

ECON 444 Problem Set 2

Joseph Sepich Feb 14

1 Problem 1

Problem Constraints

- Market Demand: $Q(p) = 1000 * p^x$
- Marginal Cost (Supply): $p = 2$
- $x = -3$

What price would a monopolist choose?

In order to maximize profits a monopolist would set marginal revenue equal to marginal cost:

$$MR = MC$$

We already have that marginal cost is a constant, but what is marginal revenue? We can determine this value by defining total revenue:

$$TR = p * Q$$

We know that marginal revenue is the first derivative of total revenue, since it is the additional revenue for each additional unit.

$$MR = \frac{dTR}{dQ}$$
$$\frac{dTR}{dQ} = \frac{d(p * Q)}{dQ}$$

Here we must use our market demand equation and we can put p in terms of Q :

$$Q = 1000 * p^x$$
$$\frac{Q}{1000} = p^x$$
$$p = \left(\frac{Q}{1000}\right)^{1/x}$$

Plugging this back in we can continue.

$$\frac{d\left(\left(\frac{Q}{1000}\right)^{1/x} * Q\right)}{dQ}$$

According to our problem constraints $x = 3$.

$$\frac{d((\frac{Q}{1000})^{-1/3} * Q)}{dQ} = \frac{d(\frac{Q^{2/3}}{1000^{-1/3}})}{dQ} = \frac{2}{3}(\frac{Q}{1000})^{-1/3}$$

$$MR = \frac{2}{3}(\frac{Q}{1000})^{-1/3}$$

Now solve for monopolist price:

$$MR = MC$$

$$\frac{2}{3}(\frac{Q}{1000})^{-1/3} = 2$$

$$\frac{Q}{1000} = 3^{-3}$$

$$Q = 1000(3)^{-3} \approx 37.04$$

To get the price of sale, plug this quantity into demand (translated into terms of price above):

$$p = (\frac{Q}{1000})^{-1/3} = (\frac{37.04}{1000})^{-1/3} = 3$$

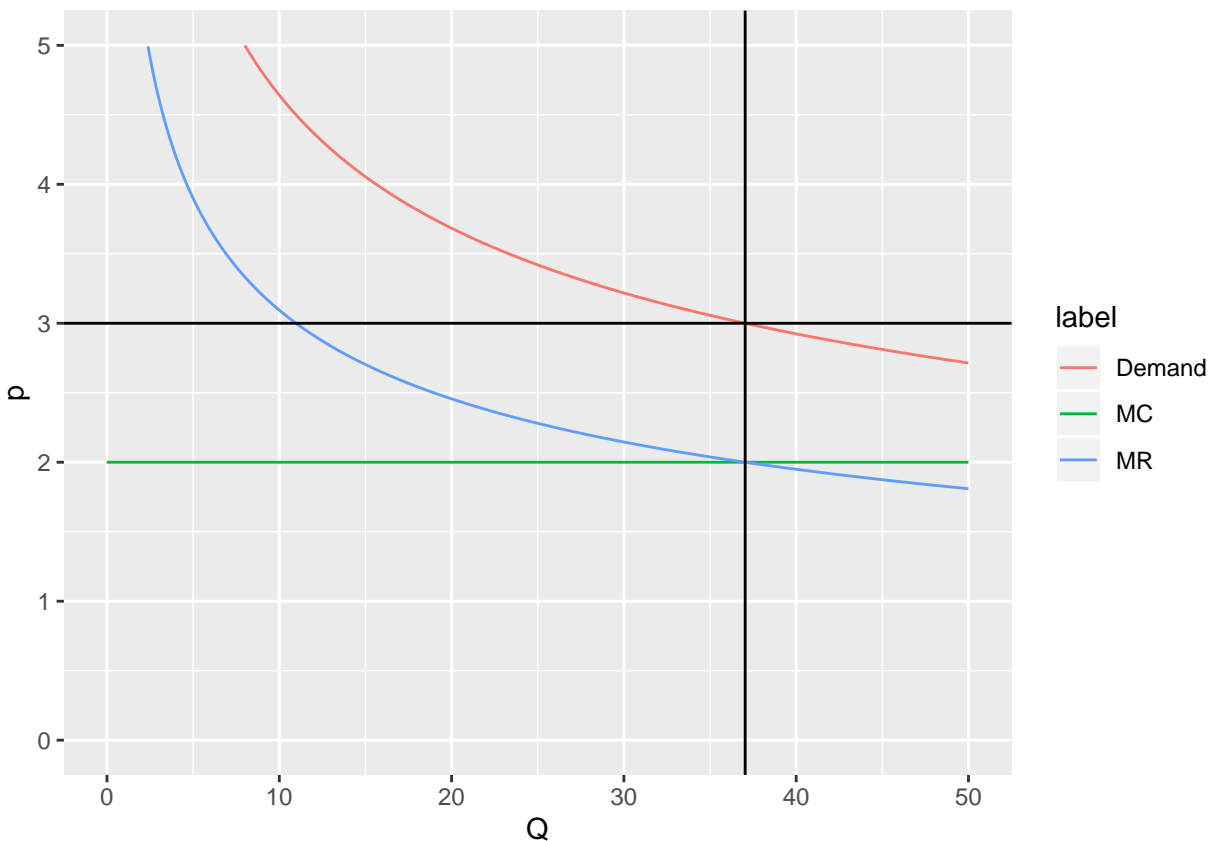
The monopolist would choose a **price of 3**. Now let's calculate profit if $FC = 25$.

$$\pi = TR - TC = p * Q - (FC + Q * MC)$$

$$\pi = 3 * 37.04 - (25 + 37.04 * 2) = 12.03704$$

The monopoly's profit level is **\$12.04**.

We can reinforce this with the following plot:



2 Problem 2

Problem Constraints (T-Shirts)

- Market Demand $Q(p) = 1000 - 50p$
- Marginal Cost (Supply and AC) $p = 10$

2.1 Part a

Calculate the market output and price under perfect competition and under monopoly.

2.1.1 Perfect Competition

In perfect competition we know that price is marginal revenue. Let's calculate marginal revenue from market demand. We will use the same method as in problem 1.

$$\begin{aligned} \text{MR} &= \frac{d(p * Q)}{dQ} \\ \text{MR} &= \frac{d\left(\left(\frac{Q-1000}{-50}\right) * Q\right)}{dQ} = \frac{d\left(\frac{Q^2-1000Q}{-50}\right)}{dQ} = 50(1000 - 2Q) \end{aligned}$$

Since firms will be at profit max, $\text{MR} = \text{MC} = p$.

$$10 = 50(1000 - 2Q)$$

$$\frac{1}{5} = 1000 - 2Q$$

$$10Q = 5000$$

$$Q = 500$$

In perfect competition **Q will be 500** and **price will be 10**.

2.1.2 Monopoly

2.2 Part b

2.3 Part c

3 Problem 3

3.1 Part a

3.2 Part b

3.3 Part c

4 Problem 4

4.1 Part a

4.2 Part b