ECON 444 Problem Set 2

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1 Problem 1

Problem Constraints

- Market Demand: $Q(p) = 1000 * p^x$
- Marginal Cost (Supply): $p = 2$
- $x = -3$

To find the elasticity of demand we can first state the elasticity defition:

$$\epsilon = \frac{dQ}{dP}\frac{P}{Q}$$

Then we can plug in using the market demand function:

$$\epsilon = \frac{d(1000 * P^x)}{dP}\frac{P}{1000 * P^x} = \frac{ln(P)1000P^{x+1}}{1000P^x} = Pln(P)$$

The elasticity of demand for coffee is $\epsilon = P * ln(P)$.

What price would a monopolist choose?

In order to maximize profits a monopolist would set marginal revenue equal to marginal cost:

$$\text{MR} = \text{MC}$$

We already have that marginal cost is a constant, but what is marginal revenue? We can determine this value by defining total revenue:

$$\text{TR} = p * Q$$

We know that marginal revenue is the first derivative of total revenue, since it is the additional revenue for each additional unit.

$$\text{MR} = \frac{d\text{TR}}{dQ}$$
$$\frac{d\text{TR}}{dQ} = \frac{d(p * Q)}{dQ}$$

Here we must use our market demand equation and we can put p in terms of Q:

$$Q = 1000 * p^x$$
$$\frac{Q}{1000} = p^x$$

$$p = (\frac{Q}{1000})^{1/x}$$

Plugging this back in we can continue.

$$\frac{d((\frac{Q}{1000})^{1/x} * Q)}{dQ}$$

According to our problem constraints $x = -3$.

$$\frac{d((\frac{Q}{1000})^{-1/3} * Q)}{dQ} = \frac{d(\frac{Q^{2/3}}{1000^{-1/3}})}{dQ} = \frac{2}{3}(\frac{Q}{1000})^{-1/3}$$

$$\text{MR} = \frac{2}{3}(\frac{Q}{1000})^{-1/3}$$

Now solve for monopolist price:

$$\text{MR} = \text{MC}$$

$$\frac{2}{3}(\frac{Q}{1000})^{-1/3} = 2$$

$$\frac{Q}{1000} = 3^{-3}$$

$$Q = 1000(3)^{-3} \approx 37.04$$

To get the price of sale, plug this quantity into demand (translated into terms of price above):

$$p = (\frac{Q}{1000})^{-1/3} = (\frac{37.04}{1000})^{-1/3} = 3$$

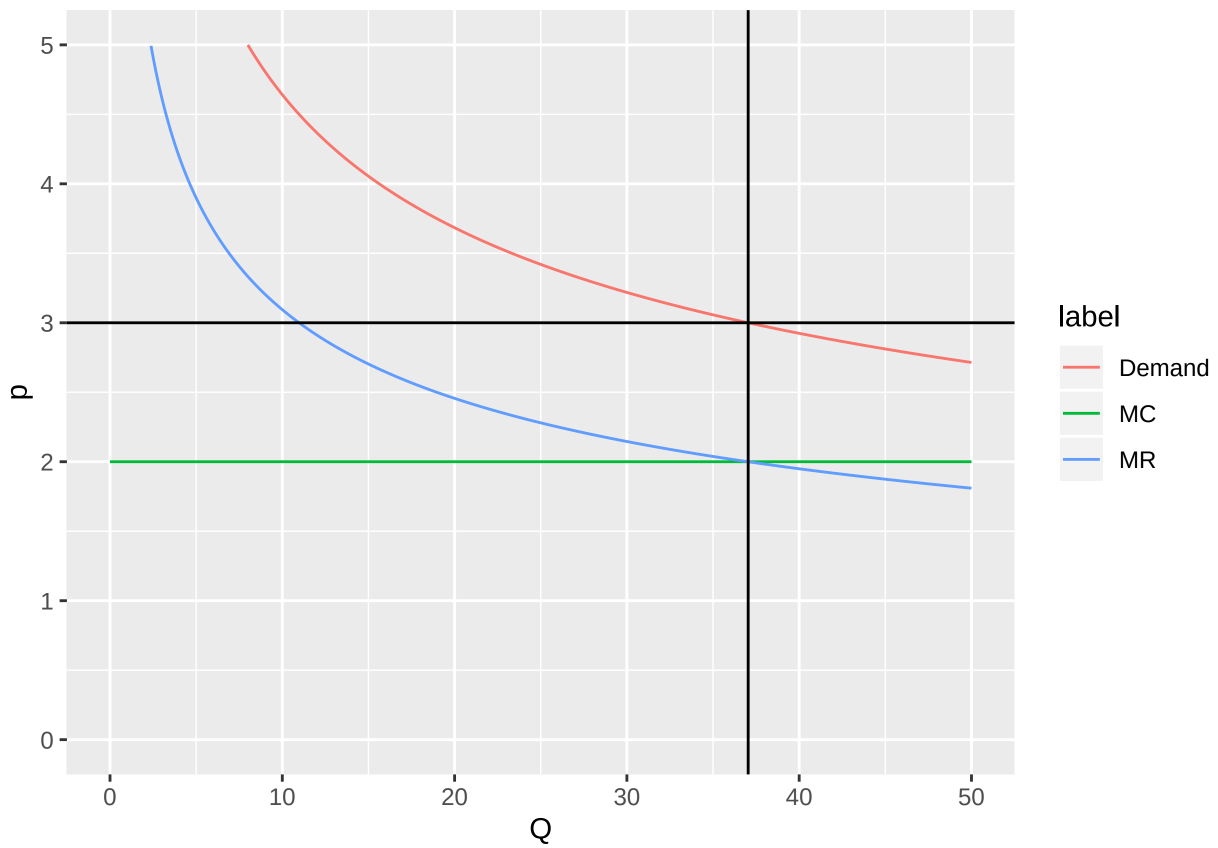
The monopolist would choose a **price of 3**. Now let's calculate profit if FC = 25.

$$\pi = \text{TR} - \text{TC} = p * Q - (FC + Q * \text{MC})$$

$$\pi = 3 * 37.04 - (25 + 37.04 * 2) = 12.03704$$

The monopoly's profit level is **$12.04**.

We can reinforce this with the following plot:



2 Problem 2

Problem Constraints (T-Shirts)

- Market Demand $Q(p) = 1000 - 50p$
- Marginal Cost (Supply and AC) $p = 10$

2.1 Part a

Calculate the market output and price under perfect compeition and under monopoly.

2.1.1 Perfect Competition

In perfect competition the price is dictated by the market. Since price is equal to marginal cost (or else firms will not be in the market), the price in perfect competition is 10. We can plug this into our demand equation and get Q.

$$Q(p) = 1000 - 50p$$
$$Q(10) = 1000 - 50 * 10$$
$$Q(10) = 1000 - 500 = 500$$

Under perfect competition **Q will be 500** and **price will be 10**.

2.1.2 Monopoly

For a monopolist they will choose a price to optimize profit. This is where marginal revenue and marginal cost are equivalent. Let's calculate marginal revenue from market demand. We will use the same method as in problem 1.

$$\text{MR} = \frac{d(p * Q)}{dQ}$$

$$\text{MR} = \frac{d((\frac{Q-1000}{-50}) * Q)}{dQ} = \frac{d(\frac{Q^2-1000Q}{-50})}{dQ} = \frac{(1000-2Q)}{50}$$

Since firms will be at profit max, MR = MC.

$$10 = \frac{(1000-2Q)}{50}$$

$$500 = 1000 - 2Q$$

$$2Q = 500$$

$$Q = 250$$

Let's plug this back into demand.

$$p = \frac{Q-1000}{-50} = \frac{250-1000}{-50} = 15$$

Under a monopoly **Q will be 250** and **price will be 15**.

We can see these two values below:

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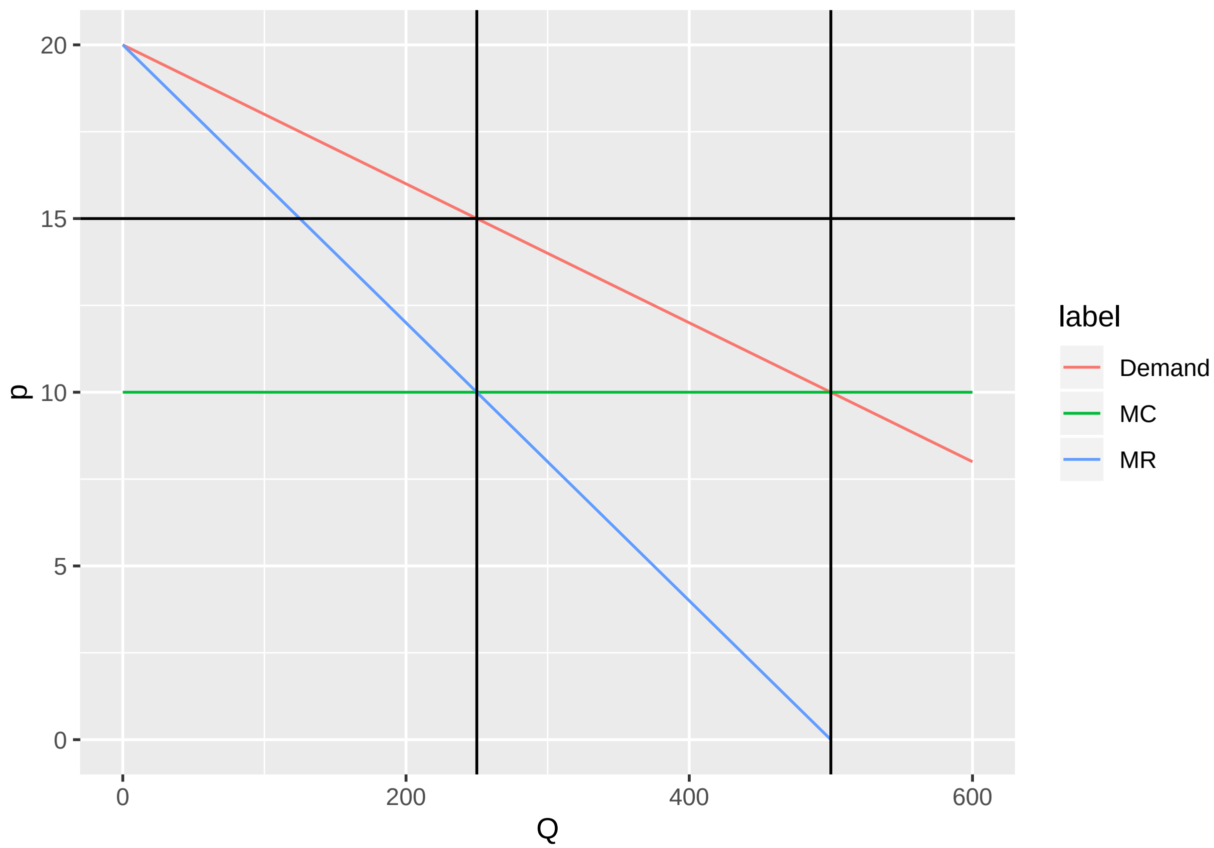

2.2 Part b

What is the elasticity of demand at competitive equilibrium?

Recall elasticity definition

$$\epsilon = \frac{dQ}{dP}\frac{P}{Q}$$

So we find the derivative and multiply by the equilibrium values from part a.

$$\frac{dQ}{dP}\frac{P}{Q} = \frac{d(1000-50p)}{dP}\frac{P}{Q} = \frac{-50*10}{500} = -1$$

The elasticity of demand at competitive equilibrium is **unit elastic**.

What is the elasticity of demand at monopoly equilibrium?

We find the derivative and multiply by the equilibrium values from part a.

$$\frac{dQ}{dP}\frac{P}{Q} = \frac{d(1000-50p)}{dP}\frac{P}{Q} = \frac{-50*15}{250} = -3$$

The elasticity of demand at monopoly equilibrium is **-3**.

2.3 Part c

Let's check if the markup rule is satisfied at our monopoly equilibrium. The markup rule is:

$$\frac{p - MC}{p} = \frac{1}{-\epsilon}$$
$$\frac{15 - 10}{15} = \frac{1}{3}$$
$$\frac{1}{3} = \frac{1}{3}$$

The markup rule is satisfied at our monopoly equilibrium.

3 Problem 3

Problem Constraints (Clothes Hangars)

- Inverse Market Demand: $p = 3 - \frac{Q}{16{,}000}$
- Marginal Cost $p = 1$

3.1 Part a

In competition the price is dictated by the intersection of supply and demand. This will occur where margincal costs intersects demand. The price therefore must equal marginal cost, since any firm will produce as long as price is at least one and consumers will consume as much as they can. The price in competition is 1. We can plug this into our demand equation and get Q.

$$1 = 3 - \frac{Q}{16,000}$$
$$16000 = 48000 - Q$$
$$Q = 48000 - 16000 = 32000$$

Under perfect competition **Q will be 32,000** and **price will be 1**.

3.2 Part b

In a monopolized market the price will be dictated by the firm's optimal profit point. This occurs when MR = MC. First we find marginal revenue as in previous problems.

$$\text{MR} = \frac{d(p * Q)}{dQ}$$
$$\text{MR} = \frac{d((3 - \frac{Q}{16,000}) * Q)}{dQ} = \frac{d(3Q - \frac{Q^2}{16,000})}{dQ} = 3 - \frac{Q}{8,000}$$

$$\text{MR} = \text{MC}$$
$$1 = 3 - \frac{Q}{8,000}$$

$$8000 = 24000 - Q$$
$$Q = 24000 - 8000 = 16000$$

We need to get price by plugging our quanity back into market demand.

$$p = 3 - \frac{Q}{16,000} = 3 - \frac{16,000}{16,000} = 3 - 1 = 2$$

Under a monopoly **Q will be 16,000** and **price will be 2**.

We can see these two values below:

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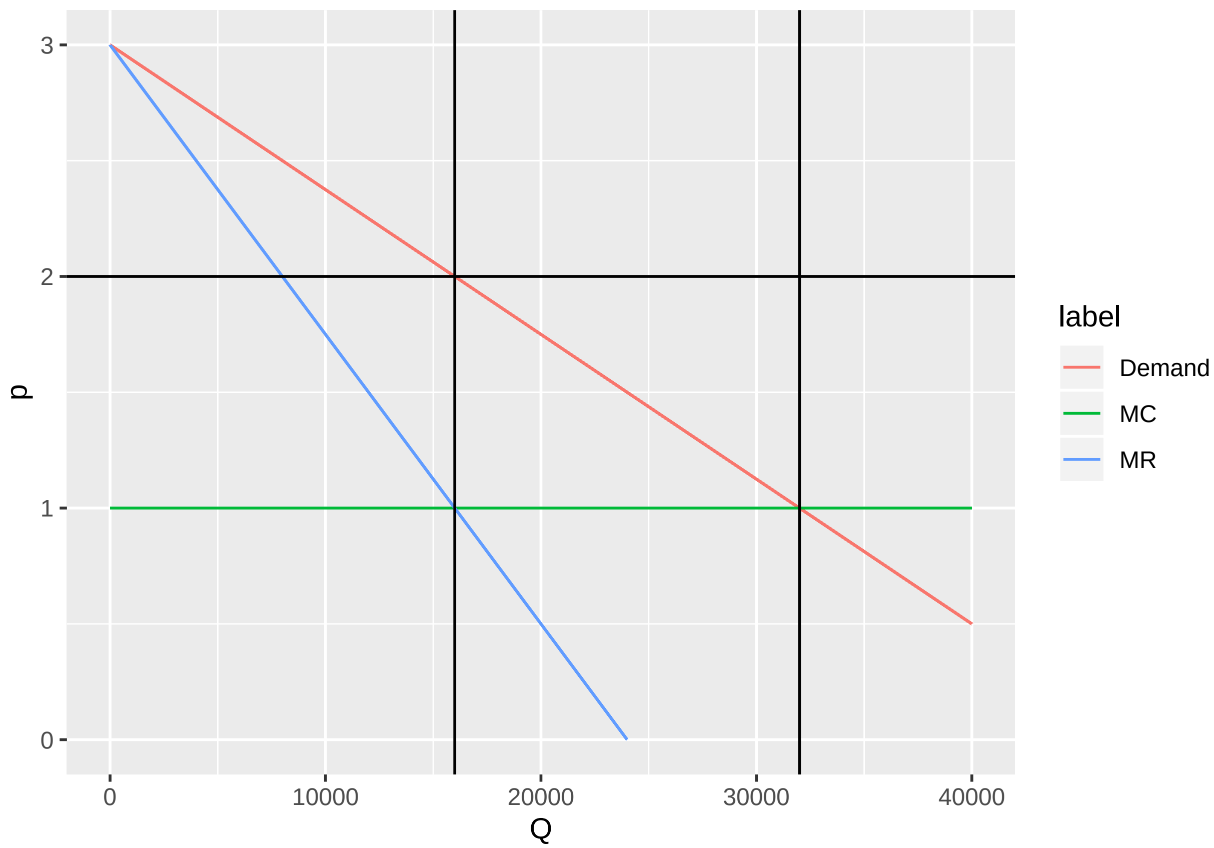

3.3 Part c

What is the deadweight loss of a monopoly in this market?

The deadweight loss in this case is the triangle between the Q a monopoly produces, market demand, and marginal cost. This is defined visually below:

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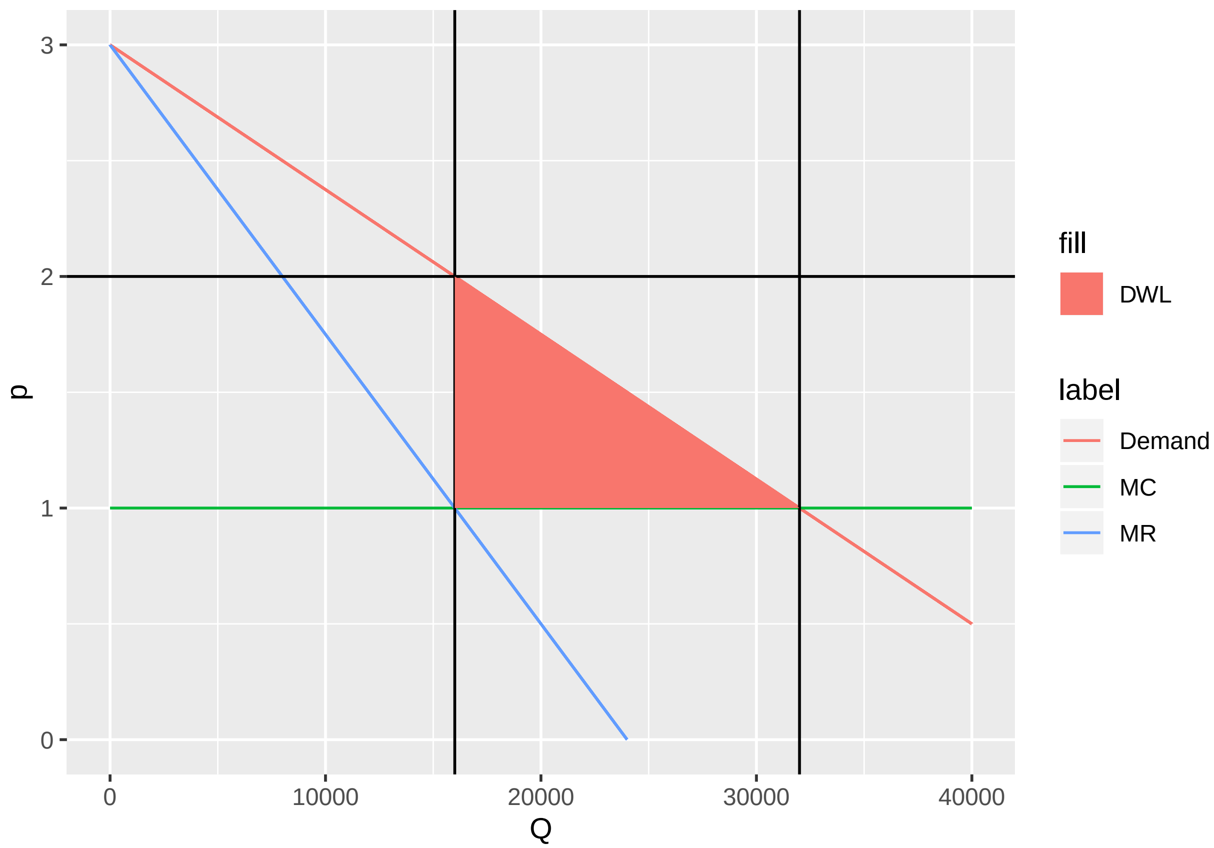

In order to determine the deadweight loss we merely have to calculate the area of the triangle.

$$\text{DWL} = \frac{1}{2}(Q_1 - Q_0)(p_0 - p_1) = \frac{1}{2}(32000 - 16000)(2 - 1) = 16000/2 = 8,000$$

The value of the deadweight loss from a monopoly is **8,000**.

4 Problem 4

Problem Constraints (Pizza production)

- Total Cost $C(q) = 2q + 2q^2$

4.1 Part a

Show that the competitive supply behavior of the typical pizza firm is $q = \frac{p}{4} - \frac{1}{2}$

Any firm looks to optimize their profits. This means that MR = MC. In this example the marginal cost is:

$$MC = \frac{dC(q)}{dq} = \frac{d(2q + 2q^2)}{dq} = 2 + 4q$$

Since we are looking at competitive behavior we assume that MR = MC = p.

$$p = 2 + 4q$$
$$4q = p - 2$$
$$q = \frac{p}{4} - \frac{1}{2}$$

This means the firm will select their supply relative to the given price.

4.2 Part b

We now have that n = 100 firms. We can determine total market supply. Since we have 100 firms with identical behavior in this compeitive market. A single firm's inverse form would be:

$$q = \frac{p}{4} - \frac{1}{2}$$
$$q + \frac{1}{2} = \frac{p}{4}$$
$$4q + 2 = p$$

Now if instead of having a single firm we had 100 firms we would have $q = \frac{Q}{n} = \frac{Q}{100}$:

$$p = q4 + 2 = \frac{Q}{100}4 + 2 = \frac{Q}{25} + 2$$