Homework 06 - STAT416

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Chapter 4 Problem 17

Here we look at the infinite random walk in Example 4.19 from the book. The random variable Y_i is an indicator of whether we went to the state 1 higher than the current state or not: $Y_i = \{1, -1\}$. Suppose $p = P(Y_i = 1) > \frac{1}{2}$. We know that the state at time n can be written as $\Sigma_{i=1}^n Y_i$. The expected value of Y_i is

$$E[Y_i] = p + (p - 1) = 2p - 1$$

When $p = \frac{1}{2}$ this expected value is $2 * \frac{1}{2} - 1 = 0$, so when $p > \frac{1}{2}$ the expected value is $1 > E[Y_i] > 0$ (equal to $2p - 1$). We are looking at the value $\Sigma_{i=1}^n Y_i = n\overline{Y_n}$

Recall the strong law of large numbers which states as $n \to \infty$, then $\overline{X_n} \to \mu$.

Similarly we can say as $n \to \infty$ then $\overline{Y_n} \to 2p - 1$, so $n\overline{Y_n} = \Sigma_{i=1}^n Y_i \to \infty$, since we have infinity multiplied by a constant we get infinity. Since we just concluded that the state at time n will go to ∞ if $p > \frac{1}{2}$, then we can say 0 is only visited finitely often, and therefore must be transient. Since transient is a class property and there is only one class the whole chain must be transient.

Note that we can use similar logic when $p < \frac{1}{2}$. Here we merely get a negative constant $0 > E[Y_i] > -1$. Therefore this proof holds for $p \neq \frac{1}{2}$.

Chapter 4 Problem 18

Chapter 4 Problem 21

Part a

Chapter 4 Problem 23

Part a

Part b

Part c

Chapter 4 Problem 33

Chapter 4 Problem 36