Homework 04 - STAT440

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```r
set.seed(42)
```

Problem 1

Use R to simulate samples from a normal distribution. Let Y be a random variable with chi-squared distribution with 5 degrees of freedom.

Part a

Since we can only sample from the standard normal distribution, I would use the transformation that the Chi-square distribution is really the sum of squares of standard normal variables. The degrees of freedom is the number of standard normal random variables that make up the Chi-square distribution. In this case we would use the following transformation:

$$\chi^2 = \Sigma_{i=1}^{5} Z_i^2$$

Part b

Variance of Y:

$$Var(Y) = (\int x f(x) dx)^2 - (\int x^2 f(x) dx)$$

Foruth moment of Y:

$$\int x^4 f(x) dx$$

We can approximate all these integrals using Monte Carlo, where $h(x) = x^n$ (depending on the moment) and $f(x)$ is the density of Y that we are sampling from. Therefore we can use our samples X_i from the standard normal and transform them (5 for 5 degrees of freedom) into a chi-square sample Y_i. These samples can give us the value $\frac{1}{n}\Sigma_{i=1}^{n} h(Y_i) = \frac{1}{n}\Sigma_{i=1}^{n} Y_i^k$, which can approximate the k^{th} moment integral.

Part c

Use R to estimate the above quantities using Monte Carlo with N = 10,000 samples and report the results.

```r
n <- 10000
df <- 5
norm_samples <- matrix(rnorm(n*df), nrow=n, ncol=df)
norm_samples <- apply(norm_samples, c(1,2), function(x) {x^2})
chi_samples <- rowSums(norm_samples)

# variance
second <- sum(chi_samples^2) / n
expect_square <- (sum(chi_samples) / n) ^ 2
variance <- second - expect_square
print(variance)
```

```
## [1] 9.983276
```

```r
# compare approx with sd function
print(sd(chi_samples)^2)
```

```
## [1] 9.984275
```

```r
# fourth moment
fourth <- sum(chi_samples^4) / n
print(fourth)
```

```
## [1] 3449.642
```