

STATISTICS

ASSIGNMENT-3

Solve at least 3 exercises.

EXERCISE 1.

Answer the following questions by calculating the number of ways of obtaining particular arrangements of objects and events.

1. An athlete has eight different trophies, but only has room for four trophies in a display cabinet. How many different ways is it possible to display just four trophies out of eight, assuming that the display order is important?

$P_1 =$ *different ways to display four trophies*

$$P_1 = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{4!} = 1680$$

2. A football manager has a squad of 20 players. How many different teams of 11 players could be selected from the squad? (Hint: Assume that positions of the players are not important)

$P_2 =$ *different teams of 11 players could be selected from the squad without position order*

$$P_2 = \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11!}{11! \cdot (20-11)!} = 167,960$$

EXERCISE 2.

Are people happy in their marriages? The table shows results from the 2008 General Social Survey for married adults classified by gender and level of happiness.

Gender	Very Happy	Pretty Happy	Not too Happy	Total
Male	183	243	43	469
Female	215	247	38	500
Total	398	490	81	969

1. Estimate the probability that a married adult is very happy.

$P_3 = \text{Probability of a married adult is "very happy"}$.

$$P_3 = 398/969 = 0.4107$$

2. Estimate the probability that a married adult is very happy,
 - (i) given that their gender is male and
 - (ii) given that their gender is female.

$P_{4i} = \text{Possibility of choosing a married adult is very happy} \mid \text{a male}$

$$P_{4i} = 183/469 = 0.39$$

$P_{4ii} = \text{Possibility of choosing a married adult is very happy} \mid \text{a female}$

$$P_{4ii} = 215/500 = 0.43$$

3. For these subjects, are the events being very happy and being a male independent?

P_A = being very happy

P_B = being a male

If two events A and B are independent, then the probability of happening of both A and B is:

$$P(B|A) = P(B)$$

$$P(B \cap A) = P(B) \times P(A)$$

$$P_A = \frac{398}{969} \cong 0.41$$

$$P_B = \frac{469}{969} \cong 0.48$$

$$P(A \cap B) = \frac{183}{969} \cong 0.18$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.18}{0.41} \cong 0.44$$

$$P(B|A) \neq P(B)$$

As we can see, the probability of two events are different. $P(B|A) \neq P(B)$. Thus, we can say that the events of being very happy and being male are not independent.

EXERCISE 3.

The Triple Blood Test screens a pregnant woman and provides as estimated risk of her baby being born with the genetic disorder Down syndrome. A study of 5282 women aged 35 or over analyzed the Triple Blood Test to test its accuracy.

A contingency table for Triple Blood Test of Down syndrome shown below.

Down	POS	NEG	Total
D (Down)	48	6	54
D _c (unaffected)	1307	3921	5228
Total	1355	3927	5282

1. Given that a test result is negative, show that the probability the fetus actually has Down syndrome is $P(D | NEG) = 0.0015$.

$$P(D | NEG) = \frac{P(D \cap NEG)}{P(NEG)} = \frac{6}{3927} = 0.00152788 \cong 0.0015$$

2. Is $P(D | NEG)$ equal to $P(NEG | D)$? If so, explain why. If not, find $P(NEG | D)$.

$$P(D | NEG) = \frac{P(D \cap NEG)}{P(NEG)} = \frac{6}{3927} = 0.00152788 \cong 0.0015$$

$$P(NEG | D) = \frac{P(NEG \cap D)}{P(D)} = \frac{6}{54} = 0.11111111 \cong 0.1111$$

These similar presentations have dissimilar meanings, that's why they are different events.

$P(D | NEG)$ means that While our test result is negative, the child with down syndrome is likely to be born in sample space.

$P(NEG | D)$ means that When a child with Down syndrome is born, the test result is likely to be negative in sample space.

EXERCISE 4.

Males and females are observed to react differently to a given set of circumstances.

It has been observed that 70% of the females react positively to these circumstances, whereas only 40% of males react positively -.

A group of 20 people, 15 female and 5 male, was subjected to these circumstances, and the subjects were asked to describe their reactions on a written questionnaire.

A response picked at random from the 20 was negative. What is the probability that it was that of a male?

Abbreviations as follows; Negative=N, Positive=P, Male=M, Female=F.

AIM;

$P(M|N)$ = the probability that it was that of a male when the response is negative.

So we need to calculate Bayes formula according to binary values.

$$P(N|F) = 1 - 70\% = 0.30 = 30\%$$

$$P(N|M) = 1 - 40\% = 0.60 = 60\%$$

$$P(M|N) = \frac{P(N|M)P(M)}{P(N|F)P(F) + P(N|M)P(M)} = \frac{0.6 * (1/4)}{0.3*(3/4)+0.6*(1/4)} = 0.4 = 40\%$$