Homework 02 - STAT416

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Chapter 2 Problem 33

Let X be a random variable with the following probability density from $-1 < x < 1$:

$$f(x) = c(1 - x^2)$$

Part a

What is the value of c? The value of c must make the density valid. For this to be true the pdf must be positive and also integrate to 1 over its support, so let's integrate:

$$\int_{-1}^{1} c(1 - x^2)dx = c(x - \frac{1}{3}x^3)|_{-1}^{1} = c((1 - \frac{1}{3}) - (-1 + \frac{1}{3})) = c(\frac{2}{3} + \frac{2}{3}) = c\frac{4}{3}$$

This means that $c = \frac{3}{4}$ for the density to be valid. This also maintains a positive function, since $1 - x^2$ will be positive between -1 and 1 and c is also a positive number.

Part b

What is the CDF of X?

To find the CDF of X we merely need to integrate the function over its support. We already did this in the last part, so we will just paste the result here:

$$F_X(a) = c(x - \frac{1}{3}x^3)|_{-1}^{a} = \frac{3}{4}x - \frac{1}{4}x^3|_{-1}^{a} = \frac{3}{4}a - \frac{1}{4}a^3 - (-\frac{3}{4} + \frac{1}{4}) = \frac{3}{4}a - \frac{1}{4}a^3 + \frac{1}{2}$$

This gives us the CDF:

$$F_X(x) = \frac{3}{4}x - \frac{1}{4}x^3 + \frac{1}{2}$$

Chapter 2 Problem 37

Let $X_1, X_2, ..., X_n$ be independent random variables, each having a uniform distribution over (0,1). Let $M = maximum(X_1, X_2, ..., X_n)$. Show that the distribution function of M is $F_M(x) = x^n$.

We want to find $F_M(x) = P(M \leq x) = P(X_1, X_2, ..., X_N \leq x)$. This must hold true, since x in this case represents the max value. Since each random variable is independent we can multiply them to find their intersect:

$$P(X_1, X_2, ..., X_N \leq x) = \Pi_{i=1}^{n} P(X_i \leq n) = F_X(x)^n$$

Since each random variable has the same distribution we can plug in the normal distribution for parameters 0 and 1: $F_X(x; a = 0, b = 1) = \frac{x-a}{b-a} = \frac{x-0}{1-0} = x$. Plugging this into result found above we get:

$$F_M(x) = F_X(x)^n = x^n$$

Chapter 2 Problem 44

Let Y denote the number of red balls chosen after the first but before the second black ball has been chosen.

Part a

Express Y as the sum of n random variables, each of which is either 0 or 1.

Let Y_i be a random variable whose value is 1 if red ball i is taken between the first and second black ball and zero otherwise. This makes $Y = \Sigma_{i=1}^{n} Y_i$.

Part b

$$E[Y] = E[\Sigma_{i=1}^{n} Y_i] = \Sigma_{i=1}^{n} E[Y_i]$$

Let's define $p(Y_i)$. Y_i means that you are drawing a single red ball out of $m - 1$ black balls and 1 red ball. This gives us $p(Y_i) = \frac{1}{m}$. This would make $E[Y_i] = 1 * p(Y_i) + 0 * (1 - p(Y_i)) = p(Y_i)$

$$\Sigma_{i=1}^{n} \frac{1}{m} = \frac{n}{m} = E[Y]$$