ECON 444 Problem Set 5

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1 Problem 1

Problem Constrains:

- Cournot Competition (quantity static)
- Market Demand: $P = 10 - 2Q$
- $\text{MC}_1 = 2$
- $\text{MC}_2 = 4$
- $Q = q_1 + q_2$

1.1 Part a

To find best response behavior of each firm, we acknowledge that the firms wish to set MR equal to MC to optimize profits.

$$MR = p * q$$

$$MR_1 = \frac{d((10 - 2(q_1 + q_2)) * q_1)}{dq_1} = \frac{d(10q_1 - 2q_1^2 - 2q_1q_2)}{dq_1} = 10 - 4q_1 - 2q_2$$

$$MR_2 = \frac{d((10 - 2(q_1 + q_2)) * q_2)}{dq_2} = \frac{d(10q_2 - 2q_2^2 - 2q_2q_1)}{dq_2} = 10 - 4q_2 - 2q_1$$

Best Response for firm 1:

$$MR_1 = MC_1$$
$$10 - 4q_1 - 2q_2 = 2$$
$$4q_1 = 10 - 2q_2 - 2$$
$$q_1^* = \frac{1}{4}(8 - 2q_2)$$

Best Response for firm 2:

$$MR_2 = MC_2$$
$$10 - 4q_2 - 2q_1 = 4$$
$$4q_2 = 10 - 2q_1 - 4$$
$$q_2^* = \frac{1}{4}(6 - 2q_1)$$

1.2 Part b

To find equilibrium we must solve our system of best response equations.

$$4q_1 = 8 - 2(\frac{1}{4}(6 - 2q_1))$$
$$16q_1 = 32 - 2(6 - q_1)$$
$$16q_1 = 32 - 12 + 2q_1$$
$$14q_1 = 20$$
$$q_1^* = \frac{20}{14} = \frac{10}{7}$$

We can plug this back in to solve for firm 2.

$$q_2 = \frac{1}{4}(6 - 2\frac{20}{14})$$
$$56q_2 = 84 - 40$$
$$56q_2 = 44$$
$$q_2* = \frac{11}{14}$$

$$Q^* = q_1^* + q_2^* = \frac{20 + 11}{14} = \frac{31}{14}$$
$$P^* = 10 - 2Q^* = 10 - \frac{31}{7} = \frac{39}{7} \approx 5.57$$