

# point-neuron-network-simulator

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## 1 Simulator Overview

- simple  
Spike interaction happens at the end of each dt. Poisson input still injected at the exact timing.
- SSC  
Spike-spike correction.
- SSC-Sparse  
Spike-spike correction, optimized for sparse network.
- SSC-Sparse2  
Spike-spike correction, optimized for sparse network, version 2.
- big-delay  
Network with constant delay. The delay must larger than 1dt (hence the name “big”). Use option “-synaptic-delay” to set the delay.
- big-net-delay  
Network with delay. The delay is set by a “delay matrix”, by passing a text numerical matrix file path to the option “-synaptic-net-delay”.
- cont-syn  
Simulate the network with continuous synaptic interaction. Specially for model “HH-GH-cont-syn”.
- auto  
Auto choose a simulator according to the neuronal model.

Table 1: Compatibility matrix

	simple	SSC/SSC-Sparse/ SSC-Sparse2	cont-syn	big-delay	big-net-delay	auto
LIF-G/LIF-GH	y	y	-	-	-	SSC
HH-G/HH-GH	y	y	-	-	-	SSC
HH-GH-cont-syn	-	-	y	-	-	cont-syn
-synaptic-delay	-	-	-	y	-	big-delay
-synaptic-net-delay	-	-	-	-	y	big-net-delay
-sine						
-extI						

## 2 Neuron Model Used

### 2.1 HH-GH

It is the classical Hodgkin-Huxley (HH) neuron model. For neuron  $i$ , its membrane potential  $V_i$  obey

$$\begin{cases} C \frac{dV_i}{dt} = -(V_i - V_{Na})G_{Na}h_i m_i^3 - (V_i - V_K)G_K n_i^4 - (V_i - V_L)G_L + I_i^{\text{input}} \\ \frac{dm_i}{dt} = (1 - m_i)\alpha_m(V_i) - m_i\beta_m(V_i) \\ \frac{dh_i}{dt} = (1 - h_i)\alpha_h(V_i) - h_i\beta_h(V_i) \\ \frac{dn_i}{dt} = (1 - n_i)\alpha_n(V_i) - n_i\beta_n(V_i) \end{cases} \quad (1)$$

where

$$\begin{aligned} \alpha_n(V_i) &= \frac{0.1 - 0.01V_i}{\exp(1 - 0.1V_i) - 1} & \beta_n(V_i) &= 0.125 \exp(-V_i/80) \\ \alpha_m(V_i) &= \frac{2.5 - 0.1V_i}{\exp(2.5 - 0.1V_i) - 1} & \beta_m(V_i) &= 4 \exp(-V_i/18) \\ \alpha_h(V_i) &= 0.07 \exp(-V_i/20) & \beta_h(V_i) &= \frac{1}{\exp(3 - 0.1V_i) + 1} \end{aligned}$$

$V_i, m_i, n_i, h_i, I_i^{\text{input}}$  are functions of  $t$ , and others are constants:  $V_{Na} = 115 \text{ mV}$ ,  $V_K = -12 \text{ mV}$ ,  $V_L = 10.6 \text{ mV}$  (resting potential set to 0 mV),  $G_{Na} = 120 \text{ mS} \cdot \text{cm}^{-2}$ ,  $G_K = 36 \text{ mS} \cdot \text{cm}^{-2}$ ,  $G_L = 0.3 \text{ mS} \cdot \text{cm}^{-2}$  and membrane capacity  $C = 1 \mu\text{F} \cdot \text{cm}^{-2}$ .

The interaction between neurons and external inputs come from  $I_i^{\text{input}}$

$$I_i^{\text{input}} = I_i^E + I_i^I, \quad (2)$$

$$I_i^E = -(V_i - V_G^E)G_i^E, \quad I_i^I = -(V_i - V_G^I)G_i^I \quad (3)$$

$I_i^E, I_i^I$  are excitatory and inhibitory input respectively, and  $V_G^E, V_G^I$  is their reversal potential. The conductances  $G_i^Q$  ( $Q \in \{E, I\}$ ) evolves according to

$$\frac{dG_i^Q}{dt} = -\frac{G_i^Q}{\sigma_r^Q} + H_i^Q, \quad (4)$$

$$\frac{dH_i^Q}{dt} = -\frac{H_i^Q}{\sigma_d^Q} + \sum_k F_i^Q \delta(t - T_{i,k}^F) + \sum_{j \neq i} S_{ij} \delta(t - T_j^S) \quad (5)$$

where  $F_i^Q$  is the strength of external input to neuron  $i$ ,  $T_{i,k}^F$  is its time of  $k$ -th input event, which is a Poisson process with rate  $\mu_i$ . We call this term the Poisson input.  $S_{ij}$  is the coupling strength from  $j$ -th neuron to  $i$ -th neuron.  $\sigma_r^Q, \sigma_d^Q$  are the fast rising and slow decaying timescales in the  $\alpha$  function.  $V_G^E = 65 \text{ mV}$ ,  $V_G^I = -15 \text{ mV}$ ,  $\sigma_r^E = 0.5$ ,  $\sigma_d^E = 3.0$ ,  $\sigma_r^I = 0.5$ ,  $\sigma_d^I = 7.0$ .

We use adjacency matrix  $A = (A_{ij})$  to denote the neural network structure, i.e.  $S_{ij} = A_{ij}S^{Q_i Q_j}$ , and  $S^{Q_i Q_j}$  is one of  $S^{EE}, S^{EI}, S^{IE}, S^{II}$ , depends on the type of corresponding neuron pair (E for excitatory, I for inhibitory).  $A_{ij} \neq 0$  means there is a direct affection to  $i$ -th neuron from  $j$ -th neuron.

$F, \mu, A, S^{Q_i Q_j}, \sigma_r^Q, \sigma_d^Q$  are parameters relate to synaptic and input to neurons. For all neurons  $F_i^E = F, F_i^I = 0, \mu_i = \mu$ . During one simulation, these parameters are all constant.

The time delay due to long dendrite or axon are ignored. The threshold is 20 mV above resting potential (0 mV).

In numerical simulation, we use explicit fourth-order Runge-Kutta method with time step 1/32 ms. When we talk about spike train data, we mean  $x_t = 1$  if  $V_i(t)$  just pass through the threshold from low to high, otherwise  $x_t = 0$ .

## 2.2 HH-G

Eq. (4)(5) change to

$$\frac{dG_i^Q}{dt} = -\frac{G_i^Q}{\sigma_r^Q} + \sum_k F_i^Q \delta(t - T_{i,k}^F) + \sum_{j \neq i} S_{ij} \delta(t - T_j^S), \quad (6)$$

## 2.3 HH-GH-cont-syn

Eq. (5) change to

$$\frac{dH_i^Q}{dt} = -\frac{H_i^Q}{\sigma_d^Q} + \sum_k F_i^Q \delta(t - T_{i,k}^F) + \sum_{j \neq i} S_{ij} g(V_j^{\text{pre}}), \quad (7)$$

$$g(V_j^{\text{pre}}) = 1 / \left( 1 + \exp(-(V_j^{\text{pre}} - 85 \text{ mV})/2) \right). \quad (8)$$

$V_j^{\text{pre}}$  is the (presynaptic) membrane potential of  $j$ -th neuron.

Note: In this model, the voltage is scaled as  $V/10$ .

## 3 Neuron Properties

### 3.1 Single neuron property (HH-G, HH-GH, HH-GH-cont-syn)

#### 3.1.1 Setting threshold

A common setting for threshold is 15 mV above the resting state.

From Fig.(1) we can see that there is some non-spike been counted as spike.

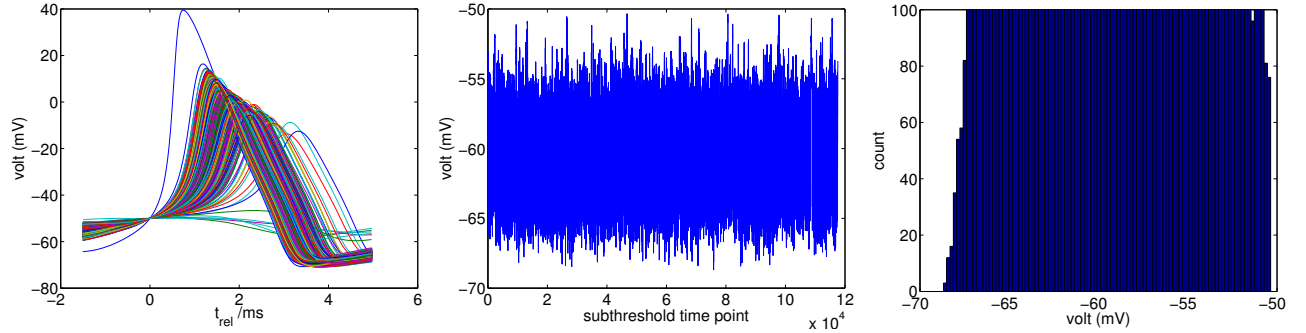


Figure 1: Threshold =  $-65 + 15$  mV, HH-GH, pr = 10.0 kHz, ps =  $0.05 \Omega^{-1} \text{ms}^{-2}$  (strong input, 97 Hz)

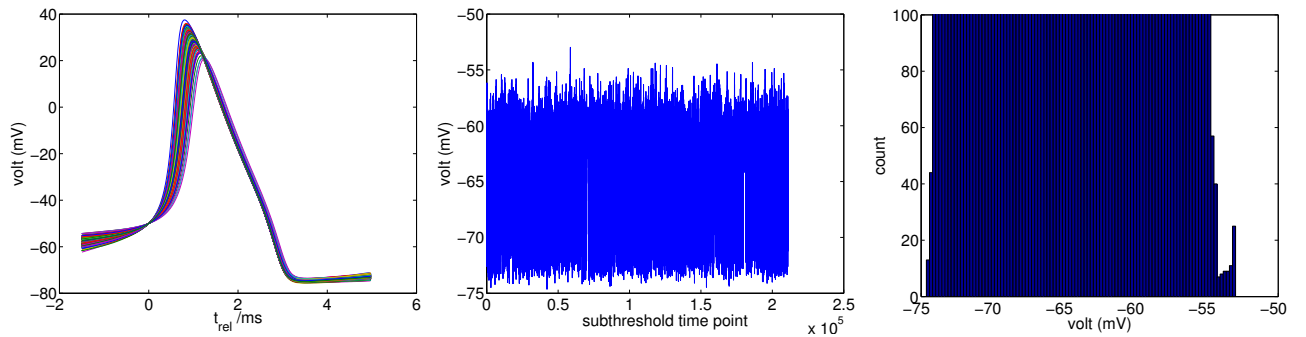


Figure 2: Threshold =  $-65 + 15$  mV, HH-GH, pr = 2.0 kHz, ps =  $0.05 \Omega^{-1} \text{ms}^{-2}$  (median input, 52 Hz)

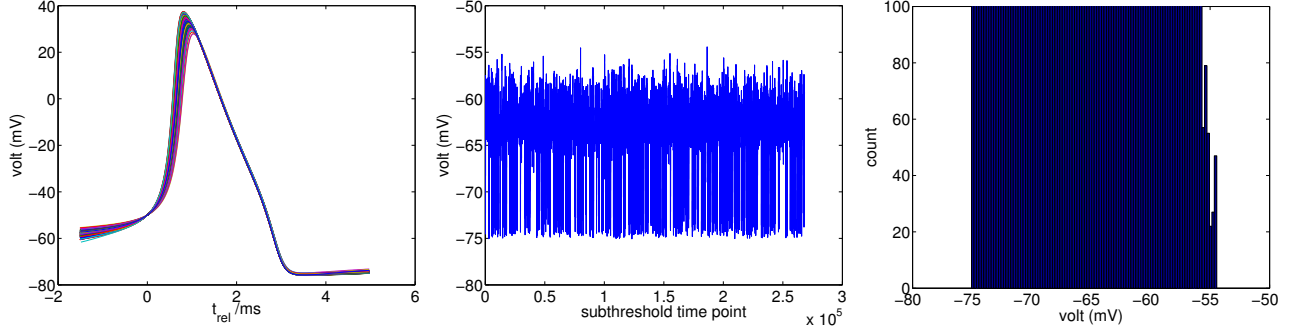


Figure 3: Threshold= $-65 + 15$  mV, HH-GH,  $pr=1.0$  kHz,  $ps=0.04 \Omega^{-1}ms^{-2}$  (low input, 24 Hz)

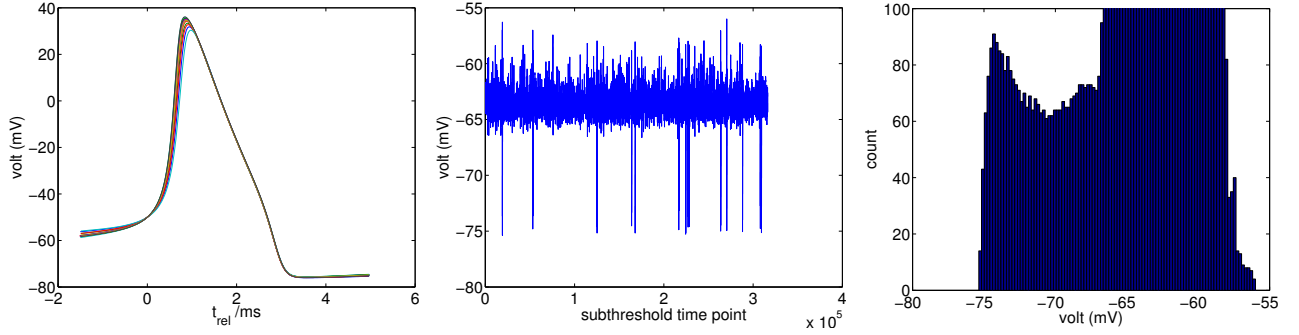


Figure 4: Threshold= $-65 + 15$  mV, HH-GH,  $pr=1.0$  kHz,  $ps=0.02 \Omega^{-1}ms^{-2}$  (tiny input, 1.4 Hz)

### 3.1.2 Higher threshold

Use 20 mV above resting as threshold.

(Fig.(5)) Compare to Fig.(1), we see no mis-counted spikes, and the timing is more accurate.

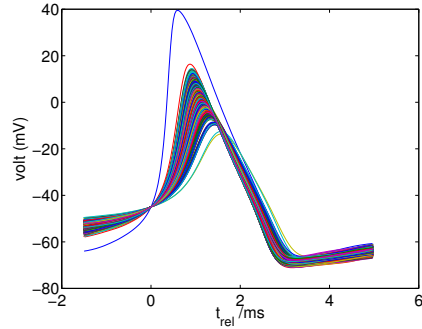


Figure 5: Threshold= $-65 + 20$  mV, HH-GH,  $pr=10.0$  kHz,  $ps=0.05 \Omega^{-1}ms^{-2}$  (strong input, 97 Hz).

Use 84 mV above resting as threshold.

Reason: In “HH-GH-cont-syn” model, only when volt above around 85 mV will the synaptic transmit a signal. In function Eq.(8),  $g(84 \text{ mV}) \approx 0.0067$ ,  $g(85 \text{ mV}) = 0.5$ .

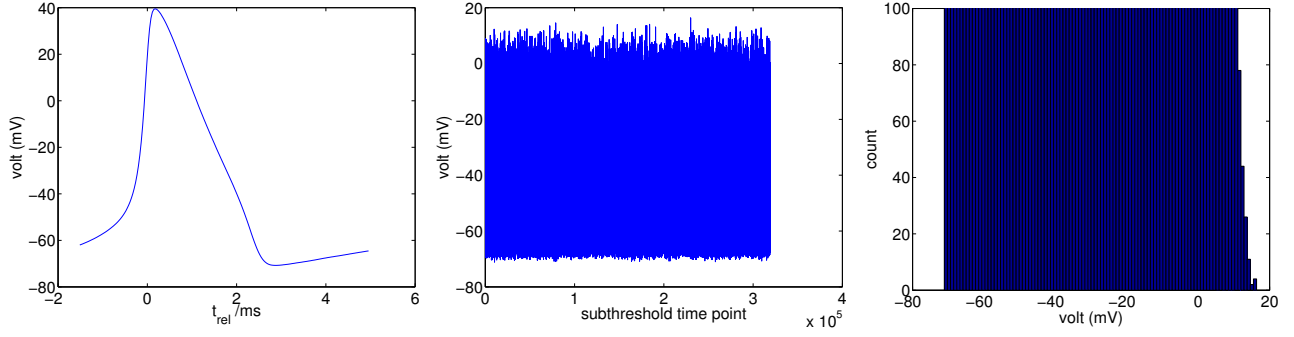


Figure 6: Threshold =  $-65 + 84$  mV. Strong input, (see Fig. (1)). Essentially all spikes are “missed”.

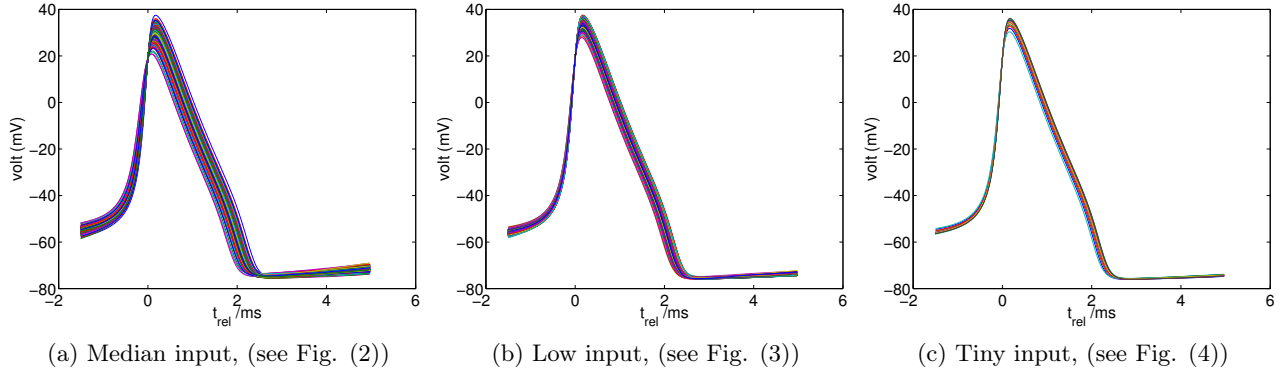


Figure 7: Threshold =  $-65 + 84$  mV.

### 3.1.3 Discussion of the “threshold”

The concept of the threshold actually has two distinct meanings:

1. Once the voltage pass it (from low to high), then the neuron will fire (sooner or later).
2. The neuron will sent a signal to it down stream around this time.

A related fact is that (in this model, with Poisson input), the action potential is not in a fixed size, there are big and small action potentials. This is what makes the above two concepts differ.

In the simulation and exact understanding of spiking dynamics. The concept 2 is desired. The proper timing should be the peak time.

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