Homework 01 - STAT440

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1. What is the best way to contact the instructor?

2. Describe a resource listed in the syllabus (do not copy and paste).

3. When do you have to complete the assigned readings per lecture?

4. With the exception of the first homework, when are homeworks due?

5. What is the general policy regarding collaboration?

6. What are the different elements of the class that will be graded, and what is their percentage contribution to the total grade?

7. What are the two elements of the final project, and what format can they be delivered?

8. In your own words, describe two etiquette principles for zoom meetings.

9. Have you read the syllabus in its entirety and understood it?

10. Do you agree to abide by the principles listed in the syllabus?

11. Basic definitions.

Let X be an exponential random variable with rate parameter $\lambda > 0$.

(a) What is the range of X, its PDF, and its CDF?

(b) What is the nth moment of X?

(c) What are the mean and variance of X?

(d) Let $\epsilon > 0$. What should a be so that $(P(X > a) = \epsilon$?

12. Transformations of random variables.

Let $\{X_i\}_{i=1}^N$ be i.i.d. exponential random variables with parameter λ. Let $(Y, Z) = T(X_1, X_2)$, where $T : D \to R$ is the transformation.

$$T(x, x') := (x + x', x - x')$$

(a) What is the joint PDF of $\{X_i\}_{i=1}^N$?

(b) What is the domain D of T?

(c) What is the inverse transformation of T^{-1}, and its Jacobian determinant?

(d) What is the range R of T?

(e) What is the joint PDF of Y and Z? Provide the support also.

(f) What are the marginal PDFs of Y and Z?

13. Important Theorems

State the following.

(a) The Central Limit Theorem

(b) The Weak Law of Large Numbers

14. MLE

Suppose that $X_1, X_2, ..., X_n$ are i.i.d. random variables with PDF:

$$f_x(x) = \frac{x}{a} exp\{\frac{-x^2}{2a}\}, x \geq 0$$

(a) Find the maximum likelihood estimate of the parameter a.

(b) Find the Fisher Information of $X_1, X_2, ..., X_n$ and use it to estimate a 95% confidence interval on the MLE of a.

(c) Explain how the central limit theorem relates to the previous quesiton.

15. Good 'ol Bayes.

A lab blood test is 95% effective in detecting a certain disease when the disease is present. However, if a healthy individual is tested, there is a 1% chance that the test result will imply that this individual has the disease. If it is known that 5% of the population have the disease, what is the probability that a person has the disease, given that the test says they do?