

Homework 08 - STAT416

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Chapter 4 Problem 70

Part a

In this Markov chain the state is the number of m black balls in the first urn. This means that there are $m + 1$ total states: $0, 1, 2, \dots, m$.

The state i can only transition from i to either $i - 1$, i , or $i + 1$. Losing one black ball means pulling a black ball from urn 1 and a white ball from urn 2. This occurs with probability $P_{i,i-1} = \frac{i}{m} \frac{i}{m} = \frac{i^2}{m^2}$. Gaining one black ball means pulling a white ball from urn 1 and a black ball from urn 2. This occurs with probability $P_{i,i+1} = \frac{m-i}{m} \frac{m-i}{m} = \frac{(m-i)^2}{m^2}$. Maintaining the same number of black balls in urn 1 involves pulling the same color from both urns. This happens with probability $P_{i,i} = \frac{i}{m} \frac{m-i}{m} + \frac{m-i}{m} \frac{i}{m} = \frac{2i(m-i)}{m^2}$.

Part b

I think that the limiting probabilities of this chain would be the possible ways to have i black balls in urn 1 out of the possible ways to divide the balls.

Part c

$$\pi_i = \pi_{i-1} \frac{(m-1)^2}{m^2} + \pi_i \frac{2i(m-i)}{m^2} + \pi_{i+1} \frac{i^2}{m^2}$$

This chain is isolated to going up or down in the current state and does not involve loops, so it is reversible. If that is true then:

$$\begin{aligned} \pi_i P_{i,i+1} &= \pi_{i+1} P_{i+1,i} \\ \frac{\pi_{i+1}}{\pi_i} &= \frac{P_{i,i+1}}{P_{i+1,i}} = \left(\frac{m-i}{i+1} \right)^2 \end{aligned}$$

If this is true we can then go back and prove π_i by induction.

$$\begin{aligned} \frac{\pi_1}{\pi_0} &= m^2 \\ \frac{\pi_2}{\pi_0} &= \left(\frac{m}{1} \frac{m-1}{2} \right)^2 = \binom{m}{2}^2 \end{aligned}$$

Continuing this by induction we get

$$\frac{\pi_i}{\pi_0} = \binom{m}{i}^2$$

From this result we then know how often π_i will happen. We just add in the denominator, which includes all possible options:

$$\pi_i = \frac{\binom{m}{i}^2}{\binom{m}{0}^2 + \binom{m}{1}^2 + \dots + \binom{m}{m}^2}$$

Chapter 4 Problem 72

Chapter 4 Problem 73

Chapter 5 Problem 1

Chapter 5 Problem 4

Chapter 5 Problem 12

Part a

Chapter 5 Problem 14

Part a

Chapter 5 Part 19

Problem A