Midterm 03 - STAT440

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```r
set.seed(42)
```

Problem 1 Bernoulli Bayes

Part a: Likelihood Parameters

The Bernoulli likelihood takes one parameter often referred to as p. This parameter can take a value between 0 and 1, which is often referred to as the probability of a successful event. Note the likelihood function:

$$f(X_i|\theta) = \Pi_{i=1}^n \theta^{X_i}(1-\theta)^{(1-X_i)}$$

Part b: Conjugate Prior

The conjugate prior to a Bernoulli likelihood is the Beta distribution. The domain of this distribution lies between 0 and 1. This distribution takes two parameters α and β. These two parameters help to shape the distribution. Since these parameters shape the distribution that lie between 0 and 1 (which is the values our parameter can take on), the parameters dictate the distribution of the parameter θ or p. Higher values of alpha creates a decaying tail towards zero and lower values (less than 1) create a growing tail toward zero. The opposite occurs near 1 for beta. In this way a high alpha would mean our parameter is more likely to be higher (towards 1), while the same value of alpha and beta would create a symmetric distribution for our parameter.

Part c: Posterior Parameters

The posterior parameters based off the data and prior are as follows:

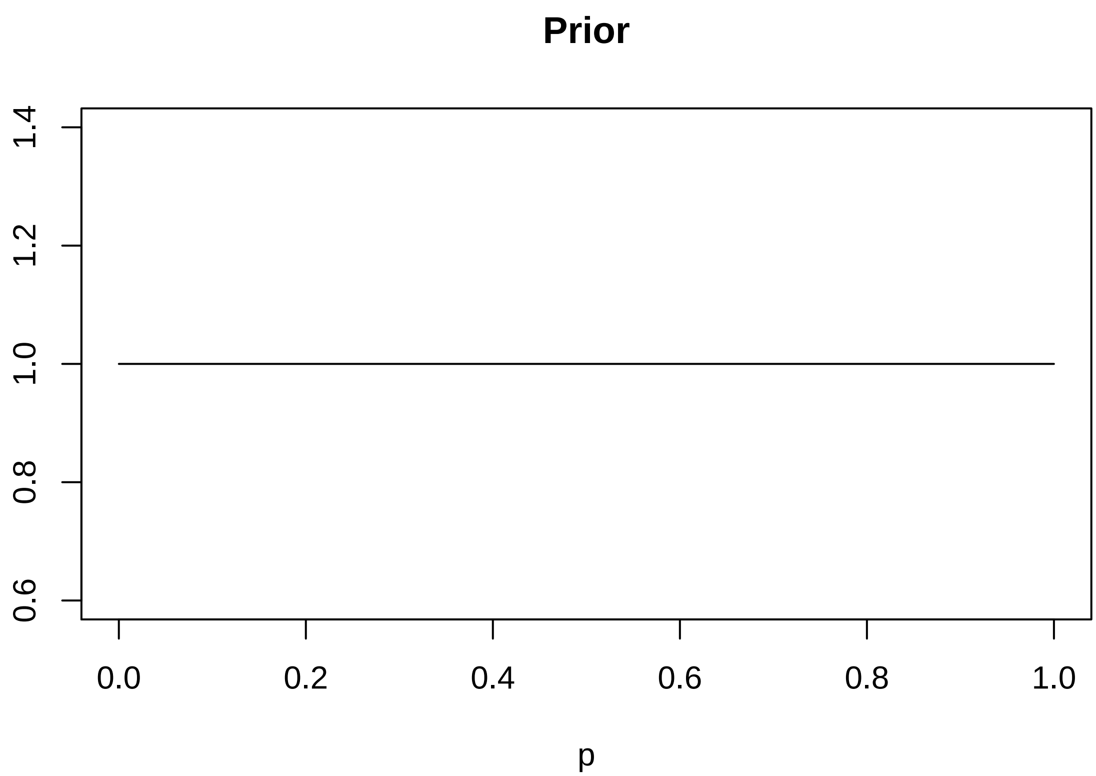
$$\alpha \rightarrow \alpha + n\overline{X}$$
$$\beta \rightarrow \beta + n - n\overline{X}$$

Both parameters depend on the term that is the summation of the events. α will become larger if there are many successes, which makes the beta distribution show that it is more likely for the success rate to be higher. β will become larger if there are many failures, which makes the beta distribution show that it is more likely for the success rate to be lower.

Part d: Plots

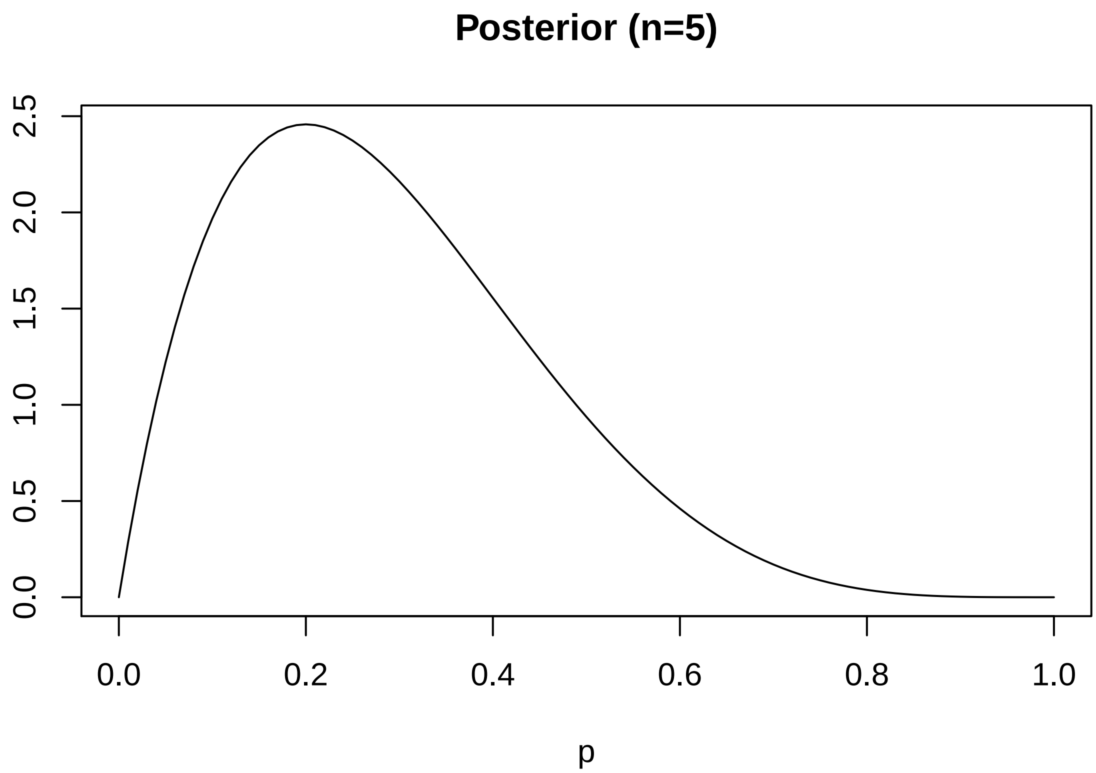
Below is our prior distribution with initial values of 1 each to show a initial belief that any parameter values from 0 to 1 is equally likely.

```
alpha_prior <- 1
beta_prior <- 1

curve(dbeta(x,alpha_prior,beta_prior),from=0,to=1,xlab="p",ylab="",main='Prior')
```



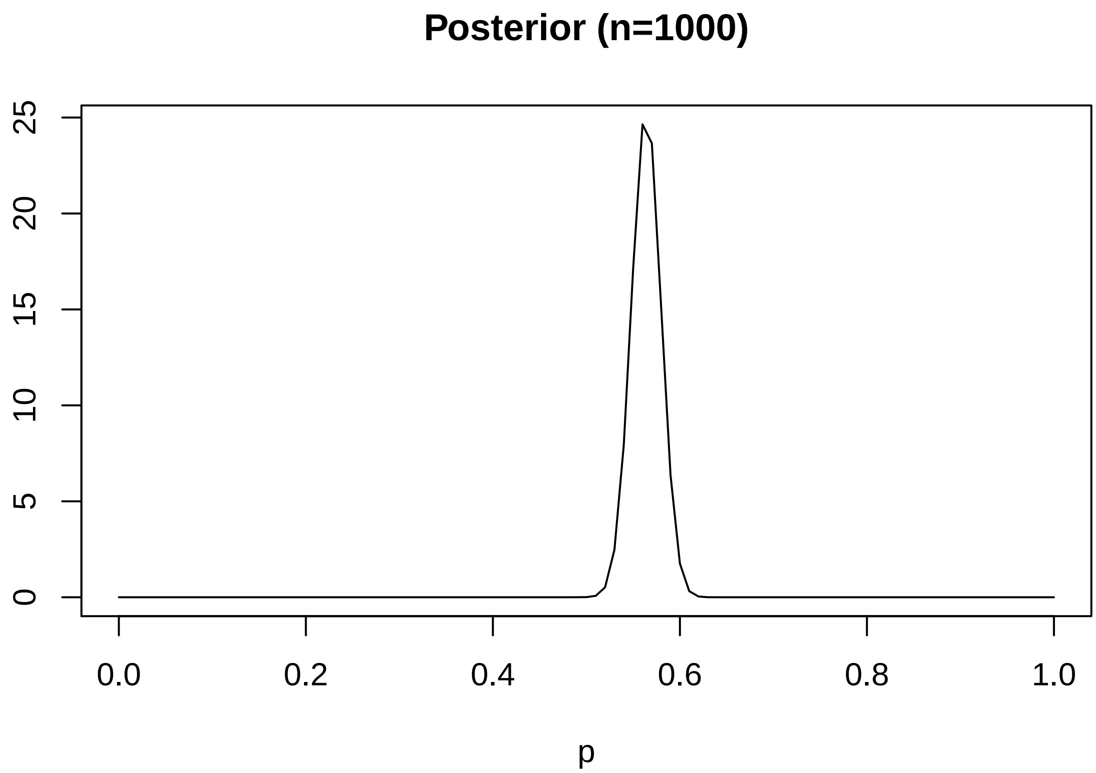
For this first example we have a small data sample size with very few successes.

```
X <- c(0, 0, 0, 0, 1)
n <- length(X)
alpha_post <- alpha_prior + n * mean(X)
beta_post <- beta_prior + n - n*mean(X)

curve(dbeta(x,alpha_post,beta_post),from=0,to=1,xlab="p",ylab="",main='Posterior (n=5)')
```



The second example we have many samples, but roughly half (p=0.55) are successes.

```
X <- rbinom(1, 1000, 0.55)
n <- 1000
alpha_post <- alpha_prior + X[1]
beta_post <- beta_prior + n - X[1]

curve(dbeta(x,alpha_post,beta_post),from=0,to=1,xlab="p",ylab="",main='Posterior (n=1000)')
```


Posterior (n=1000)
25
20
15
10
5
0
0.0
0.2
0.4
0.6
0.8
1.0
p

Problem 2

Part a

Part b

Part c

Part d

Problem 3

Part a

Part b

Part c

Part d

Problem 4

Part a

Part b

Part c

Part d

Problem 5

Part a

Part b

Part c

Part d

Part e