Lesson 7: Economic Growth

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0.1 What is Economic Growth?

Economic growth is an increase in living standards, which has traditionally been measured using Real GDP per capita.

1 Chapter Six: Solow Growth Model

1.1 Notation

- $\mathrm{L_t}$ - Number of workers
- n - growth rate of work force
  - population growth rate
- $\mathrm{K_t}$ - capital stock
- δ - depreciation rate
- $\mathrm{Y_t}$ - output (GDP)
- $\mathrm{C_t}$ - consumption
- $\mathrm{I_t}$ - gross investment
  - $C_t = Y_t - I_t$
- $y_t = \frac{Y_t}{L_t}$ - ouput per worker
- $c_t = \frac{C_t}{L_t}$ - consumption per worker
- $k_t = \frac{K_t}{L_t}$ - capital per worker (capital-labor ratio)
- $i_t = \frac{I_t}{L_t}$ - investment per worker

1.2 Big Finding of Solow Model

In the solow model, if the is no productivity growth (an increase in TFP A), then the economy reaches a steady state. At a steady state y_t, k_t, and c_t are all constant while Y, K, and C will grow at the rate n (population growth rate).

1.3 Steady-State of Investment

I_t (Investment) serves two main purposes:

1. Expand the size of capital stock (K_t)
2. Replace depreciated capital (δK_t)

Since in steady state we know that capital stock K_t will grow at the rate n, we can conclude that $I_t = nK_t + \delta K_t$, which brings per worker steady state of investment to:

$$i = (n + \delta)k$$

1.4 Per-Worker Production Function

Recall that we stated $Y = AF(K, L)$. We can add the time aspect in to get $Y_t = AF(K_t, L_t)$. Now let's find the per-worker equation:

$$\frac{Y_t}{L_t} = \frac{AF(K_t, L_t)}{L_t}$$

$$\frac{Y_t}{L_t} = A\frac{1}{L_t}F(K_t, L_t)$$

From here, if the function $F(K_t, L_t)$ has constant return to scales we can conclude:

$$\frac{Y_t}{L_t} = AF(\frac{K_t}{L_t}, \frac{L_t}{L_t})$$

Which in our notation is

$$y_t = Af(k_t)$$

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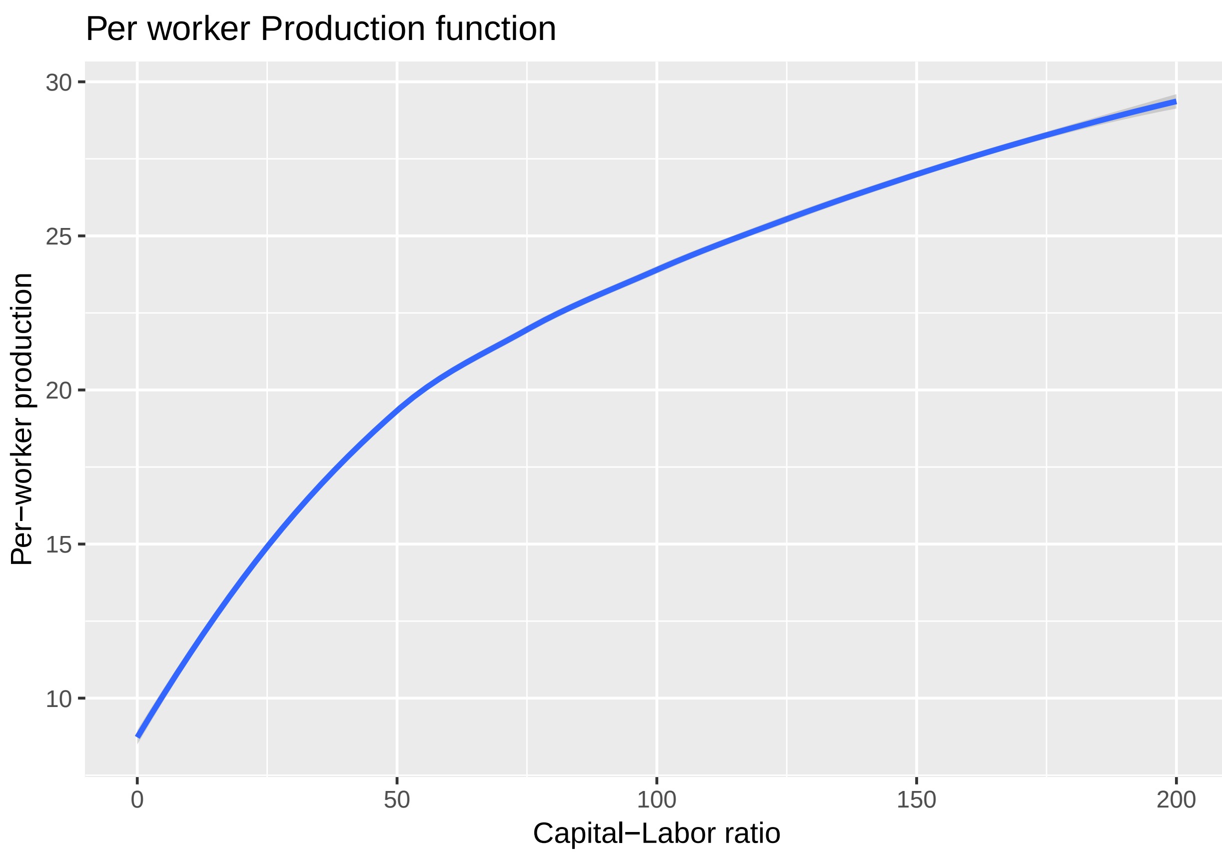

1.5 Steady-State of Consumption

We know two things:

1. $C_t = Y_t - I_t$
2. $I_t = (n + \delta)K_t$ at steady state

So we can conlude that $C_t = Y_t - (n + \delta)K_t$ and per worker we get:

$$c = y - (n + \delta)k$$

$$c = Af(k) - (n + \delta)k$$

If capital increases what happens to production?

- If k increases, then $f(k)$ increases, but at a decreasing rate, so c (consmumption increases)
- If k increases, then $(n + \delta)k$ increases, but at a rate of $(n + \delta)$, so c (consumption decreases)

What happens if we try to maximize consumption? (not related to steady state)

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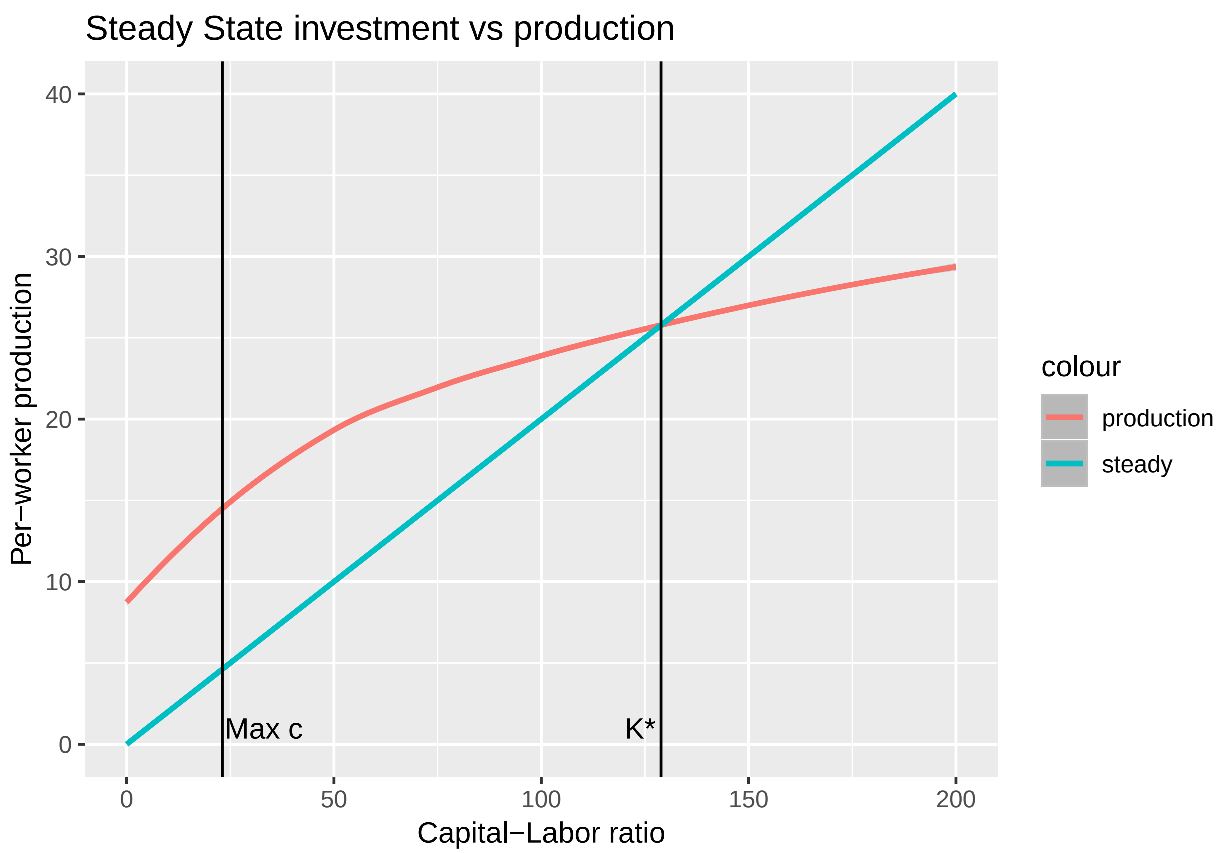


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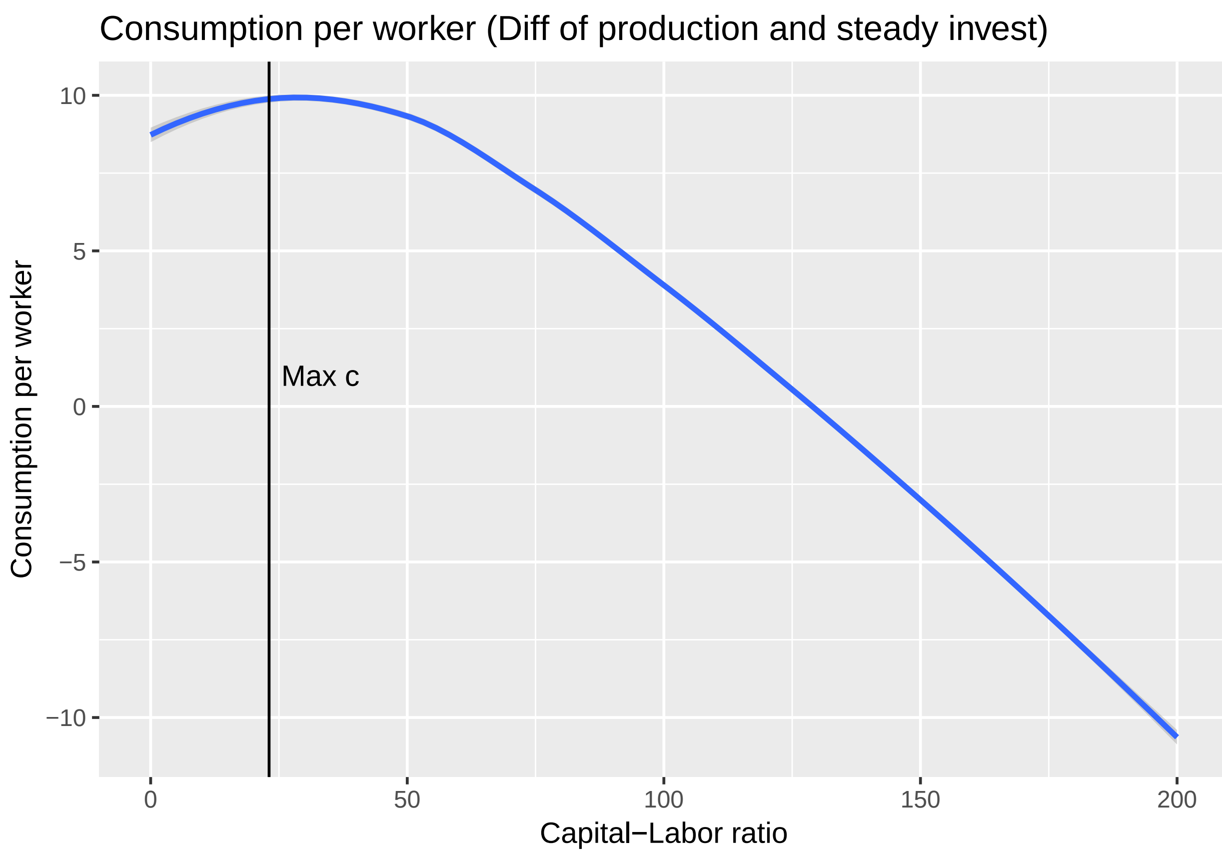

The maximum of consumption happens at the value of k with the biggest difference of y and $(n+\delta)k$. Consumption comes back down to zero where production is equal to $(n+\delta)k$.

1.6 Reaching the Steady-State

At steady state let us assume there is no change in TFP (A). We know that in steady state $I_t = (n+\delta)K_t$ and that Savings = Investment in equlibrium. Let us define s as the savings rate.

$$S_t = sY_t$$

Since we stated savings = investment:

$$sY_t = (n+\delta)K_t$$

Which gives us the per worker steady-state condition:

$$sf(k) = (n+\delta)k$$

$$sy = (n+\delta)k$$

Add in savings rate to compare steady state and actual investment:

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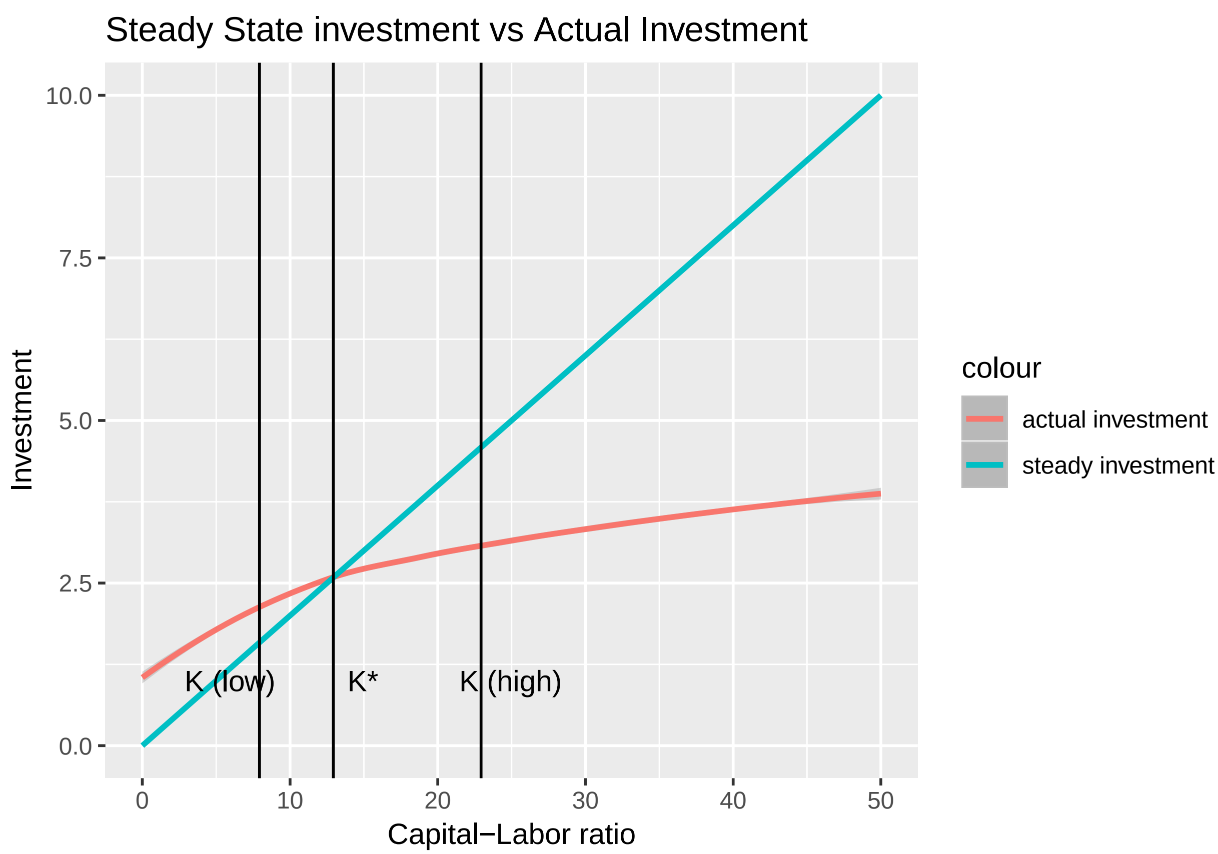

When the level of capital is less than K^* (K (low)), then actual investment is greater than that required for steady state, so the capital-labor ratio incrases until it hits steady state, which also increases per worker production. In the other case when the level of capital is greater than K^* (K (high)), then the actual investment is less than that required for steady state, so the capital-labor ratio decreases until it hits the steady state, which decrease per worker production.

1.7 Changes to the Solow Model

There are three main factors that change the solow model

1. Changes in TFP (A) change $sf(k)$
2. Cahnges in savings rate (s) change $sf(k)$
3. Changes in the population growth rate (n) change $(n + \delta)k$

1.7.1 Changes in A

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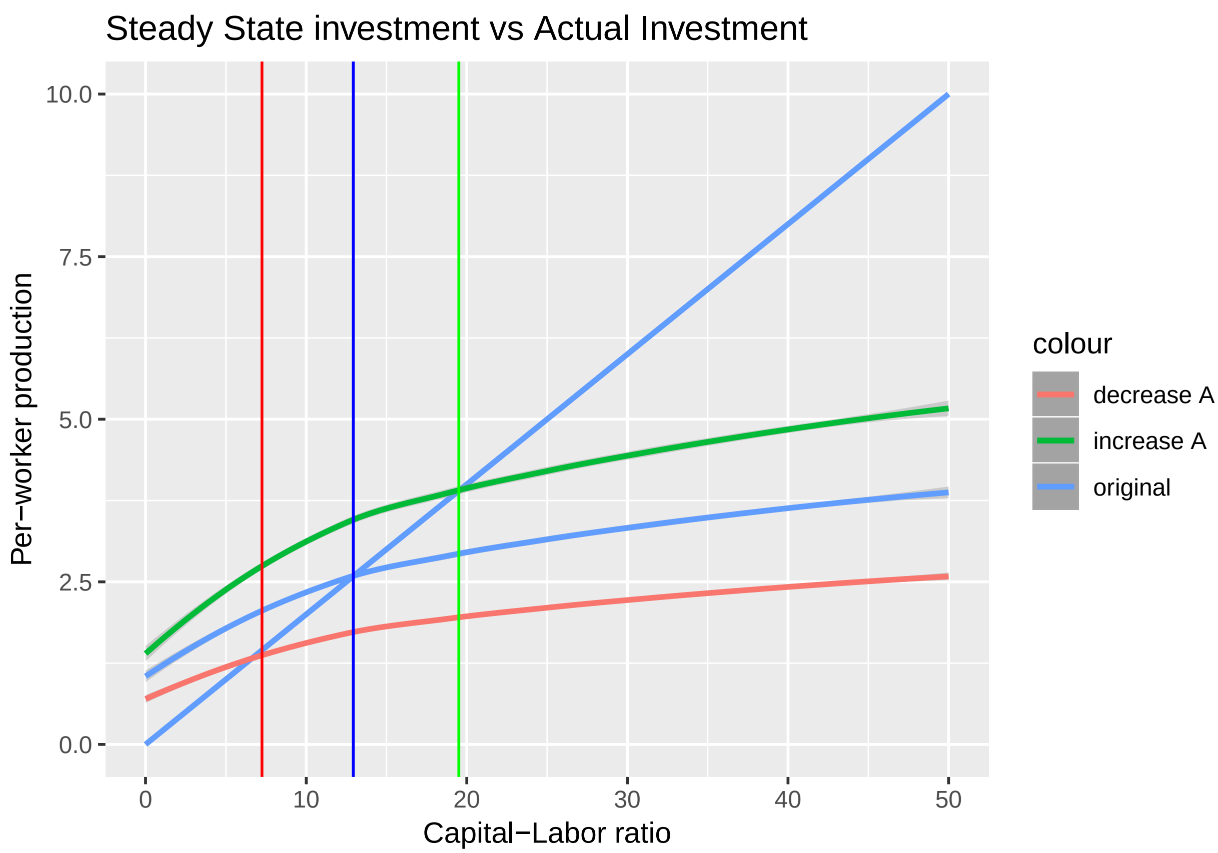

1.7.2 Changes in s

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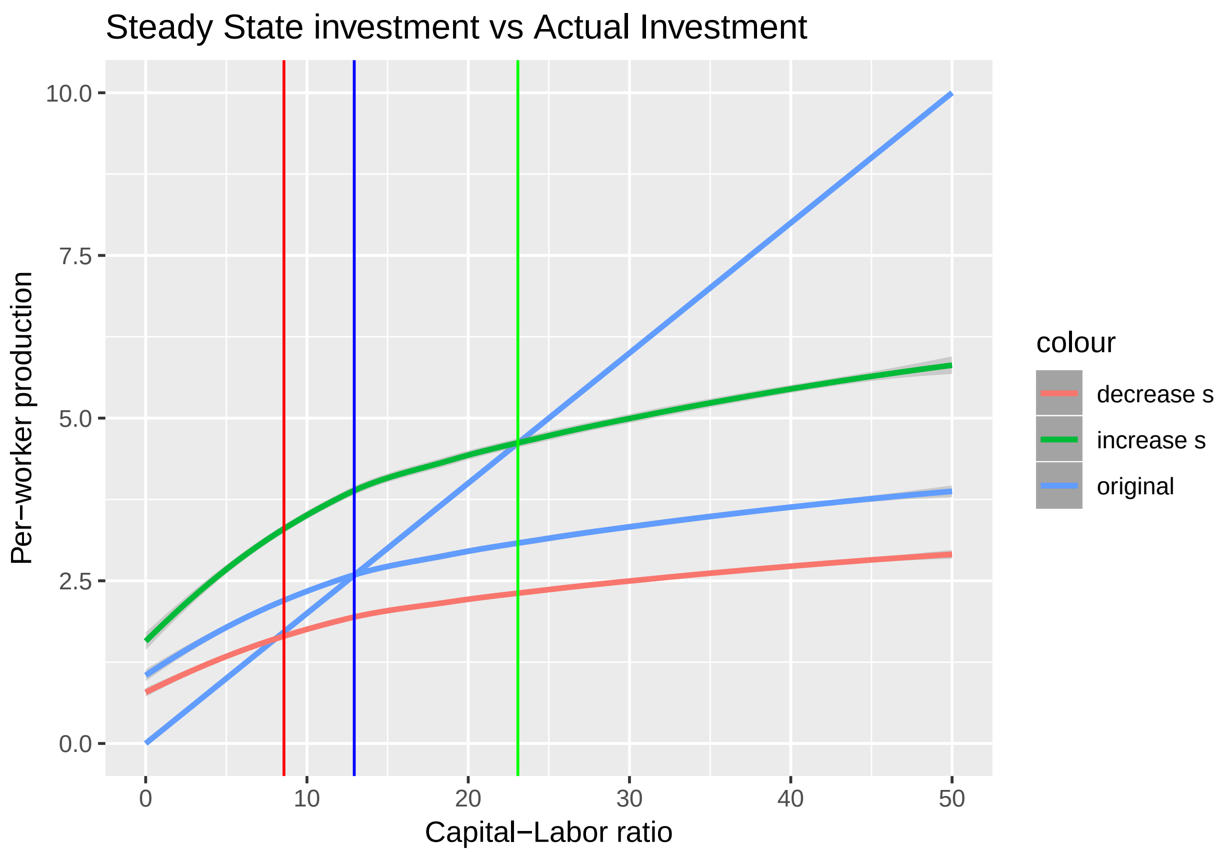

1.7.3 Changes in n

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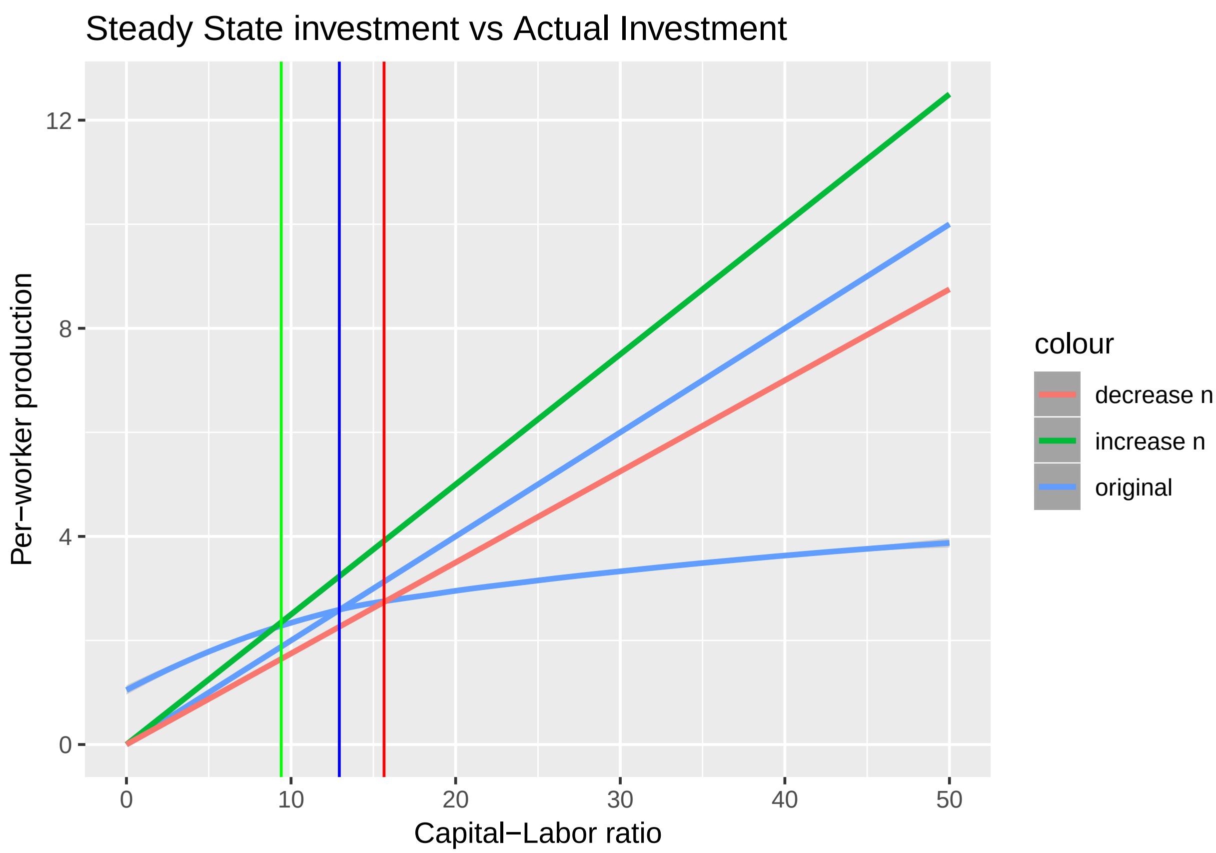

2 Insert growth paths after Romer model