1. Find a polynomial P(x) of degree 3 or less whose graph passes through the four data points *(−2,8), (0,4), (1,2), (3,−2)*. Use Newton's divided difference interpolation method.

x	f(x)
-2	8
0	4
1	2
3	-2

$x = 0$ Newton's divided difference interpolation is

$f(x) = y_0 + (x - x_0) \cdot f[x_0, x_1] + (x - x_0)(x - x_1) \cdot f[x_0, x_1, x_2]$

$f(x) = 8 + (x+2) \cdot -2 + \cancel{(x+2)\cdot(x-0)\cdot 0}$

$f(x) = 8 - 2 \cdot (x+2)$ //

Newton's divided difference table is;

x	y	1st order	2nd order
-2	8		
		-2	
			0
0	4		
		-2	
			0
1	2		
		-2	
3	-2		

2. Use Newton-Raphson Method to find the root to four correct decimal places for $3x^3 + x^2 = x + 5$ where $x_0 = -1$.

$f(x) = 3x^3 + x^2 - x - 5 \rightarrow f'(x) = 9x^2 + 2x - 1$ $\quad x_0 = -1$

1st iteration : $f(x_0) = f(-1) = 3.(-1)^3 + (-1)^2 - (-1) - 5 = -6$ $\quad x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$f'(x_0) = f'(-1) = 9.(-1)^2 + 2.(-1) - 1 = 6$ $\quad x_1 = -1 - \frac{-6}{6} = \boxed{0}$ //

2nd iteration : $f'(x_1) = f(0) = 30^3 + 0^2 . -0 - 5 = 5$ $\quad x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \boxed{-5}$ //

$f'(x_1) = f'(0) = 90^2 + 2.0 - 1 = -1$

3rd iteration : $f(x_2) = f(-5) = 3.(-5)^3 + (-5)^2 - (-5) - 5 = -350$ $\quad x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \rightarrow -5 - \frac{-350}{214}$

$f'(x_2) = f'(-5) = 9.(-5)^2 + 2.(-5) . -1 = 214$ $\quad x_3 = \boxed{-3.364}$ //

4th iteration : $f(x_3) = f(-3.364) = 3.(-3.364)^3 + (-3.364)^2 - (-3.364) - 5 = -104.571$ $\quad x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

$f'(x_3) = f'(-3.364) = 9.(-3.364)^2 + 2.(-3.364) - 1 = 94.149$

$\boxed{-2.254}$ //

5th iteration : $f(x_4) = f(2.254) = 3.(-2.254)^3 + (-2.254)^2 - (-2.254) - 5 = -32.011$

$f'(x_4) = f'(2.254) = 9.(-2.254)^2 + 2(-2.254) - 1 = 40.208$ $\quad x_5 = -2.254 - \frac{-32.011}{40.28}$

$= \boxed{-1.458}$ //

6th iteration : $f(x_5) = f(-1.458) = 3.(-1.458)^3 + (-1.458)^2 - (-14.58) - 5 = -10.709$ $\quad x_6 = -1.458 - \frac{-10.709}{15.207} =$

$f'(x_5) = f'(-1.458) = 9.(-1458)^2 + (-1.458) - 1 = 15.207$

$= \boxed{-0.753}$ //

7th iteration : $f(x_6) = f(-0.753) = 3.(-0.753)^3 + (-0.753)^2 - (-0.753) - 5 = -4.962$ $\quad x_7 = -0.753 - \frac{-4.962}{2.602}$

$f'(x_6) = f'(-0.753) = 9.(-0.753)^2 + 2(-0.753) - 1 = 2.602$

$= \boxed{1.153}$ //

8th iteration : $f(x_7) = f(1.153) = 3.(1.153)^3 + (1.153)^2 - (1.153) - 5 = -0.221$ $\quad x_8 = 1.153 - \frac{-0.221}{13.278} = \boxed{1.17}$ //

$f'(x_7) = f'(1.153) = 9.(1.153)^2 + 2(1.153) - 1 = 13.278$

9th iteration : $f(x_8) = f(1.17) = 3.(1.17)^3 + 1.(17)^2 - (1.17) - 5 = 0.003$ $\quad x_9 = 1.17 - \frac{0.003}{13.655} \approx \boxed{1.17}$ //

$f'(x_8) = f'(1.17) = 9.(1.17)^2 + 2.(1.7) - 1 = 13.659$

10th iteration : $f(x_9) = f(1.17) = 3.(1.17)^3 + (1.17)^2 - (1.17) - 5 = 0$

$f'(x_9) = f'(1.17) = 9.(1.17)^2 + 2.(1.17) - 1 = 13.654$ $\quad x_{10} = 1.17 - \frac{0}{13.654} = \boxed{1.17}$ //

Approximate root of the equation $3x^2 + x^2 - x - 5 = 0$ using Newton Raphson method is 1.17 (After 10 iterations)

3. Use finite difference approximations of $O(h_2)$ among forward difference (FD), backward difference (BD) and central difference (CD) to compute $f'(2.38)$ and $f''(2.38)$ from the following data:

X	2.36	2.37	2.38	2.39
F(x)	0.85866	0.86289	0.86710	0.87129

Central Difference (CD)

$$f'(2.38) = \frac{f(2.39) - f(2.37)}{2\,(0.1)} = \frac{0.87129 - 0.86289}{0.2} = 0.042$$

$$f''(2.38) = \frac{f(2.39) + f(2.37) - 2f(2.38)}{(0.1)^2} = \frac{0.87129 + 0.86289 - 2.(0.86710)}{(0.1)^2} = -0.00002$$

Forward Difference (FD)

$$f'(2.38) = \frac{-3f(2.38) + 4f(2.39) - f(2.40)}{2.(0.1)}$$ = we can not evalute this difference approach to $f'(2.38)$.

$$f''(2.38) = \frac{2f(2.38) - 5f(2.39) + 4f(2.40) - f(2.41)}{(0.1)^2}$$ = we can not evalute this difference approach to $f''(2.38)$.

Backword Difference (BD)

$$f'(2.38) = \frac{f(2.36) - 4f(2.37) + 3f(2.38)}{2.(0.1)} = \frac{0.85866 - 4.(0.86289) + 3.(0.86710)}{0.2} = 0.042$$

$$f''(2.38) = \frac{-f(2.35) + 4f(2.36) - 5f(2.37) + 2f(2.38)}{(0.1)^2}$$ = we can not evalute this difference approach to $f''(2.38)$.

4. Evaluate with Simpson's 1/3 rule using two, four, and six panels.

Formula $$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right)$$

$$\Delta x = \frac{b-a}{n}$$

two panel $a = 0,\ b = 1,\ n = 2$

$$\Delta x = \frac{1-0}{2} = \frac{1}{2}$$

$f(x_0) = f(a) = f(0) = 0$
$4f(x_1) = 4f(\frac{1}{2}) = 1$
$f(x_2) = f(b) = f(1) = 1$

Finally, $\frac{\Delta x}{3} = \frac{1}{6}$; $\frac{1}{6}(0+1+1) = 0.333\ldots \to 0$

four panel $a = 0,\ b = 1,\ n = 4$

$f(x_0) = f(a) = f(0) = 0$
$4f(x_1) = 4f(\frac{1}{4}) = \frac{1}{4} = 0.25$
$2f(x_2) = 2f(\frac{1}{2}) = \frac{1}{2} = 0.5$
$4f(x_3) = 4f(\frac{3}{4}) = \frac{9}{4} = 2.25$
$f(x_4) = f(b) = f(1) = 1$

Finally, $\frac{\Delta x}{3} = \frac{1}{12}$; $\frac{1}{12}(0 + 0.25 + 0.5 + 2.25 + 1) = 0.33\ldots \to 0$

six panel $a = 0,\ b = 1,\ n = 6$

$$\Delta x = \frac{1-0}{6} = \frac{1}{6}$$

$f(x_0) = f(a) = f(0) = 0$
$4f(x_1) = 4f(\frac{1}{6}) = \frac{1}{9} = 0.111$
$2f(x_2) = 2f(\frac{1}{3}) = \frac{2}{9} = 0.22$
$4f(x_3) = 4f(\frac{1}{2}) = 1 = 1$
$2f(x_4) = 2f(\frac{2}{3}) = \frac{8}{9} = 0.888\ldots9$
$4f(x_5) = 4f(\frac{5}{6}) = \frac{25}{9} = 2.777\ldots8$
$f(x_6) = f(b) = f(1) = 1$

Finally, $\frac{\Delta x}{3} = \frac{1}{18}$; $\frac{1}{18}(0 + 0.111.. + 0.22.. + 2.77 + 1) = 0.333\ldots \to 0$