A Modular Framework for Annual Averaged Power Output Generation of Wind Turbines

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Abstract

Wind energy represents an important future energy source due to rising global interest in renewable energies. For this reason, power output prediction of wind turbines is a prominent task for supporting decisions regarding future sites. The aim of this study is therefore the development of a general framework for annual averaged power output generation of wind turbines. This modular framework relies on general large wind speed data sets, general power curve modeling and general wind speed distributions - examples are Weibull, Kappa or Wakeby distributions. Cubic spline interpolation or logistic power curves and the three aforementioned wind speed distributions are applied to one weather station located at List, Germany in detail. Cubic spline interpolation for power curves and different wind speed distributions are finally adapted to weather stations from California and Germany for annual averaged wind power output predictions. As a main result of the computational study, comparison of semi-empirical power output predictions and estimated power output predictions showed that Kappa and Wakeby distributions are superior to two-parameter Weibull distributions.

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Summarizing, the proposed modular framework proves to be a flexible, unifying and useful tool for future assessment and future comparative studies of different prediction combinations.

Keywords: Big Data Analysis, Energy Analysis, Power Curve Modeling, Power Output Generation, Wind Energy, Wind Speed Probability Distributions

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Nomenclature

$\boldsymbol{\theta}_{\text{Kap}}$ Summarizing vector of all Kappa distribution parameters

$\boldsymbol{\theta}_{\text{Power}}$ Arbitrary power curve parameters

$\boldsymbol{\theta}_{\text{Wak}}$ Summarizing vector of all Wakeby distribution parameters

$\boldsymbol{\theta}_{\text{Wind}}$ Arbitrary wind speed probability distribution parameters

$\boldsymbol{\theta}$ Arbitrary optimization parameters

$\Delta\overline{P_{Hourly,Th.}}$ Absolute error of estimated averaged hourly power output generation value

δf Uncertainty of function $f\left(x_1,\ldots,x_n\right)$

ΔP_{Values} Absolute difference between semi-empirical and estimated annual averaged wind power output generation values

Δv Absolute uncertainty in wind speed measurement

δx_j Uncertainty of variable x_j

γ_{Wak} Second Wakeby scale parameter

$\mathcal{J}$ Optimization cost function

μ_{Wak} Wakeby location parameter

μ_{Kap} Kappa location parameter

$\overline{P_{\text{Ann., Th.}}}$ Estimated averaged annual wind power output generation value

$\overline{p_{\text{Emp.}}(v_i)}$ Arithmetic mean of all empirical wind speed probabilities

$\overline{P_{\text{Hourly, Th.}}}$ Estimated averaged hourly wind power output generation value

$\sigma_{A_{\text{Wei}}}$ Standard deviation of A_{Wei}

$\sigma_{k_{\text{Wei}}}$ Standard deviation of $\sigma_{k_{\text{Wei}}}$

a Cubic curve parameter

A_{Kap} Kappa scale parameter

A_{Wak} Wakeby scale parameter

A_{Wei} Weibull scale parameter

B Logistic curve parameter

b Cubic curve parameter

C Logistic curve parameter

c Cubic curve parameter

D Logistic curve parameter

d Cubic curve parameter

E Logistic curve parameter

F Logistic curve parameter

F_{Kap} Kappa cumulative distribution function

F_{Wak} Wakeby cumulative distribution function

G Logistic curve parameter

H Logistic curve parameter

h_{Kap} Second Kappa shape parameter

h_{Wak} Second Wakeby shape parameter

k_{Kap} Kappa shape parameter

k_{Wak} Wakeby shape parameter

k_{Wei} Weibull shape parameter

$p_{\text{Emp.}}(v_i)$ Empirical wind speed probabilities

p_{Kap} Four-parameter Kappa probability distribution

P_{Power} General power curve function

P_{rated} Rated power output

p_{Wak} Five-parameter Wakeby probability distribution

p_{Wei} Two-parameter Weibull probability distribution

p_{Wind} Arbitrary wind speed probability distribution function

P_j Measured power curve output data

q Arbitrary power curve model

q_{cub} Cubic interpolation function

q_{glog} Generalized logistic function

q_{log} Symmetric logistic function

R^2 Coefficient of determination

$v_{\text{cut-in}}$ Cut-in wind speed

$v_{\text{cut-off}}$ Cut-off wind speed

v_{rated} Rated wind speed

v_j Measured wind speeds

x_j Arbitrary variables

v Wind speed

1. Introduction

1.1. Motivation

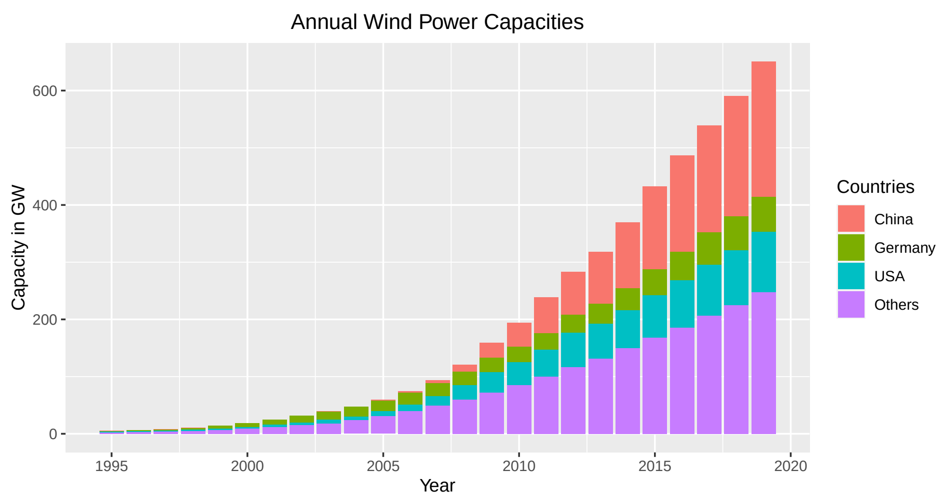

Figure 1: Barplot of the annual development of installed wind energy capacities since 1995. The three leading countries China, Germany and the United States with respect to installed wind energy capacities are distinguished from all other countries. Data are made available by Wind Energy International [5] and Volker Quaschning [6].

Fossil fuels are limited and reduction of carbon dioxide emissions is indispensable [1]. Expansion of renewable and sustainable energy sources can contribute to sustainable development [2]. In addition to that, future electricity must face the task of reducing human carbon and water footprint [3]. One major renewable energy source is wind energy [4]. As depicted in Figure 1, installed wind energy capacities have grown constantly worldwide since 1995.

Analysis of wind speed characteristics is therefore an important task due to the growing interest in renewable and sustainable energy sources. It helps to detect possible locations for future wind turbines. Thus, development of prediction tools for power output generation can be seen as a key instrument to support decision-making processes in this regard. Since modeling approaches are widely ramified, a brief overview of the literature about the two important steps of power curve modeling and wind speed probability distribution modeling is given in the following two subsections.

1.2. Literature Review of Power Curve Modeling

Here, contents of some reviews are reported before a non-exhaustive overview on the vast literature about power curve modeling is presented. Techniques for power curve modeling have largely emerged in recent years. Especially with regard to wind turbines, Carrillo and co-authors summarized parametric techniques like cubic power curves, polynomial power curves and exponential power curves [7]. Cubic power curves can be easily implemented. However, it is well-known that these functions might violate monotonicity restrictions with respect to power curves of wind turbines. Later, a review of Sohoni and co-authors was published that extended the aforementioned work by providing information on developments in non-parametric machine learning techniques [8]. The advantage of these methods is the elimination of a-priori knowledge regarding forms of such power curves. In practice, quality of such power curve highly depend on reliable data and robust optimization techniques [9]. It can be concluded that modeling approaches can be mainly divided into parametric and non-parametric methods.

Parametric modeling has been applied widely in different sciences. One type of curve, that is heavily used in power curve modeling, is the logistic function model. This flexible growth function was introduced by Richards to describe growth of plants [10]. After its invention in 1959, Pyke and Sheridan answered questions from higher education research with its usage [11]. Cramer shared some historical insights into the origins of logistic regression [12]. Some well-received introductions to this topic were written by Peng and co-authors [13] as well as Park [14].

Many research studies about power curve modeling have been published and only a small selection of them can be discussed. In their review, Lydia and co-authors described parametric approaches such as linearized segmented power curves, polynomial power curves, four-parameter logistic power curves or five-parameter logistic power curves and cubic spline interpolation techniques [15]. Logistic power curves and cubic spline interpolation techniques are relatively flexible. Especially, cubic spline interpolation passes all given points exactly by a smooth curve. Taslimi-Renani and co-authors established an enhanced parametric model by modified hyperbolic tangent functions [16]. Pei and co-authors summarized these parametric approaches as well [17]. Mehrjoo and co-authors presented a monotonic spline regression method for power curve modeling [18]. As mentioned in [17] and [9], the quality of power curve models heavily relies on optimization procedures. Cubic spline interpolation techniques are mostly applied in this work in order to compromise between parametric and non-parametric approaches.

As computer power has risen over recent years, different non-parametric machine learning approaches with an emphasis on clustering, data mining algorithms, fuzzy methods and neural networks have gained further attention [15].

Yesilbudak developed an approach based on clustering and filtering [19]. Pei and Li suggested a hybrid machine learning approach [17]. One main advantage of these techniques is no need for a-priori information on possible curve courses. However, training data sets are necessary in many of these algorithms.

1.3. Literature Review of Wind Speed Distribution Modeling and Wind Power Output Prediction

As the second main ingredient, identification of suitable wind speed distribution models is a critical task. Again, it can be differentiated between parametric and non-parametric techniques. In 1977, Hennessey mainly discussed two-parameter Weibull distributions to model wind speed probability distributions [20]. Weibull distributions have become one of most used probability distributions across different sciences [21]. Recently, Jung and Schindler recognized that two-parameter Weibull distribution are mainly applied in wind speed distribution modeling, but four-parameter Kappa probability or five-parameter Wakeby probability distributions are nearly better fits in all cases [22]. These authors also compared sixteen different wind speed probability distributions in a German case study [23].

Despite those findings, two-parameter Weibull distributions have been established as one main wind speed probability distribution since its invention [24]. For example, Carrillo and co-authors used two-parameter Weibull distribution for a study with respect to wind energy in Spain [25]. Ozay and Celiktas examined wind speed characterics of Alaçati region with two-parameter Weibull distributions [26]. Mahmood and co-authors carried out a study in Iraq using two-parameter Weibull distributions for wind speed characteristics [27]. In 2016, Akgül and co-authors presented the modified inverse Weibull distribution [28].

Many studies have been recently appeared with respect to wind speed distributions and power forecasts in Energy Conversion and Management. In 2018, Aries and co-authors assessed different wind speed probability distributions in a case study considering Algeria [29]. Miao and co-authors developed a selection algorithm for wind speed distributions based on score-radar maps [30]. Additionally, Jung and Schindler verified their findings on suitability of four-parameter Kappa distributions and five-parameter Wakeby distributions in their study of the influence of future global climate on changing wind speed distributions [31].

Machine learning techniques have been applied recently on a more regular basis for wind power prediction. Yan and Ouyang presented a hybrid approach based on data-driven error correction [32]. Modelli and co-authors compared five different machine learning algorithms for long-term wind power forecasting [33]. Combining wavelet transforms, echo state networks and ensemble techniques, Wang and co-authors applied this hybrid approach to reduce uncertainties in wind power data [34]. Using a bootstrap aggregating trees machine learning approach, Harrou, Saidi and Sun obtained power curves for wind power prediction [35]. One kernel density estimation method for wind speed forecasting was outlined by Zhang and co-authors [36]. Zhao and co-authors presented a neural network method for short-term wind speed forecast [37]. Another technique, based on multi-learner ensemble and adaptive model selection, was established by Chen and Liu [38]. Lately, Jung and Schindler offered a least-squares boosting approach where four-parameter Kappa probability distributions and five-parameter Wakeby probability distributions were applied [39].

Parametric approaches have the advantage that they are easier to interpret. However, application requires a-priori knowledge and they are not as flexible as non-parametric approaches. In contrast to parametric methods, non-parametric techniques are really flexible, but overfitting as well as hard interpretation of results might be a problematic issue to deal with in applications.

1.4. Contributions

Regarding our literature review, we have the impression that an abstract framework for wind power output generation is needed. As this is our main goal, we mainly apply parametric techniques in our simulation studies due to their ease in interpretation. Conclusively, we want to summarize our main contributions in the following list.

1) We propose an abstract framework for estimation of general annual averaged power output generation of wind turbines in Subsection 2.1. In this approach, we take all possible data like e.g. humidity, wind directions or wind speed from weather stations into account.

2) We reduce the aforementioned scheme and developed general framework for annual averaged power output prediction of wind turbines in Subsection 2.2. It relies on wind speed data sets, arbitrary power curve modeling techniques and arbitrary wind speed distribution choices. This can be regarded as our main contribution.

3) In the subsequence of Section 2, we describe non-exhaustive combinatorial possibilities of power curves and wind speed probability distributions. We restrict ourselves mainly to cubic spline interpolation of power curves because these types of curves naturally pass all given power curve data points exactly. In brief, we present other power curve modeling concepts. Additionally, we present some examples of wind speed probability distributions like two-parameter Weibull, four-parameter Kappa and five-parameter Wakeby distributions.

4) In Subsections 2.7 and 2.8, a general formulation based on arbitrary power curves, arbitrary wind speed probability distributions and finite integrals is developed for calculation of annual averaged power output prediction

of wind turbines. This method involves the gained power curve and wind speed probability function.

5) Some chosen combinations are applied to different wind speed data sets from weather stations located in California (United States) and Germany in Section 3.

This list highlights the main achievements in this article.

1.5. Structure

Our work is organized as follows. After the motivational introduction, reviews of former works and a listing of our contributions in Section 1, Section 2 contains descriptions of our proposed general framework of annual averaged power output generation for wind turbines. For that purpose, wind speed data processing techniques, power curve modeling approaches and wind speed distributions applied in this article are presented. Results of non-exhaustive combinations of these techniques and a discussion of the findings are illustrated in Section 3. Finally, conclusive remarks are given in Section 4.

2. Materials and Methods

This section contains brief descriptions of data and used methods in this article.

2.1. General Framework for Annual Averaged Power Output Generation of Wind Turbines

In Figure 2, an abstract, general work flow for calculation of averaged annual wind power output generation is illustrated. This framework is heavily based on power curve data from manufacturers and available weather station data. The latter might even include measurements such as air density, atmospheric pressure, relative humidity, temperature, wind direction or wind speed. Farkas demonstrated that these factors can influence wind power generation [40]. The main aim of this framework is the determination of averaged annual wind power output generation values based on arbitrary power curve modeling techniques and arbitrary user-chosen wind speed probability distributions.

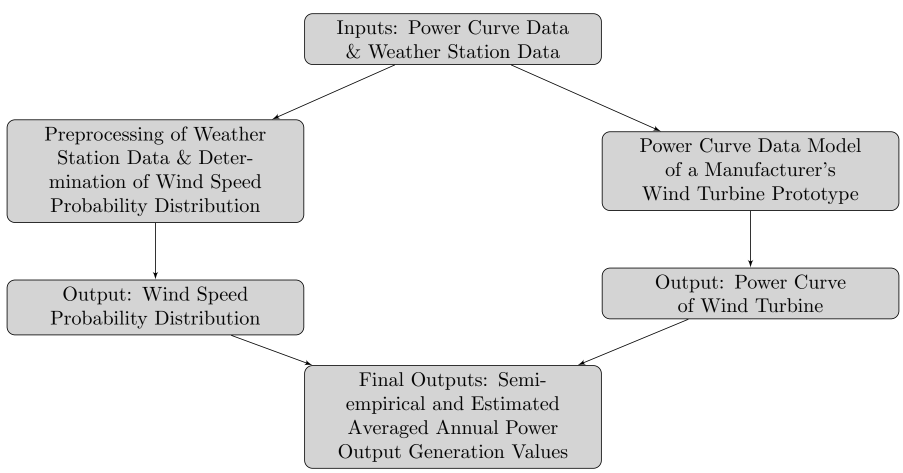

Figure 2: A flowchart of our abstract, general framework for averaged annual wind power output generation values. This method relies on weather station data and power curve data as inputs.

Since this work's practical interest lies in illustration of abstract frameworks, some minor simplifications are considered in the following.

2.2. General Framework Reduced to Wind Speed Data Sets

In contrast to the general framework in Figure 2, factors such as air density, atmospheric pressure, relative humidity, temperature or wind direction are neglected. Commonly, time series of wind speed data are applied in wind power prediction based on given power curve and wind speed data. This framework pursues the same goal as the former one in Figure 2.

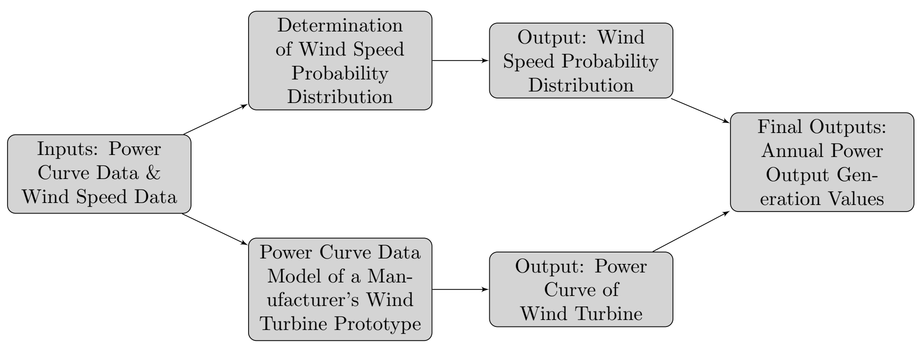

Figure 3: A flowchart of our reduced abstract, general framework for averaged annual wind power output generation values. This method is based on wind speed data and power curve data as inputs.

This reduced general framework is presented in Figure 3. However, this setting still allows arbitrary power curve modeling techniques and arbitrary wind speed probability distributions. Lines of the algorithmic procedure are given in Algorithm 1. Note that, on the one hand, Step 2 and Step 3 can be interchanged, as well as, on the other hand, this can be done with Step 4 and

Step 5.

Algorithm 1: Pseudo-code for reduced, abstract general framework as depicted in Figure 3.

Inputs: Power Curve Data Set & Wind Speed Data Set

Step 1: Preparation of Power Curve Data Set & Wind Speed Data Set

Step 2: Determination of Power Curve

Step 3: Calculation of Wind Speed Probability Distribution

Intermediate Outputs: Power Curve & Wind Speed Probability Distribution

Step 4: Calculation of Semi-empirical Annual Averaged Wind Power Output Generation Value Based on Wind Speed Data Set and Power Curve

Step 5: Calculation of Estimated Annual Averaged Wind Power Output Generation Value Based on Power Curve and Wind Speed Probability Distribution

Final Outputs: Semi-empirical & Estimated Annual Wind Power Output Generation Values

2.3. Power Curve Data

From the wind turbine manufacturer Vestas [41], power curve data of its wind turbine are presented in Table 1. Wind speeds and hourly power outputs are measured in physical units $\frac{\mathrm{m}}{\mathrm{s}}$ and $\frac{\mathrm{kW}}{\mathrm{h}}$ respectively.

Table 1: Hourly Power Output Data For Wind Turbine Vestas V112

Wind Speed	Power Output	Wind Speed	Power Output
0.0	0	13.5	3075
0.5	0	14.0	3075
1.0	0	14.5	3075

1.5	0	15.0	3075
2.0	0	15.5	3075
2.5	0	16.0	3075
3.0	26	16.5	3075
3.5	73	17.0	3075
4.0	133	17.5	3075
4.5	207	18.0	3075
5.0	302	18.5	3075
5.5	416	19.0	3075
6.0	554	19.5	3075
6.5	717	20.0	3075
7.0	907	20.5	3075
7.5	1126	21.0	3075
8.0	1375	21.5	3075
8.5	1652	22.0	3075
9.0	1985	22.5	3075
9.5	2282	23.0	3075
10.0	2585	23.5	3075
10.5	2821	24.0	3075
11.0	2997	24.5	3075
11.5	3050	25.0	3075
12.0	3067	25.5	0
12.5	3074	26.0	0
13.0	3075	26.5	0

2.4. Wind Speed Data Processing

Now, it follows a brief description of the wind speed data from the German Weather Service [42]. Data have the following format.

```
STATIONS_ID;MESS_DATUM;  QN_3;     F;    D;eor
      3032;1950010100;     5;   7.4;-999;eor
      3032;1950010101;     5;   6.6;-999;eor
      3032;1950010102;     5;   7.4;-999;eor
      ...
```

Wind speed data from station number 3032 are chosen and its data are taken from the file PRODUKT_FF_STUNDE_19500101_20181231_03032.TXT of the corresponding ZIP-archive. This weather station is located at List, Germany. Additionally, meta data is loaded from the file with the name METADATEN_GERAETE_WINDGESCHWINDIGKEIT_03032.TXT of the previously mentioned ZIP-archive. It is suggested to save those different data files from the German Weather Service within two different directories - one for hourly acquired data and one for meta data.

The algorithmic approach of Figure 3 heavily relies on the fourth column of the hourly acquired wind speed data.

2.5. Power Curve Modeling

Figure 4 portrays the general graph of a power curve function for wind turbines. Since this function is only non-constant on the interval $v \in [v_{\text{cut-in}}, v_{\text{rated}}]$, a piecewise defined function

$$P_{\text{Power}}(v) = \begin{cases} 0 & , v \in [0, v_{\text{cut-in}}) \\ q(v) & , v \in [v_{\text{cut-in}}, v_{\text{rated}}] \\ P_{\text{rated}} & , v \in (v_{\text{rated}}, v_{\text{cut-off}}) \\ 0 & , v \in [v_{\text{cut-off}}, \infty) \end{cases} \tag{1}$$

for power output functions, as reviewed by Pei & Li [17], generally describes its course. q defines an arbitrary power curve model on $[v_{\text{cut-in}}, v_{\text{rated}}]$ and P_{rated} is the rated power output. $v_{\text{cut-in}}$, v_{rated} and $v_{\text{cut-off}}$ denote the cut-in wind speed, the rated wind speed and the cut-off wind speed respectively.

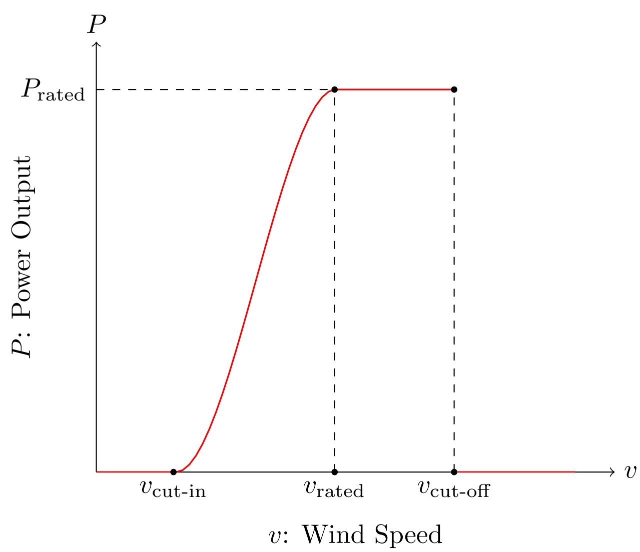

Figure 4: Schematic plot of a general power curve for wind turbines with cut-in wind speed $v_{\text{cut-in}}$, rated wind speed v_{rated}, cut-off wind speed $v_{\text{cut-off}}$ and rated power output P_{rated}.

Due to better interpretability, application of parametric approaches is preferred. Possible curves may be cubic polynomials

$$q_{\text{cub}}\left(v\,|a,b,c,d\right) = a\cdot v^3 + b\cdot v^2 + c\cdot v + d \tag{2}$$

or generalized logistic regression functions

$$q_{\text{glog}}\left(v\,|B,C,D,E,F,G,H\right) = B + \frac{C-B}{\left(D + E\cdot\exp\left(F\cdot(v-G)\right)\right)^{\frac{1}{H}}}, \tag{3}$$

also known as Richards' curves in the literature [10] where parameters a,b,c,d and parameters B,C,D,E,F,G,H are interpolation parameters. As a compromise between parametric and non-parametric techniques, it is also possible to subdivide the interval $[v_{\text{cut-in}}, v_{\text{rated}}]$ into subintervals $[v_k, v_{k+1}]$ for $k \in \{1,\ldots,M-1\}$ if M wind speed points are taken into account for this subdivision. Equation (2) can then be reconsidered on every subinterval with different parameters a,b,c,d on each subinterval. As the power curve data seem to be relatively symmetric, it seems reasonable to restrict to symmetric logistic functions

$$q_{\text{log}}\left(v\,|B,C,D,E,F\right) = \frac{B}{C + D\cdot\exp\left(-E\cdot v + F\right)}. \tag{4}$$

Equations (2), (3) and (4) are particular cases of functions available for piecewise power curve modeling in (1). Optimization methods are based on so-called cost functions $\mathcal{J}$ such as

$$\mathcal{J}\left(\boldsymbol{\theta}\right) = \sum_{j=1}^{M} \left|q\left(v_j\,|\boldsymbol{\theta}\right) - P_j\right|^p \tag{5}$$

for M given power curve data points (v_j, P_j) with $j \in \{1,\ldots,M\}$, $p \geq 1$. Here, the vector $\boldsymbol{\theta}$ summarizes all parameters which need to be determined by an appropriate optimization procedure. The exponent p represents different l^p-

norms where $p = 2$ is the case of least-squares regression and $p = 1$ is robust optimization of sum of absolute differences [43]. Typical choices of optimization methods are regression methods, gradient-based methods or Newton methods [44].

2.6. Wind Speed Distribution Modeling

There are many possible probability distributions that can be applied to wind speed distribution modeling. An exemplary plot of an empirical wind speed distribution is illustrated in Figure 5.

In this work, the most common or more advanced wind speed probability distributions such as two-parameter Weibull distributions, four-parameter Kappa distributions or five-parameter Wakeby distributions are used. See [45] for further one-component as well as mixture wind speed probability distributions.

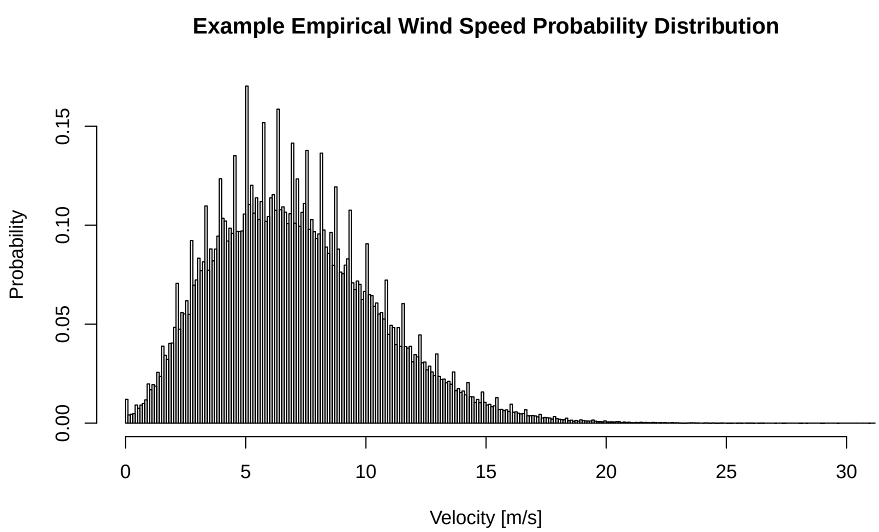

Figure 5: An exemplary histogram plot of empirical wind speed data from one weather station located at List, Germany.

The two-parameter Weibull distribution reads

$$p_{\mathrm{Wei}}\left(v\,|A_{\mathrm{Wei}},k_{\mathrm{Wei}}\right)=\frac{k_{\mathrm{Wei}}}{A_{\mathrm{Wei}}}\cdot\left(\frac{v}{A_{\mathrm{Wei}}}\right)^{k_{\mathrm{Wei}}-1}\cdot\exp\left(-\left(\frac{v}{A_{\mathrm{Wei}}}\right)^{k_{\mathrm{Wei}}}\right) \tag{6}$$

for all $v > 0$ where A_{Wei} denotes the scale parameter and k_{Wei} the shape parameter of the corresponding distribution [21].

Let $\boldsymbol{\theta}_{\mathrm{Kap}} = (A_{\mathrm{Kap}}, k_{\mathrm{Kap}}, \mu_{\mathrm{Kap}}, h_{\mathrm{Kap}})$ be the summarizing vector of all four parameters for the Kappa distribution. The four-parameter Kappa distribution is then defined by

$$p_{\mathrm{Kap}}\left(v\,|\boldsymbol{\theta}_{\mathrm{Kap}}\right)=\frac{1}{A_{\mathrm{Kap}}}\cdot\left\{1-\frac{k_{\mathrm{Kap}}\cdot(v-\mu_{\mathrm{Kap}})}{A_{\mathrm{Kap}}}\right\}^{\frac{1}{k_{\mathrm{Kap}}-1}}\cdot\left\{F_{\mathrm{Kap}}\left(v\right)\right\}^{1-h_{\mathrm{Kap}}} \tag{7}$$

for all $v \geq 0$ with scale parameter A_{Kap}, shape parameter k_{Kap}, location parameter μ_{Kap} and second shape parameter h_{Kap}. Here, the cumulative distribution function is given by

$$F_{\mathrm{Kap}}\left(v\,|\boldsymbol{\theta}_{\mathrm{Kap}}\right)=\left\{1-h_{\mathrm{Kap}}\cdot\left\{1-\frac{k_{\mathrm{Kap}}\cdot(v-\mu_{\mathrm{Kap}})}{A_{\mathrm{Kap}}}\right\}^{\frac{1}{k_{\mathrm{Kap}}}}\right\}^{\frac{1}{h_{\mathrm{Kap}}}}. \tag{8}$$

See [46] for further details.

Let $\boldsymbol{\theta}_{\mathrm{Wak}} = (A_{\mathrm{Wak}}, \gamma_{\mathrm{Wak}}, k_{\mathrm{Wak}}, \mu_{\mathrm{Wak}}, h_{\mathrm{Wak}})$ be the summarizing vector of all five parameters for the Wakeby distribution. The five-parameter Wakeby distribution is then defined by

$$\begin{aligned}p_{\mathrm{Wak}}\left(v\,|\boldsymbol{\theta}_{\mathrm{Wak}}\right)=\Big\{&A_{\mathrm{Wak}}\cdot\left\{1-F_{\mathrm{Wak}}\left(v\right)\right\}^{\gamma_{\mathrm{Wak}}-1}\\&+k_{\mathrm{Wak}}\cdot\left\{1-F_{\mathrm{Wak}}\left(v\right)\right\}^{-k_{\mathrm{Wak}}-1}\Big\}^{-1}\end{aligned} \tag{9}$$

for all $v \geq 0$ with scale parameter A_{Wak}, second scale parameter γ_{Wak}, shape parameter k_{Wak}, location parameter μ_{Wak} and second shape parameter h_{Wak}. Here, the cumulative distribution function is implicitly given by

$$\begin{aligned} F_{\mathrm{Wak}}^{-1}\left(v\,|\boldsymbol{\theta}_{\mathrm{Wak}}\right) = \mu_{\mathrm{Wak}} + \frac{A_{\mathrm{Wak}}}{\mu_{\mathrm{Wak}}} \cdot \{1-(1-F_{\mathrm{Wak}}(v))^{\gamma_{\mathrm{Wak}}}\} \\ - \frac{k_{\mathrm{Wak}}}{h_{\mathrm{Wak}}} \cdot \left\{1-(1-F_{\mathrm{Wak}}(v))^{-h_{\mathrm{Wak}}}\right\}. \end{aligned} \tag{10}$$

See [47] for further information.

There are many optimization methods to determine the unknown parameters. Maximum log-likelihood parameter estimation is often used. Additionally, moment methods or least-squares approaches are commonly applied [45].

2.7. Calculation of Semi-Empirical Annual Averaged Power Output

From given wind speed data, semi-empirical annual averaged wind power output generation values can be calculated by applying one estimated power curve function from Subsection 2.5. The semi-empirical averaged hourly wind power output generation value reads

$$\overline{P_{Hourly,Semi-Emp.}} \approx \sum_{j=1}^{N} \frac{P_{\mathrm{Power}}\left(v_j\right)}{N} \tag{11}$$

for all wind speed data $v_j \geq 0$ for all $j \in \{1,\ldots,N\}$ with physical unit kWh^{-1}. This integral is approximated by a sum of step functions. For more sophisticated numerical integration techniques, interested readers are referred to the book of Davis and Rabinowitz on this vast topic [48]. This leads to a semi-empirical averaged annual power output value. It is obtained by

$$\overline{P_{\text{Ann., Semi-Emp.}}} = \frac{365 \cdot 24 \cdot \overline{P_{\text{Hourly, Semi-Emp.}}}}{1000000} \tag{12}$$

with physical unit $\dfrac{\mathrm{GW}}{\mathrm{year}}$.

2.8. Calculation of Estimated Annual Averaged Power Output

The current goal is the calculation of averaged hourly wind power output generation values from arbitrary power curves P_{Power} and arbitrary wind speed probability distributions p_{Wind} by modifying (11). For this purpuse, the integral is approximated by appropriate step functions. Hence, the calculation reads

$$\begin{aligned}
\overline{P_{\text{Hourly, Th.}}} &= \int_{0}^{\infty} P_{\text{Power}}\left(v\,|\boldsymbol{\theta_{Power}}\right) \cdot p_{\text{Wind}}\left(v\,|\boldsymbol{\theta}_{\text{Wind}}\right)\,\mathrm{d}v \\
&\approx \int_{v_{\text{cut-in}}}^{v_{\text{cut-off}}} P_{\text{Power}}\left(v\,|\boldsymbol{\theta_{Power}}\right) \cdot p_{\text{Wind}}\left(v\,|\boldsymbol{\theta}_{\text{Wind}}\right)\,\mathrm{d}v \\
&\approx \sum_{m=10\cdot v_{\text{cut-in}}}^{10\cdot v_{\text{cut-off}}} p_{\text{Wind}}\left(\frac{m}{10}\,|\boldsymbol{\theta}_{\text{Wind}}\right) \cdot P_{\text{Power}}\left(\frac{m}{10}\,|\boldsymbol{\theta}_{\text{Power}}\right) \cdot \frac{1}{10}
\end{aligned} \tag{13}$$

because wind speeds are normally measured in 0.1 steps. $\boldsymbol{\theta}_{\text{Power}}$ and $\boldsymbol{\theta}_{\text{Wind}}$ represent the optimized parameters for power curves and wind speed probability distribution curves respectively. Furthermore, $v_{\text{cut-in}}$ and $v_{\text{cut-off}}$ are the cut-in and cut-off wind speeds. Note that non-parametric curves can be used as well in this approach. This integral yields one hourly averaged wind power output generation value with physical unit kWh^{-1}. In this case, the evaluation becomes

$$\overline{P_{\text{Ann., Th.}}} = \frac{365 \cdot 24 \cdot \overline{P_{Hourly,Th}}}{1000000} \tag{14}$$

and this yields the annual averaged wind power output generation value with physical unit $\frac{\text{GW}}{\text{year}}$.

2.9. Goodness-of-fit Measures and Error and Uncertainty Quantification

A short discussion on goodness-of-fit measures and uncertainty quantification is given.

2.9.1. Goodness-of-fit Measures

Coefficients of determination are applied to compare parametric models. Let $v_i \in [v_{\text{cut-in}}, v_{\text{cut-off}}]$ be all measured wind speeds which are larger than the cut-in wind speed $v_{\text{cut-in}}$ and which are smaller than the cut-off wind speed $v_{\text{cut-off}}$. Denote empirical wind speed probabilities by $p_{\text{Emp.}}(v_i)$ and estimated wind speed probabilities of certain wind speed distribution models by $p_{\text{Wind}}(v_i)$. The mean of all empirical wind speed probabilities is represented by $\overline{p_{\text{Emp.}}(v_i)}$. The coefficient of determination reads

$$R^2 = 1 - \frac{\sum\limits_{v_i} \left(p_{\text{Emp.}}(v_i) - p_{\text{Wind}}(v_i)\right)^2}{\sum\limits_{v_i} \left(p_{\text{Emp.}}(v_i) - \overline{p_{\text{Emp.}}(v_i)}\right)^2} \tag{15}$$

where summations are performed over all measured wind speeds which are larger than $v_{\text{cut-in}}$ and which are smaller than $v_{\text{cut-off}}$.

2.9.2. Error and Uncertainty Quantification

A brief statement of error and uncertainty quantification for estimations from two-parameter Weibull distributions is presented. For further reading, readers are referred to Taylor's book on error analysis [49].

The starting equation is (13). Assume both functions p_{Wind} and P_{Power} to be uncertain. Here, the wind speed probability distribution function is the two-parameter Weibull distribution. Assume that the variables $x_1, \ldots, x_n$ are measured with uncertainties $\delta x_1, \ldots, \delta x_n$ and these values are used to compute a function value $f(x_1, \ldots, x_n)$. If formula (3.48)

$$\delta f \leq \sum_{j=1}^{n} \left| \frac{\partial f}{\partial x_j} \right| \cdot \delta x_j$$

for the uncertainty δf of f from [49] is applied, the absolute error $\Delta\overline{P_{Hourly,Th.}}$

reads

$$\begin{aligned}
\Delta\overline{P_{Hourly,Th.}} = &\sum_{m=10\cdot v_{\text{cut-in}}}^{10\cdot v_{\text{cut-off}}} \left(\frac{1}{10}\cdot p_{\text{Wind}}\left(\frac{m}{10}\,|\boldsymbol{\theta}_{\text{Wind}}\right)\cdot P_{\text{Power}}\left(\frac{m}{10}\,|\boldsymbol{\theta}_{\text{Power}}\right)\right.\\
&\times\left\{\left|\frac{\partial p_{\text{Wind}}}{\partial v}\left(\frac{m}{10}\,|\boldsymbol{\theta}_{\text{Wind}}\right)\right|\cdot\Delta v+\left|\frac{\partial p_{\text{Wind}}}{\partial A_{\text{Wei}}}\left(\frac{m}{10}\,|\boldsymbol{\theta}_{\text{Wind}}\right)\right|\cdot\sigma_{A_{\text{Wei}}}\right.\\
&+\left.\left|\frac{\partial p_{\text{Wind}}}{\partial k_{\text{Wei}}}\left(\frac{m}{10}\,|\boldsymbol{\theta}_{\text{Wind}}\right)\right|\cdot\sigma_{k_{\text{Wei}}}\right\}+\left|\frac{\partial P_{\text{Power}}}{\partial v}\left(\frac{m}{10}\,|\boldsymbol{\theta}_{\text{Power}}\right)\right|\cdot\Delta v\\
&-\left|\frac{\partial P_{\text{Power}}}{\partial v}\left(\frac{m}{10}\,|\boldsymbol{\theta}_{\text{Power}}\right)\right|\cdot\Delta v\cdot\left\{\left|\frac{\partial p_{\text{Wind}}}{\partial v}\left(\frac{m}{10}\,|\boldsymbol{\theta}_{\text{Wind}}\right)\right|\cdot\Delta v\right.\\
&+\left.\left.\left|\frac{\partial p_{\text{Wind}}}{\partial A_{\text{Wei}}}\left(\frac{m}{10}\,|\boldsymbol{\theta}_{\text{Wind}}\right)\right|\cdot\sigma_{A_{\text{Wei}}}+\left|\frac{\partial p_{\text{Wind}}}{\partial k_{\text{Wei}}}\left(\frac{m}{10}\,|\boldsymbol{\theta}_{\text{Wind}}\right)\right|\cdot\sigma_{k_{\text{Wei}}}\right\}\right)
\end{aligned} \tag{16}$$

where Δv is the absolute uncertainty in wind speed, $\sigma_{A_{\text{Wei}}}$ and $\sigma_{k_{\text{Wei}}}$ are standard deviations of both Weibull parameters obtained, for example, from maximum log-likelihood parameter estimation. Formula (3.48) from [49] is always valid as both p_{Wind} and P_{Power} are not necessarily assumed to be independent.

Since one main goal of this article is the prediction of annual averaged wind power output generation values, absolute differences of such values are suitable comparative measures. With regard to Equations (12) and (14), the absolute difference between semi-empirical and estimated annual averaged wind power output generation values reads

$$\Delta P_{\text{Values}} = \left|\overline{P_{\text{Ann., Semi-Emp.}}} - \overline{P_{\text{Ann., Th.}}}\right|. \tag{17}$$

3. Results and Discussion

The codes for production of the results were mainly written in R [50] and GNU Octave [51]. Since Weibull distributions assume positive wind speeds, all zero wind speed values are deleted from given wind speed data. We mainly use

R-packages EnvStats [52] and fitdistrplus [53].

3.1. Detailed Results for List, Germany

The aim of this subsection is a thorough description of the estimation procedure explained using one weather station located at List, Germany as an example.

3.1.1. Power Curve Models

A closer look at Table 1 suggests setting cut-in and rated wind speeds to $v_{\text{cut-in}} = 2.5$ and $v_{\text{rated}} = 13.0$. Least-squares logistic regression and cubic spline interpoluation are applied to obtain two models. The resulting curves are portrayed in Figure 6.

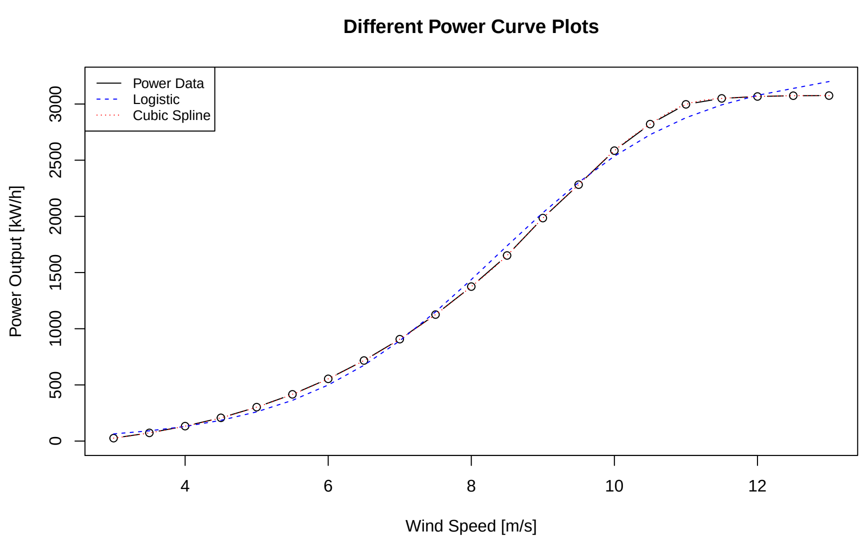

Figure 6: Plot of different power curves models.

Circles represent given power curve data points in Figure 6. The colored curves portray both parametric models. It is clear that cubic splines interpolate all data points whereas the logistic regression curve undershoots and overshoots given power output data values. For this reason, cubic spline interpolation is used in the following.

3.1.2. Wind Speed Distribution Models

Different probability distributions are applied to approximate given wind speed data from List, Germany. At first, the parameters of the two-parameter Weibull distribution are computed by maximum log-likelihood parameter estimation. This method preserves asymptotic normality, consistency and it is attractive from a theoretical point of view because a unique global maximizer is guaranteed [54]. Additionally, parameters of both four-parameter Kappa and five-parameter Wakeby distributions are estimated by L-moments. Details of these methods can be found in the book by Hosking and Wallis [55]. The resulting wind speed distributions are plotted in Figure 7 with the histogram of the given data.

The histogram presents given wind speed data. It can be seen that the four-parameter Kappa distribution and the five-parameter Wakeby distribution can be regarded as better fits for higher wind speeds. However, the five-parameter Wakeby distribution does not suit low wind speed well in this case. An explanation might be the change in measurements which lowered difference steps in wind speed from 0.5 to 0.1. This example demonstrates that an important first step in evaluation is a first look at a plot of the probability distributions as all R^2-values are close together.

3.1.3. Annual Averaged Wind Power Output Generation Values

Wind speed data at List, Germany were recorded from 1950/01/01 until 2018/12/31. Results are summarized in Table 2.

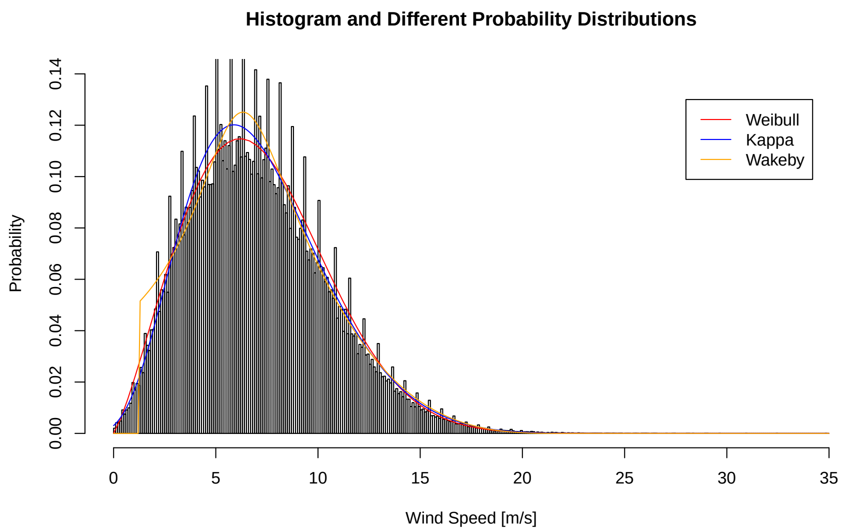

Figure 7: Plot of a histogram of given wind speed data and different wind speed probability distributions.

Table 2: Results for List, Germany. All values have physical units $\frac{\text{GW}}{\text{year}}$.

Distribution	R^2	$\overline{P_{Ann.,Semi-Emp.}}$	$\overline{P_{Ann.,Th.}}$	ΔP_{Values}
Weibull	0.956	10.48	10.64 ± 0.39	0.16
Kappa	0.958	10.48	10.47	0.01
Wakeby	0.953	10.48	10.45	0.03

The summary in Table 2 indicates that better fits of wind speed distributions yield more appropriate wind power output generation values. In this case, the four-parameter Kappa distribution and the five-parameter Wakeby distribution gave similar results.

3.2. Results for Germany

Results for semi-empirical annual averaged power output generation values, estimated annual averaged power output generation values for all three wind speed probability distributions, errors for the Weibull wind speed distributions and differences between semi-empirical and estimated annual averaged annual power output generation values for all three wind speed probability distributions are illustrated in Figures 8–9.

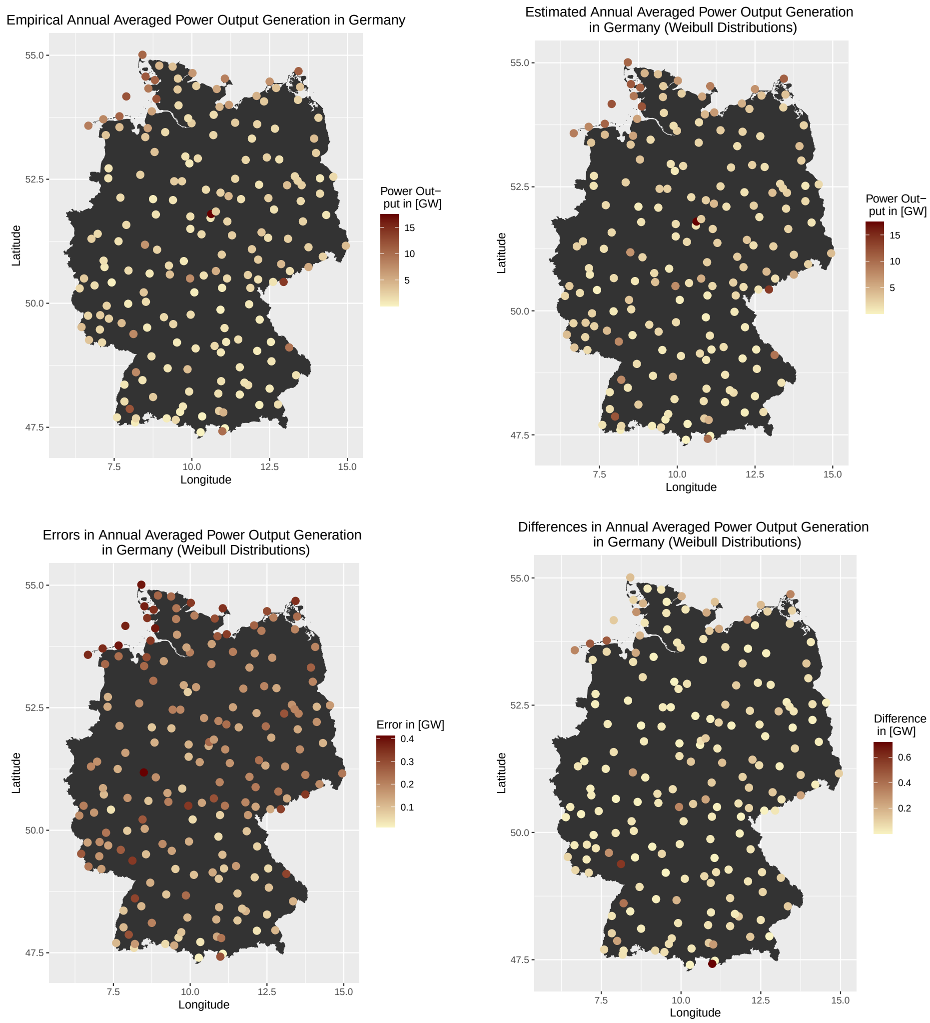

Figure 8: All results from Weibull distributions for Germany. Top left: Semi-empirical annual averaged power output generation values. Top right: Estimated annual averaged power output generation values by Weibull distributions. Bottom left: Errors for annual averaged power output generation estimates by Weibull distributions. Bottom right: Absolute differences between semi-empirical and estimated power output generation values by Weibull distributions.

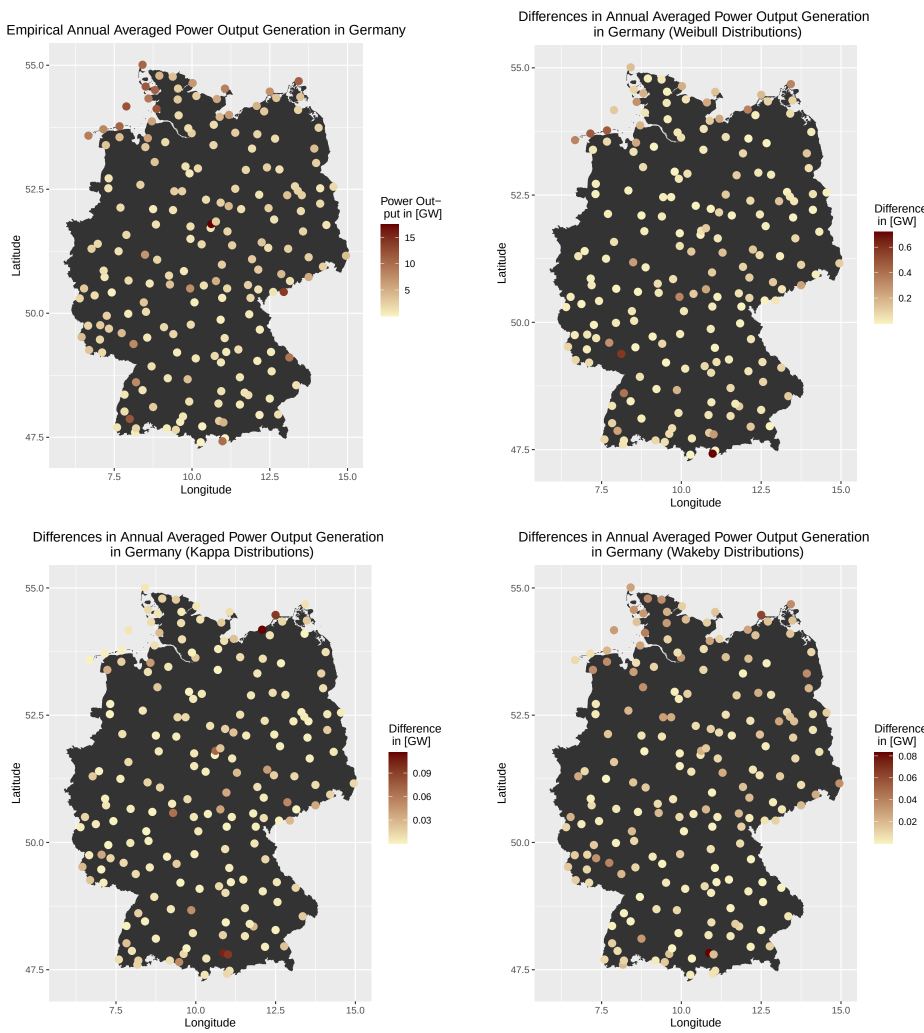

Figure 9: All difference results for Germany. Top left: Semi-empirical annual averaged power output generation values. Top right: Absolute differences between semi-empirical and estimated power output generation values by Weibull distributions. Bottom left: Absolute differences between semi-empirical and estimated power output generation values by Kappa distributions. Bottom right: Absolute differences between semi-empirical and estimated power output generation values by Wakeby distributions.

3.3. Results for California (United States)

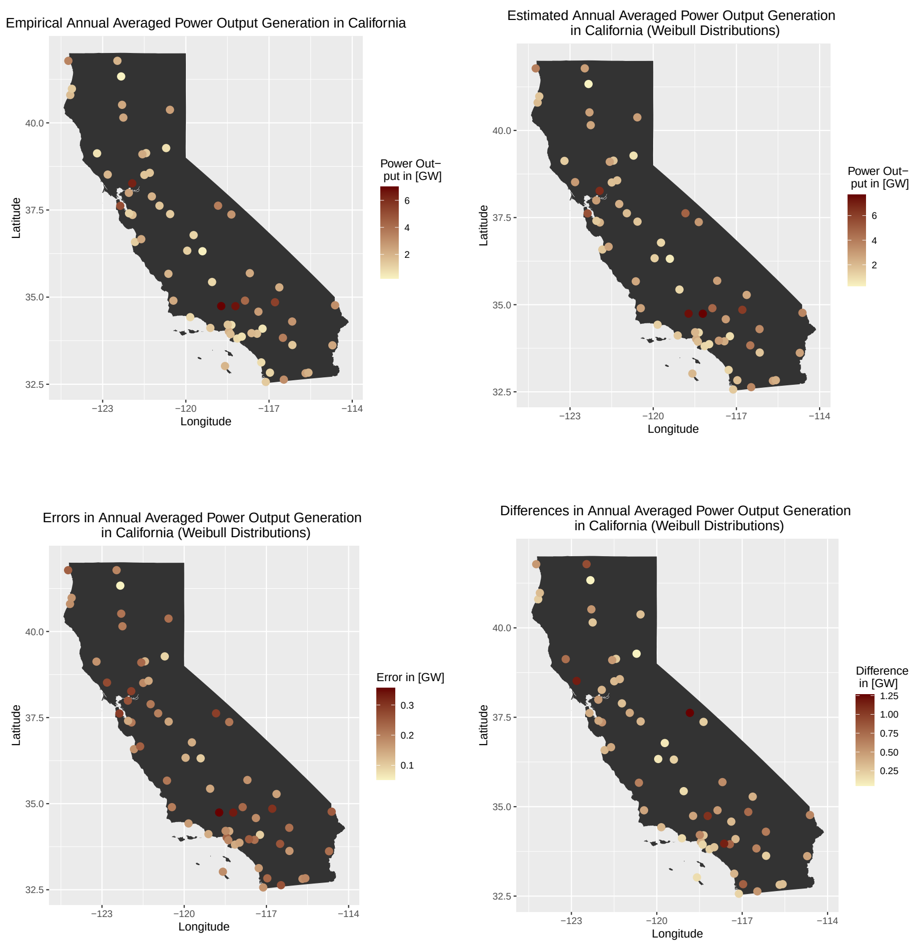

Figure 10: All results from Weibull distributions for California (United States). Top left: Semi-empirical annual averaged power output generation values. Top right: Estimated annual averaged power output generation values by Weibull distributions. Bottom left: Errors for annual averaged power output generation estimates by Weibull distributions. Bottom right: Absolute differences between semi-empirical and estimated power output generation values by Weibull distributions.

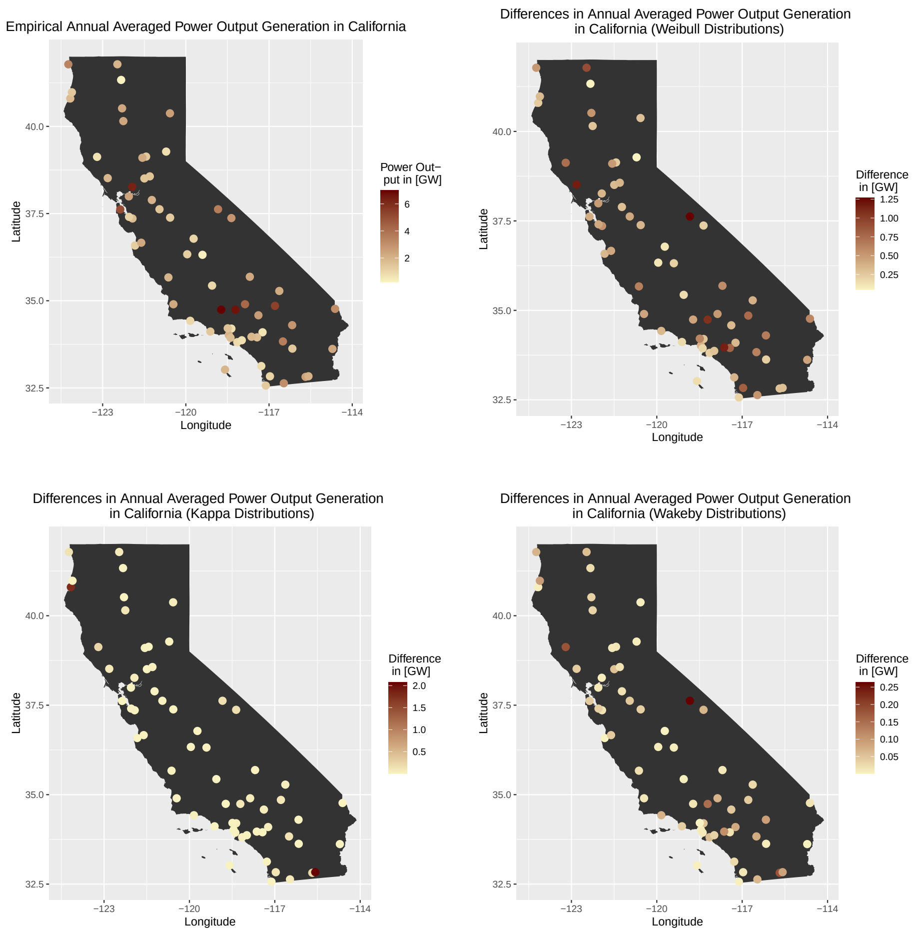

Figure 11: All difference results for California (United States). Top left: Semi-empirical annual averaged power output generation values. Top right: Absolute differences between semi-empirical and estimated power output generation values by Weibull distributions. Bottom left: Absolute differences between semi-empirical and estimated power output generation values by Kappa distributions. Bottom right: Absolute differences between semi-empirical and estimated power output generation values by Wakeby distributions.

Results for semi-empirical annual averaged power output generation values, estimated annual averaged power output generation values for all three wind speed probability distributions, errors for the Weibull wind speed distributions and differences between semi-empirical and estimated annual averaged annual power output generation values for all three wind speed probability distributions regarding California (United States) are illustrated in Figures 10–11.

3.4. Discussion

We first consider results for List, Germany. Surprisingly, all R^2-values in Table 2 for the three wind speed probability distributions are relatively close together. However, four-parameter Kappa and five-parameter Wakeby distributions yield better predictions for annual averaged power output generation values compared to two-parameter Weibull distributions. More optimization parameters in contrast to two-parameter Weibull distributions explain this fact. Hence, this flexibility implies better approximations of wind speed probability distributions in nearly all cases.

Figures 9 and 11 especially underline this fact. All differences between semi-empirical and modeled estimations for annual averaged power output generation values from different wind speed probability distributions are depicted. In the case of California, there were two exceptional cases where two-parameter Weibull distributions led to better predictions than four-parameter Kappa distributions.

For many stations, time periods that last longer than twenty years were applied. This is accordance with one study by Jung and Schindler [23]. Hence, we suggest to use time series that span at least twenty years.

We note that used data were acquired near surface. Since this study focuses on algorithmic and theoretical aspects, these near surface wind speeds were applied to calculate all values. It is worth mentioning that wind speeds in hub height can be obtained by Hellmann's power law. However, this needs careful consideration with respect to the exponent [56].

Conclusively, this computational study stresses observations by Jung and Schindler that four-parameter Kappa and five-parameter Wakeby distributions nearly always prove to be better univariate fits than two-parameter Weibull distributions [22]. In contrast to many other works mentioned in our literature review, we tried to establish a structuring framework for comparison purpose on wind speed distributions and power output prediction of wind turbines.

4. Conclusion

A modular framework for annual averaged power output generation of wind turbines was developed. It is a flexible and unifying approach to wind power output prediction due to allowing every power curve modeling technique and every parametric or non-parametric procedure of estimating arbitrary wind speed probability distributions.

Only a few combinations of power curve approaches and wind speed distribution models were presented because of the non-exhaustive combinatorial possibilities in this field. Thus, this work restricts itself to application of cubic spline interpolation for power curves and to two-parameter Weibull, four-parameter Kappa and five-parameter Wakeby distributions for wind speed probability distributions. Detailed results were portrayed for one German weather station located at List, Germany. A comparative study between weather stations from California (United States) and Germany were included where cubic spline interpolation for power curves and Wakeby distributions for wind speed probability functions were applied. It was shown that four-parameter Kappa distributions and five-parameter Wakeby distributions are almost always superior to two-parameter Weibull distributions.

In practice, this combinatorial framework allows comparison of future sites for wind turbines and it can be seen as an integrable tool for part of decision processes regarding determination of such future locations. Our work is therefore not only interesting to theorists, but it equips practitioners with a structuring approach in the widely branched area of wind power output prediction as well.

Conclusively, our work generalizes vast specialized methods on wind power output prediction in the literature and combines them in one unifying approach. Hence, it establishes a flexible approach to the field and it appears to be a valuable tool for future comparative studies of different wind power output prediction methods.

Author contribution statement

Conceptualization: Jan Christian Schlüter and Benjamin Wacker; **Materials and Methods:** Benjamin Wacker; **Data Preparation:** Johann Seebaß and Benjamin Wacker; **Implementation:** Benjamin Wacker; **Data Visualization:** Benjamin Wacker; **Discussion:** Jan Christian Schlüter and Benjamin Wacker; **Writing - First Draft:** Benjamin Wacker; **Writing - Editing & Review**: Jan Christian Schlüter, Johann Seebaß and Benjamin Wacker

Code availability, data availability and further results

The R [50] and GNU Octave [51] codes can be downloaded from `https://github.com/bewa87/2020-Energy-AAPOGFWT`. Data for the presented wind turbine from Vestas can be obtained from `https://www.wind-turbine-models.com/turbines/7-vestas-v112-onshore#datasheet`. Wind speed data for all German weather stations are available under [42] and wind speed data for worldwide weather stations can be accessed under [57]. Our supplementary material contains additional computational results for a number of weather stations from California (United States) and Germany.

Declaration of interest

The authors declare that they have no potential conflict of interest.

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