Stat/Math 415 Homework 5

Due Friday Oct 25; Joseph Sepich (jps6444)

1 Problem 7.4-3

Problem information:

- $\overline{x} = 6.09$
- $s = 0.02$

1.1 Part a

Sample size calculation: sample size needed for $\epsilon = 0.001$ with confidence of $\alpha = 0.1$

Assuming standard deviation is the same for the population we can use the Z value of 1.645 for alpha of 0.1 and plug the numbers into our equation based off a confidence interval:

$$n \geq \frac{z_{\alpha/2} S^2}{\epsilon^2} = \frac{1.645^2 * 0.02^2}{0.001^2} = 1082.41$$

Remember we must round up, so we need the sample size to be at least **1,083** samples.

1.2 Part b

With the following numbers we have a decent sample size, and we don't know the distribution, so we use central limit theorem and approximate with a normal distribution:

- $n = 1219$
- $\overline{x} = 6.048$
- $s = 0.022$

Since we looking at the confidecnce level from the last problem, we are still using the same Z value of 1.645. We plug this into our confidence interval formula:

$$\overline{x} + -Z_{\alpha/2} \frac{s}{\sqrt{n}} = 6.048 + -1.645 * \frac{0.022}{\sqrt{1219}}$$

This gives us a confidence interval for mu of **[6.0470, 6.0490]**

1.3 Part c

The problem states that for every 0.01 pounds less the company would save $14,000 per year. the original sample mean was 6.09 and the new one is around 6.048. This would be a savings of roughly 4.2 * 14,000 = $58,800.

1.4 Part d

To estimate the proportion of boxes that will weight less than 6 pounds, we want to know P(X < 6). We can use our estimated normal distribution from CLT to come up with a z score of $\frac{6-6.048}{0.022} \approx -2.18$.

$$P(X < 6) = P(Z < -2.18) = 1 - P(Z < 2.18) = 1 - 0.9854 = 0.0146$$

The proportion of boxes measured to be under 6 pounds is now **0.0146**.

2 Problem 7.4-7

2.1 Part a

What we know:

- $\epsilon = 0.03$
- $\alpha = 0.05$

What we don't have is any idea of what the actual proportion or a pilot sample proportion would be. We would have to assume the worst with p = 0.5 to maximize possible variance. Z score for a value of 0.025 (half of alpha) is 1.96. We plug this into our formula:

$$n = \frac{Z_{\alpha/2}^2 * p(1-p)}{\epsilon^2} = \frac{1.96^2 * 0.25}{0.03^2} = 1067.11$$

Rounding up we would require a sample size of at least **1,068**.

2.2 Part b

What we know:

- $\epsilon = 0.02$
- $\alpha = 0.05$

What we don't have is any idea of what the actual proportion or a pilot sample proportion would be. We would have to assume the worst with p = 0.5 to maximize possible variance. Z score for a value of 0.025 (half of alpha) is 1.96. We plug this into our formula:

$$n = \frac{Z_{\alpha/2}^2 * p(1-p)}{\epsilon^2} = \frac{1.96^2 * 0.25}{0.02^2} = 2401$$

We would require a sample size of at least **2,401**.

2.3 Part c

What we know:

- $\epsilon = 0.03$
- $\alpha = 0.1$

What we don't have is any idea of what the actual proportion or a pilot sample proportion would be. We would have to assume the worst with p = 0.5 to maximize possible variance. Z score for a value of 0.5 (half of alpha) is 1.645. We plug this into our formula:

$$n = \frac{Z^2_{\alpha/2} * p(1-p)}{\epsilon^2} = \frac{1.645^2 * 0.25}{0.03^2} = 751.674$$

Rounding up we would require a sample size of at least **752**.

3 Problem 7.4-8

What we know:

- $n = 137$
- $y = 54$
- $p^* = 0.3942$
- $\epsilon = 0.04$
- $\alpha = 0.1$

Since we have a study already we can figure out how many samples we need using the variance based off the point estime from the first study. With a Z score value of 1.645 for alpha /2 of 0.5 we can plug into our formula:

$$n = \frac{Z^2_{\alpha/2} * p(1-p)}{\epsilon^2} = \frac{1.645^2 * 0.3942 * 0.6058}{0.04^2} = 403.885$$

Rounding up we would require a sample size of at least **404**.

4 Problem 6.5-3

5 Problem 6.5-5