

ECON 444 Problem Set 5

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1 Problem 1

Problem Constrains:

- Cournot Competition (quantity static)
- Market Demand: $P = 10 - 2Q$
- $MC_1 = 2$
- $MC_2 = 4$
- $Q = q_1 + q_2$

1.1 Part a

To find best response behavior of each firm, we acknowledge that the firms wish to set MR equal to MC to optimize profits.

$$\begin{aligned}MR &= p * q \\MR_1 &= \frac{d((10 - 2(q_1 + q_2)) * q_1)}{dq_1} = \frac{d(10q_1 - 2q_1^2 - 2q_1q_2)}{dq_1} = 10 - 4q_1 - 2q_2 \\MR_2 &= \frac{d((10 - 2(q_1 + q_2)) * q_2)}{dq_2} = \frac{d(10q_2 - 2q_2^2 - 2q_2q_1)}{dq_2} = 10 - 4q_2 - 2q_1\end{aligned}$$

Best Response for firm 1:

$$\begin{aligned}MR_1 &= MC_1 \\10 - 4q_1 - 2q_2 &= 2 \\4q_1 &= 10 - 2q_2 - 2 \\q_1^* &= \frac{1}{4}(8 - 2q_2)\end{aligned}$$

Best Response for firm 2:

$$\begin{aligned}MR_2 &= MC_2 \\10 - 4q_2 - 2q_1 &= 4 \\4q_2 &= 10 - 2q_1 - 4 \\q_2^* &= \frac{1}{4}(6 - 2q_1)\end{aligned}$$

1.2 Part b

To find equilibrium we must solve our system of best response equations.

$$4q_1 = 8 - 2\left(\frac{1}{4}(6 - 2q_1)\right)$$

$$16q_1 = 32 - 2(6 - q_1)$$

$$16q_1 = 32 - 12 + 2q_1$$

$$14q_1 = 20$$

$$q_1^* = \frac{20}{14} = \frac{10}{7}$$

We can plug this back in to solve for firm 2.

$$q_2 = \frac{1}{4}\left(6 - 2\frac{10}{7}\right)$$

$$56q_2 = 84 - 40$$

$$56q_2 = 44$$

$$q_2^* = \frac{11}{14}$$

$$Q^* = q_1^* + q_2^* = \frac{20 + 11}{14} = \frac{31}{14}$$

$$P^* = 10 - 2Q^* = 10 - \frac{31}{7} = \frac{39}{7} \approx 5.57$$

1.3 Part c

$$\text{Markup: } \frac{p - c}{p}$$

$$m_1 = \frac{p - MC_1}{p} = \frac{5.57 - 2}{5.57} \approx 0.64$$

$$m_2 = \frac{p - MC_2}{p} = \frac{5.57 - 4}{5.57} \approx 0.28$$

The markup in equilibrium for firm 1 is **0.64** and **0.28** for firm 2.

1.4 Part d

$$P = 10 - 2Q = 10 - 2(q_1 + q_2)$$

Firm 1 residual demand according to best response:

$$P = 10 - 2(q_1 + q_2^*) = 10 - 2q_1 - 2\left(\frac{11}{14}\right) = 10 - \frac{11}{7} - 2q_1$$

$$q_1 = \frac{1}{2}(8.43 - P)$$

Firm 2 residual demand according to best response:

$$P = 10 - 2(q_2 + \frac{20}{14}) = 10 - \frac{20}{7} - 2q_2$$

$$q_2 = \frac{1}{2}(7.14 - P)$$

The slope of residual demand is **-2** for both firm 1 and 2. (Inverse market demand curve)

1.5 Part e

Recall the elasticity of demand.

$$\epsilon = \frac{dQ}{dP} \frac{P}{Q}$$

$$\epsilon = \frac{-P}{2Q} = \frac{-5.57}{2(q)}$$

$$\epsilon_1 = \frac{-5.57}{2(1.43)} = -1.95$$

$$\epsilon_2 = \frac{-5.57}{2(0.786)} = -3.54$$

The residual demand elasticity for firm 1 is **-1.95** and **-3.54** for firm 2. The firm with the smaller market share has larger residual demand elasticity. If the firm supplying less decides to supply more, then there will be a greater effect on decrease in prices.

1.6 Part f

The markup does not equal the inverse of residual demand elasticity for either firm.

2 Problem 2

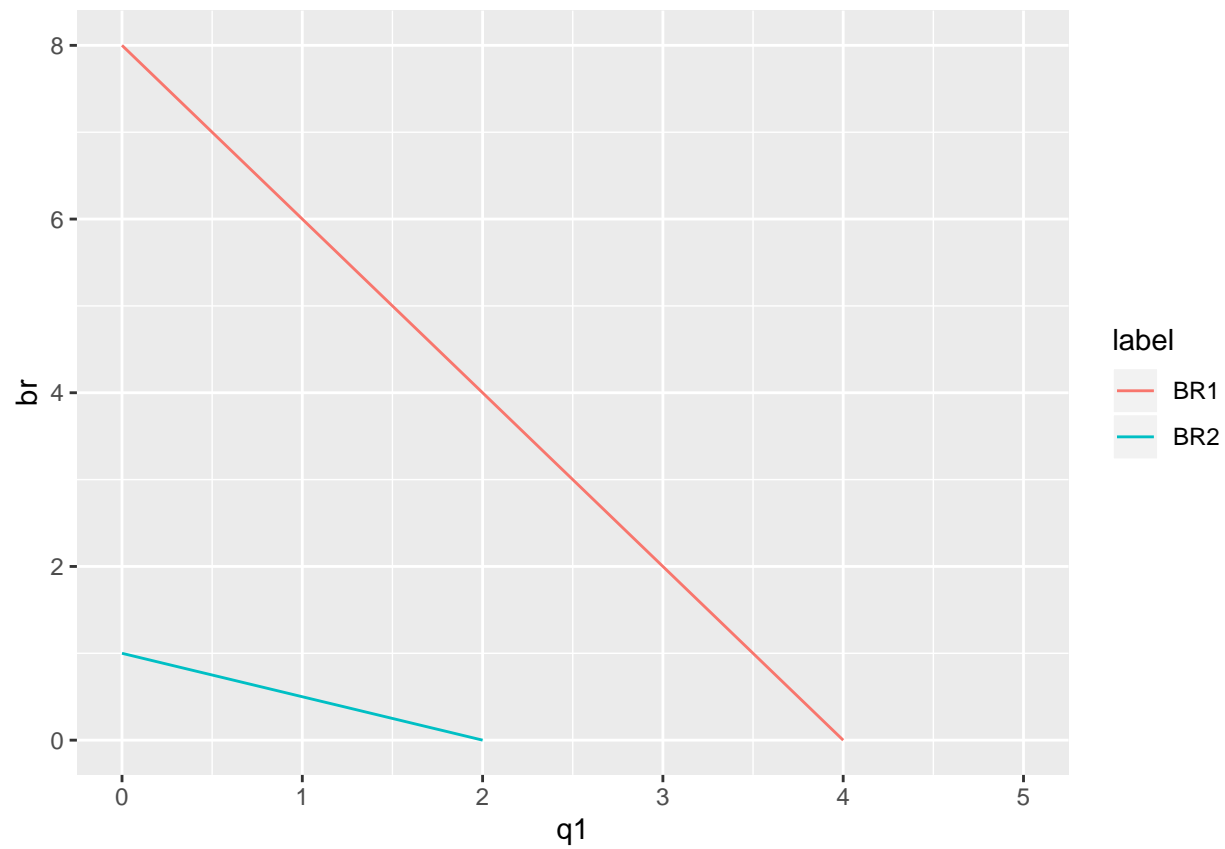
Using the equations from derivation we can create the sample demand curves:

$$BR_1(q_2) = \frac{a - c_1 - bq_2}{2b} = q_1^m - \frac{1}{2}q_2$$

$$BR_2(q_1) = \frac{a - c_2 - bq_1}{2b} = q_2^m - \frac{1}{2}q_1$$

The q_1 intercept for firm 1's BR curve would be q_1^m and for firm two would be $2q_2^m$. The q_2 intercept for each curve would be $2q_1^m$ and q_2^m respectively. Firm 2 will never produce if their best response curve lies under the best response curve for firm 1. This means that the monopoly quantity amount of firm 1 must be more than double the equilibrium quantity of firm 2. An example can be seen below with $a = 10$, $b = 1$, $c_1 = 2$, $c_2 = 8$.

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3 Problem 3

3.1 Part a

3.2 Part b

4 Problem 4

4.1 Part a

4.2 Part b

4.3 Part c

5 Problem 5

5.1 Part a

5.2 Part b