

1-) [20 Points] Seven Sided Dices

Three fair **seven-sided dice**, each numbered from 1 to 7, are simultaneously rolled, and the numbers on the top face of each die are observed. Let random variable X represent the number observed on the top face of the first die, Y represent the number observed on the top face of the second die, and Z represent the number observed on the top face of the third die.

- a-) Event A is defined as “the maximum of the numbers observed at the end of the process is less than 5”. Find the probability $P[A]$.
- b-) Event B is defined as “the minimum of the numbers observed at the end of the process is greater than 3”. Find the probability $P[B]$.
- c-) Find the probability $P[B | A]$.
- d-) Event C is defined as “the sum of numbers being greater than 5”. Find the probability $P[C]$.
- e-) Event D is defined as “the maximum of the numbers observed at the end of the process is less than 2”. Find the probability $P[C | D]$.

2-) [20 Points] Circuit System

A circuit system between X and Y lines is given in Figure 1. Additionally, the probabilities of each component functioning correctly are provided. It is known that;

- The circuit components A, B, C and D fails independently.
- If component A is functional, then the operation of either line 1 or line 2 is sufficient for the circuit system between X and Y to function.

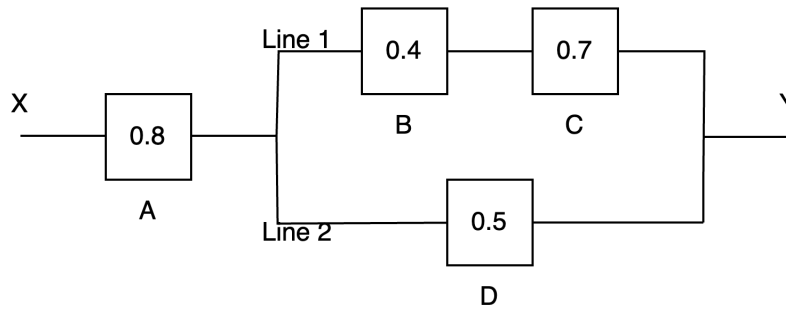


Figure 1: The Circuit System

- a-) What is the probability of the circuit system works?
- b-) Given that the system is not working, what is the probability that the component A is working?
- c-) Given that the system is working, what is the probability that the component C is working?
- d-) Given that the system is working, what is the probability that the component D is not working?



3-) [20 Points] Software Practice

Mehmet is a computer engineering student at TOBB-ETÜ. He has a habit of practicing his coding skills every day. If he practices coding on a given day, the probability that he will practice (or skip) on the next day is 0.7 (or 0.3, respectively). If he skips practicing on a given day, the probability that he will practice (or skip) on the next day is 0.9 (or 0.1, respectively). Mehmet (by default) practices when he starts his routine.

Let's denote;

- P_k : the event that Mehmet practices at the end of the k th day, for $k=1, 2, \dots$ respectively.
- S_k : the event that Mehmet skips practice at the end of the k th day, for $k=1, 2, \dots$ respectively.

a-) What is the probability of $P(P_3)$?

b-) What is the probability of $P(S_4)$?

c-) What is the likelihood of him practicing after two days, considering that he skipped practice at the end of the first day?

d-) What is the likelihood of him having skipped practice at the end of the first day, given that he is practicing at the end of the third day?

4-) [20 Points] Shopping Center

Random variables P and Q represent the number of customers that enter 2 separate stores during a certain 10-minute period. These stores are located in the same shopping center so it is important that store managers coordinate their operations if necessary.

The joint distribution of P and Q is known to be

$$f(p, q) = \frac{1}{2^{(p+q+2)}}; p \in \{\mathbb{Z}^+ \cup \{0\}\} \text{ and } q \in \{\mathbb{Z}^+ \cup \{0\}\}$$

a-) Find the marginal distribution of P .

b-) Are the two random variables P and Q independent or not independent? Explain the reason mathematically.

c-) What is the probability that, during the time period in question, less than a total of 6 customers enter the two stores given that P is more than 2?

d-) What is the expected value of customers that enter store Q during the 10-minute period?



5-) [20 Points] Lifetime of Components at a Computer System

Let X and Y be the lifetime of two components in a computer system. If the joint density functions of these variables is

$$f(x, y) = \begin{cases} cxy^2 & \text{if } x \geq 0, y \geq 0, x + y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- a-) Find c .
- b-) Find **CDF** of X and draw it.
- c-) Find **PDF** of X and draw it.
- d-) Find $P(Y \leq 1/3 \mid X = 0.4)$.
- e-) Find $P(X + Y \geq 1/3)$.
- f-) Find $E[X]$.

Submission (Deadline: 16.02.2024 Friday 15.00)

- Bring your stapled paper at Teknoloji-Merkezi-217 (can be a box in front of the door, if you can't find anyone or there is no box at the door, throw it under the door.)
- Submit a scanned pdf of your answers to the <https://uzak.etu.edu.tr> site.

***Important Note:** Both are required. Be aware that your answers should be in-order.*