Homework 01 - STAT416

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09/08/2020

Chapter 1 Problem 4

Let E, F, G be three events. Find expressions for the events.

Part a

Only F occurs:

$$E^c \cap F \cap G^c$$

Part b

Both E and F but not G occur:

$$(E \cup F) \cap G^c$$

Part c

At least one event occurs:

$$E \cup F \cup G$$

Part d

At least two events occur:

$$(E \cap F) \cup (E \cap G) \cup (F \cap G)$$

Part e

All three events occur:

$$E \cap F \cap G$$

Part f

None occurs:

$$(E \cup F \cup G)^c$$

Part g

At most one occurs (Also the complement of at least two):

$$((E \cap F) \cup (E \cap G) \cup (F \cap G))^c$$

Part h

At most two occur (Also the complement of all three):

$$(E \cap F \cap G)^c$$

Chapter 1 Problem 11

If two fair dice are tossed, what is the probability that the sum is $i, i = 2, 3, ...12$.

Recall the pmf of a single die. The likelihood of each digit is $p(x) = \frac{1}{6}$. Since each digit has an equal likelihood of returning, we only have to count the total number of ways to sum to i and then divide by the total number of combinations $6 * 6 = 36$ (size of sample space) to get the probability of the event to be that sum.

For example, for i = 7, we have 2 ways to get 1 and 6, 2 ways to get 2 and 5, and 2 ways to get 3 and 4. This would give i = 7 6 instances out of 36 total instances for a $p(i) = \frac{1}{6}$.

i	p(i)
2	$\frac{1}{36}$
3	$\frac{2}{36} = \frac{1}{18}$
4	$\frac{3}{36} = \frac{1}{12}$
5	$\frac{4}{36} = \frac{1}{9}$
6	$\frac{5}{36}$
7	$\frac{6}{36} = \frac{1}{6}$
8	$\frac{5}{36}$
9	$\frac{4}{36} = \frac{1}{9}$
10	$\frac{3}{36} = \frac{1}{12}$
11	$\frac{2}{36} = \frac{1}{18}$
12	$\frac{1}{36}$

We can see that the pmf outline above is valid, because all probabilities are between 0 and 1, and the sum of all probabilities is 1.

Chapter 1 Problem 20

Three dice are thrown. What is the probability the same number appears on exactly two of the three dice?

To solve this we can look at a sub event: the probability of a one appearing on exactly two of the three dice.

There is exactly $\frac{1}{6}$ ways for two of the dice to land on 1 and there is exactly $\frac{5}{6}$ ways for the third die to land in order to not land on the same number. This gives us a probability of $\frac{1}{6} * \frac{1}{6} * \frac{5}{6} = \frac{5}{216}$ of exactly two to land on one. We can multiply the probabilities like this, because the events are independent.

Since this is the exact same case for each of the six digits we can multiply the value we found by 6, or add it together six times:

$$p(x) = 6 * \frac{5}{216} = \frac{15}{108}$$

Chapter 1 Problem 21

Suppose that 5 percent of men and 0.25 percent of women are colorblind. A randomly chosen person is colorblind. What is the probability of this person being a male? Assume that there are an equal number of males and females.

Problem Constraints:

- $M :=$ Is a Male
- $B :=$ Is Colorblind
- $P(B|M) = 0.05$
- $P(B|M^c) = 0.0025$
- $P(M) = 0.5$

We want to find $P(M|B)$. For this we can use Bayes Theorem.

$$P(M|B) = \frac{P(B|M)P(M)}{P(B)}$$

The law of total probability will give us:

$$P(B) = P(B|M)P(M) + P(B|M^c)P(M^c) = 0.05 * 0.5 + 0.0025 * 0.5 = 0.02625$$

We can take this back to solve:

$$P(M|B) = \frac{0.05 * 0.5}{0.02625} = 0.9524$$

We can conclude that the probability of a randomly chosen person being a male, given that they are colorblind, is 95.24%.

Chapter 1 Problem 26

A deck of 52 playing cards, containing 4 aces, is randomly divided into 4 piles of 13 cards each. Define events $E_1, E_2, E_3, \text{and} E_4$ such that each pile has E_i has exactly 1 ace. Find the probability that each pile has an ace: $P(E_1 E_2 E_3 E_4)$.

$$P(E_1E_2E_3E_4) = P(E_1)P(E_2|E_1)P(E_3|E_1E_2)P(E_4|E_1E_2E_3)$$

$P(E_1) = \frac{\binom{4}{1}\binom{48}{12}}{\binom{52}{13}}$, since we have the sample space of choosing 13 cards from 52 cards, but only want to consider the instances where we draw 1 ace and 12 non-ace cards. We can continue this for each probability to get:

$$P(E_1)P(E_2|E_1)P(E_3|E_1E_2)P(E_4|E_1E_2E_3) = \frac{\binom{4}{1}\binom{48}{12}}{\binom{52}{13}} \frac{\binom{3}{1}\binom{36}{12}}{\binom{39}{13}} \frac{\binom{2}{1}\binom{24}{12}}{\binom{26}{13}} \frac{\binom{1}{1}\binom{12}{12}}{\binom{13}{13}}$$

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p_e_1 <- choose(4, 1) * choose(48, 12) / choose(52, 13)
p_e_2 <- choose(3, 1) * choose(36, 12) / choose(39, 13)
p_e_3 <- choose(2, 1) * choose(24, 12) / choose(26, 13)
p_e_4 <- choose(1, 1) * choose(12, 12) / choose(13, 13)

p_e_1 * p_e_2 * p_e_3 * p_e_4
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## [1] 0.1054982
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$$P(E_1E_2E_3E_4) \approx 0.1055$$

The probability of each pile having an ace is approximately **10.55%**.

Chapter 1 Problem 29

Suppose that $P(E) = 0.6$. What can you say about $P(E|F)$?

Part a

If events E and F are mutually exculusive, then we know that $P(E \cap F) = 0$ by definition of disjoint events. We can plug this into the formula of conditional probability:

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0}{P(F)} = 0$$

Part b

If $E \subset F$, then $P(E \cap F) = P(E) = 0.6$, so we can say that

$$P(E|F) = \frac{P(E)}{P(F)} = \frac{0.6}{P(F)}; 0 < P(E|F) \leq 1$$

Part c

If $F \subset E$, then $P(E \cap F) = P(F)$, so we can say that

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{P(F)}{P(F)} = 1$$

Chapter 1 Problem 38

Chapter 1 Problem 42

Chapter 1 Problem 46

Chapter 2 Problem 5

Chapter 2 Problem 9

Chapter 2 Problem 10