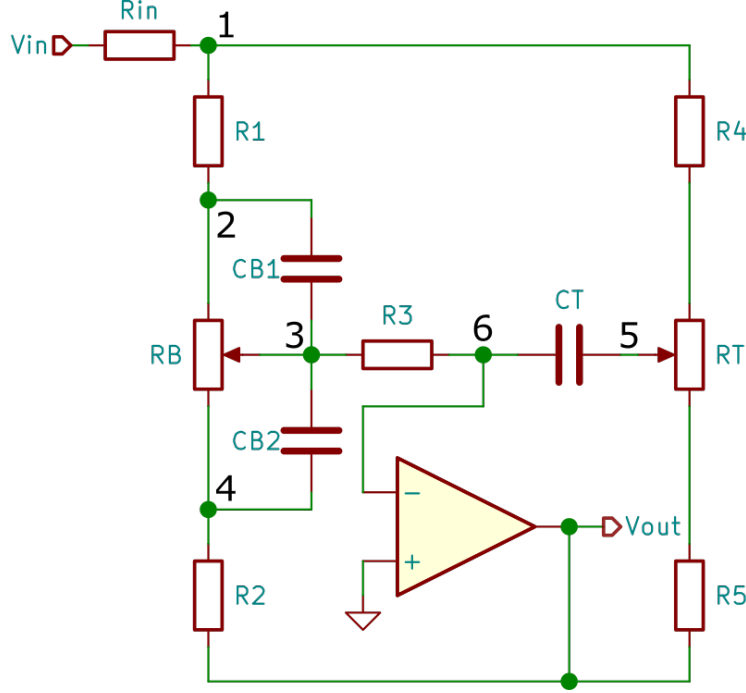


# Circuit Analysis of Baxandall Tone Control

Variation: Active, dual bass capacitor, single treble capacitor

To find the frequency response of the circuit, the ratio  $\frac{V_{out}}{V_{in}}$  needs to be determined. Nodal analysis is performed to yield a system of linear equations, which are then placed in matrix form and solved using Cramer's rule.



## Nodal Analysis

**Node #1:** Treble potentiometer  $R_T$  is modeled as resistors  $R_{T1}$  and  $R_{T2}$ , connected at the wiper. The values of  $R_{T1}$  and  $R_{T2}$  are then increased to include series resistors  $R_4$  and  $R_5$  respectively. Using Kirchhoff's current law (KCL),

$$v_1\left(\frac{1}{R_{in}} + \frac{1}{R_{T1}} + \frac{1}{R_1}\right) + v_2\left(-\frac{1}{R_1}\right) + v_5\left(-\frac{1}{R_{T1}}\right) = \frac{V_{in}}{R_{in}}$$

**Node #2:** Bass potentiometer  $R_B$  is modeled as two resistors,  $R_{B1}$  and  $R_{B2}$ , connected at the wiper.

$$v_1\left(-\frac{1}{R_1}\right) + v_2\left(\frac{1}{R_1} + \frac{1}{R_{B1}} + j\omega C_{B1}\right) + v_3\left(-\frac{1}{R_{B1}} - j\omega C_{B1}\right) = 0$$

**Node #3:** The positive and negative inputs of an ideal opamp are modeled as having no voltage difference between them. Since the positive input is grounded, the negative input (and therefore the connection to  $R_3$  at node 6) is considered grounded as well.

$$v_2\left(-\frac{1}{R_{B1}} - j\omega C_{B1}\right) + v_3\left(\frac{1}{R_{B1}} + \frac{1}{R_{B2}} + \frac{1}{R_3} + j\omega C_{B1} + j\omega C_{B2}\right) + v_4\left(-\frac{1}{R_{B2}} - j\omega C_{B2}\right) = 0$$

**Node #4:**

$$v_3(-\frac{1}{R_{B2}} - j\omega C_{B2}) + v_4(\frac{1}{R_{B2}} + \frac{1}{R_2} + j\omega C_{B2}) + V_{out}(-\frac{1}{R_2}) = 0$$

**Node #5:** As with  $R_3$  above, the other side of  $C_T$  at node 6 is considered grounded since it is connected to the negative opamp input.

$$v_1(-\frac{1}{R_{T1}}) + v_5(\frac{1}{R_{T1}} + \frac{1}{R_{T2}} + j\omega C_T) + V_{out}(-\frac{1}{R_{T2}}) = 0$$

**Node #6:** The inputs of an ideal opamp are modeled as conducting no current.

$$v_3(-\frac{1}{R_3}) + v_5(-j\omega C_T) = 0$$

## Matrix Form

There are 6 node equations with 6 node voltage variables. These can be stated in matrix form

$$Ax = b$$

where  $A$  is a 6 x 6 matrix of the coefficients (admittances),  $x$  is a column vector of the variables (node voltages), and  $b$  is a column vector of the right-hand sides of the equations (inputs and constants).

$$\begin{bmatrix} \frac{1}{R_{in}} + \frac{1}{R_{T1}} + \frac{1}{R_1} & -\frac{1}{R_1} & 0 & 0 & -\frac{1}{R_{T1}} & 0 \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_{B1}} + j\omega C_{B1} & -\frac{1}{R_{B1}} - j\omega C_{B1} & 0 & 0 & 0 \\ 0 & -\frac{1}{R_{B1}} - j\omega C_{B1} & \frac{1}{R_{B1}} + \frac{1}{R_{B2}} + \frac{1}{R_3} + j\omega C_{B1} + j\omega C_{B2} & -\frac{1}{R_{B2}} - j\omega C_{B2} & 0 & 0 \\ 0 & 0 & -\frac{1}{R_{B2}} - j\omega C_{B2} & \frac{1}{R_{B2}} + \frac{1}{R_2} + j\omega C_{B2} & 0 & -\frac{1}{R_2} \\ -\frac{1}{R_{T1}} & 0 & 0 & 0 & \frac{1}{R_{T1}} + \frac{1}{R_{T2}} + j\omega C_T & -\frac{1}{R_{T2}} \\ 0 & 0 & -\frac{1}{R_3} & 0 & -j\omega C_T & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ V_{out} \end{bmatrix} = \begin{bmatrix} \frac{V_{in}}{R_{in}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The output voltage,  $V_{out}$ , can now be found using Cramer's rule.

$$V_{out} = \frac{|A_6|}{|A|}$$

where matrix  $A_6$  is formed by replacing the 6th column of  $A$  with the contents of  $b$ . Since  $b$  contains only one non-zero element, the determinant of  $A_6$  is equal to that element multiplied by its cofactor.

$$|A_6| = \frac{V_{in}}{R_{in}} C_{1,6} = \frac{V_{in}}{R_{in}} (-1)^{1+6} M_{1,6}$$

$$|A_6| = -\frac{V_{in}}{R_{in}} M_{1,6}$$

where  $M_{1,6}$  is the determinant of  $A_6$  with row 1 and column 6 removed.

$$M_{1,6} = \begin{vmatrix} -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_{B1}} + j\omega C_{B1} & -\frac{1}{R_{B1}} - j\omega C_{B1} & 0 & 0 \\ 0 & -\frac{1}{R_{B1}} - j\omega C_{B1} & \frac{1}{R_{B1}} + \frac{1}{R_{B2}} + \frac{1}{R_3} + j\omega C_{B1} + j\omega C_{B2} & -\frac{1}{R_{B2}} - j\omega C_{B2} & 0 \\ 0 & 0 & -\frac{1}{R_{B2}} - j\omega C_{B2} & \frac{1}{R_{B2}} + \frac{1}{R_2} + j\omega C_{B2} & 0 \\ -\frac{1}{R_{T1}} & 0 & 0 & 0 & \frac{1}{R_{T1}} + \frac{1}{R_{T2}} + j\omega C_T \\ 0 & 0 & -\frac{1}{R_3} & 0 & -j\omega C_T \end{vmatrix}$$

Substituting into the equation for  $V_{out}$ ,

$$V_{out} = -\frac{V_{in}}{R_{in}} \frac{M_{1,6}}{|A|}.$$

The transfer function of the circuit is then found to be

$$\frac{V_{out}}{V_{in}} = -\frac{1}{R_{in}} \frac{M_{1,6}}{|A|}$$

where the negative sign indicates that the output is inverted compared to the input.