# Minimum Required Data Length to Reconstruct GC Network

November 10, 2013

## 1 Task

Compute the minimum required data length when calculating GC. Use IF neural model as an example.

## 2 Analysis

Suppose there are two random variables $x$ and $y$.

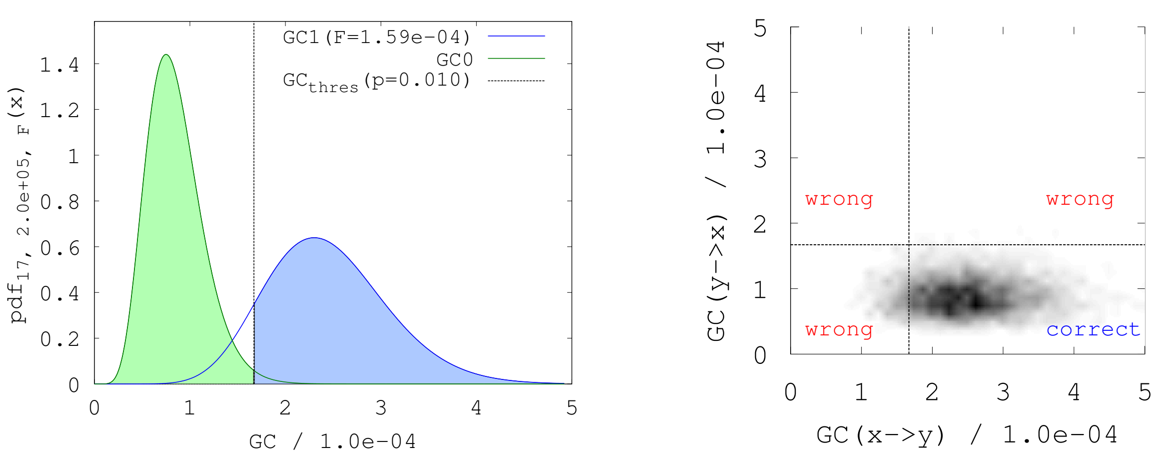
Recall: Distribution of GC obey

$$L \cdot \hat{F}_{x\to y} \overset{a}{\sim} \chi'^2(m,\ L \cdot F_{x\to y})$$

in the large $L$ limit ($\overset{a}{\sim}$). Where $L$ is number of data samples, $\hat{F}_{x\to y}$ is calculation value of true GC $F_{x\to y}$, $m$ is fitting order. $\chi'^2$ is Noncentral chi-squared distribution (see ref. [1] for definition and properties).

For convenience, we denote the probability density function (pdf) of $\hat{F}$ as $\rho_{m,L,F}(x)$.

Now we want to know: for a given $L$, $m$ and true $F_{x\to y}$, $F_{y\to x}$, what's the expected correct rate? Obviously, 25% will be the lowest bound (random guess, without any other knowledge about the two neuron network).




(a) theoretical asymptotic pdf of $\hat{F}_{x\to y}$ and $\hat{F}_{y\to x}$ (separately).

(b) density of simultaneous distribution of $\hat{F}_{x\to y}$ and $\hat{F}_{y\to x}$ . Obtained from 4000 experiment and counting by divide each axis into 40 uniform bins.

Figure 1: GC pdf under one set of typical parameter: $\mu = 1.0\,\mathrm{kHz}$, $f = 0.012$, $S = 0.01$, using $L = 2.0\times10^5 (T = 1\times10^5\,\mathrm{ms})$, $m = 17$, the true GC is $F_{x\to y} \approx 1.592\times10^{-4}$ and $F_{y\to x} \approx 0.006\times10^{-4}$ (obtained by $L = 1\times10^8$). The black line represent the GC thresholding value ($\mathrm{GC_{thres}}$) that we used to judge whether there is connection or not. Here $\mathrm{GC_{thres}}$ satisfis $P\left(\hat{F}_{y\to x} < \mathrm{GC_{thres}}\right) = 0.01$ and $F_{y\to x} = 0$ (our null hypothesis), i.e. false positive error rate is 1%.

In order to measure the correctness of GC analysis, here define the correct rate $p_{\text{correct}}$ as following: in a two-variable GC test, the true GC $F_{x\to y} > 0$, $F_{y\to x} = 0$, the GC test use a fixed p-value (false positive error rate, say 0.01) and fixed fitting order $m$, sample number $L$ to determine a GC threshold $GC_{\text{thres}}$. Then compare calculated GC $\hat{F}$ to $GC_{\text{thres}}$ to guess whether there is connection or not. We denote the ratio between number of correct guesses (network is $x- > y$) and number of total guess as $p_{\text{correct}}$.

If $\hat{F}_{x\to y}$ and $\hat{F}_{y\to x}$ are independent, then the expression for $p_{\text{correct}}$ will be very simple:

$$p_{\text{correct}} = \int_0^{F_{thres}} \rho_{m,L,F0}(F)\,\mathrm{d}F \left(1 - \int_0^{F_{thres}} \rho_{m,L,F1}(F)\,\mathrm{d}F\right), \tag{1}$$

that is the product of areas of green and blue region in Fig.(1a). Otherwise, we have to count the volumn of lower right part of Fig.(1b).

## 2.1 Is $\hat{F}_{x\to y}$ and $\hat{F}_{y\to x}$ are independent?

Geweke said (Ref.[2]) $\hat{F}_{x\to y}$ and $\hat{F}_{y\to x}$ are asymptotically independent. But how "asymptotically".

First, is the asymptotic pdf of $\rho_{m,L,F}(x)$ accurate?

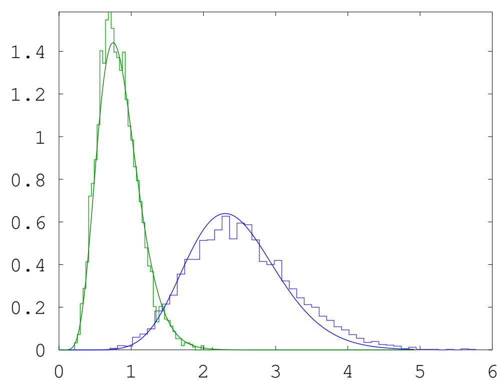
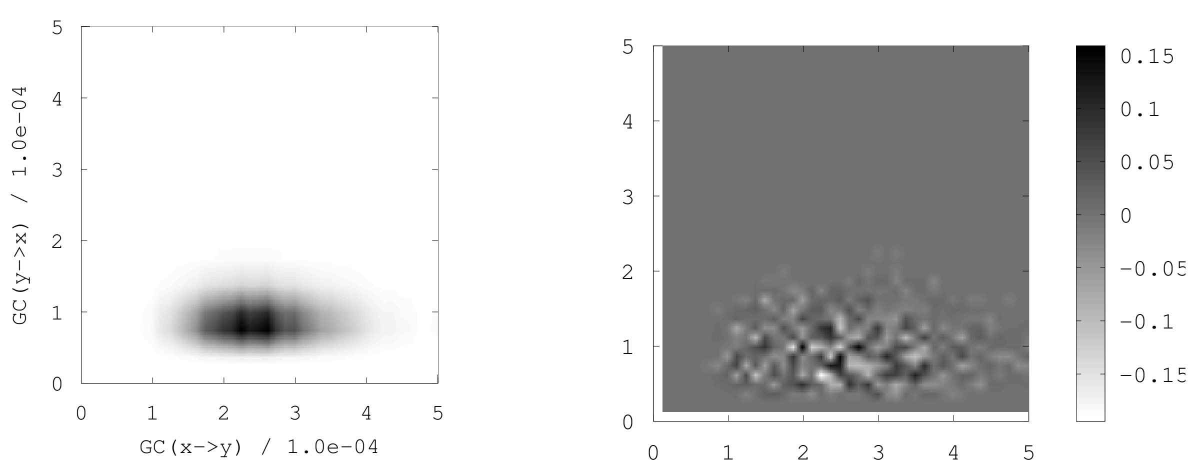


Figure 2: Comparison of statistic data and asymptotic pdf of $\rho_{m,L,F}(x)$. Same parameter as Fig.(1).

Second, by the 4000 GC data point mensioned in Fig.(1b), we can calculate the correlation of $\hat{F}_{x\to y}$ and $\hat{F}_{y\to x}$. The result is $-0.012$, which can be explaind by statistic error ($1/\sqrt{4000} \approx 0.016$).

Further, we compare the joint distribution to the product of marginal distribution.




(a) product of marginal distribution, looks similar to Fig.(1b).

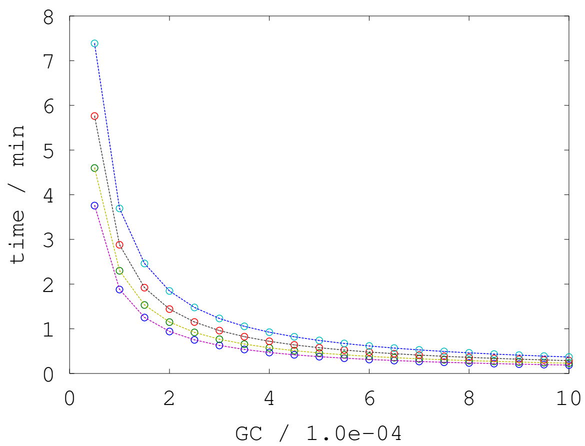
(b) pdf of $(\hat{F}_{x\to y}, \hat{F}_{y\to x})$ subtract product of marginal distribution, normalized to distribution peak equals one.

Figure 3

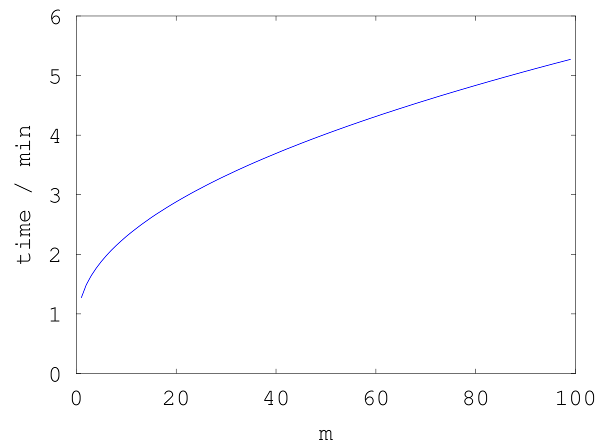
In this parameter (See description of Fig.(1)), the 4000 experiements tell us the correct rate is about 87.1%, while using asymptotic pdf and assume $\hat{F}_{x\to y}$ and $\hat{F}_{y\to x}$ are independent, we get 88.9% from Eq.(1) through $m$, $L$, $F_{x\to y}$, $F_{y\to x} = 0$ in Fig.(1). Several more experiments are done, they're all matched good enough($< 5\%$).

## 2.2 Minimum required data length

From Eq.(1) it's now possible to solve the minimum required data length $L_{\min}$ or data time length $T_{\min}$ (instead of doing a lot of numerical experiements), if $m$ and $F_{x\to y}$ are known.




(a) Required time length v.s. GC value. Four curves from up to down corresponding to $m = \{5, 10, 20, 40\}$, circle dot is obtained by solving Eq.(1), dashed line is obtained from Eq.(2)(see below).




(b) Required time length v.s. fitting order. Fix GC value to $F_{\text{true}} = 1.0\times 10^{-4}$

Figure 4: False positive error rate set to 0.01, required correct ratio set to 90%.

In the case of false positive error rate set to 0.01, required correct ratio set to 90%, there is a good approximation of minimum length (relative error of $T_{\min}$ is about 0.1%):

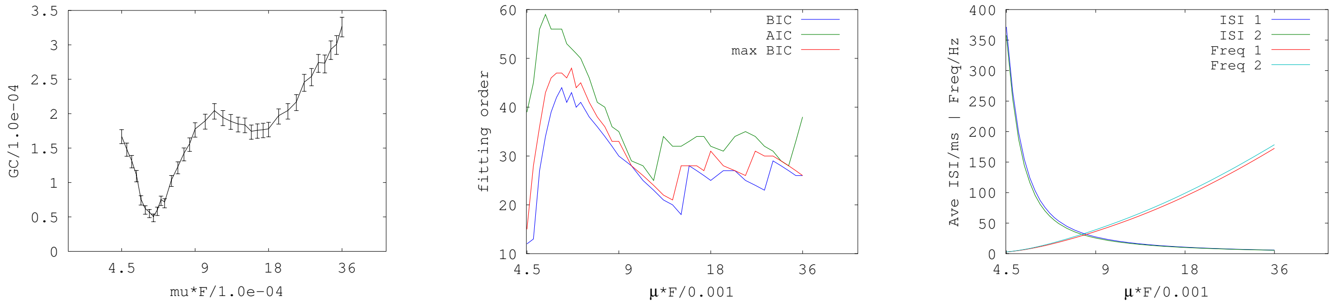
$$T_{\min} \approx \frac{10.00}{F_{\text{true}}}\left(1.153 + \frac{\sqrt{m-0.513}}{1.917}\right)\Delta t, \ (m \in \{1, 2, \dots, 100\}) \tag{2}$$

where $1/\Delta t$ is sample rate, $\Delta t = 0.5\,\text{ms}$ in all above cases.

Recall that $F_{\text{true}}/\Delta t \to$ const. in the limit $\Delta t \to 0$, so Eq.(2) tell us that there is no benefit to use small $\Delta t$, because in that case $m \propto 1/\Delta t$ which make $T_{\min}$ larger.

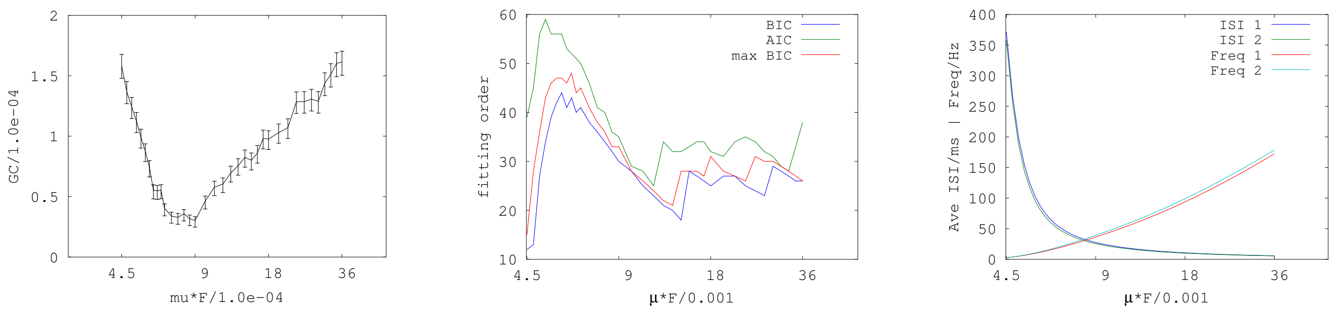
Bigger positive error rate will lead to smaller $T_{\min}$, but effect is limited. e.g. set positive error rate to 0.05 will decrease $T_{\min}$ about 12%, while set positive error rate to 0.005 will increase $T_{\min}$ about 7%.

Now remaining problem is: what is the true GC and corresponding $m$.
We once have done that:




(a) (b) (c)

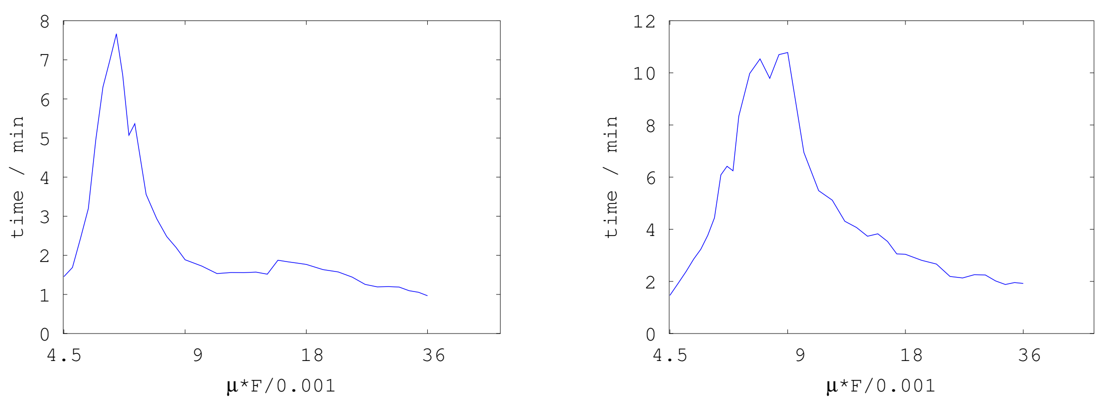
Figure 5: Scan $F$. $\mu = 0.01$, $S = 0.01$, $T = 1.0 \times 10^4$ sec, $\Delta t = 0.5$ ms




(a) (b) (c)

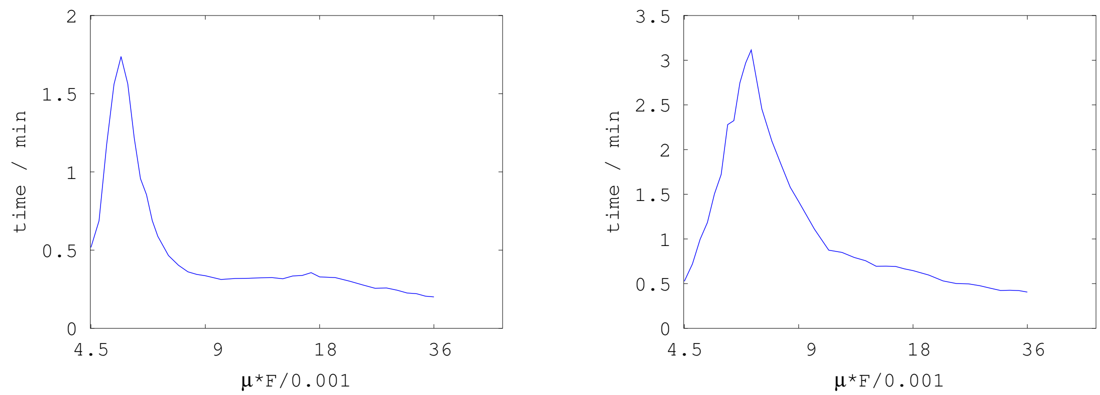
Figure 6: Scan $F$. $\mu = 0.02$, $S = 0.01$, $T = 1.0 \times 10^4$ sec, $\Delta t = 0.5$ ms

Importing these data to Eq.(2) we get:




(a) $\mu = 0.01$, $S = 0.01$. (Case of Fig.(5)) (b) $\mu = 0.02$, $S = 0.01$. (Case of Fig.(6))

Figure 7: Required time length

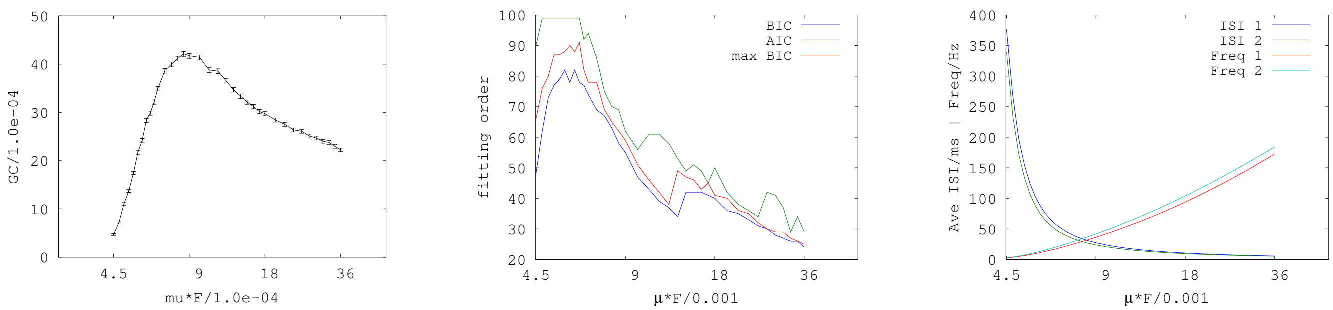



(a) $\mu = 0.01$, $S = 0.02$ (b) $\mu = 0.02$, $S = 0.02$

Figure 8: Required time length

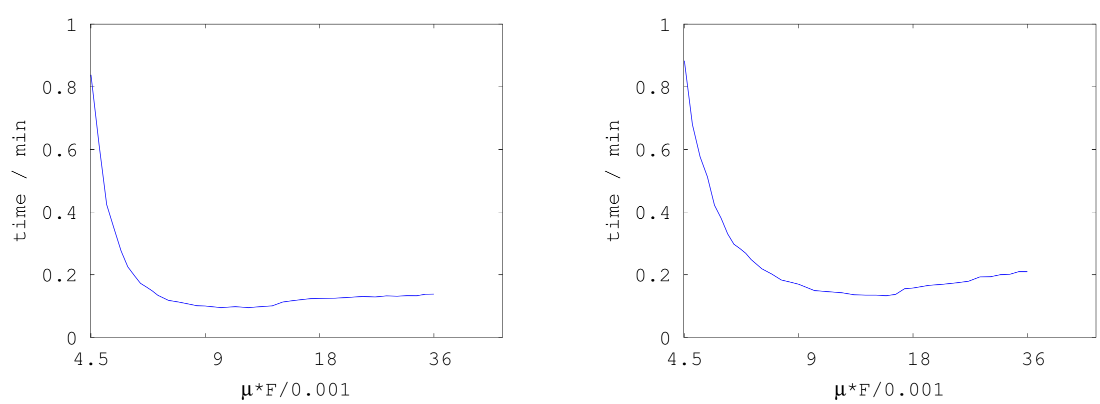
As what we expected, twice the $S$, roughly quarter the required time.

Case of using spike train:




(a) (b) (c)

Figure 9: Using spike train. Scan $F$. $\mu = 0.01$, $S = 0.02$, $T = 1.0 \times 10^4$ sec, $\Delta t = 0.5$ ms

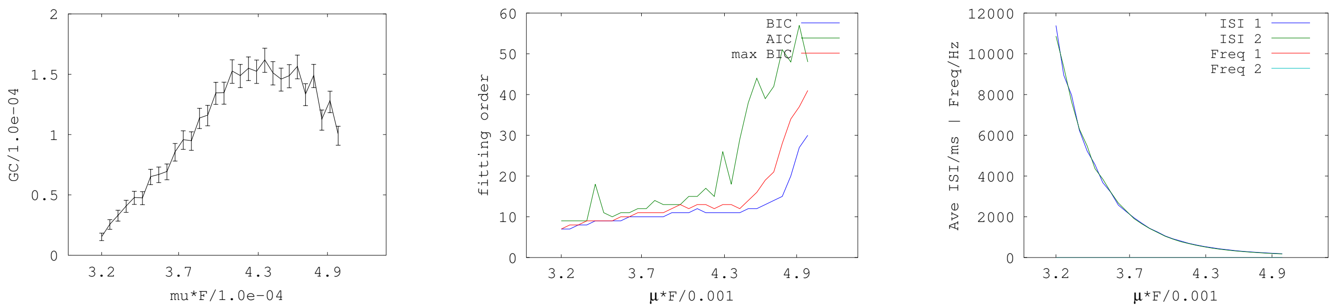



(a) $\mu = 0.01$, $S = 0.02$ (Case of Fig.(9)) (b) $\mu = 0.02$, $S = 0.02$

Figure 10: Required time length. (Use spike train)

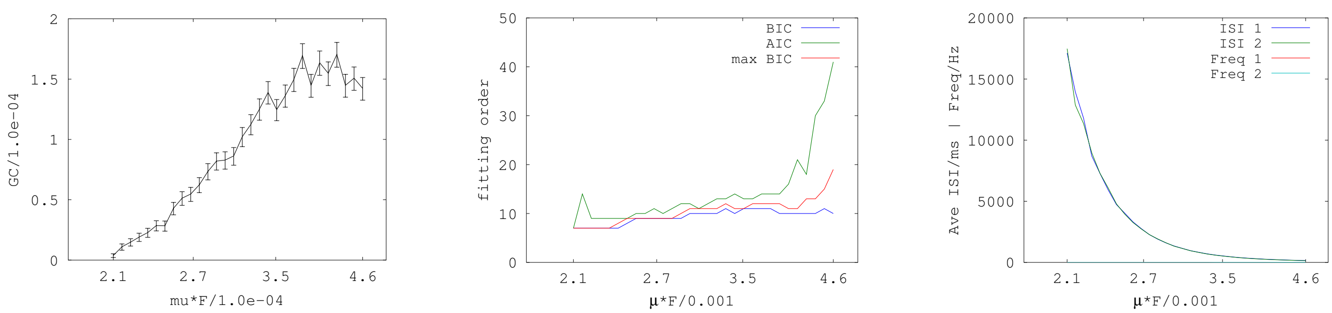
Note: $EPSP \approx 107\,S$ mV or $EPSP \approx 7\,S$ in the normalized unit.

# 3 Big ISI Case



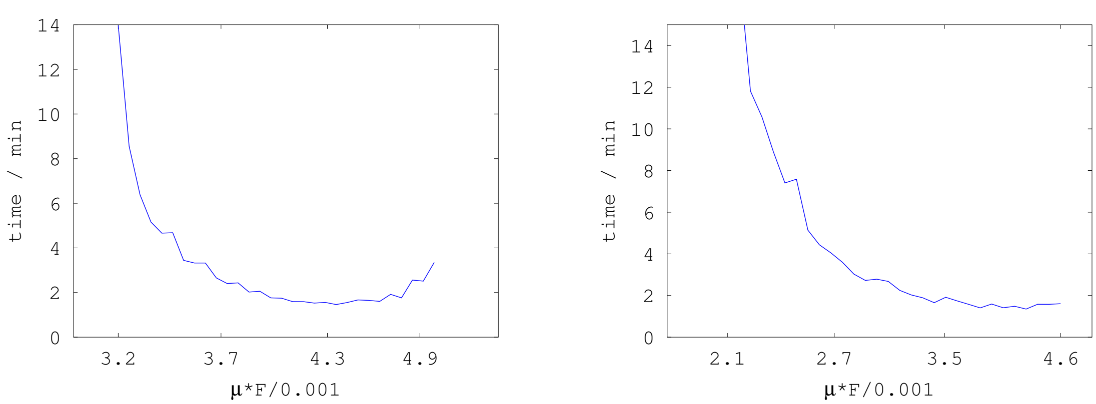

(a) (b) (c)

Figure 11: Scan $F$. $\mu = 0.01$, $S = 0.01$, $T = 1.0 \times 10^4$ sec, continue Fig.(5)




(a) (b) (c)

Figure 12: Scan $F$. $\mu = 0.02$, $S = 0.01$, $T = 1.0 \times 10^4$ sec, continue Fig.(6)




(a) $\mu = 0.01$, $S = 0.01$ (b) $\mu = 0.02$, $S = 0.01$

Figure 13: Required time length

# References

[1] http://en.wikipedia.org/wiki/Non-central_chi-square_distribution

[2] Measurement of Linear Dependence and Feedback Between Multiple Time Series, John Geweke, Journal of the American Statistical Association, Vol.77, No.378 (1982)