Mathematical Modeling Approach

In this study, we propose a single objective, mixed-integer linear programming (MILP) model to determine the minimum cost route for a vehicle traveling between two cities. The cities are defined as the source and destination, and the customer specifies the countries that the vehicle is allowed to travel through, each of which contains its own set of cities. When the vehicle travels from one city to another, specific costs such as toll fees and bridge fees are incurred, which are represented as a toll price matrix for each city-to-city route, taking into account the exchange rate. The model utilizes a binary decision variable ($x_{i,j}$) in conjunction with the cost matrix to obtain the minimum cost route.

In addition to toll fees and bridge fees, fuel prices also vary between cities, and the model incorporates fuel prices as a matrix. By multiplying the fuel price matrix with the amount of fuel purchased for each city, the model can determine the cities in which fuel should be purchased and the corresponding amounts required, thereby minimizing fuel-related costs. Moreover, the model computes employee costs that are time-dependent, by multiplying the hourly employee costs with the travel time between cities.

Overall, our proposed MILP model optimizes the routing process by taking into account various factors such as toll fees, fuel prices, and employee costs, leading to a cost-effective and efficient transportation plan.

Since the proposed algorithm is a mixed integer programming model is hard to obtain optimal value in a reasonable time. The dataset that we try to handle grows rapidly when the number of countries and cities increases. For the small dataset, it is suitable to implement Exact Algorithm in order to solve the problem optimally, on the other hand for the larger dataset in realistic models, the heuristic algorithm is more appropriate.

Sets

C = set of cities

Parameters:

$c_{i,j}$ = toll cost of traveling city $i \epsilon C$ to city $j \epsilon C$

$d_{i,j}$ = distance between city $i \epsilon C$ to $j \epsilon C$

p_i = fuel price in city i, $i \in C$

a = hourly employee cost

F = vehicle fuel capacity

I = initial fuel amount

$k_{i,j}$ = fuel consumption amount for per kilometer for city i to j, $i, j \in C$

$v_{i,j}$ = average velocity for traveling city i to city j, $i, j \in C$

$t_{i,j}$ = time spent on traveling city i to city j, $i, j \in C$

$$t_{i,j} = \frac{d_{i,j}}{v_{i,j}}$$

Decision Variable:

$x_{i,j}$ = Binary variable indicating whether vehicle travel city i to city j or not, $i, j \in C$

l_i = Amount of fuel is bought in city i (liters) $i \in C$

f_i = Level of fuel available when vehicle arrives city i (liters), $i \in C$

Objective:

$$minimize \quad a \sum_{i=0}^{C} \sum_{j=0}^{C} (t_{i,j} \, x_{i,j}) \; + \; \sum_{i=0}^{C} \sum_{j=0}^{C} c_{i,j} x_{i,j} + \; \sum_{i=0}^{C} p_i l_i \qquad (1)$$

The objective function (1) aims to find the optimal route that minimizes the total cost, which includes refueling, hourly employees, and travel expenses between cities. By considering the hourly employee cost as a function of travel time and the refueling cost as a function of the travel distance and time, the model indirectly minimizes the total time required for the travel.

Constraints:

$$\sum_i^C x_{j,i} \ = \ 1 \quad j = s \tag{2}$$

$$\sum_i^C x_{i,j} \ = \ 1 \quad j = t \tag{3}$$

$$\sum_i^C x_{j,i} - \sum_i^C X_{k,j} = 0 \ \ j \neq s \ , j \neq t \tag{4}$$

$$x_{i,j} + x_{i,j} \leq \ 1 \ \ \forall i \epsilon C, \forall j \epsilon C \tag{5}$$

$$f_j \ \leq F(1 - x_{i,j} \ ) + l_i \ + \ f_i \ - kd_{i,j} \, x_{i,j} \quad \forall i \epsilon C, \forall j \epsilon C \tag{6}$$

$$f_i = I \qquad\qquad i = s \tag{7}$$

$$F \geq f_i \geq \ 0 \qquad \forall i \epsilon C \tag{8}$$

$$F - f_i \geq \ l_i \ \geq 0 \ \forall i \epsilon C \tag{9}$$

$$X_{i,i} = 0 \qquad\qquad \forall i \epsilon C \tag{10}$$

$$X_{i,j} \epsilon \ \{0,1\} \tag{11}$$

Constraints (2)(3) guarantees that a vehicle starts from the source city and ends in the destination city that is specified by the customers. Constraints on the city-to-city passage (4) impose that where the cities are not the source and destination if the vehicle enters the city, it must leave from that city. Constraint (5) ensures that the cities visited will not be visited again. Constraint (6) guarantees the fuel balance. Constraint (7) defines initial fuel amount. Constraint (8) imposes that the fuel amount available when a vehicle arrives in any city cannot exceed the fuel capacity of the vehicle. Constraint (9) enforces that the amount of fuel purchased from each city cannot exceed the fuel capacity of the vehicle. It also considers the possibility of buying any amount of fuel as long as it does not exceed the fuel capacity. Constraint (10) ensures that travel from the same city to the same city is not possible. Constraint (11) defines the binary variable.