)$



亮度還行，但 washed-out

(e.g. 低對比)

m 代表平均 intensity

4. 低對比: bar chart 窄、集中在中間，代表黯淡

高對比: intensity 跨度大、分布均勻，突出的 bar 很少，代表呈現的細節多

5. Continuous Histogram Equalization 證明 範例:

$$p_r(r) = \begin{cases}\dfrac{2r}{(L-1)^2}, & \text{for } 0 \le r \le L-1\\ 0, & \text{otherwise}\end{cases}.$$

令左式，求$p_s(s)$

Sol: $\boldsymbol{s = T(r) = (L-1)\int_0^r p_r(w)dw} = \frac{2}{L-1}\int_0^r wdw = \frac{r^2}{L-1}$

$$\boldsymbol{p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|} = \frac{2r}{(L-1)^2}\left|\left[\frac{ds}{dr}\right]^{-1}\right|$$

$$= \frac{2r}{(L-1)^2}\left|\left[\frac{d}{dr}\frac{r^2}{L-1}\right]^{-1}\right| = \frac{2r}{(L-1)^2}\left|\frac{(L-1)}{2r}\right| = \frac{1}{L-1}$$

→ $p_s(s)$ will always be uniform 且 independent 於 $p_r(r)$

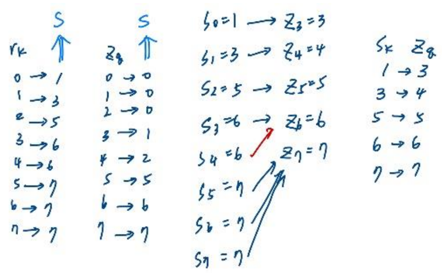
6. 但在 Discrete HE 就不是 uniform & independent 了

$$\boldsymbol{s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j), k = 0, 1 \ldots, L-1}$$

給各 intensity 的 pixel 數n_k → 算$p_r(r_k) = n_k/MN$ → 算累加的 CDF → $round((L-1)CDF)$ → 列新的 intensity 的 pixel 數n_s、$p_s(r_s)$

7. Histogram Matching (Specification)

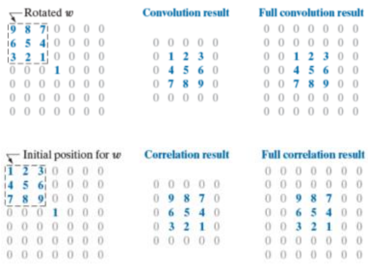
$$s = (L-1)\int_0^r p_r(w)dw \qquad G(z) = (L-1)\int_0^z p_z(t)dt = s \qquad z = G^{-1}(s)$$

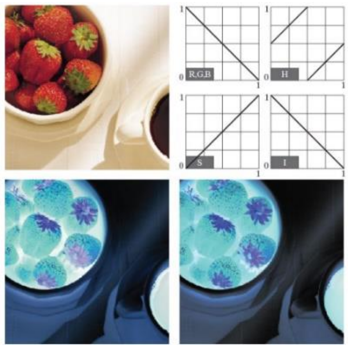


8. Histogram Statistics for image enhancement

求 Local avg(代表明暗程度) & local variance(告訴 detail)

$$m = \sum_{i=0}^{L-1} r_i p(r_i) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y) \qquad \sigma^2 = u_2(r) = \sum_{i=0}^{L-1}(r_i - m)^2 p(r_i) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[f(x,y) - m]^2$$





	Box				Gaussian		
	1	1	1		0.3679	0.6065	0.3679
$\frac{1}{9}\times$	1	1	1	$\frac{1}{4.8976}\times$	0.6065	1.0000	0.6065
	1	1	1		0.3679	0.6065	0.3679

LP spatial filter 們:

Box 的 edge 在 ramp 頂端的部分有

hard transition

9. Gaussian: $G(s,t) = Ke^{\frac{-(s^2+t^2)}{2\sigma^2}}$ (size>[6σ]就看不出差異了)

10. First order derivative:

$\frac{\partial f}{\partial x} = f(x+1) - f(x)$ 、 $\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$

11. Laplacian for sharpening:

$g(x,y) = f(x,y) + c[\nabla^2 f(x,y)]$, $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$

12. Edge enhancement

−1	0	0	−1	Roberts cross-gradient operators
0	1	1	0	

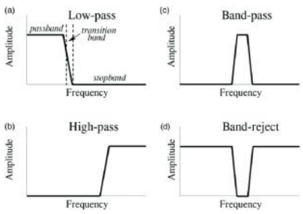
−1	−2	−1	−1	0	1	
0	0	0	−2	0	2	Sobel operators
1	2	1	−1	0	1	

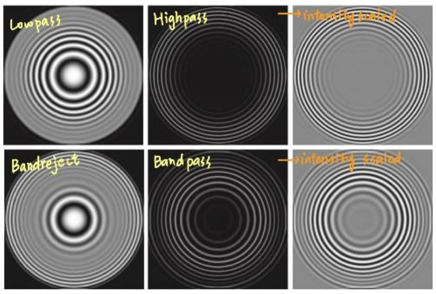
最後取

Gradient Image=

$M(x,y) = [g_x{}^2 + g_y{}^2]^{1/2}$

13.





CH4 Frequency Domain Filtering

1. 2D Continuous:

FT: $F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,z)e^{-j2\pi(ut+vz)}dtdz$

IFT: $f(t,z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)e^{2j\pi(ut+vz)}du\,dv$

2. 2D Discrete:

FT: $F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi(ux/M + vy/N)}$

IFT: $f(x,y) = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)\, e^{j2\pi(ux/M + vy/N)}$

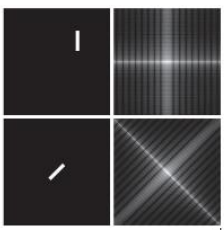
3. 取的 sample rate 需要 $\frac{1}{\Delta T} > 2\mu_{max}$ (兩倍最高 freq)，否則會有 Aliasing

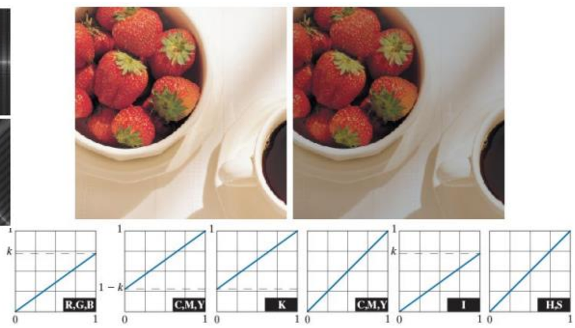
4. Symmetry properties (兩奇兩偶相乘=偶，一奇一偶相乘=奇)

$w_e(x,y) = \frac{w(x,y) + w(-x,-y)}{2}$, $w_o(x,y) = \frac{w(x,y) - w(-x,-y)}{2}$

(1) 對稱中心: $w_e(x,y) = w_e(M-x, N-y)$, $w_o(x,y) = -w_o(M-x, N-y)$

(2) $w_o(0,0) = 0$





Transform	Input Signal		Output Spectrum	
	Periodicy	Image domain	Periodicy	Spectral domain
CT FS	periodic	continuous	aperiodic	discrete
DT FS	periodic	discrete	periodic	discrete
CT FT	aperiodic	continuous	aperiodic	continuous
DT FT	aperiodic	discrete	periodic	continuous
DFT	aperiodic	discrete	periodic	discrete

5. Circular convolution 沒 pad 的話會有 wrap around error

6. LPF: (Gaussian 的是 less smoothing, but no ringing)

Ideal	Gaussian	Butterworth
$H(u,v)=\begin{cases}1 & \text{if } D(u,v)\le D_0\\ 0 & \text{if } D(u,v)>D_0\end{cases}$	$H(u,v)=e^{-D^2(u,v)/2D_0^2}$	$H(u,v)=\frac{1}{1+[D(u,v)/D_0]^{2n}}$

7. Freq 的 Laplacian $H(u,v) = -4\pi^2 D^2(u,v)$

$g(x,y) = IDFT\{F - HF\} = IDFT\{[1-H]F\}$，結果會比 spatial 好

8. Freq 的 unsharp masking (k>1: highboost filtering)

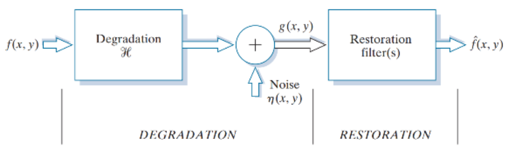
$g(x,y) = f(x,y) + kg_{mask}(x,y) = IDFT\{(1 + k[1 - H_{LP}(u,v)])F(u,v)\}$

9. Homomorphic filtering ($\gamma_H \ge 1$放大高頻, $\gamma_L < 1$衰減低頻)

Illumination has slow spatial variation → low frequency components
Reflectance tends to vary abruptly → high frequency components

$H(u,v) = (\gamma_H - \gamma_L)\left[1 - e^{-c[D^2(u,v)/D_0^2]}\right] + \gamma_L$

CH5 Image Reconstruction $g(x,y) = f(x,y) + n(x,y)$



右下胡椒鹽有 4 個 bar

1. Mean filter:

(1) Geometric：比 arithmetic 更能保留細節，都適合處理 random noise

(2) Harmonic: salt noise 有用，pepper 沒用，gaussian 有用

(3) Contraharmonic: Q>0 時處理 pepper，Q<0 時處理 salt

$\hat{f}(x,y) = \frac{\sum_{(r,c)\in S_{xy}} g(r,c)^{Q+1}}{\sum_{(r,c)\in S_{xy}} g(r,c)^{Q}}$ Q=0 變 mean，Q=-1 變 harmonic

適合處理 impulse noise

2. Order statistics filter

(1) Max: 處理 pepper，Min: 處理 salt (各自會侵蝕黑和白)

(2) Midpoint: work best for Gaussian, uniform 等 random noise

(3) Alpha-Trimmed Mean Filter: 去頭尾各 d/2 val，對剩下的做處理

3. Adaptive local noise reduction filter: **前提** $\boldsymbol{\sigma_\eta{}^2 \le \sigma_{S_{xy}}{}^2}$, 處理 random 噪音

若$\sigma_\eta{}^2 = 0$，代表 zero noise。若 $\sigma_{S_{xy}}{}^2$ 較高，代表有想要保留的 edge。$\sigma_\eta{}^2 \approx \sigma_{S_{xy}}{}^2$，代表是 noise。$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta{}^2}{\sigma_{S_{xy}}{}^2}[g(x,y) - \bar{z}_{S_{xy}}]$, z 是區域平均

4. Adaptive median filter: 處理 impulse 噪音，也不會改變物體的 boundary

Level A: If $z_{min} < z_{med} < z_{max}$, go to Level B
Else, increase the size of S_{xy}
If $S_{xy} \le S_{max}$, repeat level A
Else, output z_{med}.

Level B: If $z_{min} < z_{xy} < z_{max}$, output z_{xy}
Else output z_{med}.

甚至可以 smooth 一些不是 impulse 的噪音

比一般 median filter 效果更好

5. Periodic noise 要用 freq domain 的去噪: notch filter

6. Inverse filter: (H: degradation function) $H(u,v) = \int_0^T e^{-j2\pi[ux_0(t) + vy_0(t)]}dt$

$\hat{F}(u,v)$復原圖 $=\frac{G(u,v)\text{未處理}}{H(u,v)}=F(u,v)+\frac{N(u,v)}{H(u,v)}$

N 未知，且當 H 很小時，會 dominate F → zero problem

7. Min mean square error (wiener) filter: 處理 motion blur

沒有 zero prob. $\hat{F}(u,v)=\left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2+K}\right]G(u,v)$

8. Constrained least square filter: 只需要 mean & variance

$\hat{F}(u,v)=\left[\frac{H^*(u,v)}{|H(u,v)|^2+\gamma|P(u,v)|^2}\right]G(u,v)$ $\quad p(x,y)=\begin{bmatrix}0&-1&0\\-1&4&-1\\0&-1&0\end{bmatrix}$

9. Randon transform:

公式: $\boldsymbol{g(\rho,\theta)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(x,y)\delta(x\cos\theta+y\sin\theta-\rho)dxdy}$

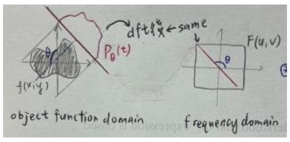
給一圓物體求投影 $f(x,y)=\begin{cases}A, & x^2+y^2\le r^2\\ 0, & otherwise\end{cases}$

Sol: $g(\rho,\theta)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(x,y)\delta(x-\rho)dxdy=\int_{-\infty}^{\infty}f(\rho,y)dy$

$$=\int_{-\sqrt{r^2-\rho^2}}^{\sqrt{r^2-\rho^2}}f(\rho,y)dy=\int_{-\sqrt{r^2-\rho^2}}^{\sqrt{r^2-\rho^2}}Ady=g(\rho)=\begin{cases}2A\sqrt{r^2-\rho^2}, & |\rho|\le r\\ 0, & otherwise\end{cases}$$

10. Backprojection (laminogram)

(1) Fourier slice theorem: $\boldsymbol{G(\omega,\theta)=F(\omega\cos\theta,\omega\sin\theta)}$



現實中只能取 discrete 個數的 lines，加上取出來的 DFT 值 lies in circle (常見的 IDFT 方法如 IFFT 都是 based in square)，硬轉成方形會使結果模糊

(2) Parallel-beam filtered: $u=\omega\cos\theta, v=\omega\sin\theta, dudv=\omega d\omega d\theta$ 代入連續 FT

$f(x,y)=\int_0^{2\pi}\int_0^{\infty}F(\omega\cos\theta,\omega\sin\theta)\,e^{j2\pi\omega(x\cos\theta+y\sin\theta)}\omega d\omega d\theta=\cdots G(\omega,\theta)\ldots$且$G(\omega,\theta+180°)=G(-\omega,\theta)$

$$=\int_0^{\pi}\int_{-\infty}^{\infty}|\omega|G(\omega,\theta)\,e^{j2\pi\omega(x\cos\theta+y\sin\theta)}\omega d\omega d\theta=\int_0^{\pi}\left[\int_{-\infty}^{\infty}|\omega|\,\boldsymbol{G(\omega,\theta)e^{j2\pi\omega\rho}d\omega}\right]_{\rho=x\cos\theta+y\sin\theta}\boldsymbol{d\theta}$$

$$=\int_0^{\pi}[s(\rho)*g(\rho,\theta)]_{\rho=x\cos\theta+y\sin\theta}\,d\theta=\int_0^{\pi}\left[\int_{-\infty}^{\infty}\boldsymbol{g(\rho,\theta)s(x\cos\theta+y\sin\theta-\rho)d\rho}\right]\boldsymbol{d\theta}$$

(3) Fan-beam filtered: ∵fourier slice 是設計給 parallel beam ∴要找出轉換

W.L.O.G., suppose objects are encompassed within a circular area of radius T about the origin of the plane, or $g(\rho,\theta)=0$ for $|\rho|>T$

$$f(x,y)=\int_0^{\pi}\left[\int_{-\infty}^{\infty}g(\rho,\theta)s(x\cos\theta+y\sin\theta-\rho)d\rho\right]d\theta$$
$$=\frac{1}{2}\int_0^{2\pi}\int_{-T}^{T}g(\rho,\theta)s(x\cos\theta+y\sin\theta-\rho)d\rho d\theta$$

Let $x=r\cos\varphi$ and $y=r\sin\varphi$

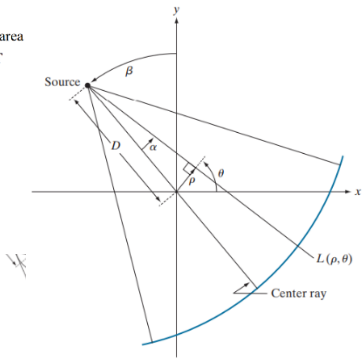
$$x\cos\theta+y\sin\theta=r\cos\varphi\cos\theta+r\sin\varphi\sin\theta$$
$$=r\cos(\varphi-\theta)$$

代入

$$f(x,y)=\frac{1}{2}\int_0^{2\pi}\int_{-T}^{T}g(\rho,\theta)s[r\cos(\varphi-\theta)-\rho]d\rho d\theta$$

$x=r\cos\varphi,\ y=r\sin\varphi$

$\theta=\alpha+\beta,\ \rho=D\sin\alpha\ \Rightarrow\ d\rho d\theta=D\cos\alpha d\alpha d\beta$



$$f(x,y)=\frac{1}{2}\int_0^{2\pi}\int_{-T}^{T}g(\rho,\theta)s[r\cos(\varphi-\theta)-\rho]d\rho d\theta$$
$$f(r,\phi)=\frac{1}{2}\int_{-\alpha}^{2\pi-\alpha}\int_{-\sin^{-1}(-T/D)}^{\sin^{-1}(T/D)}g(D\sin\alpha,\alpha+\beta)s[r\cos(\alpha+\beta-\varphi)-D\sin\alpha]D\cos\alpha d\alpha d\beta$$
$$f(r,\phi)=\frac{1}{2}\int_0^{2\pi}\int_{-\alpha_m}^{\alpha_m}g(D\sin\alpha,\alpha+\beta)s[r\cos(\alpha+\beta-\varphi)-D\sin\alpha]D\cos\alpha d\alpha d\beta$$

Let $p(\alpha,\beta)$ denote a fan-beam projection. Becasue a raysum is the sum of all values along a line, it follows that regardless of the coordinate system in which it is expressed, $p(\alpha,\beta)=g(\rho,\theta)$. Thus

$$\boxed{f(r,\varphi)=\frac{1}{2}\int_0^{2\pi}\int_{-\alpha_m}^{\alpha_m}p(\alpha,\beta)s\left(r\cos(\alpha+\beta-\varphi)-D\sin\alpha\right)D\cos\alpha d\alpha d\beta}$$

From Fig. 5.47,

$$f(r,\varphi)=\frac{1}{2}\int_0^{2\pi}\int_{-\alpha_m}^{\alpha_m}p(\alpha,\beta)s\left(R\sin\left(\alpha'-\alpha\right)\right)D\cos\alpha d\alpha d\beta$$
$$s(R\sin\alpha)=\left(\frac{\alpha}{R\sin\alpha}\right)^2s(\alpha)$$
$$h(\alpha)\triangleq\frac{1}{2}\left(\frac{\alpha}{\sin\alpha}\right)^2s(\alpha)$$
$$q(\alpha,\beta)\triangleq p(\alpha,\beta)D\cos\alpha$$
$$f(r,\varphi)=\int_0^{2\pi}\frac{1}{R^2}\left[\int_{-\alpha_m}^{\alpha_m}q(\alpha,\beta)h\left(\alpha'-\alpha\right)d\alpha\right]d\beta$$

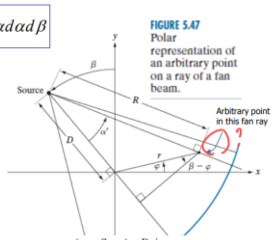


FIGURE 5.47 Polar representation of an arbitrary point on a ray of a fan beam.

$r\cos(\alpha+\beta-\varphi)-D\sin\alpha$
$=r\cos(\beta-\varphi)\cos\alpha-[r\sin(\beta-\varphi)+D]\sin\alpha$
$=R\sin\alpha'\cos\alpha-R\cos\alpha'\sin\alpha$
$=R\sin(\alpha'-\alpha)$

Prove $s(R\sin\alpha)=\left[\frac{\alpha}{R\sin\alpha}\right]^2s(\alpha)$, where $s(\rho)$ is the inverse FT of $|\omega|$.

Proof:

$s(\rho)=\int_{-\infty}^{\infty}|\omega|e^{j2\pi\omega\rho}d\omega.$ Let $\rho=R\sin\alpha$. Note that $\frac{\alpha}{R\sin\alpha}$ is always positive. We have

$\Rightarrow s(R\sin\alpha)=\int_{-\infty}^{\infty}|\omega|e^{j2\pi\omega R\sin\alpha}d\omega.$

Let $\omega'=\frac{\omega R\sin\alpha}{\alpha}$. Then

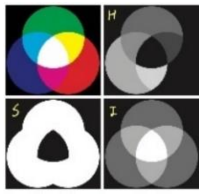
$d\omega=\frac{\alpha}{R\sin\alpha}d\omega'$

$\Rightarrow s(R\sin\alpha)=\left[\frac{\alpha}{R\sin\alpha}\right]^2\int_{-\infty}^{\infty}|\omega'|e^{j2\pi\omega'\alpha}d\omega'=\left[\frac{\alpha}{R\sin\alpha}\right]^2s(\alpha).$

CH6 Color Models

1. CMY (CMYK, K for black) 需 normalize RGB [0,1] $\begin{bmatrix}C\\M\\Y\end{bmatrix}=\begin{bmatrix}1\\1\\1\end{bmatrix}-\begin{bmatrix}R\\G\\B\end{bmatrix}$

2. HSI



Conversion from CMY to CMYK:

$K=\min(C,M,Y)$

If $K=1\ \Rightarrow\ C=M=Y=0$

Otherwise,

$$\begin{bmatrix}C\\M\\Y\end{bmatrix}=\frac{1}{1-K}\begin{bmatrix}C-K\\M-K\\Y-K\end{bmatrix}$$

3. 用 RGB channel 各自做 histogram equalization 會導致 erroneous color，所以要用 HIS 的 I 去 spread the intensities uniformly，保持 hue 不變

4. 在 color edge detection 不能用 CH3 的 gradient 做，要用 Di Zenzo (重新定義一個 vector 可用的 gradient)，但其實直接用 sobel 在 RGB 做，也不會太差(but fast)，需取捨結果與計算量

CH8-1 Lossless Compression

需要幾個 bit 去 encode 符號

1. Compression ration= 壓縮前/壓縮後的 bit 數

2. Entropy: $\eta=\sum_{i=1}^{n}p_i\log_2\frac{1}{p_i}=-\sum_{i=1}^{n}p_i\log_2 p_i$

3. Sannon-Fano Algo (top-down): 機率由小排到大，每輪都分成機率最接近的兩堆

4. Huffman coding (bottom-up): 每輪都找機率最小的兩個合併

它是 unique prefix & optimal in integer length & $\eta\le\bar{l}\le\eta+1$

5. Arithmetic coding: 整段 msg 視為一個 unit，使 symbol 的 bit 數可為小數

解碼時: 下一個 value= (目前 value-low)/range

CH8-2 JPEG

1. 動機:

(1) Spatial redundancy(單一圖片間有相似性可省略)，frame 之間是 temporal

(2) High spatial freq.的部分可略

(3) 人眼對色彩敏銳度(acuity)低 → Chroma sub-sampling

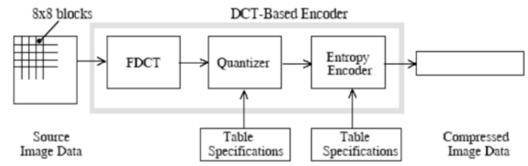
Cb Cr		Cb Cr	

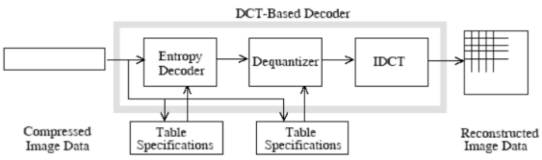
2. DCT (切成 8*8 的 block)

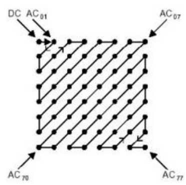
FDCT: $\boldsymbol{F(u,v)=\frac{1}{4}C(u)C(v)\left[\sum_{x=0}^{7}\sum_{y=0}^{7}f(x,y)\cos\frac{(2x+1)u\pi}{16}\cos\frac{(2y+1)v\pi}{16}\right]}$

IDCT: $\boldsymbol{f(x,y)=\frac{1}{4}\left[\sum_{u=0}^{7}\sum_{v=0}^{7}C(u)C(v)F(u,v)\cos\frac{(2x+1)u\pi}{16}\cos\frac{(2y+1)v\pi}{16}\right]}$

where $\begin{cases}C(u)=C(v)=\frac{1}{\sqrt{2}}, & for\ u,v=0\\ C(u)=C(v)=1, & otherwise\end{cases}$







3. Encode

(1) 切成 8*8 的 block → 全部減 128 (把原點設 0)

(2) 算 DCT 得$F(u,v)$

(3) $round(F(u,v)\div Luminance\ table)=\hat{F}(u,v)$, chrominance 同理

(4) Encode DC $(size, amplitude)$=(110, 00101)

Diff= 50(現) – 30(前)= -26 = 100101，則 amplitude= 00101

Size= amplitude 的長度= 5，查表 category(size) 5 的 codeword 是 110

(5) Encode AC $(run\ length, size, amplitude)$，使用 bit 數(4,4,1~10)

取出那 63 個 AC 係數

(a) $\begin{cases} (L(run\ len), F(amplitude)): value\ F前面有L個0 \\ (0,0): EOB, 剩下的AC全是0 \\ (15,0): 代表\mathbf{16}個0 \end{cases}$

(b) 加上 Size= amplitude 的長度

(c) (run len, size)查表

ex. 57, 45, 0, 0, 0, 0, 23, 0, -30, -16, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, ..., 0

⇒ (0,57), (0,45), (4,23), (1,-30), (0,-16), (2,1), (0,0)

⇒ (0, 6, 111001), (0.6, 101101)(4.5, 10111)(1.5, 00001), (0, 5, 01111)(2, 1, 1)(0,0)

Table K.5 Table for luminance AC coefficients

Run/Size	Code length	Code word
0/0 (EOB)	4	1010
0/1	2	00
0/2	2	01
0/3	3	100
0/4	4	1011
0/5	5	11010
0/6	7	1111000
0/7	8	11111000
0/8	10	1111110110
0/9	16	1111111110000010
0/A	16	1111111110000011
1/1	4	1100
1/2	5	11011
1/3	7	1111001
1/4	9	111110110
1/5	11	11111110110
1/6	16	1111111110000100
1/7	16	1111111110000101
1/8	16	1111111110000110
1/9	16	1111111110000111
1/A	16	1111111110001000
2/1	5	11100
2/2	8	11111001
2/3	10	1111110111
2/4	12	111111110100
2/5	16	1111111110001001
2/6	16	1111111110001010
2/7	16	1111111110001011
2/8	16	1111111110001100
2/9	16	1111111110001101
2/A	16	1111111110001110
3/1	6	111010
3/2	9	111110111
3/3	12	111111110101
3/4	16	1111111110001111
3/5	16	1111111110010000
3/6	16	1111111110010001
3/7	16	1111111110010010
3/8	16	1111111110010011
3/9	16	1111111110010100
3/A	16	1111111110010101
4/1	6	111011
15/0 (ZRL)	4	11111111001
⋮		

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ex. ($\boxed{0, 6}$, $\boxed{111001}$), ($\boxed{0.6}$, $\boxed{101101}$)($\boxed{4,5}$, $\boxed{10111}$)(1.5, 00001), (0, 5 01111)(2, 1, 1)$\boxed{(0,0)}$

⇒ 1111000 111001 1111000 101101 1111111110001000 10111 .....

⇒ final output of a single block:

$\underbrace{110\ 00101}_{DC部份}$ 1111000 111001 1111000 101101 1111111110001000 10111 .....

4. Decoder

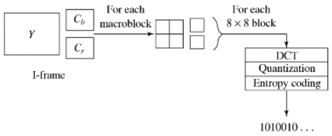
(1) 查表反推回 quantization 後的值$\hat{F}(u,v)$

(2) $\hat{F}(u,v) \times Luminance\ table = \tilde{F}(u,v)$, chrominance 同理

(3) $IDCT\left(\tilde{F}(u,v)\right) = \tilde{f}(i,j) \rightarrow$ decode 後的最終結果

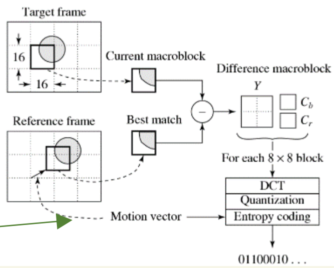
CH8-3 Video Compression

1. H.261 I-frame coding



2. H.261 P-frame coding

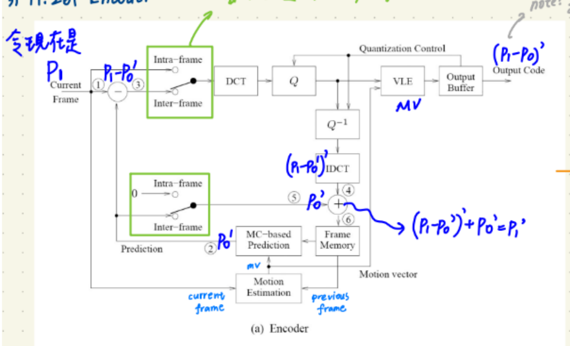
MVD= MV 前 — MV 現



3. H.261 Encoder

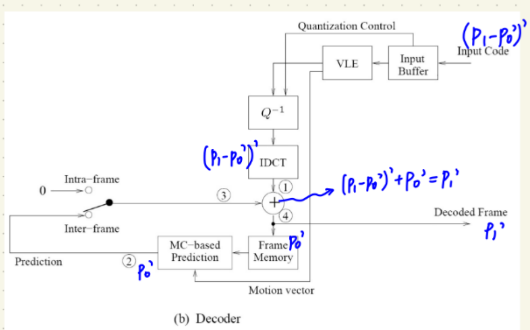
看現在是 I frame / P frame

note: 是 encode block diff



→ encoder 裡也有 decoder 來取得 previous frame

4. H.261 Decoder



5. Frame F_0 F_1 F_2 F_3 F_4 F_5

encoder: I P_1 P_2 P_3 P_4 I

計算 predict: I' P_1' P_2' P_3' P_4' ...

送出給 decoder: I' P_1' P_2' P_3'

decode: $\tilde{I}$ $\tilde{P}_1$ $\tilde{P}_2$

P': prediction data

$\tilde{P}$: decoded data

CH9 Morphology

1. Reflect: $\hat{B} = \{w | w = -b,\ \ for\ b \in B\}$

   Translation: $(B)_z = \{c | c = b + z,\ for\ b \in B\}$

2. Erosion: $A \ominus B = \{z | (B)_z \subseteq A\}$

   Dilation: $A \oplus B = \left\{z \middle| \left(\hat{B}\right)_z \cap A \neq \emptyset\right\} = \left\{z \middle| \left[\left(\hat{B}\right)_z \cap A\right] \subseteq A\right\}$

Duality:

$(\boldsymbol{A} \ominus \boldsymbol{B})^c = \{z | (B)_z \subseteq A\}^c = \{z | ((B)_z \cap A^c) = \emptyset\}^c = \{z | ((B)_z \cap A^c) \neq \emptyset\} = \boldsymbol{A^c} \oplus \hat{\boldsymbol{B}}$

$(\boldsymbol{A} \oplus \boldsymbol{B})^c = \left\{z \middle| \left(\left(\hat{B}\right)_z \cap A\right) \neq \emptyset\right\}^c = \left\{z \middle| \left(\left(\hat{B}\right)_z \cap A\right) = \emptyset\right\} = \boldsymbol{A^c} \ominus \hat{\boldsymbol{B}}$

3. Opening: $A \circ B = (A \ominus B) \oplus B = \cup\{(B)_z | (B)_z \subseteq A\}$

   Closing: $A \bullet B = (A \oplus B) \ominus B = [\cup\{(B)_z | ((B)_z \cap A) = \emptyset\}]^c$

Duality:

$(\boldsymbol{A} \circ \boldsymbol{B})^c = [(A \ominus B) \oplus B]^c = (A \ominus B)^c \ominus \hat{B} = \left(A^c \oplus \hat{B}\right) \ominus \hat{B} = \boldsymbol{A^c} \bullet \hat{\boldsymbol{B}}$

$(\boldsymbol{A} \bullet \boldsymbol{B})^c = [(A \oplus B) \ominus B]^c = (A \oplus B)^c \oplus \hat{B}] = \left(A^c \ominus \hat{B}\right) \oplus \hat{B} = \boldsymbol{A^c} \circ \hat{\boldsymbol{B}}$

4. Hit-or-Miss transform (HMT)

$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2) = \{z | (B)_z \subseteq I\},\ \ B_2$是 foreground

5. Boundary extraction: $\beta(A) = A - (A \ominus B)$

Hole filling

start: 從 Hole 中挑一點開始

公式: $X_k = (X_{k-1} \oplus B) \cap I^c$

until $X_k = X_{k-1}$

output $= X_k \cup I$

找 connected components

和 hole filling 類似

start: A 的其中一點

$X_k = (X_{k-1} \oplus B) \cap A,\ \ k = 1, 2, 3, \ldots$

until $X_k = X_{k-1}$. Then $Y = X_k$.

Thinning

$A \otimes B = A - (A \circledast B)$

$\quad = A \cap (A \circledast B)^c$

$\{B\} = \{B^1, B^2, \ldots, B^n\}$

$A \otimes \{B\} = \left(\left(\ldots\left(\left(A \otimes B^1\right) \otimes B^2\right)\ldots\right) \otimes B^n\right)$

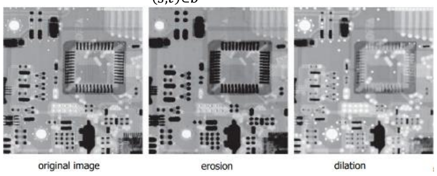
Thickening

$A \odot B = A \cup (A \circledast B)$

$A \odot \{B\} = \left(\left(\ldots\left(\left(A \odot B^1\right) \odot B^2\right)\ldots\right) \odot B^n\right)$

6. Gray-scale morphology

$[f \ominus b](x,y) = \min_{(s,t) \in b}\{f(x+s, y+t)\}$、$[f \oplus b](x,y) = \max_{(s,t) \in b}\{f(x-s, y-t)\}$



$$H(u,v) = \int_0^T e^{-j2\pi u x_0(t)}\,dt$$
$$= \int_0^T e^{-j2\pi u a t/T}\,dt$$
$$= \frac{T}{\pi u a}\sin(\pi u a)e^{-j\pi u a}$$