

Homework 05 - STAT416

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Chapter 4 Problem 5

We are given both the initial probability of starting at state i along with the 1 step transition probability matrix. We can use both of these to determine $P(X_3) = \alpha P^3$.

$$\alpha_1 = \alpha_0 P = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{3}{8} & \frac{1}{6} & \frac{11}{24} \end{bmatrix}$$

$$\alpha_2 = \alpha_1 P = \begin{bmatrix} \frac{3}{8} & \frac{1}{6} & \frac{11}{24} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{5}{12} & \frac{13}{72} & \frac{29}{72} \end{bmatrix}$$

$$\alpha_3 = \alpha_2 P = \begin{bmatrix} \frac{5}{12} & \frac{13}{72} & \frac{29}{72} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{59}{144} & \frac{43}{216} & \frac{169}{432} \end{bmatrix}$$

$$E[X_3] = 0 + \frac{43}{216} + \frac{169}{216} = \frac{212}{216} \approx 0.9815$$

Chapter 4 Problem 8

Part c

To find the probability that the second ball selected is red we need to know the probability of the state at time 2 (from which the ball will be selected). We use our initial probabilities combined with our transition probability matrix:

$$\alpha_2 = \alpha_1 P = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.15 & 0.6 & 0.25 \\ 0 & 0.3 & 0.7 \end{bmatrix} = \begin{bmatrix} 0.15 & 0.6 & 0.25 \end{bmatrix}$$

So the probability of drawing a red ball second when there is 0 red balls is 0, 0.5 when there is 1 red ball, and 1 when there is 2 red balls. This gives us the probability $0 * 0.15 + 0.5 * 0.6 + 1 * 0.25 = 0.25 + 0.3 = 0.55$. The probability of selecting a red ball second is **55%**.

Part d

Calculating the probability of selecting a red ball fourth follows the same logic as the previous part. We can pickup with α_2 .

$$\alpha_3 = \alpha_2 P = \begin{bmatrix} 0.15 & 0.6 & 0.25 \end{bmatrix} \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.15 & 0.6 & 0.25 \\ 0 & 0.3 & 0.7 \end{bmatrix} = \begin{bmatrix} 0.165 & 0.51 & 0.325 \end{bmatrix}$$
$$\alpha_4 = \alpha_3 P = \begin{bmatrix} 0.165 & 0.51 & 0.325 \end{bmatrix} \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.15 & 0.6 & 0.25 \\ 0 & 0.3 & 0.7 \end{bmatrix} = \begin{bmatrix} 0.159 & 0.486 & 0.355 \end{bmatrix}$$

This gives us the probability $0 * 0.159 + 0.5 * 0.486 + 1 * 0.355 = 0.243 + 0.355 = 0.598$. The probability of selecting a red ball fourth is **59.8%**. It makes sense that the likelihood of selecting red goes up with time as you are more likely to replace a red ball with a red ball while replace blue ball has equal probability for each.

Chapter 4 Problem 10

Chapter 4 Problem 15

Chapter 4 Problem 16

Problem A

Problem B