point-neuron-network-simulator

xyy

October 9, 2016

1 Simulator Overview

- simple

  Spike interaction happens at the end of each dt. Poisson input still injected at the exact timing.

- SSC

  Spike-spike correction.

- SSC-Sparse

  Spike-spike correction, optimized for sparse network.

- SSC-Sparse2

  Spike-spike correction, optimized for sparse network, version 2.

- big-delay

  SSC for network with constant delay. The delay must larger than 1dt (hence the name "big"). Use option "`--synaptic-delay`" to set the delay.

- big-net-delay

  SSC for network with delay. The delay is set by a "delay matrix", by passing a text numerical matrix file path to the option "`--synaptic-net-delay`".

- cont-syn

  Simulate the network with continuous synaptic interaction. Specially for model "HH-GH-cont-syn".

- auto

  Auto choose a simulator according to the neuronal model.

Table 1: Compatibility matrix

	simple	SSC/SSC-Sparse/ SSC-Sparse2	cont-syn	big-delay	big-net-delay	auto
LIF-G/LIF-GH	y	y	-	-	-	SSC
HH-G/HH-GH	y	y	-	-	-	SSC
HH-GH-cont-syn	-	-	y	-	-	cont-syn
–synaptic-delay	-	-	-	y	-	big-delay
–synaptic-net-delay	-	-	-	-	y	big-net-delay
-sine						
-extI						

2 Neuron Model Used

2.1 HH-GH

It is the classical Hodgkin-Huxley (HH) neuron model. For neuron i, its membrane potential V_i obey

$$\begin{cases} C\dfrac{\mathrm{d}V_i}{\mathrm{d}t} = -(V_i - V_{\mathrm{Na}})G_{\mathrm{Na}}h_i m_i^3 - (V_i - V_{\mathrm{K}})G_{\mathrm{K}}n_i^4 - (V_i - V_{\mathrm{L}})G_{\mathrm{L}} + I_i^{\mathrm{input}} \\ \dfrac{\mathrm{d}m_i}{\mathrm{d}t} = (1-m_i)\alpha_m(V_i) - m_i\beta_m(V_i) \\ \dfrac{\mathrm{d}h_i}{\mathrm{d}t} = (1-h_i)\alpha_h(V_i) - h_i\beta_h(V_i) \\ \dfrac{\mathrm{d}n_i}{\mathrm{d}t} = (1-n_i)\alpha_n(V_i) - n_i\beta_n(V_i) \end{cases} \tag{1}$$

where

$$\begin{aligned} \alpha_n(V_i) &= \frac{0.1 - 0.01V_i}{\exp(1-0.1V_i) - 1} & \beta_n(V_i) &= 0.125\exp(-V_i/80) \\ \alpha_m(V_i) &= \frac{2.5 - 0.1V_i}{\exp(2.5-0.1V_i) - 1} & \beta_m(V_i) &= 4\exp(-V_i/18) \\ \alpha_h(V_i) &= 0.07\exp(-V_i/20) & \beta_h(V_i) &= \frac{1}{\exp(3-0.1V_i)+1} \end{aligned}$$

V_i, m_i, n_i, h_i, I_i^{input} are functions of t, and others are constants: $V_{\mathrm{Na}} = 115\,\mathrm{mV}$, $V_{\mathrm{K}} = -12\,\mathrm{mV}$, $V_{\mathrm{L}} = 10.6\,\mathrm{mV}$ (resting potential set to $0\,\mathrm{mV}$), $G_{\mathrm{Na}} = 120\,\mathrm{mS}\cdot\mathrm{cm}^{-2}$, $G_{\mathrm{K}} = 36\,\mathrm{mS}\cdot\mathrm{cm}^{-2}$, $G_{\mathrm{L}} = 0.3\,\mathrm{mS}\cdot\mathrm{cm}^{-2}$ and membrane capacity $C = 1\,\mu\mathrm{F}\cdot\mathrm{cm}^{-2}$.

The interaction between neurons and external inputs come from I_i^{input}

$$I_i^{\mathrm{input}} = I_i^{\mathrm{E}} + I_i^{\mathrm{I}}, \tag{2}$$

$$I_i^{\mathrm{E}} = -(V_i - V_G^{\mathrm{E}})G_i^{\mathrm{E}}, \quad I_i^{\mathrm{I}} = -(V_i - V_G^{\mathrm{I}})G_i^{\mathrm{I}} \tag{3}$$

I_i^{E}, I_i^{I} are excitatory and inhibitory input respectively, and V_G^{E}, V_G^{I} is their reversal potential. The conductances G_i^Q ($Q \in \{\mathrm{E}, \mathrm{I}\}$) evolves according to

$$\frac{\mathrm{d}G_i^Q}{\mathrm{d}t} = -\frac{G_i^Q}{\sigma_r^Q} + H_i^Q, \tag{4}$$

$$\frac{\mathrm{d}H_i^Q}{\mathrm{d}t} = -\frac{H_i^Q}{\sigma_d^Q} + \sum_k F_i^Q\delta(t - T_{i,k}^F) + \sum_{j\neq i} S_{ij}\delta(t - T_j^S) \tag{5}$$

where F_i^Q is the strength of external input to neuron i, $T_{i,k}^F$ is its time of k-th input event, which is a Poisson process with rate μ_i. We call this term the Poisson input. S_{ij} is the coupling strength from j-th neuron to i-th neuron. σ_r^Q, σ_d^Q are the fast rising and slow decaying timescales in the α function. $V_G^{\mathrm{E}} = 65\,\mathrm{mV}$, $V_G^{\mathrm{I}} = -15\,\mathrm{mV}$, $\sigma_r^E = 0.5$, $\sigma_d^E = 3.0$, $\sigma_r^I = 0.5$, $\sigma_d^I = 7.0$.

We use adjacency matrix $A = (A_{ij})$ to denote the neural network structure, i.e. $S_{ij} = A_{ij}S^{Q_iQ_j}$, and $S^{Q_iQ_j}$ is one of S^{EE}, S^{EI}, S^{IE}, S^{II}, depends on the type of corresponding neuron pair (E for excitatory, I for inhibitory). $A_{ij} \neq 0$ means there is a direct affection to i-th neuron from j-th neuron.

F, μ, A, $S^{Q_iQ_j}$, σ_r^Q, σ_d^Q are parameters relate to synaptic and input to neurons. For all neurons $F_i^{\mathrm{E}} = F$, $F_i^{\mathrm{I}} = 0$, $\mu_i = \mu$. During one simulation, these parameters are all constant.

The threshold is $65\,\mathrm{mV}$ above the resting potential ($0\,\mathrm{mV}$). And the synaptic interaction is performed at the time of this crossing.

In numerical simulation, we use explicit fourth-order Runge-Kutta method with time step $1/32\,\mathrm{ms}$. When we talk about spike train data, we mean $x_t = 1$ if $V_i(t)$ just pass through the threshold from low to high, otherwise $x_t = 0$.

2.2 HH-G

Eq. (4)(5) change to

$$\frac{\mathrm{d}G_i^Q}{\mathrm{d}t} = -\frac{G_i^Q}{\sigma_r^Q} + \sum_k F_i^Q \delta(t - T_{i,k}^F) + \sum_{j \neq i} S_{ij} \delta(t - T_j^S), \tag{6}$$

2.3 HH-GH-cont-syn

Eq. (5) change to

$$\frac{\mathrm{d}H_i^Q}{\mathrm{d}t} = -\frac{H_i^Q}{\sigma_d^Q} + \sum_k F_i^Q \delta(t - T_{i,k}^F) + \sum_{j \neq i} S_{ij} g(V_j^{\mathrm{pre}}), \tag{7}$$

$$g(V_j^{\mathrm{pre}}) = 1/\left(1 + \exp(-(V_j^{\mathrm{pre}} - 85\,\mathrm{mV})/2)\right). \tag{8}$$

V_j^{pre} is the (presynaptic) membrane potential of j-th neuron.
Note: In this model, the voltage is scaled as $V/10$.

2.4 HH-PT-GH

Same as HH-GH, except that the spike timing is at the spike peak.

3 Neuron Properties

3.1 Single neuron property (HH-G, HH-GH, HH-GH-cont-syn)

3.1.1 Setting threshold

A common setting for threshold is 15 mV above the resting state.

From Fig.(1) we can see that some non-spike trajectory been counted as spikes.

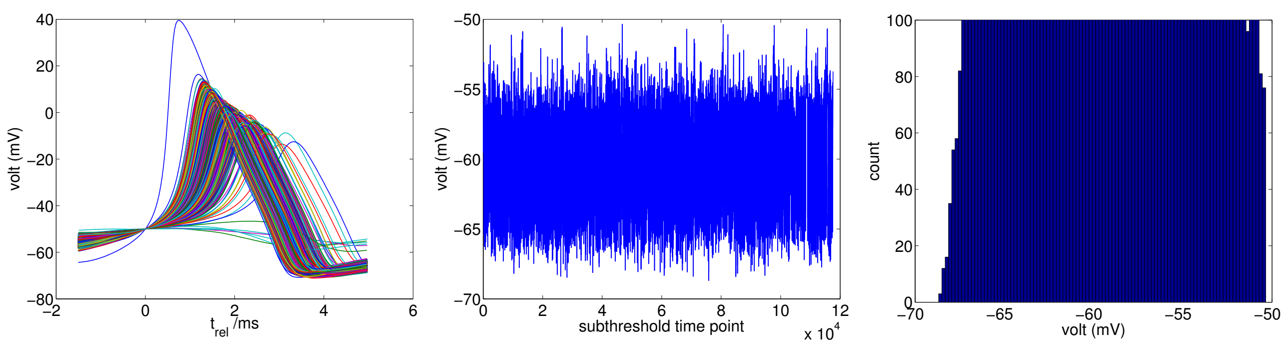


Figure 1: Threshold$=-65+15\,\mathrm{mV}$, HH-GH, pr=10.0 kHz, ps=0.05 $\Omega^{-1}\mathrm{ms}^{-2}$ (strong input, 97 Hz)

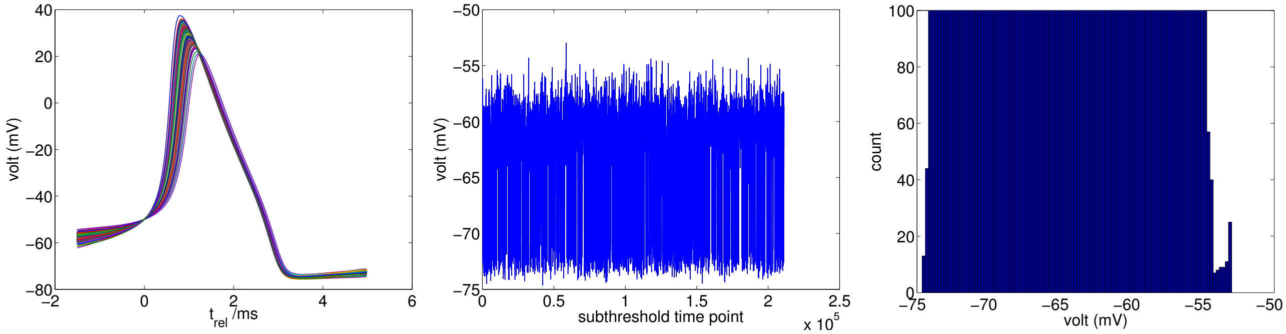


Figure 2: Threshold$=-65+15\,$mV, HH-GH, pr$=2.0\,$kHz, ps$=0.05\,\Omega^{-1}\mathrm{ms}^{-2}$ (median input, 52 Hz)

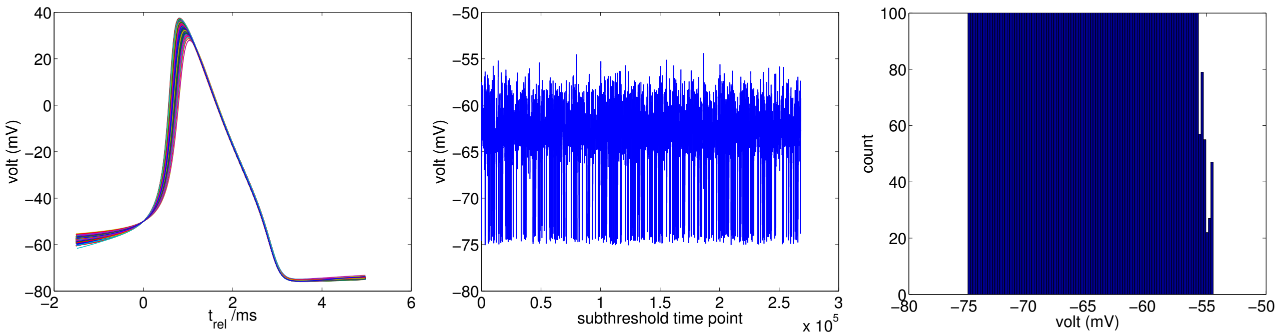


Figure 3: Threshold$=-65+15\,$mV, HH-GH, pr$=1.0\,$kHz, ps$=0.04\,\Omega^{-1}\mathrm{ms}^{-2}$ (low input, 24 Hz)

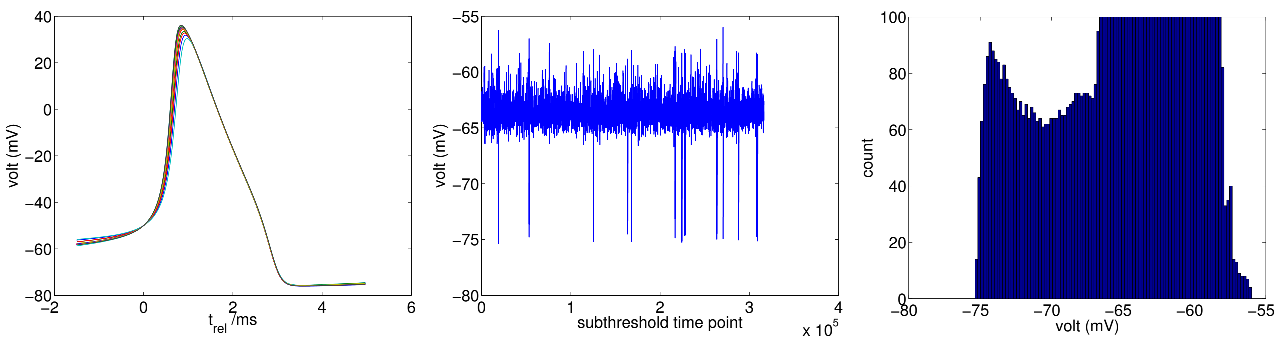


Figure 4: Threshold$=-65+15\,$mV, HH-GH, pr$=1.0\,$kHz, ps$=0.02\,\Omega^{-1}\mathrm{ms}^{-2}$ (tiny input, 1.4 Hz)

3.1.2 Higher threshold

Use 20 mV above resting as threshold.

(Fig.(5)) Compare to Fig.(1), we see no mis-counted spikes, and the timing is more accurate.

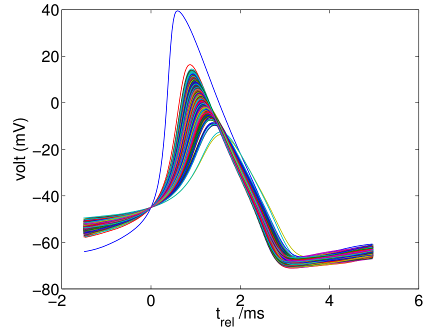


Figure 5: Threshold$=-65+20\,\mathrm{mV}$, HH-GH, pr$=10.0\,\mathrm{kHz}$, ps$=0.05\,\Omega^{-1}\mathrm{ms}^{-2}$ (strong input, $97\,\mathrm{Hz}$).

Use $84\,\mathrm{mV}$ above resting as threshold.

Reason: In "HH-GH-cont-syn" model, only when volt above around $85\,\mathrm{mV}$ will the synaptic transmit a signal. In function Eq.(8), $g(84\,\mathrm{mV}) \approx 0.0067$, $g(85\,\mathrm{mV}) = 0.5$.

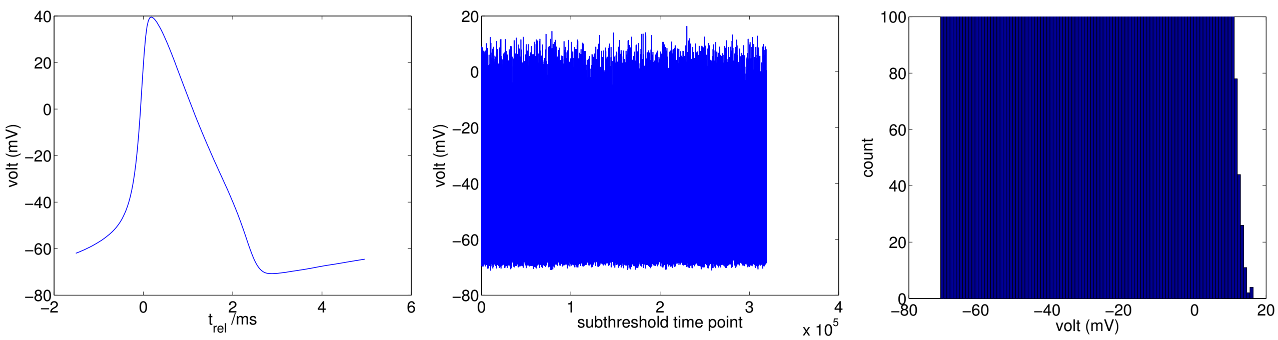
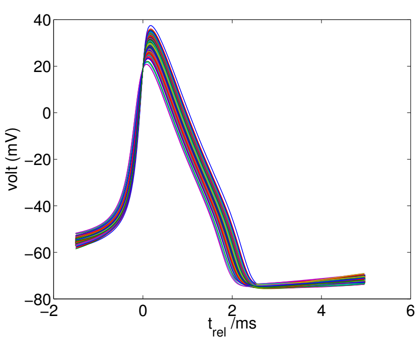
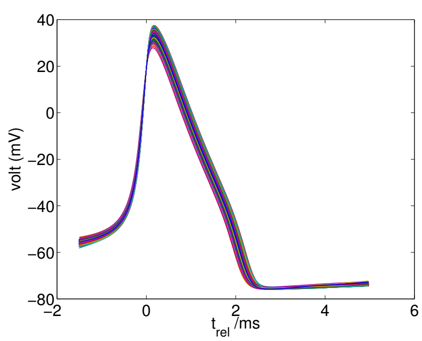


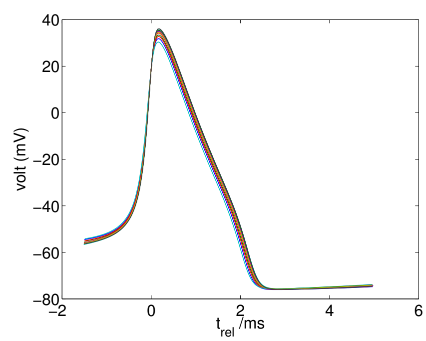
Figure 6: Threshold$=-65+84\,\mathrm{mV}$. Strong input, (see Fig. (1)). Essentailly all spikes are "missed".



(a) Median input, (see Fig. (2))



(b) Low input, (see Fig. (3))



(c) Tiny input, (see Fig. (4))

Figure 7: Threshold$=-65+84\,\mathrm{mV}$.

3.1.3 Discussion of the "threshold"

The concept of the threshold actually has two distinct meanings:

1. Once the voltage pass it (from low to high), then the neuron will fire (sooner or later).

2. The neuron will sent a signal to it down stream around this time.

A related fact is that (in this model, with Poisson input), the action potential is not in a fixed size, there are big and small action potentials. This is what makes the above two concepts essentially differ.

In the simulation and exact understanding of spiking dynamics. The concept 2 is desired. The proper timing should be the peak time.

4 Convergence

Refer to code.

```
mfile/HH_test_convergence.m
```

.
.
.
.
.
.
.
.
.
.
.
.