ECON 444 Problem Set 5

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1 Problem 1

Problem Constrains:

- Cournot Competition (quantity static)
- Market Demand: $P = 10 - 2Q$
- $MC_1 = 2$
- $MC_2 = 4$
- $Q = q_1 + q_2$

1.1 Part a

To find best response behavior of each firm, we acknowledge that the firms wish to set MR equal to MC to optimize profits.

$$MR = p * q$$

$$MR_1 = \frac{d((10 - 2(q_1 + q_2)) * q_1)}{dq_1} = \frac{d(10q_1 - 2q_1^2 - 2q_1q_2)}{dq_1} = 10 - 4q_1 - 2q_2$$

$$MR_2 = \frac{d((10 - 2(q_1 + q_2)) * q_2)}{dq_2} = \frac{d(10q_2 - 2q_2^2 - 2q_2q_1)}{dq_2} = 10 - 4q_2 - 2q_1$$

Best Response for firm 1:

$$MR_1 = MC_1$$

$$10 - 4q_1 - 2q_2 = 2$$

$$4q_1 = 10 - 2q_2 - 2$$

$$q_1^* = \frac{1}{4}(8 - 2q_2)$$

Best Response for firm 2:

$$MR_2 = MC_2$$

$$10 - 4q_2 - 2q_1 = 4$$

$$4q_2 = 10 - 2q_1 - 4$$

$$q_2^* = \frac{1}{4}(6 - 2q_1)$$

1.2   Part b

To find equilibrium we must solve our system of best response equations.

$$4q_1 = 8 - 2(\frac{1}{4}(6 - 2q_1))$$
$$16q_1 = 32 - 2(6 - q_1)$$
$$16q_1 = 32 - 12 + 2q_1$$
$$14q_1 = 20$$
$$q_1^* = \frac{20}{14} = \frac{10}{7}$$

We can plug this back in to solve for firm 2.

$$q_2 = \frac{1}{4}(6 - 2\frac{20}{14})$$
$$56q_2 = 84 - 40$$
$$56q_2 = 44$$
$$q_2* = \frac{11}{14}$$

$$Q^* = q_1^* + q_2^* = \frac{20 + 11}{14} = \frac{31}{14}$$
$$P^* = 10 - 2Q^* = 10 - \frac{31}{7} = \frac{39}{7} \approx 5.57$$

1.3   Part c

$$\text{Markup:}\frac{p - c}{p}$$

$$m_1 = \frac{p - MC_1}{p} = \frac{5.57 - 2}{5.57} \approx 0.64$$
$$m_1 = \frac{p - MC_2}{p} = \frac{5.57 - 4}{5.57} \approx 0.28$$

The markup in equilibrium for firm 1 is **0.64** and **0.28** for firm 2.

1.4   Part d

$$P = 10 - 2Q = 10 - 2(q_1 + q_2)$$

Firm 1 residual demand according to best response:

$$P = 10 - 2(q_1 + q_2*) = 10 - 2q_1 - 2(\frac{11}{14}) = 10 - \frac{11}{7} - 2q_1$$
$$q_1 = \frac{1}{2}(8.43 - P)$$

Firm 2 residual demand according to best response:

$$P = 10 - 2(q_2 + \frac{20}{14}) = 10 - \frac{20}{7} - 2q_2$$

$$q_2 = \frac{1}{2}(7.14 - P)$$

The slope of residual demand is **-2** for both firm 1 and 2. (Inverse market demand curve)

1.5   Part e

Recall the elasticity of demand.

$$\epsilon = \frac{dQ}{dP}\frac{P}{Q}$$

$$\epsilon = \frac{-P}{2Q} = \frac{-5.57}{2(q)}$$

$$\epsilon_1 = \frac{-5.57}{2(1.43)} = -1.95$$

$$\epsilon_2 = \frac{-5.57}{2(0.786)} = -3.54$$

The residual demand elasticity for firm 1 is **-1.95** and **-3.54** for firm 2. The firm with the smaller market share has larger residual demand elasticity. If the firm supplying less decides to supply more, then there will be a greater effect on decrease in prices.

1.6   Part f

The markup does not equal the inverse of residual demand elasticity for either firm.

2 Problem 2

3 Problem 3

3.1 Part a

3.2 Part b

4 Problem 4

4.1 Part a

4.2 Part b

4.3 Part c

5 Problem 5

5.1 Part a

5.2 Part b