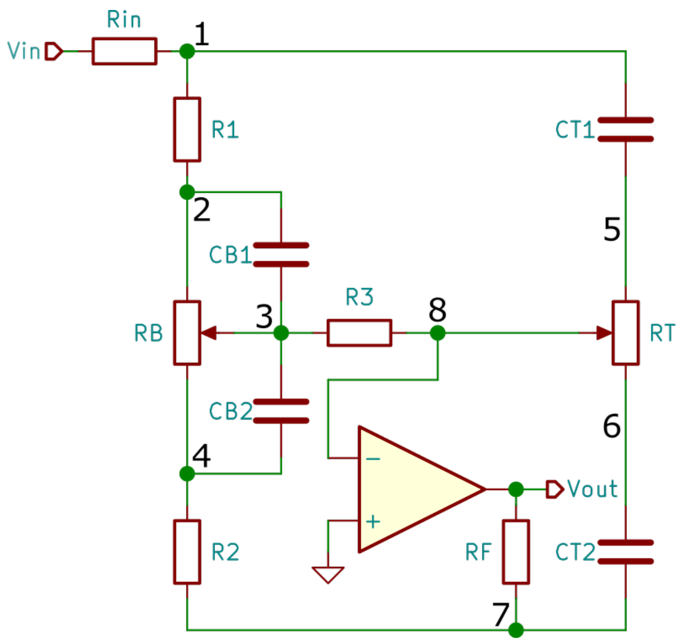
Analysis of James & Baxandall Tone Control Circuits

Variation: Active, dual bass capacitor, dual treble capacitor

To find the frequency response of the circuit, the ratio $\frac{V_{out}}{V_{in}}$ needs to be determined. Nodal analysis is performed to yield a system of linear equations, which are then placed in matrix form and solved using Cramer's rule.



Nodal Analysis

Node #1: Using Kirchhoff's current law (KCL),

$$v_1(\tfrac{1}{R_{in}} + \tfrac{1}{R_1} + j\omega C_{T1}) + v_2(-\tfrac{1}{R_1}) + v_5(-j\omega C_{T1}) = \tfrac{V_{in}}{R_{in}}.$$

Node #2: Bass potentiometer R_B is modeled as two resistors, R_{B1} and R_{B2}, connected at the wiper.

$$v_1(-\tfrac{1}{R_1}) + v_2(\tfrac{1}{R_1} + \tfrac{1}{R_{B1}} + j\omega C_{B1}) + v_3(-\tfrac{1}{R_{B1}} - j\omega C_{B1}) = 0$$

Node #3: The positive and negative inputs of an ideal opamp are modeled as having no voltage difference between them. Since the positive input is grounded, the negative input (and therefore the connection to R_3 at node 7) is considered grounded as well.

$$v_2(-\tfrac{1}{R_{B1}} - j\omega C_{B1}) + v_3(\tfrac{1}{R_{B1}} + \tfrac{1}{R_{B2}} + \tfrac{1}{R_3} + j\omega C_{B1} + j\omega C_{B2}) + v_4(-\tfrac{1}{R_{B2}} - j\omega C_{B2}) = 0$$

Node #4:

$$v_3(-\tfrac{1}{R_{B2}} - j\omega C_{B2}) + v_4(\tfrac{1}{R_{B2}} + \tfrac{1}{R_2} + j\omega C_{B2}) + v_7(-\tfrac{1}{R_2}) = 0$$

Node #5: Treble potentiometer R_T is modeled as resistors R_{T1} and R_{T2}, connected at the wiper. As with R_3 above, the wiper of R_T at node 7 is considered grounded since it is connected to the negative opamp input.

$$v_1(-j\omega C_{T1}) + v_5(\tfrac{1}{R_{T1}} + j\omega C_{T1}) = 0$$

Node #6:

$$v_6(\tfrac{1}{R_{T2}} + j\omega C_{T2}) + v_7(-j\omega C_{T2}) = 0$$

Node #7:

$$v_4(-\tfrac{1}{R_2}) + v_6(-j\omega C_{T2}) + v_7(\tfrac{1}{R_2} + \tfrac{1}{R_F} + j\omega C_{T2}) + V_{out}(-\tfrac{1}{R_F}) = 0$$

Node #8: The inputs of an ideal opamp are modeled as conducting no current.

$$v_3(-\tfrac{1}{R_3}) + v_5(-\tfrac{1}{R_{T1}}) + v_6(-\tfrac{1}{R_{T2}}) = 0$$

Matrix Form

There are 8 node equations with 8 node voltage variables. These can be stated in matrix form

$$Ax = b$$

where A is a 8 x 8 matrix of the coefficients (admittances), x is a column vector of the variables (node voltages), and b is a column vector of the right-hand sides of the equations (inputs and constants).

$$\begin{bmatrix}
\frac{1}{R_{in}} + \frac{1}{R_1} + j\omega C_{T1} & -\frac{1}{R_1} & 0 & 0 & -j\omega C_{T1} & 0 & 0 & 0 \\
-\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_{B1}} + j\omega C_{B1} & -\frac{1}{R_{B1}} - j\omega C_{B1} & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{R_{B1}} - j\omega C_{B1} & \frac{1}{R_{B1}} + \frac{1}{R_{B2}} + \frac{1}{R_3} + j\omega C_{B1} + j\omega C_{B2} & -\frac{1}{R_{B2}} - j\omega C_{B2} & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{R_{B2}} - j\omega C_{B2} & \frac{1}{R_{B2}} + \frac{1}{R_2} + j\omega C_{B2} & 0 & 0 & -\frac{1}{R_2} & 0 \\
-j\omega C_{T1} & 0 & 0 & 0 & \frac{1}{R_{T1}} + j\omega C_{T1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{R_{T2}} + j\omega C_{T2} & -j\omega C_{T2} & 0 \\
0 & 0 & 0 & -\frac{1}{R_2} & 0 & -j\omega C_{T2} & \frac{1}{R_2} + \frac{1}{R_F} + j\omega C_{T2} & -\frac{1}{R_F} \\
0 & 0 & -\frac{1}{R_3} & 0 & -\frac{1}{R_{T1}} & -\frac{1}{R_{T2}} & 0 & 0
\end{bmatrix}
\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ V_{out} \end{bmatrix}
=
\begin{bmatrix} \frac{V_{in}}{R_{in}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The output voltage, V_{out}, can now be found using Cramer's rule.

$$V_{out} = \frac{|A_8|}{|A|}$$

where matrix A_8 is formed by replacing the 8th column of A with the contents of b. Since b contains only one non-zero element, the determinant of A_8 is equal to that element multiplied by its cofactor.

$$\begin{aligned}
|A_8| &= \frac{V_{in}}{R_{in}} C_{1,8} = \frac{V_{in}}{R_{in}} (-1)^{1+8} M_{1,8} \\
|A_8| &= -\frac{V_{in}}{R_{in}} M_{1,8}
\end{aligned}$$

where $M_{1,8}$ is the determinant of A_8 with row 1 and column 8 removed.

$$M_{1,8} = \begin{vmatrix} -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_{B1}} + j\omega C_{B1} & -\frac{1}{R_{B1}} - j\omega C_{B1} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{R_{B1}} - j\omega C_{B1} & \frac{1}{R_{B1}} + \frac{1}{R_{B2}} + \frac{1}{R_3} + j\omega C_{B1} + j\omega C_{B2} & -\frac{1}{R_{B2}} - j\omega C_{B2} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{R_{B2}} - j\omega C_{B2} & \frac{1}{R_{B2}} + \frac{1}{R_2} + j\omega C_{B2} & 0 & 0 & -\frac{1}{R_2} \\ -j\omega C_{T1} & 0 & 0 & 0 & \frac{1}{R_{T1}} + j\omega C_{T1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{R_{T2}} + j\omega C_{T2} & -j\omega C_{T2} \\ 0 & 0 & 0 & -\frac{1}{R_2} & 0 & -j\omega C_{T2} & \frac{1}{R_2} + \frac{1}{R_F} + j\omega C_{T2} \\ 0 & 0 & -\frac{1}{R_3} & 0 & -\frac{1}{R_{T1}} & -\frac{1}{R_{T2}} & 0 \end{vmatrix}$$

Substituting into the equation for V_{out},

$$V_{out} = -\frac{V_{in}}{R_{in}} \frac{M_{1,8}}{|A|}.$$

The transfer function of the circuit is then found to be

$$\frac{V_{out}}{V_{in}} = -\frac{1}{R_{in}} \frac{M_{1,8}}{|A|}.$$