Stat/Math 415 Homework 7

Due Friday Nov 8; Joseph Sepich (jps6444)

1 Problem 8.1-1

Problem constraints:

- X ~ N(μ, 100)
- H_0: $\mu = 110$
- H_1: $\mu > 110$
- n = 16
- $\overline{x} = 113.5$

1.1 Part a

Given a significance level of $\alpha = 0.05$, we want to determine if we reject the null hypothesis. Let us use a critical region approach where we will only reject the null hypothesis if observed $Z > Z_\alpha$. The $Z_\alpha = Z_{0.05}$ is 1.645. Let's transform our sample mean into a test statistic using the known variance of 100.

$$Z = \frac{\overline{x} - \mu}{\sqrt{\frac{\sigma^2}{n}}} = \frac{113.5 - 110}{\sqrt{100/16}} = 1.40$$

Since 1.40 < 1.645, our test statistic is **not** within the critical region, therefore we fail to reject the null hypothesis of $\mu = 110$.

1.2 Part b

Given a significance level of $\alpha = 0.10$, we want to determine if we reject the null hypothesis. Let us use a critical region approach where we will only reject the null hypothesis if observed $Z > Z_\alpha$. The $Z_\alpha = Z_{0.10}$ is 1.28. Let's transform our sample mean into a test statistic using the known variance of 100.

$$Z = \frac{\overline{x} - \mu}{\sqrt{\frac{\sigma^2}{n}}} = \frac{113.5 - 110}{\sqrt{100/16}} = 1.40$$

Since $1.40 > 1.28$, our test statistic is within the critical region, therefore we will reject the null hypothesis of $\mu = 110$.

1.3 Part c

The p-value for this test corresponds to the probability of having our test statistic or higher given our distribution: $P(Z > 1.40)$. Looking at the Z score table the p-value would be 1-0.9192 = **0.0808**.

2 Problem 8.1-3

Problem constraints:

- X ~ N(μ, 100)
- H_0: $\mu = 170$
- H_1: $\mu > 170$
- n = 25

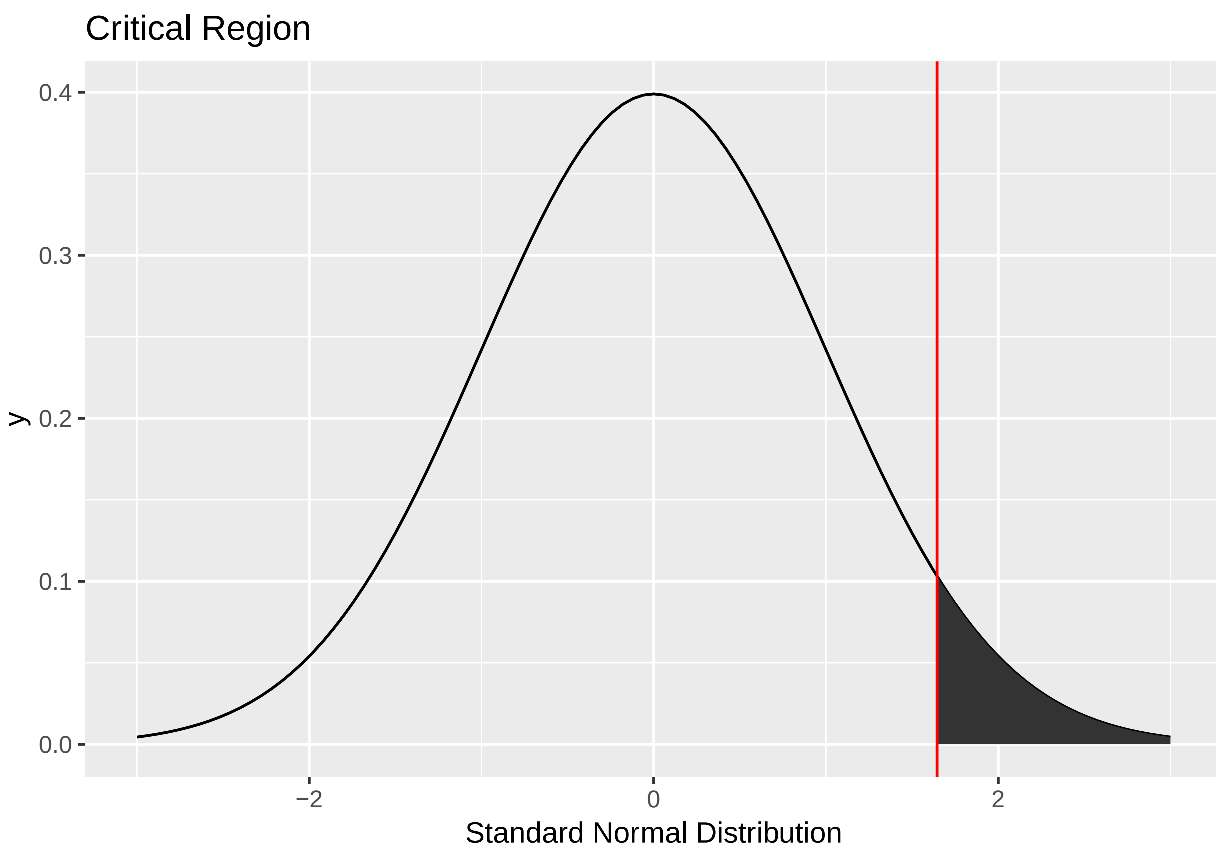
2.1 Part a

The test statistic for our distribution (varaince of 100) is as follows:

$$Z = \frac{\overline{x} - \mu}{\sqrt{\frac{\sigma^2}{n}}} = \frac{\overline{x} - 170}{\sqrt{100/25}} = \frac{\overline{x} - 170}{2}$$

Given a significance level of $\alpha = 0.05$, the criticial region is $Z > Z_\alpha = 1.645$

```
ggplot(data.frame(x = c(-3, 3)), aes(x)) +
  stat_function(fun = dnorm) +
  stat_function(fun = dnorm,
                xlim = c(1.645,3),
                geom = "area") +
  geom_vline(xintercept=1.645, color="red") +
  ggtitle('Critical Region') +
  xlab('Standard Normal Distribution')
```



2.2 Part b

To calculate the value of the test statisic we must first calculate the value of the sample mean: $\overline{x} = \frac{1}{n}\Sigma_{i=1}^{25} x_i$.

```
x_data <- c(170, 167, 174, 179, 179, 156, 163, 156, 187, 156, 183, 179, 174, 179,
            170, 156, 187, 179, 183, 174, 187, 167, 159, 170, 179)
n <- 25

x_mean <- sum(x_data) / n
paste0('The vale of the sample mean is: ', x_mean)
```

```
## [1] "The vale of the sample mean is: 172.52"
```

$$Z = \frac{\overline{x} - 170}{2} = \frac{172.52 - 170}{2} = 1.26$$

Given the value of our test statistic and our critical region, we **cannot** reject the null hypothesis $\mu = 170$, because the test statistic 1.26 is not in the critical region $Z > 1.645$.

2.3 Part c

The p-value for this test corresponds to the probability of having our test statistic or higher given our distribution: $P(Z > 1.26)$. Looking at the Z score table the p-value would be 1-0.8962 = **0.1038**.

3 Problem 8.1-6

Problem constraints:

- X $\approx N(\mu = 3.4, \sigma^2)$
- n = 9

3.1 Part a

The null hypothesis is H_0: $\mu = 3.4$.

3.2 Part b

3.3 Part c

3.4 Part d

3.5 Part e

3.6 Part f

3.7 Part g

4 Problem 8.2-3

5 Problem 8.2-5

6 Problem 8.2-9