Lesson 7: Economic Growth

Contents

0.1 What is Economic Growth?

Economic growth is an increase in living standards, which has traditionally been measured using Real GDP per capita.

1 Chapter Six: Solow Growth Model

1.1 Notation

- $\mathrm{L_t}$ - Number of workers
- n - growth rate of work force
  - population growth rate
- $\mathrm{K_t}$ - capital stock
- δ - depreciation rate
- $\mathrm{Y_t}$ - output (GDP)
- $\mathrm{C_t}$ - consumption
- $\mathrm{I_t}$ - gross investment
  - $C_t = Y_t - I_t$
- $y_t = \frac{Y_t}{L_t}$ - ouput per worker
- $c_t = \frac{C_t}{L_t}$ - consumption per worker
- $k_t = \frac{K_t}{L_t}$ - capital per worker (capital-labor ratio)
- $i_t = \frac{I_t}{L_t}$ - investment per worker

1.2 Big Finding of Solow Model

In the solow model, if the is no productivity growth (an increase in TFP A), then the economy reaches a steady state. At a steady state $\mathrm{y_t}$, $\mathrm{k_t}$, and $\mathrm{c_t}$ are all constant while Y, K, and C will grow at the rate n (population growth rate).

1.3 Steady-State of Investment

I_t (Investment) serves two main purposes:

1. Expand the size of capital stock (K_t)
2. Replace depreciated capital (δK_t)

Since in steady state we know that capital stock K_t will grow at the rate n, we can conclude that $I_t = nK_t + \delta K_t$, which brings per worker steady state of investment to:

$$i = (n + \delta)k$$

1.4 Per-Worker Production Function

Recall that we stated $Y = AF(K, L)$. We can add the time aspect in to get $Y_t = AF(K_t, L_t)$. Now let's find the per-worker equation:

$$\frac{Y_t}{L_t} = \frac{AF(K_t, L_t)}{L_t}$$

$$\frac{Y_t}{L_t} = A\frac{1}{L_t}F(K_t, L_t)$$

From here, if the function $F(K_t, L_t)$ has constant return to scales we can conclude:

$$\frac{Y_t}{L_t} = AF(\frac{K_t}{L_t}, \frac{L_t}{L_t})$$

Which in our notation is

$$y_t = Af(k_t)$$

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## `geom_smooth()` using method = 'loess' and formula 'y ~ x'
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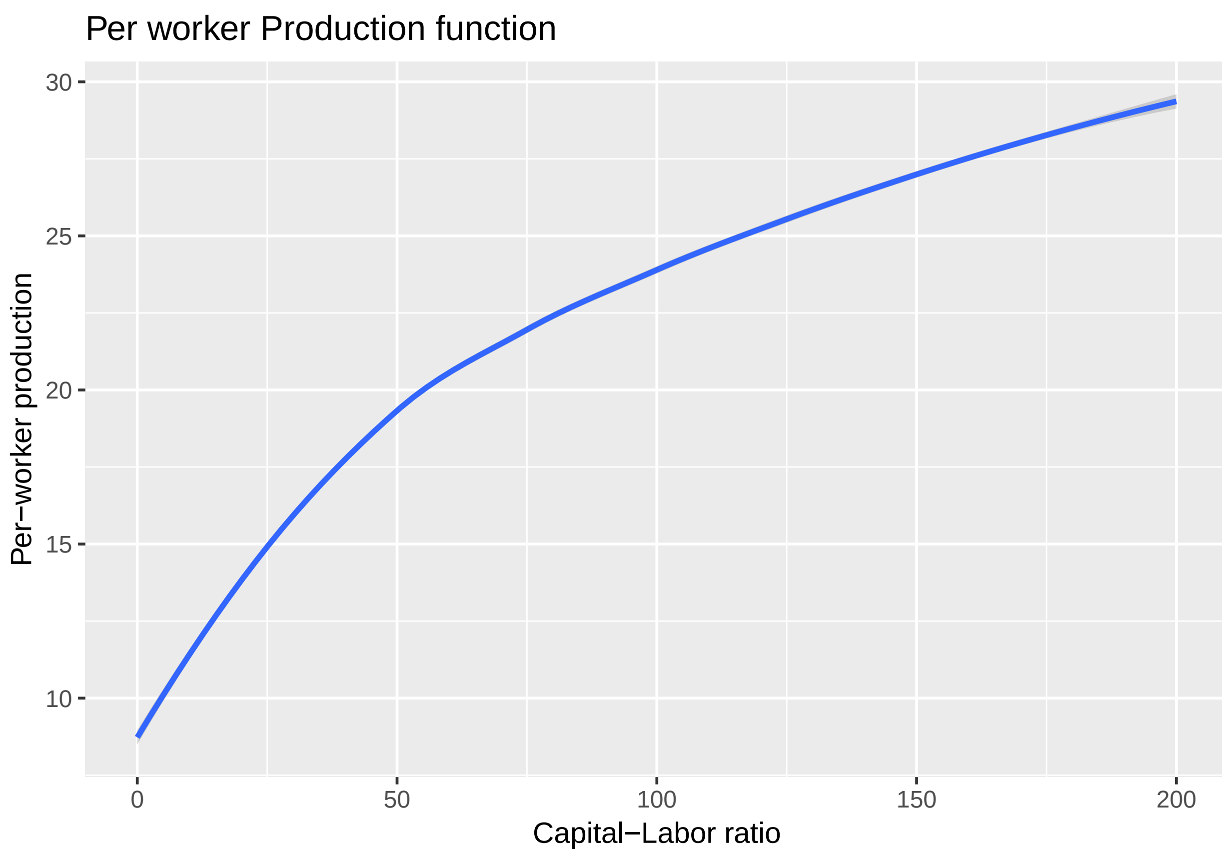

1.5 Steady-State of Consumption

We know two things:

1. $C_t = Y_t - I_t$
2. $I_t = (n + \delta)K_t$ at steady state

So we can conlude that $C_t = Y_t - (n + \delta)K_t$ and per worker we get:

$$c = y - (n + \delta)k$$

$$c = Af(k) - (n + \delta)k$$

If capital increases what happens to production?

- If k increases, then $f(k)$ increases, but at a decreasing rate, so c (consmumption increases)
- If k increases, then $(n + \delta)k$ increases, but at a rate of $(n + \delta)$, so c (consumption decreases)

What happens if we try to maximize consumption? (not related to steady state)

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## [1] 3.641825
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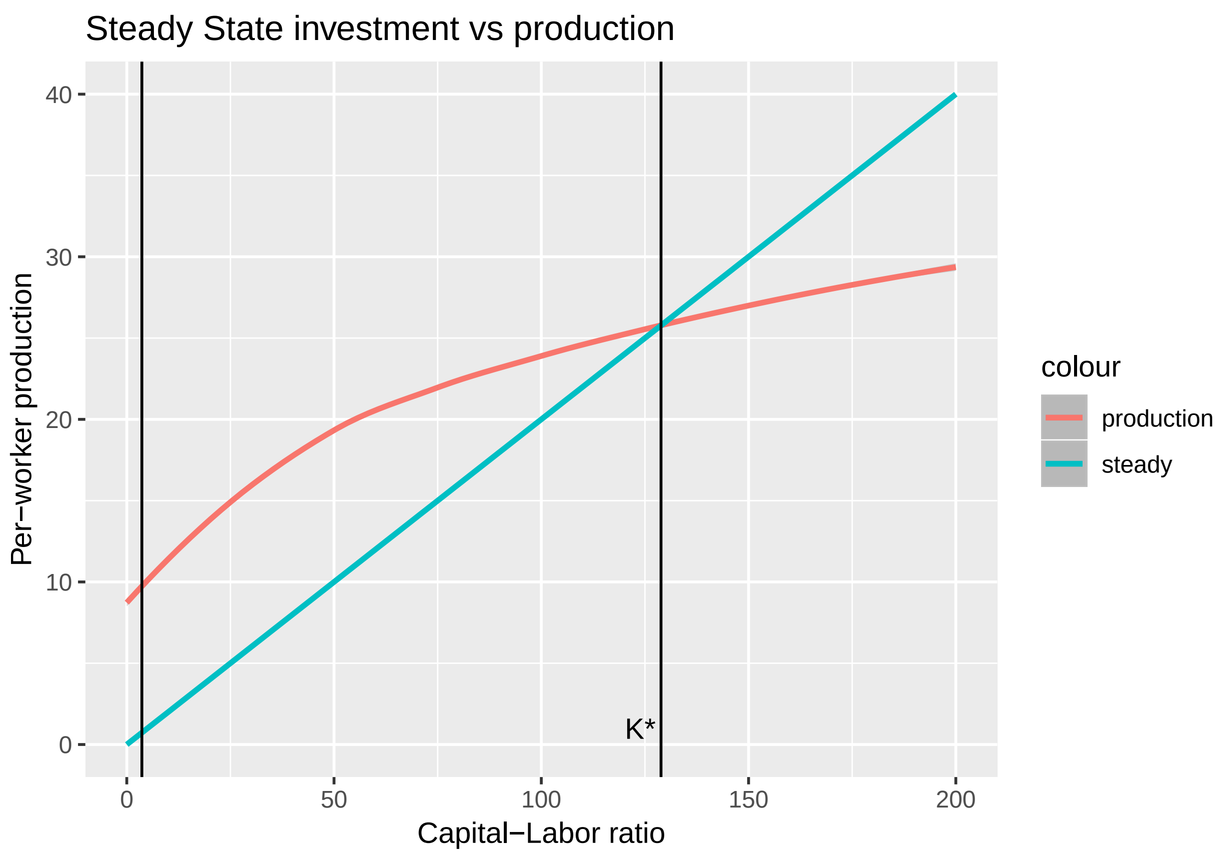


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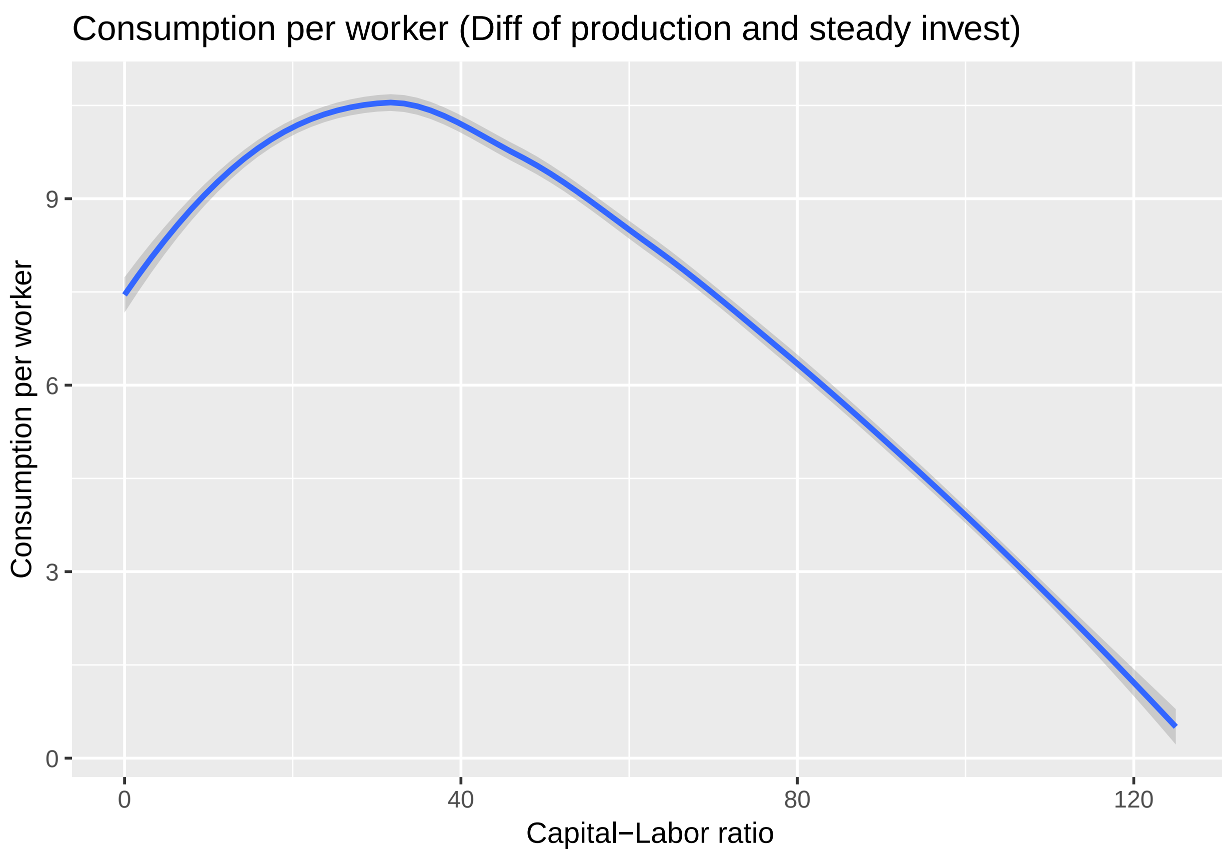

The maximum of consumption happens at the value of k with the biggest difference of y and $(n+\delta)k$. Consumption comes back down to zero where production is equal to $(n+\delta)k$.

1.6 Reaching the Steady-State

At steady state let us assume there is no change in TFP (A). We know that in steady state $I_t = (n+\delta)K_t$ and that Savings = Investment in equilbrium. Let us define s as the savings rate.

$$S_t = sY_t$$

Since we stated savings = investment:

$$sY_t = (n+\delta)K_t$$

Which gives us the per worker steady-state condition:

$$sf(k) = (n+\delta)k$$

$$sy = (n+\delta)k$$