Lesson 7: Economic Growth

Contents

0.1 What is Economic Growth?

Economic growth is an increase in living standards, which has traditionally been measured using Real GDP per capita.

1 Chapter Six: Solow Growth Model

1.1 Notation

- $\mathrm{L_t}$ - Number of workers
- n - growth rate of work force
    - population growth rate
- $\mathrm{K_t}$ - capital stock
- δ - depreciation rate
- $\mathrm{Y_t}$ - output (GDP)

- C_t - consumption
- I_t - gross investment
    - $C_t = Y_t - I_t$
- $y_t = \frac{Y_t}{L_t}$ - ouput per worker
- $c_t = \frac{C_t}{L_t}$ - consumption per worker
- $k_t = \frac{K_t}{L_t}$ - capital per worker (capital-labor ratio)
- $i_t = \frac{I_t}{L_t}$ - investment per worker

1.2 Big Finding of Solow Model

In the solow model, if the is no productivity growth (an increase in TFP A), then the economy reaches a steady state. At a steady state y_t, k_t, and c_t are all constant while Y, K, and C will grow at the rate n (population growth rate).

1.3 Steady-State of Investment

I_t (Investment) serves two main purposes:

1. Expand the size of capital stock (K_t)
2. Replace depreciated capital (δK_t)

Since in steady state we know that capital stock K_t will grow at the rate n, we can conclude that $I_t = nK_t + \delta K_t$, which brings per worker steady state of investment to:

$$i = (n + \delta)k$$

1.4 Per-Worker Production Function

Recall that we stated $Y = AF(K, L)$. We can add the time aspect in to get $Y_t = AF(K_t, L_t)$. Now let's find the per-worker equation:

$$\frac{Y_t}{L_t} = \frac{AF(K_t, L_t)}{L_t}$$

$$\frac{Y_t}{L_t} = A\frac{1}{L_t}F(K_t, L_t)$$

From here, if the function $F(K_t, L_t)$ has constant return to scales we can conclude:

$$\frac{Y_t}{L_t} = AF(\frac{K_t}{L_t}, \frac{L_t}{L_t})$$

Which in our notation is

$$y_t = Af(k_t)$$

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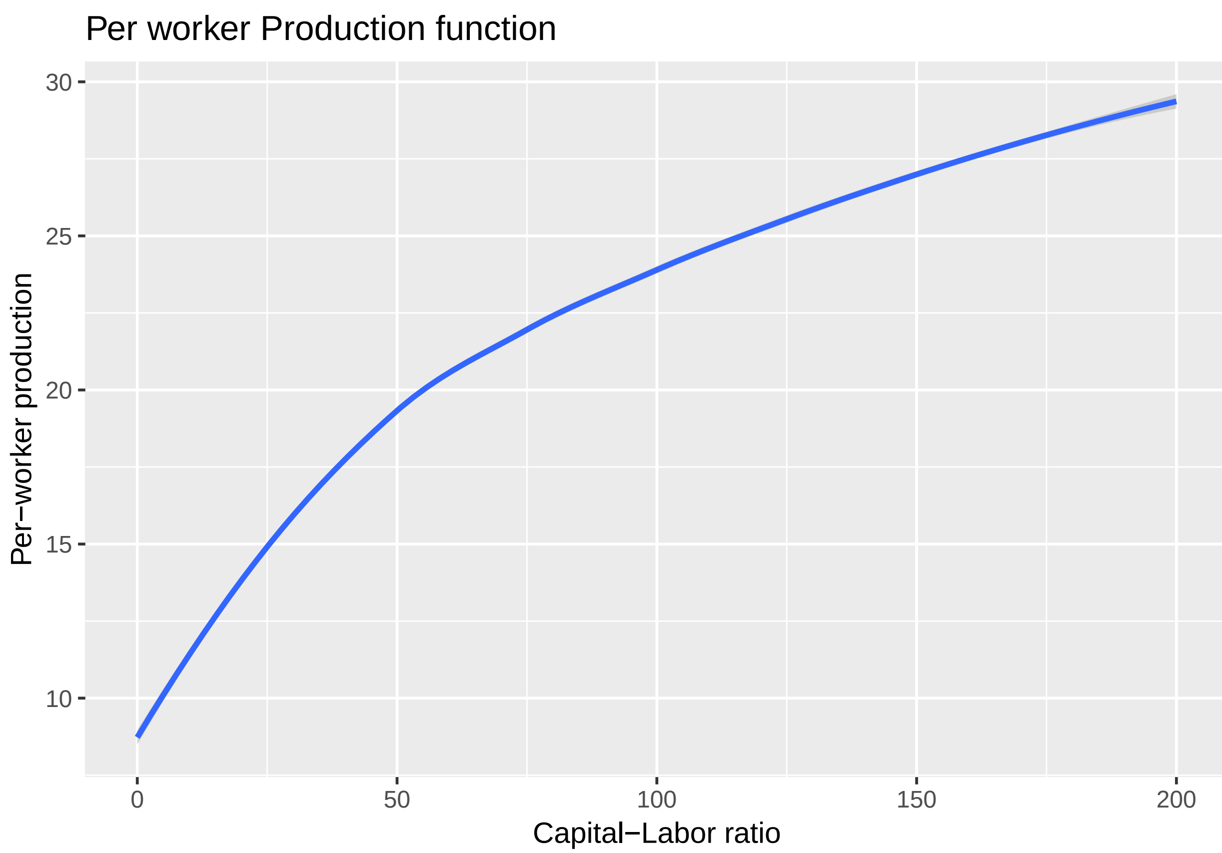

1.5 Steady-State of Consumption

We know two things:

1. $C_t = Y_t - I_t$
2. $I_t = (n + \delta)K_t$ at steady state

So we can conlude that $C_t = Y_t - (n + \delta)K_t$ and per worker we get:

$$c = y - (n + \delta)k$$

$$c = Af(k) - (n + \delta)k$$

If capital increases what happens to production?

- If k increases, then $f(k)$ increases, but at a decreasing rate, so c (consmumption increases)
- If k increases, then $(n + \delta)k$ increases, but at a rate of $(n + \delta)$, so c (consumption decreases)

What happens if we try to maximize consumption? (not related to steady state)

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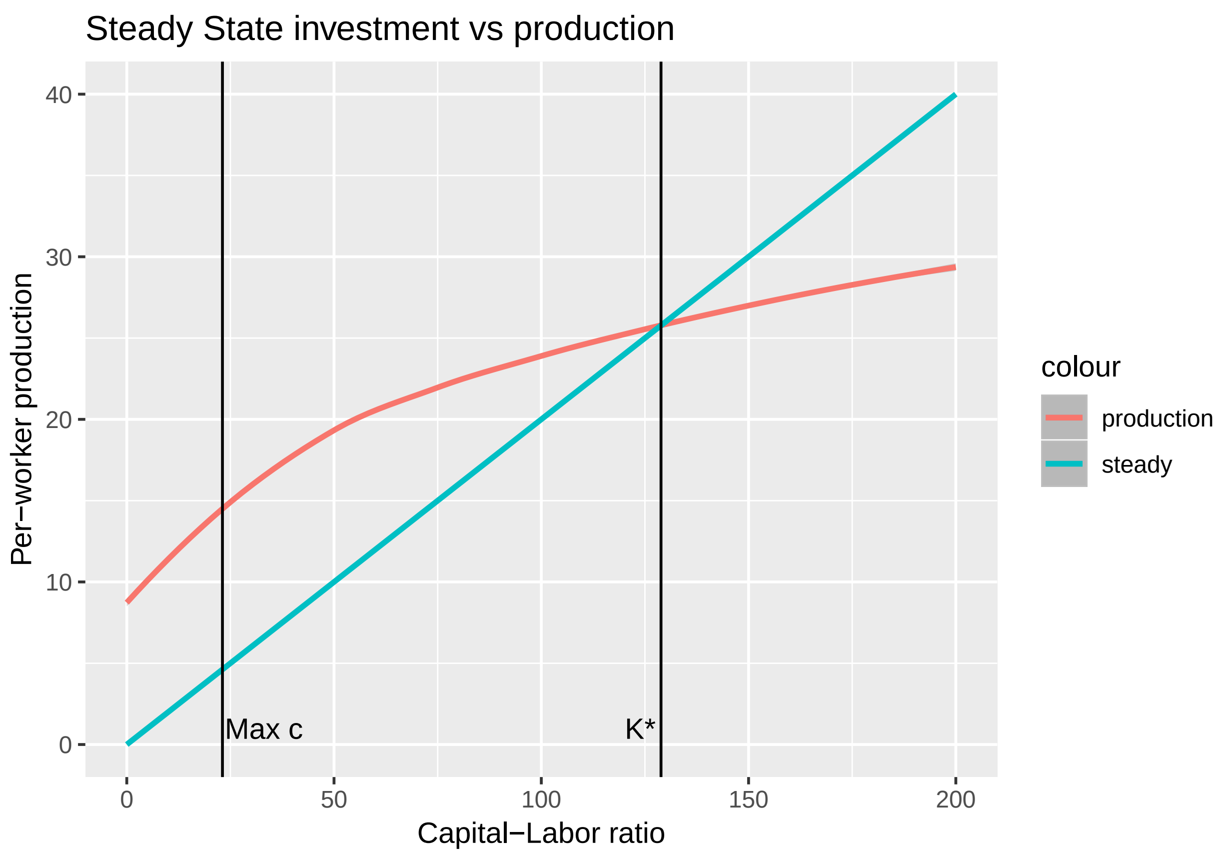


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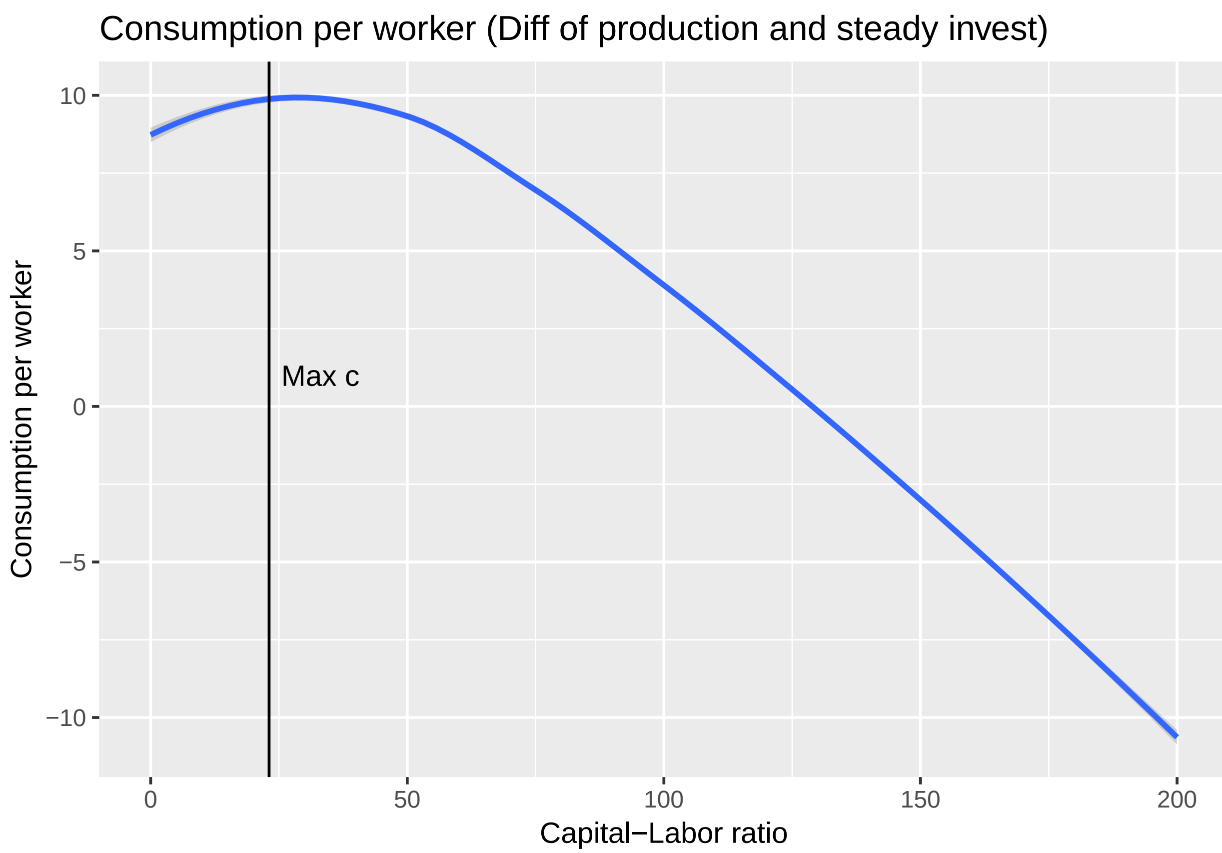

The maximum of consumption happens at the value of k with the biggest difference of y and $(n+\delta)k$. Consumption comes back down to zero where production is equal to $(n+\delta)k$.

1.6 Reaching the Steady-State

At steady state let us assume there is no change in TFP (A). We know that in steady state $I_t = (n+\delta)K_t$ and that Savings = Investment in equilbrium. Let us define s as the savings rate.

$$S_t = sY_t$$

Since we stated savings = investment:

$$sY_t = (n+\delta)K_t$$

Which gives us the per worker steady-state condition:

$$sf(k) = (n+\delta)k$$

$$sy = (n+\delta)k$$

Add in savings rate to compare steady state and actual investment:

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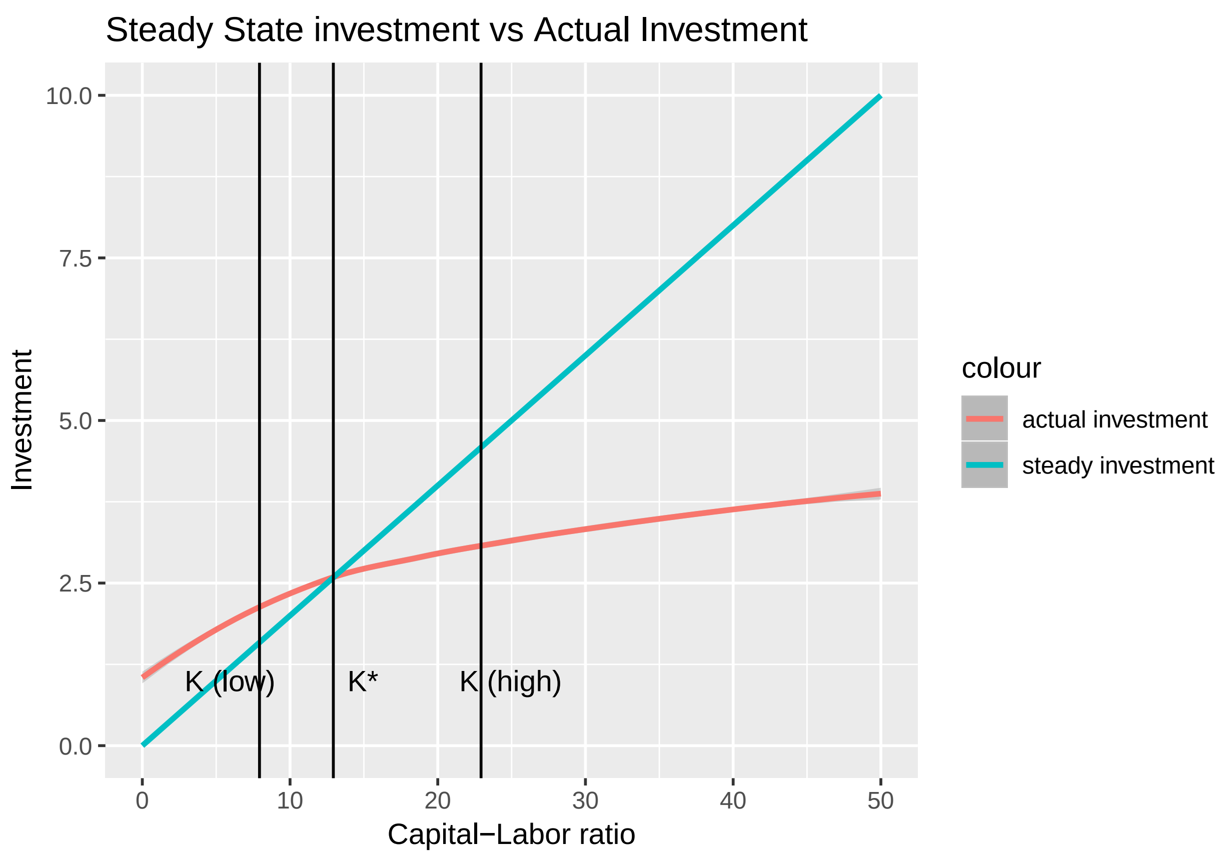

When the level of capital is less than K^* (K (low)), then actual investment is greater than that required for steady state, so the capital-labor ratio incrases until it hits steady state, which also increases per worker production. In the other case when the level of capital is greater than K^* (K (high)), then the actual investment is less than that required for steady state, so the capital-labor ratio decreases until it hits the steady state, which decrease per worker production.

1.7 Changes to the Solow Model

There are three main factors that change the solow model

1. Changes in TFP (A) change $sf(k)$
2. Cahnges in savings rate (s) change $sf(k)$
3. Changes in the population growth rate (n) change $(n+\delta)k$

1.7.1 Changes in A

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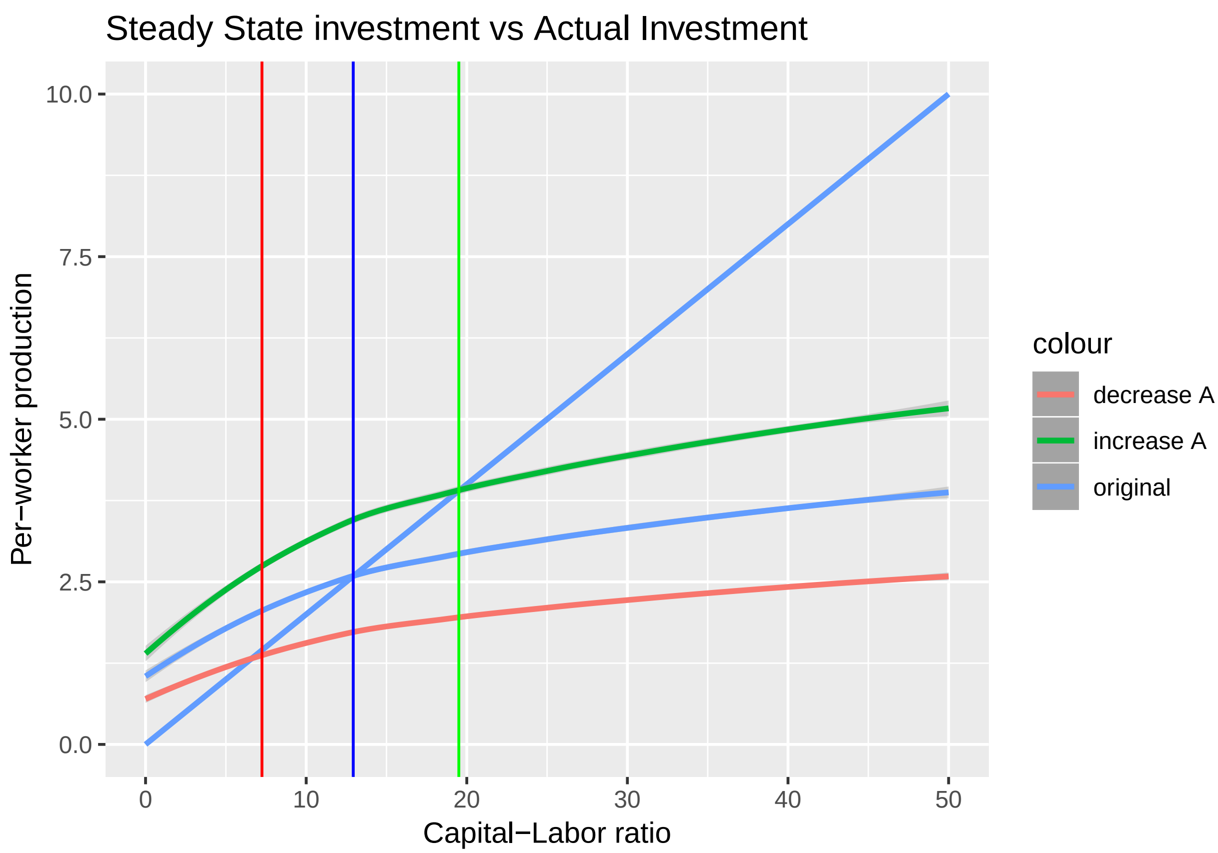

1.7.2 Changes in s

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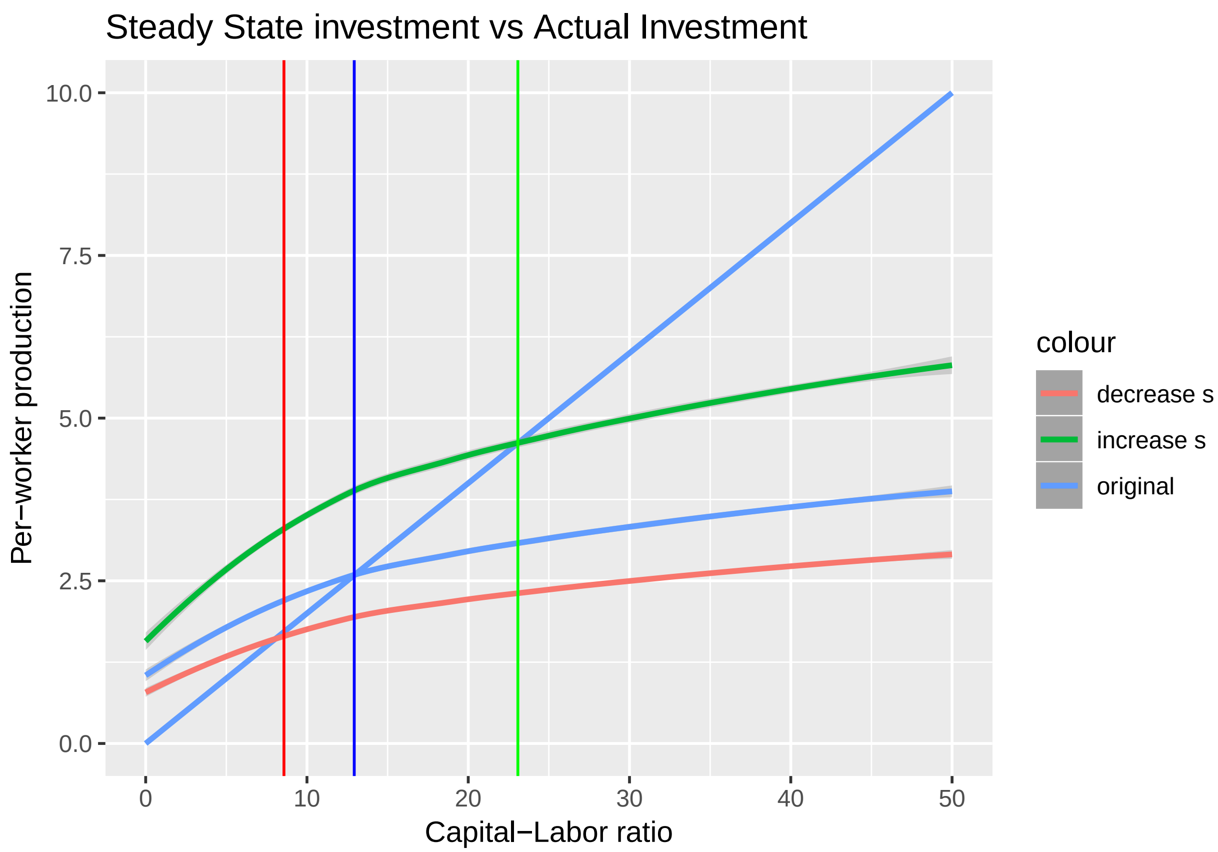

1.7.3 Changes in n

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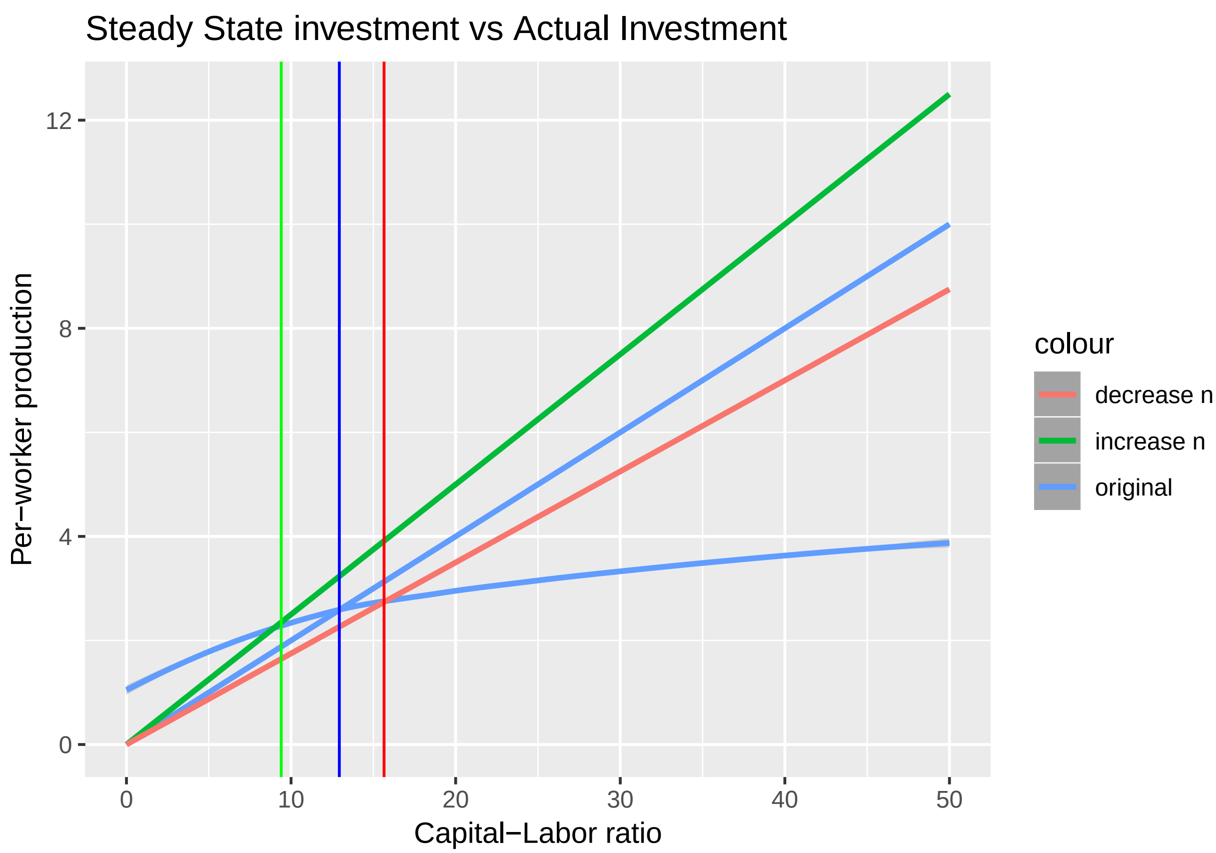

1.8 Insert growth paths after Romer model

2 Chapter 7: Romer Growth Model

2.1 Endogenous Growth Theory

This theory is merely an extension of the Solow model from chapter six.

The Solow model explains the rule of **capital accumulation** in economic growth.

- Requires an increase in technology or TFP in order to sustain growth
- Technology is exogenous
- Does **not** address **why** capital and productivity grow at different rates across different countries
  - Why is there a greater increase in technology occurr in the USA versus Croatia?

2.1.1 Solow Basis

Only Capital (K) and Labor (L) are inputs to production:

$$Y = AF(K, L)$$

2.1.2 Romer Basis

Technology or TFP (A) becomes and input alongside capital and labor. You get to choose what kind of technology you implement. Capital (K) and Labor (L) are objects, while techonology (A) is ideas, so you input both objects and ideas to make your product.

Objects:

- Rival in use
  - If someone is using machine x, then I cannot use machine x
- Diminishing returns
- Excludable
  - If someone wanted to make machine x really expensive, then I could be excluded from using it

Ideas:

- Non-Rivalrous
- Increasing returns
- Non-Excludable
  - Idea behind patents and IP protections
  - See under production of these goods - no incentive
  - There are incentives when you can protect a resource you own

2.2 Romer Model

2.2.1 Notation

In the Solow model all Labor was allocated to produce goods and services. In the Romer model some Labor producs goods and services ($\mathrm{L_p}$), but also some labor produces ideas ($\mathrm{L_A}$)

- In t-shirt example we are all working to phsyically make the t-shirt (Solow)
- If some of us were trying to figure out how to make it more effiicently then that would follow the Romer model ($\mathrm{L_A}$)
  - Sew then dye vs dye then sew? - creation of new **ideas**

$$\overline{N} = L_p + L_A$$

α ~ a constant fraction of labor devote to research and development (R + D)

- $0 \leq \alpha \leq 1$
- $L_A = \alpha\overline{N}$
- $L_p = (1-\alpha)\overline{N}$
- In USA there is a big scare about decreased manufacturing
- We still see growth in the value of our economy's production
- This is because we care about the value derived from research and development

y_{p_t} ~ output per worker with a specific number of workers devoted to production of G + S (L_p)

$$y_{p_t} = \frac{Y_t}{L_p}$$

$$k_{p_t} = \frac{K_t}{L_p}$$

We can then conclude

$$y_{p_t} = A_t k_{p_t}$$

This states that A is now time dependent, but what drives the factor A_t? We must have a production function for technology.

So how does technology change?

Change in technology over a time period can be denoted: $A_{t+1} - A_t = \Delta A_t$

$$\Delta A_t = \chi A_t L_A$$

Where χ is a factor that denotes how productive the labor (L_A) is at producing new ideas ΔA_t. This variable is exogenous.

2.3 Solow vs Romer Growth Rates

In Solow $g_A = 0$ (growth rate of tech).

In Romer $g_A = \frac{\Delta A_t}{A_t} = \frac{\chi A_t L_A}{A+t} = \chi L_A$. Since $L_A = \alpha \overline{N}$ we can state the growth rate of tech is:

$$g_A = \chi \alpha \overline{N}$$

Therefore growth rate of technology increases when:

- Produtivity of R and D increases (χ)
- Overall Population Increases ($\overline{N}$)
- Percentage of Labourers/Population producing ideas increases (α)
- All factors have a positive relationship with growth (g_A)

So now since $\alpha > 0$ we can see that $g_A > 0$ and $g_y > 0$

Romer shows **sustained** growth!!

2.4 Production Function

$$Y_t = A_t L_t^{\alpha} K_t^{\beta}$$

with the conditions:

- $\alpha + \beta = 1$
- $0 < \alpha < 1$
- $0 < \beta < 1$

$$y_t = \frac{Y_t}{L_t} = \frac{A_t L_t^{\alpha} K_t^{\beta}}{L_t} = A_t L_t^{\alpha-1} K_t^{\beta}$$

$$y_t = A_t L_t^{-\beta} K_t^{\beta} = A_t \frac{K_t^{\beta}}{L_t^{\beta}} = A_t (\frac{K_t}{L_t})^{\beta}$$

$$y_t = A_t k_t^{\beta}$$

This is for ALL levels of y when we have some level of k This is NOT for y when we are in steady-state of capital, k^*

2.5 Steady State

We know that k^* exists when $sy = (n+\delta)k$, therefore: $y^* = A{k^*}^{\beta}$

$$sA{k^*}^{\beta} = (n+\delta)k^*$$

2.5.1 Solving for Romer growth rate

Three steps:

1. Solve for k^*
2. Plug in to get y^*
3. Show the growth rates

Solve for k^*

$$sA{k^*}^{\beta} = (n+\delta)k^*$$

$$sA = (n+\delta){k^*}^{1-\beta}$$

$$\frac{sA}{(n+\delta)} = {k^*}^{1-\beta}$$

$$k^* = (\frac{sA}{(n+\delta)})^{(1/(1-\beta))}$$

Plug in for y^*

$$y^* = A{k^*}^{\beta}$$

$$y^* = A(\frac{sA}{(n+\delta)})^{(\beta/(1-\beta))}$$

$$y^* = A^{1+(\beta/1-\beta)}(\frac{s}{(n+\delta)})^{(\beta/(1-\beta))}$$

$$y^* = A^{1/1-\beta}(\frac{s}{(n+\delta)})^{(\beta/(1-\beta))}$$

Show the growth rates (which you can get using calculus)

$$g_{y_*} = \frac{1}{1-\beta} g_A + \frac{\beta}{1-\beta} g_{\frac{s}{n+\delta}}$$

But the growth of savings rate, population rate, and depreciation we assume is zero

$$g_{y_*} = \frac{1}{1-\beta} g_A$$

Which makes sense with respect to the Solow model. We saw a growth rate of zero, where we held A constant until there was a shock.

2.5.2 Romer at Steady State

$$y^*_{p_t} = A_t {k^*_p}^\beta$$

Recall $y_{p_t} = \frac{Y_t}{L_p}$ and $k_p = \frac{K}{L_p}$ and $\overline{N} = L_p + L_A$

$$y_t = \frac{Y_t}{\overline{N}} * (\frac{L_p}{L_p}) = \frac{Y_t}{L_p} * (\frac{L_p}{\overline{N}})$$

We know that $L_p = (1-\alpha)\overline{N}$

$$y_t = y_{p_t} * (1-\alpha)$$

$$y_t = (1-\alpha) A_t k^\beta_{P_t}$$

But what if y is at steady state y^*_t with the steady state of capital k^*:

$$y^*_t = (1-\alpha) A_t {k^*_{P_t}}^\beta$$

2.6 Back to Growth Paths

$$g_y = g_{1-\alpha} + \frac{1}{1-\beta} g_A + \beta k_p$$

But the growth rate of $1-\alpha$ is zero (it is a constant). If k is in steady state, its growth is zero.

$$g_y = \frac{1}{1-\beta} g_A + \beta g_{k_p}$$

If in steady state:

$$g_{y^*} = \frac{1}{1-\beta} g_A = \frac{1}{1-\beta} \chi \alpha \overline{N}$$

2.6.1 TO DO: GROWTH PATH GRAPHS FOR BOTH MODELS

2.7 Changes in $\overline{N}$ and α

$\overline{N}$ increases:

- $\mathrm{g_A}$ will increase
- $\mathrm{L_p}$ will go up
- $\mathrm{k_p}$ will go down (not enough for steady state)
- $\mathrm{k_p}$ goes up in long run
- $\frac{1}{1-\beta} g_A$ increases
- βg_{k_p} increases

α increases:

- $\mathrm{g_A}$ will increase
- $\mathrm{L_p}$ will go down
- $\mathrm{k_p}$ will go up (too much for steady state)
- $\mathrm{k_p}$ goes down in long run
- $\frac{1}{1-\beta} g_A$ increases
- βg_{k_p} decreases