# point-neuron-network-simulator

xyy

May 1, 2016

## 1 The HH model Used

It is the classical Hodgkin-Huxley (HH) neuron model. For neuron $i$, its membrane potential $V_i$ obey

$$\begin{cases} C\dfrac{\mathrm{d}V_i}{\mathrm{d}t} = -(V_i - V_{\mathrm{Na}})G_{\mathrm{Na}}h_i m_i^3 - (V_i - V_{\mathrm{K}})G_{\mathrm{K}}n_i^4 - (V_i - V_{\mathrm{L}})G_{\mathrm{L}} + I_i^{\mathrm{input}} \\ \dfrac{\mathrm{d}m_i}{\mathrm{d}t} = (1 - m_i)\alpha_m(V_i) - m_i\beta_m(V_i) \\ \dfrac{\mathrm{d}h_i}{\mathrm{d}t} = (1 - h_i)\alpha_h(V_i) - h_i\beta_h(V_i) \\ \dfrac{\mathrm{d}n_i}{\mathrm{d}t} = (1 - n_i)\alpha_n(V_i) - n_i\beta_n(V_i) \end{cases}$$

where

$$\begin{aligned} \alpha_n(V_i) &= \frac{0.1 - 0.01V_i}{\exp(1 - 0.1V_i) - 1} & \beta_n(V_i) &= 0.125\exp(-V_i/80) \\ \alpha_m(V_i) &= \frac{2.5 - 0.1V_i}{\exp(2.5 - 0.1V_i) - 1} & \beta_m(V_i) &= 4\exp\left(-V_i/18\right) \\ \alpha_h(V_i) &= 0.07\exp(-V_i/20) & \beta_h(V_i) &= \frac{1}{\exp(3 - 0.1V_i) + 1} \end{aligned}$$

$V_i$, $m_i$, $n_i$, $h_i$, $I_i^{\mathrm{input}}$ are functions of $t$, and others are constants: $V_{\mathrm{Na}} = 115\,\mathrm{mV}$, $V_{\mathrm{K}} = -12\,\mathrm{mV}$, $V_{\mathrm{L}} = 10.6\,\mathrm{mV}$ (resting potential set to $0\,\mathrm{mV}$), $G_{\mathrm{Na}} = 120\,\mathrm{mS}\cdot\mathrm{cm}^{-2}$, $G_{\mathrm{K}} = 36\,\mathrm{mS}\cdot\mathrm{cm}^{-2}$, $G_{\mathrm{L}} = 0.3\,\mathrm{mS}\cdot\mathrm{cm}^{-2}$ and membrane capacity $C = 1\,\mu\mathrm{F}\cdot\mathrm{cm}^{-2}$.

The interaction between neurons and external inputs come from $I_i^{\mathrm{input}}$

$$I_i^{\mathrm{input}} = I_i^{\mathrm{E}} + I_i^{\mathrm{I}}, \quad I_i^{\mathrm{E}} = -(V_i - V_G^{\mathrm{E}})G_i^{\mathrm{E}}, \quad I_i^{\mathrm{I}} = -(V_i - V_G^{\mathrm{I}})G_i^{\mathrm{I}}$$

$I_i^{\mathrm{E}}$, $I_i^{\mathrm{I}}$ are excitatory and inhibitory input respectively, and $V_G^{\mathrm{E}}$, $V_G^{\mathrm{I}}$ is their reversal potential. The conductances $G_i^Q$ ($Q \in \{\mathrm{E}, \mathrm{I}\}$) evolves according to

$$\frac{\mathrm{d}G_i^Q}{\mathrm{d}t} = -\frac{G_i^Q}{\sigma_r^Q} + H_i^Q, \quad \frac{\mathrm{d}H_i^Q}{\mathrm{d}t} = -\frac{H_i^Q}{\sigma_d^Q} + \sum_k F_i^Q \delta(t - T_{i,k}^F) + \sum_{j\neq i} S_{ij}\delta(t - T_j^S)$$

where $F_i^Q$ is the strength of external input to neuron $i$, $T_{i,k}^F$ is its time of $k$-th input event, which is a Poisson process with rate $\mu_i$. We call this term the Poisson input. $S_{ij}$ is the coupling strength from $j$-th neuron to $i$-th neuron. $\sigma_r^Q$, $\sigma_d^Q$ are the fast rising and slow decaying timescales in the $\alpha$ function. $V_G^{\mathrm{E}} = 65\,\mathrm{mV}$, $V_G^{\mathrm{I}} = -15\,\mathrm{mV}$, $\sigma_r^E = 0.5$, $\sigma_d^E = 3.0$, $\sigma_r^I = 0.5$, $\sigma_d^I = 7.0$.

We use adjacency matrix $A = (A_{ij})$ to denote the neural network structure, i.e. $S_{ij} = A_{ij}S^{Q_iQ_j}$, and $S^{Q_iQ_j}$ is one of $S^{\mathrm{EE}}$, $S^{\mathrm{EI}}$, $S^{\mathrm{IE}}$, $S^{\mathrm{II}}$, depends on the type of corresponding neuron pair (E for excitatory, I for inhibitory). $A_{ij} \neq 0$ means there is a direct affection to $i$-th neuron from $j$-th neuron. When we talk about "homogeneous coupling", we mean $A_{ij}$ equals either 1 or 0.

$F$, $\mu$, $A$, $S^{Q_i Q_j}$, $\sigma_r^Q$, $\sigma_d^Q$ are parameters relate to synaptic and input to neurons. For all neurons $F_i^{\mathrm{E}} = F$, $F_i^{\mathrm{I}} = 0$, $\mu_i = \mu$. During one simulation, these parameters are all constant.

The time delay due to long dendrite or axion are ignored. The threshold is 50 mV above resting potential.

(default parameters)

In numerical simulation, we use explicit fourth-order Runge-Kutta method with time step 1/32 ms. The data samples (i.e. $x_t$, $y_t$) we used are voltages obtained in sampling rate 2 kHz. When we talk about spike train data, we mean $x_t = 1$ if $V_i(t)$ just pass through the threshold (10 mV in our case) from low to high, otherwise $x_t = 0$.