

Data Structures and Algorithms Homework 1

Due Wednesday Sept 4; Joseph Sepich (jps6444)

1 Problem 1

I understand the course policies.

2 Problem 2

2.1 Part a. Prove that if a and b are even, then $\gcd(a, b) = 2\gcd(a/2, b/2)$

1. By the definition of even we can state that $a = 2n$ and $b = 2m$, where n and m are both integers.
2. Using the definition in step 1 we can write $\gcd(a, b) = \gcd(2n, 2m)$.
3. Since 2 is a common divisor we can also write $\gcd(2n, 2m) = 2\gcd(n, m)$.
4. Using step 1, we also know that $n = \frac{a}{2}$ and $m = \frac{b}{2}$.
5. Plugging step 4 into the equalities in step 2 and 3 we get $\gcd(a, b) = \gcd(2n, 2m) = 2\gcd(n, m) = 2\gcd(\frac{a}{2}, \frac{b}{2})$.

Therefore if a and b are both even, $\gcd(a, b) = 2\gcd(\frac{a}{2}, \frac{b}{2})$.

2.2 Part b. Prove that if a is even and b is odd, then $\gcd(a, b) = \gcd(a/2, b)$

1. By the definition of even we can state that $a = 2n$, where n is an integer.
2. Using the definition in step 1 we can write $\gcd(a, b) = \gcd(2n, b)$.
3. Since b is odd, it cannot be divided by 2, so the 2 in the term $2n$ is unnecessary information (cannot contribute to the gcd). We can then write $\gcd(a, b) = \gcd(n, b)$.
4. Using step 1, we also know that $n = \frac{a}{2}$.
5. Plugging step 4 into the equalities in step 2 and 3 we get $\gcd(a, b) = \gcd(2n, b) = \gcd(n, b) = \gcd(\frac{a}{2}, b)$.

Therefore if a is even and b is odd, then $\gcd(a, b) = \gcd(\frac{a}{2}, b)$.