GC in Sparse Neuronal Network

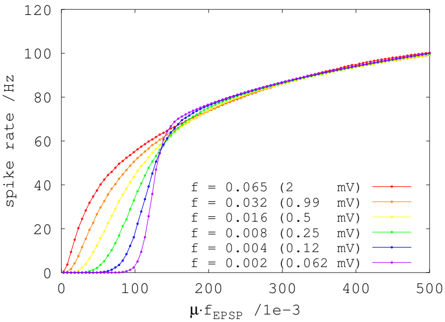
XYY

2015-04-21

Outline

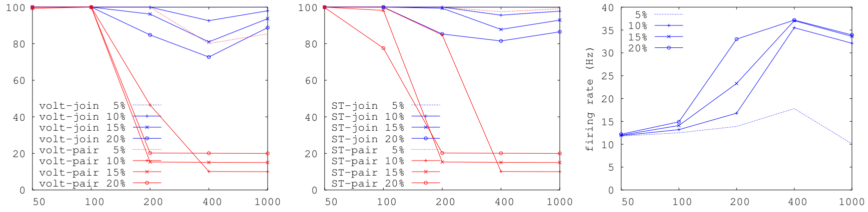
Setups

- Classical HH Model. With continuous coupling($\sim 1\,\mathrm{ms}$) and Poisson pulse external input (G, H smoothed).
- Random network with a given sparseness (#edges / #possible edges), all edges the same coupling strength.
- The gain function:



Network GC Reconstruction: Correctness

#E+#I = 30+20, 80+20, 180+20, 380+20, 750+250



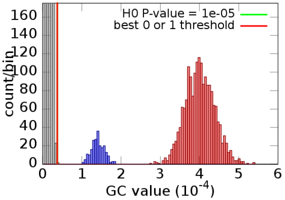
(a) Volt (b) Spike Train (c) Firing rate

S^{EE}, S^{IE}	S^{EI}, S^{II}	μ	f	T	Sample Rate
0.05 (0.7 mV)	0.09 (−0.45 mV)	1.0 kHz	0.03 (0.93 mV)	1×10^6	2.0 kHz
		0.7 kHz	for 1000 neuron		

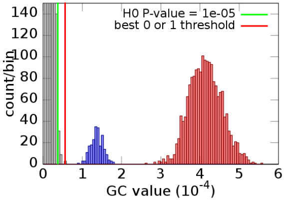
Notes: Firing rate increase coincide with pairwise correctness drop.
For 5% sparseness, pairwise is good even for 1000 neuron case.

Network GC Reconstruction: Correctness

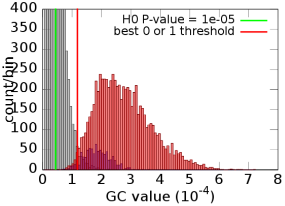
Typical GC distribution (n=200)



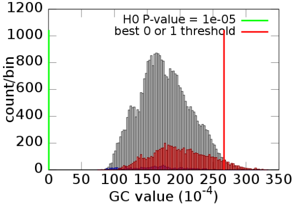
(d) **Volt**



(e) **Spike Train**



(f) **Firing rate**



(g) **Firing rate**

Calculation Speed of GC

GC analysis time cost. There are ranges because there are different networks.

n	len/ms	Fr Rate(Hz)	HH simu	od_{max}	GC (sec)
50	10^6	32.0	0.838 h	40	74.2
50	10^6	12.2	0.509 h	40	81.6
100	10^6	33.7	2.200 h	40	117.0
100	10^6	12.5	1.203 h	40	120.3
200	10^6	36.0	11.97 h	40	1391~1608
200	10^6	13.9	5.51 h	40	1562
400	10^6	35.4~37.1	25.36 h~30.52 h	40	7256~8063
1000	10^6	~35	~3 days	40	263351(old), 7200(new)

Old: $O(p^2 \cdot m \cdot L) + O(p^4 m^3)$, New: $O(p^2 \cdot m \cdot L) + O(p^3 m^3)$ or (with Levinson type inversion) $O(p^2 \cdot m \cdot L) + O(p^3 m^2 \log m)$.
PDC, plain time domain: $O(p^2 \cdot m \cdot L) + O(p^3 m^3)$ or (with Levinson type inversion) $O(p^2 \cdot m \cdot L) + O(p^3 m^2)$;
Frequency domain decomposition based (N the length of FFT):
$O(p^2 \cdot L \cdot \log N) + O(p^3 N + p^2 N \log N)$.

Approximation of GC: Linear Regression

Assume $\mathrm{E}(x_t) = \mathrm{E}(y_t) = \mathrm{E}(y_t) = 0$, let's do a 3-var AR of order m:

$$\begin{cases} x_t = \sum_{j=1}^{m} a_j^{(11)} x_{t-j} + \sum_{j=1}^{m} a_j^{(12)} y_{t-j} + \sum_{j=1}^{m} a_j^{(13)} z_{t-j} + \varepsilon_t^{(1)} & \text{(a)} \\ y_t = \sum_{j=1}^{m} a_j^{(21)} x_{t-j} + \sum_{j=1}^{m} a_j^{(22)} y_{t-j} + \sum_{j=1}^{m} a_j^{(23)} z_{t-j} + \varepsilon_t^{(2)} & \text{(b)} \\ z_t = \sum_{j=1}^{m} a_j^{(31)} x_{t-j} + \sum_{j=1}^{m} a_j^{(32)} y_{t-j} + \sum_{j=1}^{m} a_j^{(33)} z_{t-j} + \varepsilon_t^{(3)} & \text{(c)} \end{cases} \tag{1}$$

It solution (for variable x, let's focus on $y \to x$):

$$\begin{bmatrix} \vec{a}^{(11)} & \vec{a}^{(12)} & \vec{a}^{(13)} \end{bmatrix} \begin{bmatrix} R^{(xx)} & R^{(xy)} & R^{(xz)} \\ R^{(yx)} & R^{(yy)} & R^{(yz)} \\ R^{(zx)} & R^{(zy)} & R^{(zz)} \end{bmatrix} = \begin{bmatrix} \vec{r}^{(x|x)} & \vec{r}^{(x|y)} & \vec{r}^{(x|z)} \end{bmatrix}$$

where

$$R^{(uv)} = (b_{jk}) = \left(\mathrm{E}(x_{t-j}^{(u)} x_{t-k}^{(v)T})\right), \ \vec{v}^{(u|v)} = (b_k)^T = \left(\mathrm{E}(x_t^{(u)} x_{t-k}^{(v)T})\right)^T, \quad (j, k = 1 \ldots m).$$

Let's denote $R = \left(R^{(uv)}\right)$.

Approximation of GC: Residual Reduction

The variance of residuals are (2-variable and 3-variable)

$$\mathrm{SS}_{R[x,z]} = \begin{bmatrix} \vec{r}^{(x|x)} & \vec{r}^{(x|z)} \end{bmatrix} \begin{bmatrix} R^{(xx)} & R^{(xz)} \\ R^{(zx)} & R^{(zz)} \end{bmatrix}^{-1} \begin{bmatrix} \vec{r}^{(x|x)} & \vec{r}^{(x|z)} \end{bmatrix}^{T}$$

$$\mathrm{SS}_{R[x,y,z]} = \begin{bmatrix} \vec{r}^{(x|x)} & \vec{r}^{(x|y)} & \vec{r}^{(x|z)} \end{bmatrix} \begin{bmatrix} R^{(xx)} & R^{(xy)} & R^{(xz)} \\ R^{(yx)} & R^{(yy)} & R^{(yz)} \\ R^{(zx)} & R^{(zy)} & R^{(zz)} \end{bmatrix}^{-1} \begin{bmatrix} \vec{r}^{(x|x)} & \vec{r}^{(x|y)} & \vec{r}^{(x|z)} \end{bmatrix}^{T}$$

$$\mathrm{SS}_{R[x,y,z]} - \mathrm{SS}_{R[x,z]} = \vec{a}^{(12)} \left(Q^{(yy)}\right)^{-1} \left(\vec{a}_T^{(12)}\right)^{T}, \begin{bmatrix} Q^{(xx)} & Q^{(xy)} & Q^{(xz)} \\ Q^{(yx)} & Q^{(yy)} & Q^{(yz)} \\ Q^{(zx)} & Q^{(zy)} & Q^{(zz)} \end{bmatrix} = R^{-1}$$

$$F_{y\to x|z} = -\ln\left(1 - \frac{\frac{1}{\mathrm{var}(x_t)}\left(\mathrm{SS}_{R[x,y,z]} - \mathrm{SS}_{R[x,z]}\right)}{1 - \frac{1}{\mathrm{var}(x_t)}\mathrm{SS}_{R[x,z]}}\right),$$

get

$$F_{y\to x|z} \approx \frac{1}{\mathrm{var}(x_t)}\left(\mathrm{SS}_{R[x,y,z]} - \mathrm{SS}_{R[x,z]}\right) = \frac{1}{\mathrm{var}(x_t)} \vec{a}^{(12)} \left(Q^{(yy)}\right)^{-1} \left(\vec{a}^{(12)}\right)^{T}.$$

Relation of Joint and Auto Regression

For Autoregression

$$\begin{bmatrix} \vec{b}^{(11)} & \vec{b}^{(12)} \end{bmatrix} \begin{bmatrix} R^{(xx)} & R^{(xy)} \\ R^{(yx)} & R^{(yy)} \end{bmatrix} = \begin{bmatrix} \vec{r}^{(x|x)} & \vec{r}^{(x|y)} \end{bmatrix},$$

we have

$$b^{(12)} = a^{(12)} - a^{(13)} \left(Q^{(zz)}\right)^{-1} Q^{(zy)}$$

$$a^{(12)} \left(I - \left(Q^{(yy)}\right)^{-1} Q^{(yz)} \left(Q^{(zz)}\right)^{-1} Q^{(zy)}\right) = b^{(12)} + b^{(13)}_{[xz]} \left(Q^{(zz)}\right)^{-1} Q^{(zy)} \tag{2}$$

Imply: $b^{(12)} = 0,\ b^{(13)} = 0 \ \Rightarrow\ a^{(12)} = 0$, also $a^{(13)} = 0$.
Under assumption

$$\Sigma = \operatorname{diag}\left(\begin{bmatrix} \Sigma_{xx} & \Sigma_{yy} & \Sigma_{zz} \end{bmatrix}\right)$$

$$B_{xy} \approx \left[A_{xy} - A_{xz}\left(A_{zy} + \Sigma_{zz} A^{*T}_{yz} \Sigma^{-1}_{yy}\right)\right]_{+}$$

For scaler x and y (z might be vector), and further assume $\Sigma_{zz} \approx \Sigma_{yy}$ (which is the usual case), we get

$$B_{xy} \approx \left[A_{xy} - A_{xz}\left(A_{zy} + A^{*}_{yz}\right)\right]_{+}$$

For Further Reading I

Binomial inverse theorems

$$(A+UBV)^{-1} = A^{-1} - A^{-1}UB\left(B+BVA^{-1}UB\right)^{-1}BVA^{-1}$$