CMPEN 454 Project 1

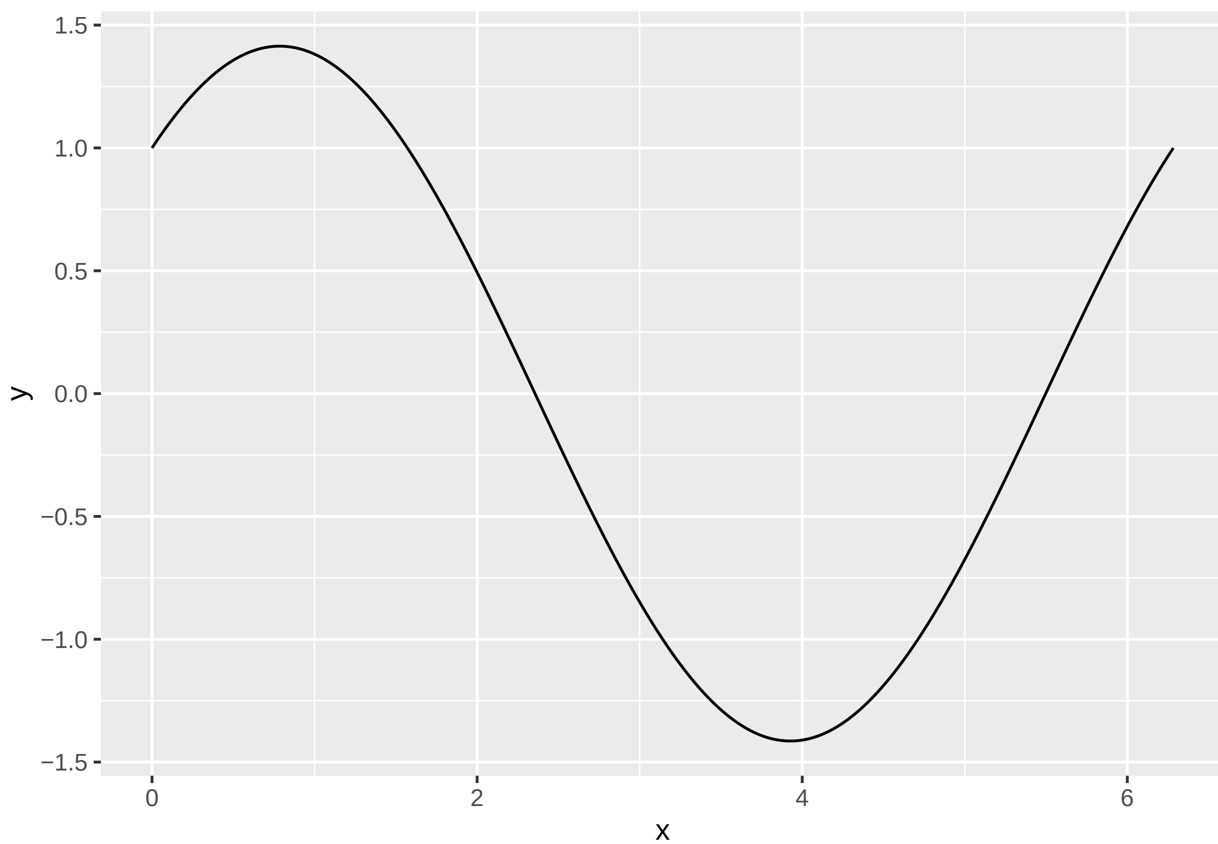
Joseph Sepich Feb 28

1 Theory Questions

1.1 Problem 1

Show that if you use the line equations $\rho = xcos\theta + ysin\theta$, each image point (x,y) results in a sinusoid in (ρ, θ) Hough space. Relate the amplitude and phase of the sinusoid to the point(x,y).

In Hough space the coordinates are ρ and θ. Say we have a simple sin formula in x,y space. This is represented as $y = sin(x)$. Similarly $y = cos(x) + sin(x)$ is also a sinusoidal equation, which can be seen below.



Since in hough space the parameters are instead ρ and θ, this means the same function in Hough space would be written as $\rho = cos(\theta) + sin(\theta)$. This makes the form of a line equation in (x,y) space $\rho = xcos\theta + ysin\theta$ show as a sinusoid in Hough space.

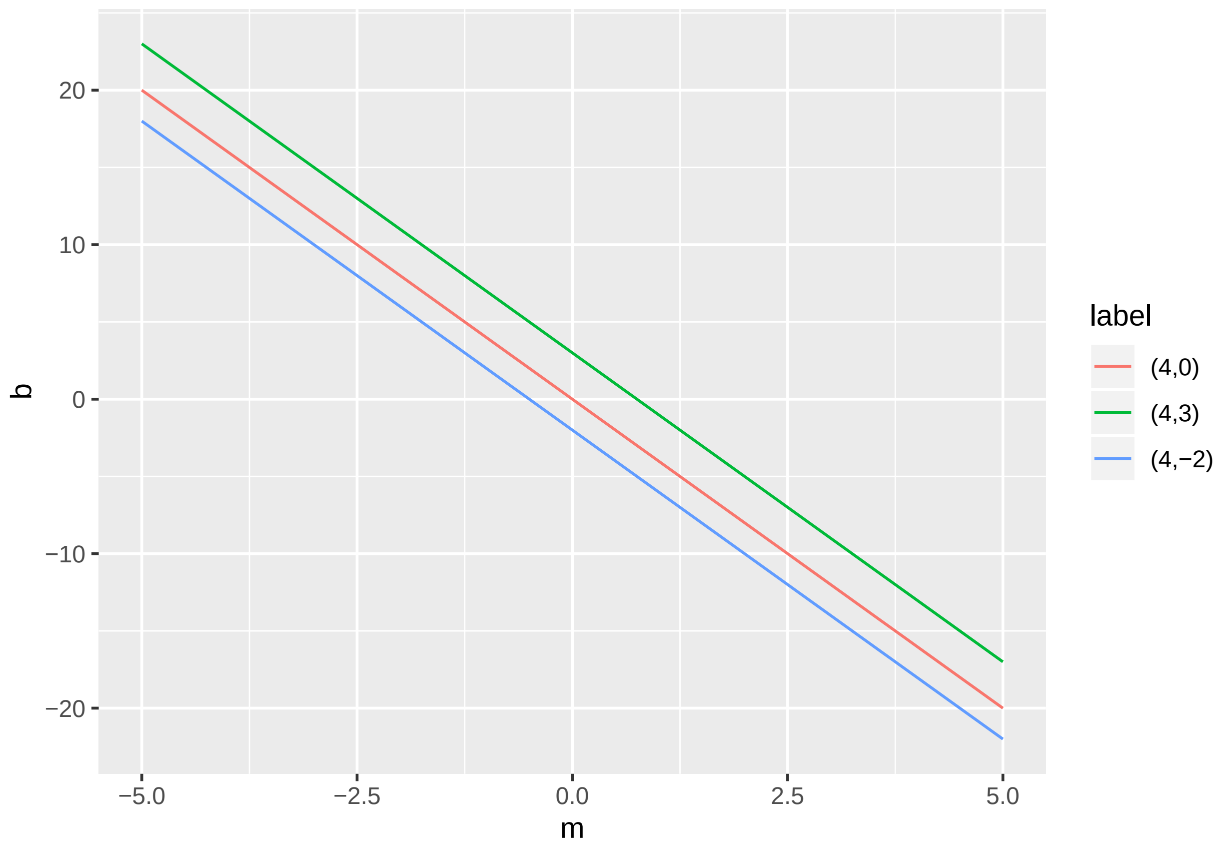
We know that in $x\cos\theta + y\sin\theta$ we can write $x = R\cos\alpha, y = R\sin\alpha$. Furthermore $R\cos\alpha\cos\theta + R\sin\alpha\sin\theta = R\cos(\alpha - \theta)$. From here we can determine that R is the amplitude of the function and α is the phase. We know that $x^2 + y^2 = R^2\cos^2\alpha + R^2\sin^2\alpha = R^2(1)$, so $R = \sqrt{x^2 + y^2}$. This would be the sinusoid **amplitude** in terms of (x,y). We also know that $\frac{R \sin \alpha}{R \cos \alpha} = \frac{y}{x} = \tan \alpha$, so the **phase** is $\alpha = \arctan \frac{y}{x}$ in terms of (x,y).

1.2 Problem 2

Why do we parameterize the line in terms of (ρ, θ) instead of the slope and intercept (m,c)? Express the slope and intercept in terms of (ρ, θ).

One of the reasons we parameterize the line in terms of (ρ, θ) instead of the slope and intercept is to be able to identify vertical lines. They appear as parallel when casting to parameter space: $b = -mx + y$, since x is constant.

Say we have the lines $x = 4$, and $y = 5$. On the first line we could have the points (4,0), (4, 3), and (4, -2). The second line we could have (0,5), (-3, 5) and (2, 5). Below we can compare the image space points to the lines in Hough Space.



So let's express the slope and intercept as (ρ, θ). This lets use see points as sinusoids instead of lines (so they will not be parallel), and it lets us see lines as unique points. Take our line equation and phrase it as $y = mx + b$.

$$\rho = xcos\theta + ysin\theta$$

$$\rho - xcos\theta = ysin\theta$$

$$y = \frac{\rho - xcos\theta}{sin\theta}$$

$$y = \frac{-xcos\theta}{sin\theta} + \frac{\rho}{sin\theta}$$

$$y = -\cot\theta * x + \csc\theta * rho$$

We can now see in the slope intercept form that the slope would be $m = -\cot\theta$ and the intercept would be $c = \csc\theta * rho$.

1.3 Problem 3

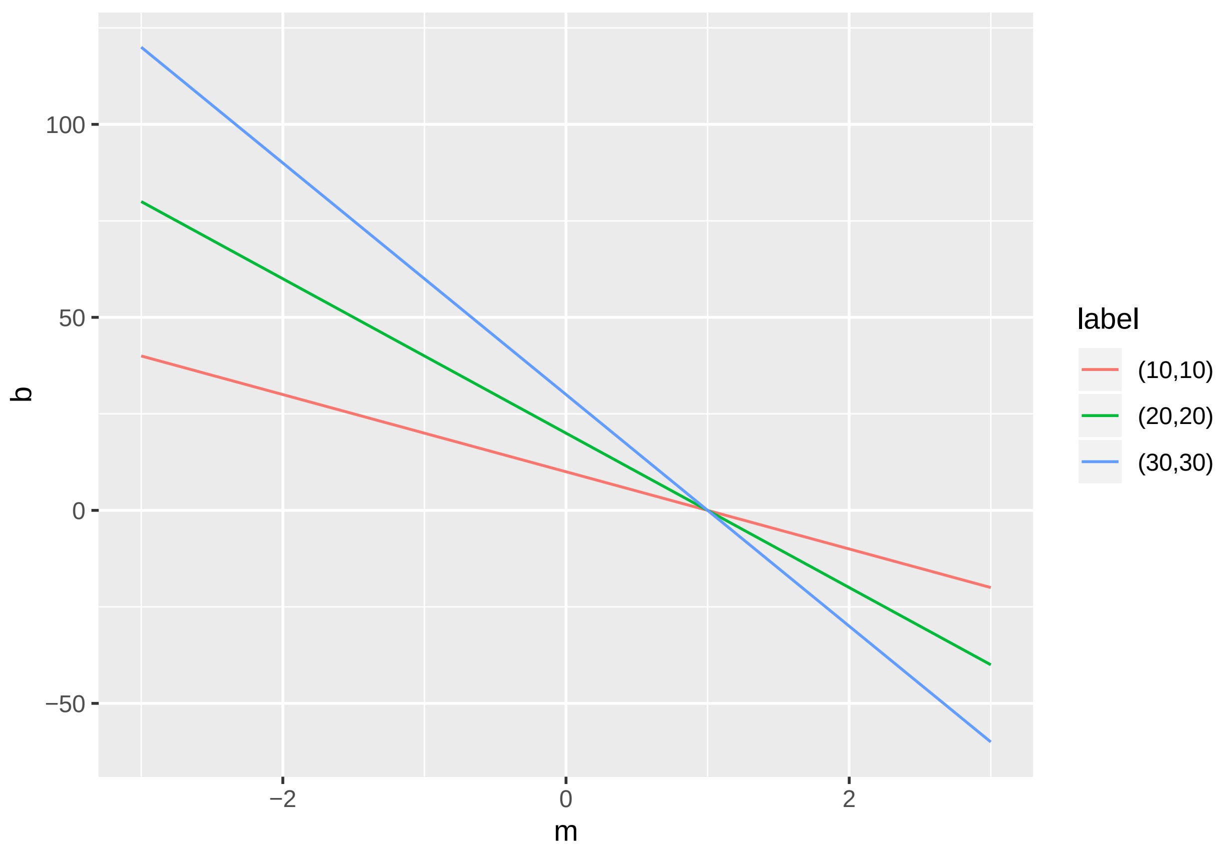
Assuming that the image points (x,y) are in an image of width W and height H, that is $x \in [1, W], y \in [1, H]$, what is the maximum absolute value of ρ, and what is the range for θ?

The maximum absolute value of $\rho = \sqrt{W^2 + H^2}$, since this is the value of the amplitude of the line function found in problem 1.

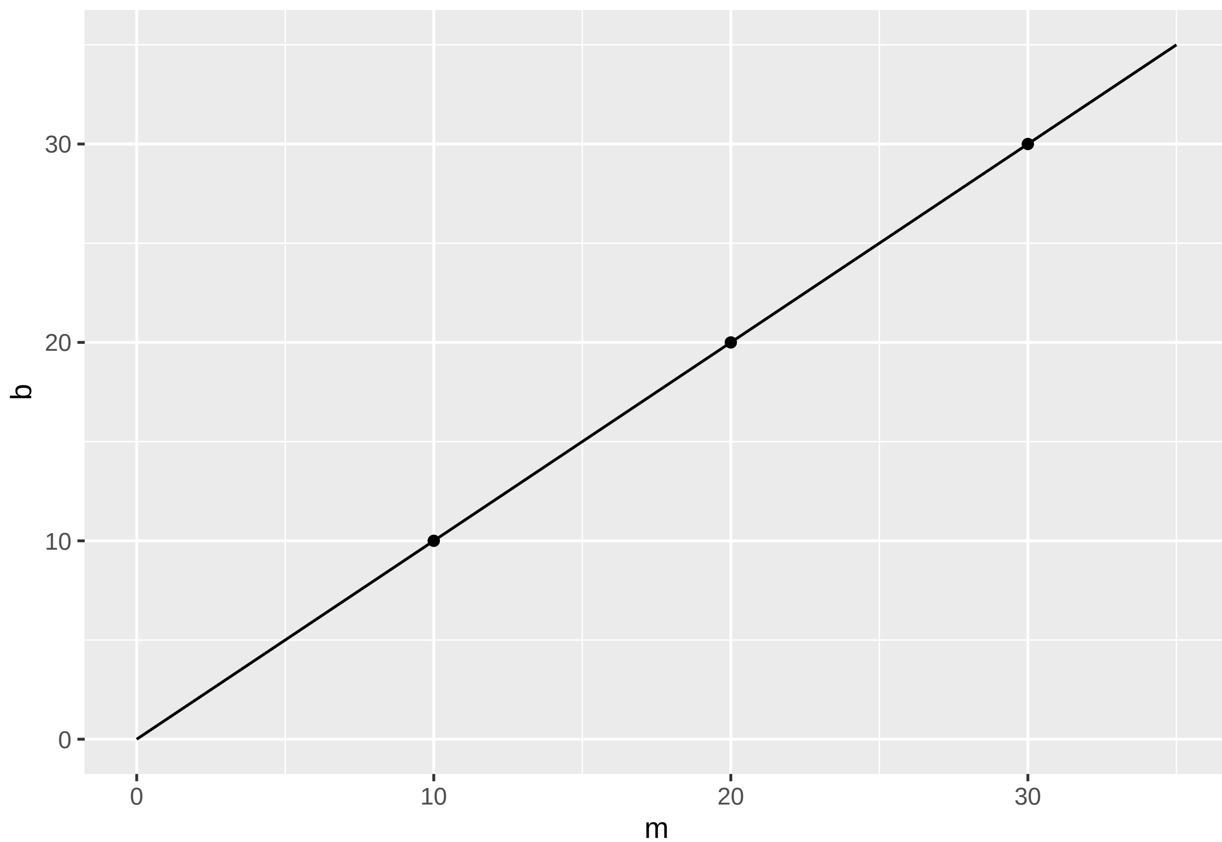
For θ we would only have to look between 0 and 2π to see all values of ρ.

1.4 Problem 4

For point (10, 10) and points (20, 20) and (30, 30) in the image, plot the corresponding sinusoid waves in Hough space, and visualize how their intersection point defines the line. What is (m, c) for this line?



It is clear that in Hough space, these three image points intersect at (m,c) = (1,0). This corresponds to the line $y = mx + b = x$. This can be seen below.


30
20
10
0
b
0
10
20
30
m