

Homework 05 - STAT416

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10/06/2020

Chapter 4 Problem 5

We are given both the initial probability of starting at state i along with the 1 step transition probability matrix. We can use both of these to determine $P(X_3) = \alpha P^3$.

$$\alpha_1 = \alpha_0 P = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{3}{8} & \frac{1}{6} & \frac{11}{24} \end{bmatrix}$$

$$\alpha_2 = \alpha_1 P = \begin{bmatrix} \frac{3}{8} & \frac{1}{6} & \frac{11}{24} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{5}{12} & \frac{13}{72} & \frac{29}{72} \end{bmatrix}$$

$$\alpha_3 = \alpha_2 P = \begin{bmatrix} \frac{5}{12} & \frac{13}{72} & \frac{29}{72} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{59}{144} & \frac{43}{216} & \frac{169}{432} \end{bmatrix}$$

$$E[X_3] = 0 + \frac{43}{216} + \frac{169}{216} = \frac{212}{216} \approx 0.9815$$

Chapter 4 Problem 8

Chapter 4 Problem 10

Chapter 4 Problem 15

Chapter 4 Problem 16

Problem A

Problem B